



# REDUCING HEAT TRANSFER BETWEEN TWO CONCENTRIC SEMI-CYLINDERS USING RADIATION SHIELDS WITH TEMPERATURE-DEPENDENT EMISSIVITY

Seyfolah Saedodin, M.S. Motaghedi Barforoush, Mohsen Torabi\*

*Department of Mechanical Engineering, Faculty of Engineering, Semnan University, Semnan, Iran*

## ABSTRACT

In this paper, a simplifying approach for calculating the radiant energy is achieved using the concept of net radiation heat transfer and provides an easy way for solving a variety of situations. This method has been applied to calculate the net radiation heat transfer between two long concentric semi-cylinders. Then this method used to calculate reduction heat transfer when radiation shields with temperature-dependent emissivity applied between these objects. Moreover, using this method the percentage reduction in heat transfer between two surfaces was calculated. The findings reveal that, one radiation shield with lower emissivity can reduce the net heat transfer even better than two radiation shields with higher emissivity. Moreover, it can be concluded that for silicon carbide radiation shield in the range of assumed temperatures, the emissivity is almost constant.

**Keywords:** Temperature-dependent emissivity, Radiation shield, Radiant energy, Net radiation method, Concentric semi-cylinders, Grey surface.

## 1. INTRODUCTION

Heat transfer by radiation is one of the basic modes of heat transfer. This model of heat transfer is not just a theoretical problem, since understanding and predicting the radiant energy becomes crucial in many practical situations. In high-performance insulating materials it is common to suppress conduction and convection heat transfer by evacuating the space between two surfaces. This leaves thermal radiation as the dominant heat loss mode even for low-temperature applications such as insulation in cryogenic storage tanks. On way of reducing radiant heat transfer between two particular surfaces is to use materials which are highly reflective. An alternative method is to use radiation shields between the heat exchange surfaces (Holman, 2009). These shields do not deliver or remove any heat from the overall system; they only place another resistance in the heat-flow path so that the overall heat transfer is retarded. Moreover use of radiation shield is recommended to and allows reproducibility of the patient positioning for daily treatments (Mantri and Bhasin, 2010; Zennick *et al.*, 2007; Brosky *et al.*, 2000).

Radiation shields constructed from low emissivity (high reflectivity) materials can be used to reduce the net radiation transfer between two surfaces. Note that the emissivity associated with one side ( $\epsilon_{shn}^+$ ) may differ from that associated with the opposite side ( $\epsilon_{shn}^-$ ) of the shield (Incropera *et al.*, 2007). Our objective consists in showing how apparently intractable problems in heat transfer by radiation can be easily solved using the concept of net radiation transfer. This method was used for all three modes of heat transfer by many researchers (Afonso and Castro, 2010; Jamalud-Din *et al.*, 2010; Zueco and Campo, 2006; Zueco *et al.*, 2004). Although this method is simple, but provides a useful tool for visualizing radiation exchange between plates in the enclosure and may be used as the basis for predicting this exchange. Moreover, Micco and Aldao (2003) generalized the method

of net transmittance to spherical and cylindrical symmetry. But, they used only one radiation shield between two main surfaces.

We do not claim to be original since the net radiation method can be found in the literature (Howell *et al.*, 2010). A careful assessment of the foregoing literature shows that most studies of radiation shields assume that the surfaces are gray. In reality, the surfaces are finite in extent, the shield emissivities are either functions of temperature or direction. In this work, the general formulation has been investigated to calculate net heat transfer between two gray, long concentric semi-cylinders, which is more challenging compare with our previous studies (Saedodin *et al.*, 2010; Saedodin *et al.*, 2010). Then, reduction heat transfer by one and two diffuse radiation shield calculated. Accordingly, by applying two radiation shields with different materials optimization was done.

