

**PROCEEDINGS**

## Improved XFEM (IXFEM): Accurate, Efficient, Robust and Reliable Analysis for Arbitrary Multiple Crack Problems

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### ABSTRACT

The extended finite element method (XFEM) has been successful in crack analysis but faces challenges in modeling multiple cracks. One challenge is the linear dependence and ill-conditioning of the global stiffness matrix, while another is the geometric description for multiple cracks. To address the first challenge, the Improved XFEM (IXFEM) [1–9] is extended to handle multiple crack problems, effectively eliminating issues of linear dependence and ill-conditioning. Additionally, to overcome the second challenge, a novel level set templated cover cutting method (LSTCCM) [10] is proposed, which combines the advantages of the level set method and cover cutting technique. The present approach offers highly accurate stress intensity factor evaluation, efficient linear system solving, and robust geometric computations. Furthermore, this approach introduces novel techniques for modeling multiple evolving cracks and proposes a prediction–correction scheme for competing cracks [11]. The developed approach demonstrates accuracy, effectiveness, robustness, and reliability in analyzing arbitrary multiple crack propagation problems in 2-D elastic solids. Ongoing work aims to investigate the approach for multi-physics fracture problems (e.g., hydraulic fracturing [12–14]) by coupling the present solid solver with a fluid flow solver (e.g., [15–17]), showing promise in such scenarios.

### KEYWORDS

Extra-DOF-free enrichment; level set templated cover cutting method (LSTCCM); multiple crack growth; stress intensity factor (SIF); extended finite element method (XFEM)

**Acknowledgement:** The authors gratefully acknowledge the financial support that they have received.

**Funding Statement:** L.X. Wang received the funding from the National Natural Science Foundation of China (Grant Nos. 12102059, 12472207) and the China Postdoctoral Science Foundation (Grant No. 2023M743604); L.F. Wen received the funding from the National Natural Science Foundation of China (Grant No. 12402231); R. Tian received the funding from the Science Challenge Project (Grant No. TZ2018002); and C. Feng received the funding from the National Key R&D Program of China (Grant No. 2023YFC3007203).

**Conflicts of Interest:** The authors declare that they have no conflicts of interest to report regarding the present study.



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