## 2. MATHEMATICAL MODELLING

Consider two long concentric semi-cylinders as shown in Fig. 1. (a). The space between these two semi-cylinders separated from outer space by plates  $A_3$  and  $A_4$ . We believe that a detailed discussion of this case brings out the thermal characteristics of the system and this case can be used as a storage tank for hot liquids or gases. For the analysis, the following simplifying assumptions were made:

1. Surfaces are diffuse and gray.
2. Space between semi-cylinders is evacuated.
3. Conduction resistance for radiation shield is negligible. Because the thickness of the shield is very small and the fact that the shield is made of highly conductive material, the temperature gradients within the shield along all its three dimensions may be assumed to be negligible. Thus it is appropriate to assume that the shield at any instant of time is at uniform temperature i.e. it behaves as a lumped capacitance system. Indeed, this assumption is commonly used in many studies on radiation heat shields (e.g. Howell *et al.*, 2010).
4. The end effects are negligible.

\* Corresponding author. Tel: +9891-28103473, E-mail: Torabi\_mech@yahoo.com

5. The temperature of the heat-transfer surfaces are maintained the same in all cases.
6. Since three surfaces  $A_2, A_3$  and  $A_4$  are assumed to be at the same temperature, the heat flux between surfaces  $A_2$  and  $A_3$ , and  $A_2$  and  $A_4$  are assumed to be negligible.
7. The two concentric semi-cylinders and all the shields are in radiant balance.
8. The emissivity associated with the inner and outer surfaces of the shield are the same.

The basic concepts related to heat transport by radiation are very well known. For a black surface the emitted thermal radiation leaving a surface, per unit time and unit area, is given by

$$E_b = \sigma T^4 \quad (1)$$

The net radiation heat transfer between inner and outer of the object can be calculated as follows

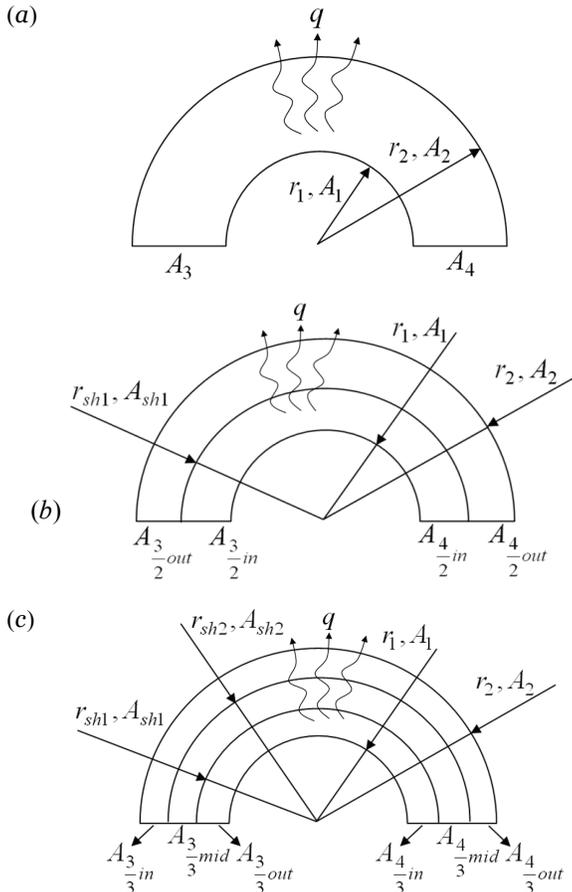
$$(Q_{net})_{without-shield} = \frac{E_{b1} - E_{b2}}{R_{12}} + 2 \frac{E_{b1} - E_{b3}}{R_{13}} \quad (2)$$

when

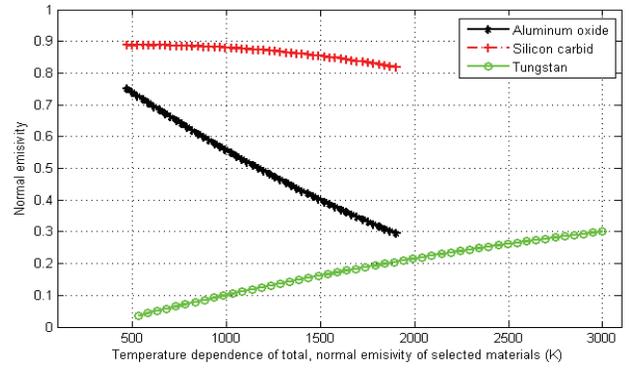
$$E_{b1} - E_{b2} = \sigma(T_1^4 - T_2^4) \quad (3)$$

$$E_{b1} - E_{b3} = \sigma(T_1^4 - T_3^4) \quad (4)$$

Using the above assumptions, the radiation heat transfer equations can be investigated by following procedures: Most real surfaces exhibit a selective emission, in the sense that the emissivity is different for different wavelengths. In general  $\varepsilon$  can be a function of the



**Fig. 1.** Top view of two long concentric semi-cylinders: (a) without radiation shield, (b) with one radiation shield, and (c) with two radiation shields



**Fig. 2.** Normal emissivity as a function of temperature (Incropera *et al.*, 2007)

and the surface temperature, i.e.  $\varepsilon = \varepsilon(\lambda, T)$ . A special type of non-black surface, called a grey body, is defined as one for which the emissivity is independent of the wavelength (Modest, 2010). For simplicity we will restrict our study to grey bodies. In addition, we will consider that surfaces are diffuse; therefore the intensity leaving a surface is independent of direction.

Using the net radiation method the total resistance between each two surfaces can be obtained by:

$$R_{12} = \frac{1 - \varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{1-2}} + \frac{1 - \varepsilon_2}{\varepsilon_2 A_2} \quad (5)$$

$$R_{13} = \frac{1 - \varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{1-3}} + \frac{1 - \varepsilon_3}{\varepsilon_3 A_3} \quad (6)$$

Therefore, the net heat transfer between inner and outer surfaces is:

$$(Q_{net})_{without-shield} = \frac{E_{b1} - E_{b2}}{\frac{1 - \varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{1-2}} + \frac{1 - \varepsilon_2}{\varepsilon_2 A_2}} + 2 \frac{E_{b1} - E_{b3}}{\frac{1 - \varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{1-3}} + \frac{1 - \varepsilon_3}{\varepsilon_3 A_3}} \quad (7)$$

By introducing  $F_{2-2}$  as cited in Howell *et al.* (2010) and using reciprocal relations and configuration-factor algebra

$$F_{1-2} = \frac{2}{\pi} \arcsin\left(\frac{r_1}{r_2}\right) \quad (8)$$

$$F_{1-3} = \frac{1}{2} - \frac{1}{\pi} \arcsin\left(\frac{r_1}{r_2}\right) \quad (9)$$

the net heat transfer between inner and outer surfaces can be obtained. To have a comparison between the amount of heat transfer with and without radiation shields, it is must to find functions as the amount of heat transfer with one and two radiation shields between inner surface and outer space. As cited before, the shields do not deliver or remove heat from the system. Therefore, the net heat transfer between inner surface and outer space, using one radiation shield, can be found as follows:

$$(Q_{net})_{with-one-shield} = 2Q_{1-\frac{3}{2}in} + 2Q_{sh1-\frac{3}{2}out} + Q_{sh1-2} \quad (10)$$

When  $Q_{1-\frac{3}{2}in}$ ,  $Q_{sh1-\frac{3}{2}out}$  and  $Q_{sh1-2}$  can be found same as follows:

$$Q_{1-\frac{3}{2}in} = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1 - \varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{1-\frac{3}{2}in}} + \frac{1 - \varepsilon_3}{\varepsilon_3 A_{\frac{3}{2}in}}} \quad (11)$$

$$Q_{sh1-\frac{3}{2}out} = \frac{\sigma(T_{sh1}^4 - T_3^4)}{\frac{1-\epsilon_{sh1}}{\epsilon_{sh1}A_{sh1}} + \frac{1}{A_{sh1}F_{sh1-\frac{3}{2}out}} + \frac{1-\epsilon_3}{\epsilon_3A_3}} \quad (12)$$

$$Q_{sh1-2} = \frac{\sigma(T_{sh1}^4 - T_2^4)}{\frac{1-\epsilon_{sh1}}{\epsilon_{sh1}A_{sh1}} + \frac{1}{A_{sh1}F_{sh1-2}} + \frac{1-\epsilon_2}{\epsilon_2A_2}} \quad (13)$$

while  $T_{sh1}$  and  $\epsilon_{sh1}$  should be found from the following equation:

$$Q_{1-sh1} = 2Q_{sh1-\frac{3}{2}out} + Q_{sh1-2} \quad (14)$$

As mentioned before the emissivity is a function of temperature; Because of the fact that emissivity and temperature of each shield are unknown, Fig. 2. has been employed for solving Eq. (14) at the same time. By following the same procedures as for one radiation shield, the net heat transfer can be found when two radiation shields applied between two main surfaces as follows:

$$(Q_{net})_{with-two-shield} = 2Q_{1-\frac{3}{3}in} + 2Q_{sh1-\frac{3}{3}mid} + 2Q_{sh2-\frac{3}{3}out} + Q_{sh2-2} \quad (15)$$

It is obvious that, for calculating  $T_{sh1}$ ,  $T_{sh2}$ ,  $\epsilon_{sh1}$  and  $\epsilon_{sh2}$ , Fig. 2 should be employed at the same time with two following equations:

$$Q_{1-sh1} = 2Q_{sh1-\frac{3}{3}mid} + Q_{sh1-sh2} \quad (16)$$

$$Q_{sh1-sh2} = 2Q_{sh2-\frac{3}{3}out} + Q_{sh2-2} \quad (17)$$

### 3. APPLICATION

Using our solution, we performed sample numerical computations of reduction heat transfer between two long concentric semi-cylinders by applying one and two radiation shields as shown in Fig. 1, based on equations derived on the previous section. Note that all the calculations performed for all three materials in Fig. 2.

**Example 1.** Consider two concentric semi-cylinders as shown in Fig. 1. (a). As mentioned before, the space between these two semi-cylinder separated from outer space by plates  $A_3$  and  $A_4$ . The inner semi-cylinder has temperature 873.15 K, radius 50 cm and emissivity of 0.28. The outer semi-cylinder has temperature 330 K, radius 100 cm and emissivity of 0.13. Also, plates  $A_3$  and  $A_4$  have temperature 330 K and emissivity of 0.13. If one shield of 75 cm radius has been applied to reduce heat transfer between inner semi-cylinder and outer space (Fig. 1. (b)), the percentage reduction in heat transfer, temperature and emissivity of the radiation shield can be calculated as follows:

$$(Q_{net})_{without-shield} = 9500.14 \text{ W}$$

For aluminum oxide shield:

Using Fig. 2. and solving Eqs. (10) and (14) together:

$$(Q_{net})_{with-one-shield} = 7941.77 \text{ W}$$

$$T_{sh1} = 725.98 \text{ K}, \quad \epsilon_{sh1} = 0.65$$

And the percentage reduction in heat transfer is:

$$\frac{(Q_{net})_{without-shield} - (Q_{net})_{with-one-shield}}{(Q_{net})_{without-shield}} \times 100 = \frac{9500.14 - 7941.77}{9500.14} \times 100 = 16.40\%$$

Similarly for silicon carbide shield:

$$(Q_{net})_{with-one-shield} = 8235.30 \text{ W}$$

$$T_{sh1} = 726.28 \text{ K}, \quad \epsilon_{sh1} = 0.88$$

And the percentage reduction in heat transfer is:

$$\frac{9500.1478 - 8235.3004}{9500.1478} \times 100 = 13.31\%$$

Finally for tungsten shield:

$$(Q_{net})_{with-one-shield} = 4436.89 \text{ W}$$

$$T_{sh1} = 715.34 \text{ }^\circ\text{K}, \quad \epsilon_{sh1} = 0.06$$

And the percentage reduction in heat transfer is:

$$\frac{9500.14 - 4436.89}{9500.14} \times 100 = 53.29\%$$

**Example 2.** Consider the two concentric semi-cylinder of Example 1. If two shields with same materials have been applied at radius 66.67 and 83.33 cm to reduce heat transfer between inner semi-cylinder and outer space (Fig. 1. (c)), the percentage reduction in heat transfer, temperatures and emissivities of the radiation shields can be calculated as follows:

$$(Q_{net})_{without-shield} = 9500.14 \text{ W}$$

For aluminum oxide shield:

Using Fig. 2. and solving Eqs. (15), (16) and (17) together:

$$(Q_{net})_{with-one-shield} = 6554.31 \text{ W}$$

$$T_{sh1} = 742.64 \text{ K}, \quad \epsilon_{sh1} = 0.64$$

$$T_{sh2} = 675.49 \text{ K}, \quad \epsilon_{sh2} = 0.67$$

And the percentage reduction in heat transfer is:

$$\frac{(Q_{net})_{without-shield} - (Q_{net})_{with-two-shields}}{(Q_{net})_{without-shield}} \times 100 = \frac{9500.14 - 6554.31}{9500.14} \times 100 = 31.00\%$$

Similarly for silicon carbide shield:

$$(Q_{net})_{with-one-shield} = 7019.52 \text{ W}$$

$$T_{sh1} = 739.18 \text{ K}, \quad \epsilon_{sh1} = 0.88$$

$$T_{sh2} = 685.36 \text{ K}, \quad \epsilon_{sh2} = 0.88$$

And the percentage reduction in heat transfer is:

$$\frac{9500.14 - 7019.52}{9500.14} \times 100 = 26.11\%$$

Finally for tungsten shield:

$$(Q_{net})_{with-one-shield} = 2971.50 \text{ W}$$

$$T_{sh1} = 758.80 \text{ K}, \quad \epsilon_{sh1} = 0.06$$

$$T_{sh2} = 585.27 \text{ K}, \quad \epsilon_{sh2} = 0.04$$

And the percentage reduction in heat transfer is:

$$\frac{9500.14 - 2971.50}{9500.14} \times 100 = 68.72\%$$

**Example 3.** Consider the two concentric semi-cylinder of Example 1. If two shields with different materials have been applied at radius 66.67 and 83.33 cm to reduce heat transfer between inner semi-cylinder and outer space (Fig. 1. (c)), the percentage reduction in heat transfer, temperature and emissivity of the radiation shields can be calculated with same procedures as Example 2. The temperatures, emissivities, net heat transfer and percentage reduction in heat transfer in all six possible models are shown in Table 1.

As it can be perceived from Table 1., model No. 5 is the best model for reducing heat transfer between two concentric hemispheres, if we want to use two radiation shields with different materials. It can be deduced from this table that, if we want to choose the best combination of two radiation shields with different materials, it is better to use the shield with lower emissivity closer to the surface with higher temperature.

**Table 1.** the percentage reduction in heat transfer, temperatures and emissivities of two radiation shields with different materials

Model	Shield at radius 66.67 cm			Shield at radius 83.33 cm			$(Q_{net})_{with-two-shields} W$	Percentage reduction in heat transfer %
	Material	Temperature K	Emissivity	Material	Temperature °K	Emissivity		
No. 1.	Aluminum oxide	737.68	0.64	Silicon carbide	674.89	0.88	6706.18	29.40
No. 2.	Aluminum oxide	809.02	0.62	Tungsten	675.27	0.05	4239.39	55.37
No. 3.	Silicon carbide	744.55	0.88	Aluminum oxide	685.72	0.66	6846.34	27.93
No. 4.	Silicon carbide	812.02	0.88	Tungsten	680.72	0.05	4334.02	54.37
No. 5.	Tungsten	710.59	0.06	Aluminum oxide	513.19	0.73	3290.32	65.36
No. 6.	Tungsten	709.51	0.06	Silicon carbide	510.92	0.88	3295.63	65.30

### 3. CONCLUSIONS

In this paper an equation for calculating heat transfer between two concentric semi-cylinder was investigated. Thanks to net radiation method the percentage reduction in heat transfer, temperature and emissivity of the radiation shield were calculated, unlike the previous literature (Micco and Aldao, 2003). It is found that, when two shields with same materials applied for reducing heat transfer, the one with lower emissivity better reduced net heat transfer. Also it was concluded that when two radiation shields with different materials have been applied to reduce heat transfer, the shield with lower emissivity should be closer to the surface with higher temperature to increase reduction in heat transfer.

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### NOMENCLATURE

- A Surface area ( $m^2$ )
- $E_b$  Blackbody emissive power ( $W / m^2$ )
- F Shape factor
- Q Net heat transfer ( $W$ )
- r Radius of hemisphere ( $m$ )
- T Absolute temperature ( $K$ )
- Greek Symbols*
- $\epsilon$  Emissivity
- $\lambda$  Wavelength
- $\sigma$  Stefan-Boltzmann constant ( $W / m^2 K^4$ )
- Superscripts*
- Outer surface
- + Inner surface

### REFERENCES

Afonso, C., and Matos, J., 2006, "The effect of radiation shields around the air condenser and compressor of a refrigerator on the temperature distribution inside it," *International Journal of Refrigeration*, **29**, 1144-1151.  
<http://dx.doi.org/10.1016/j.ijrefrig.2006.01.006>

Afonso, C., and Castro, M., 2010, "Air infiltration in domestic refrigerators: The influence of the magnetic seals conservation," *International Journal of Refrigeration* **33**, 856-867.  
<http://dx.doi.org/10.1016/j.ijrefrig.2009.12.007>

Brosky, M.E., Lee, C., Bartlett, T.S., and Lo, S., 2000, "Fabrication of a radiation bolus prosthesis for the maxillectomy patient," *Journal of Prosthetic Dentistry*, **83**, 119-121.  
[http://dx.doi.org/10.1016/S0022-3913\(00\)70097-7](http://dx.doi.org/10.1016/S0022-3913(00)70097-7)

Holman, J.P., 2009, *Heat Transfer*, 10<sup>th</sup> Edn., McGraw-Hill, New York.

Howell, J.R., Siegel, R., and Menguc, M.P., 2010, *Thermal Radiation Heat Transfer*, 5<sup>th</sup> Edn., CRC Press, New York.

Incropera, F.P., DeWitt, D.P., Bergman, T.L., and Lavine, A.S., 2007, *Fundamentals of Heat and Mass Transfer*, 6<sup>th</sup> ed., John Wiley & Sons, Hoboken, NJ.

Jamalud-Din, S.D., Rees, D.A.S., Reddy B.V.K., and Narasimhan, A., 2010, "Prediction of natural convection flow using network model and numerical simulations inside enclosure with distributed solid blocks," *Heat Mass Transfer*, **46**, 333-343.  
<http://dx.doi.org/10.1007/s00231-009-0567-9>

Mantri, S.S., and Bhasin, A.S., 2010, "Preventive prosthodontics for head and neck radiotherapy," *Journal of Clinical and Diagnostic Research*, **4**, 2958-2962.

Micco, C.D. and Aldao, C.M., 2003, "On the net radiation method for heat transfer," *European Journal of Physics*, **24**, 81-89.  
<http://dx.doi.org/10.1088/0143-0807/24/1/309>

Modest, M.F., 2003, *Radiative Heat Transfer*, 2<sup>nd</sup> Edn., Academic Press, New York.

Zemnick, C., Woodhouse, S.A., Gewanter, R.M., Raphael M., and Piro, J.D., 2007, "Rapid prototyping technique for creating a radiation shield," *J. Prosthetic Dentistry*, **97**, 236-241.  
<http://dx.doi.org/10.1016/j.prosdent.2007.02.005>

Saedodin, S., Torabi, M., Moghimi Kandelousi J., and Maghsoudlou, N., 2010, "Application of net radiation transfer method for optimization and calculation of reduction heat transfer, using spherical radiation shields," *World Applied Sciences Journal*, **11(4)**, 457-461.

Saedodin, S., Torabi, M., Maghsoudlou N., and Moghimi Kandelousi, J., 2010, "Calculation of reduction heat transfer using cylindrical radiation shields," *International Review of Mechanical Engineering*, **4(7)**, 924-928.

Zueco, J. and Campo, A., 2006, "Network model for the numerical simulation of transient radiative transfer process between the thick walls of enclosures," *Applied Thermal Engineering*, **26**, 673-679.  
<http://dx.doi.org/10.1016/j.applthermaleng.2005.09.010>

Zueco, J., Alhamab F., and Gonzalez Fernandez, C.F., 2004, "Analysis of laminar forced convection with Network Simulation in thermal entrance region of ducts," *International Journal of Thermal Sciences*, **43**, 443-451.  
<http://dx.doi.org/10.1016/j.ijthermalsci.2003.10.007>