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THE EFFECT OF MELTING ON MIXED CONVECTION HEAT AND MASS TRANSFER IN NON-NEWTONIAN NANOFLUID SATURATED IN POROUS MEDIUM

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ABSTRACT

In this paper, we investigated the influence of melting on mixed convection heat and mass transfer from the vertical flat plate in a non-Newtonian nanofluid saturated porous medium. The wall and the ambient medium are maintained at constant, but different, levels of temperature and concentration. The Ostwald–de Waele power-law model is used to characterize the non-Newtonian nanofluid behavior. A similarity solution for the transformed governing equations is obtained. The numerical computation is carried out for various values of the non-dimensional physical parameters. The variation of temperature, concentration, heat and mass transfer coefficients with the power-law index, mixed convection parameter, melting parameter, Brownian motion parameter, thermophoresis parameter, buoyancy ratio and Lewis number are discussed in a wide range of values of these parameters.

Keywords: *mixed convection, melting, non-Newtonian, nanafluid, Brownian motion, thermophoresis.*

1. INTRODUCTION

A nanofluid is a stable, uniform suspension of nanometer sized solid particles (< 100 nm) in conventional liquids such as water and ethylene glycol. An innovative procedure for improving heat transfer by using ultra fine solid particles in fluids has been used extensively during the last several years. For example, the thermal conductivity of copper oxide is about 100 times greater than that of water. Thus, the thermal conductivity of a suspension containing solid particles could be expected to be significantly greater than that of the base fluid. Xuan and and Roetzel (2000) proposed a homogeneous flow model where the convective transport equations of pure fluids are directly extended to nanofluids. The boundary layer flow in nanofluds has been analyzed recently by Nield and Kuznetsov (2009a, 2009b). Hojjat *et al.,* (2011) experimentally investigated the laminar convective heat transfer of non-Newtonian nanofluids with constant wall temperature. Gorla *et al.,* (2011) also analyzed mixed convective boundary layer flow over a vertical wedge embedded in a porous medium saturated with a nanofluid. They conclude that thermophoresis and buoyancy ratio enhanced the heat transfer rate while the same reduced the mass transfer rate. Moreover, the Brownian motion parameter enhanced mass transfer rate whereas the heat transfer rate decreased with the same. The effects of viscous dissipation on mixed convection heat and mass transfer along a vertical plate embedded in a nanofluid-saturated non-Darcy porous medium have been investigated by Rashad *et al.* (2014).

The study of melting phenomena in porous media has received much attention in recent years because of its important applications in casting, welding and magma solidification, permafrost melting and thawing of frozen ground etc. Epstein and Cho (1976) studied the melting heat transfer from a flat plate in a steady laminar case, while Kazmierczak *et al.,* (1986, 1987) analyzed melting from a vertical flat plate embedded in a porous medium in both free and forced convection processes. The heat transfer at the melting surface in the laminar boundary layer by using Karman-Pohlhausen method was discussed by Pozvonkov *et al.,* (1970). Bakier (1997) studied the melting effect on mixed convection from a vertical plate of arbitrary wall temperature both in aiding and opposing flows in a fluid saturated porous medium while Gorla *et al.,* (1999) considered a similar study with uniform wall temperature conditions.

Non-Newtonian power-law fluids are so widespread in industrial processes and in the environment that it would be no exaggeration to affirm that Newtonian shear flows are the exception rather than the rule. Natural convection in a non-Newtonian fluid about a vertical wall and that around horizontal cylinder and sphere in a porous medium was presented by Chen and Chen (1988(a),1988(b)), respectively. Nakayama and Koyama (1991) analyzed the more general case of free convection over a non-isothermal body of arbitrary shape embedded in a porous medium. Poulikakos and Spatz (1988) analyzed the melting phenomena on free convection from a vertical front in a non-Newtonian fluid saturated porous matrix. Recently Kairi and Murthy (2012) explored the melting effect of a non-Newtonian fluid on mixed convection with Soret effects in a non-Darcy porous medium. The influence of yield stress on free convective boundary layer flow of non-Newtonian nanofluids over a vertical plate in a porous medium was discussed by Hady *et al.,* (2011). Rashad *et al.,* (2011) investigated the boundary layer flow of a non-Newtonian nanofluid over a permeable vertical cone embedded in a porous medium. Since there are few investigations in the recent literature about the convective heat and mass transport in non-Newtonian nanofluids saturated with or without porous medium. This motivated us to investigate the significance of melting effect on mixed convection in a porous medium saturated with non-Newtonian nanofluids.

The main purpose of the present investigation is to analyze the effect of melting in non-Newtonian nanofluids on mixed convection in porous medium. The effects of Brownian motion and thermophoresis are included. Using Matlab BVP solver bvp4c, which is a finite difference code that implements the 3-stage Lobatto IIIa formula, a numerical solution of the boundary layers equations is obtained.

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2. Mathematical Formulation

Consider the mixed convective heat and mass transfer from a vertical plate embedded in a Darcy porous medium saturated with a non-Newtonian nanofluid. It is assumed that the plate constitutes the interface between the liquid phase and the solid phase during melting inside the porous matrix. The co-ordinate system and flow model are shown in the fig. 1.

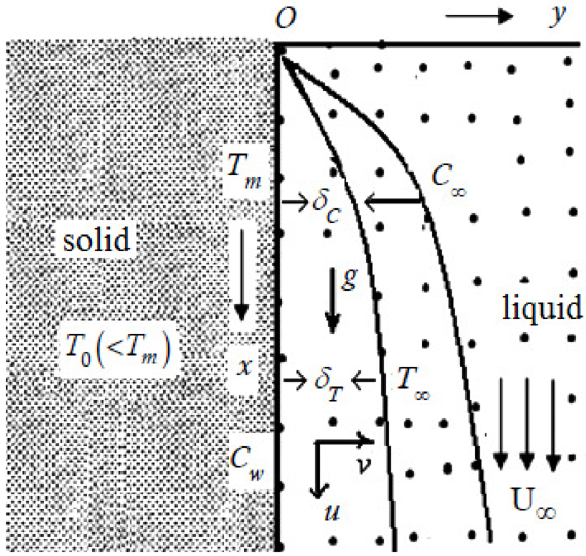

Fig. 1 Physical model and coordinate system (Kairi and Murthy 2012)

The x - coordinate is taken along the plate, the y - coordinate is measured normal to the plate, while the origin of the reference system is taken at the leading edge of the plate. The plate is at a constant temperature T_m at which the material of the porous matrix melts. The liquid phase temperature is $T_\infty(>T_m)$ and the temperature of the solid far from the interface is $T_0(<T_m)$. The concentration at the wall is C_w and the surrounding porous medium is maintained at constant concentration C_∞. The flow is steady, laminar and two dimensional. With the usual boundary layer and linear Boussinesq approximations, the governing equations, namely the equations of continuity, flow, energy and concentration for the isotropic and homogeneous porous medium may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u^n}{\partial y}=-\frac{(1-C_\infty)\rho_{f_\infty}\beta gK}{\mu}\frac{\partial T}{\partial y}+\frac{(\rho_p-\rho_{f_\infty})gK}{\mu}\frac{\partial C}{\partial y} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{3}$$

$$\frac{1}{\varepsilon}\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where $\alpha=\frac{k}{(\rho c)_f}$ and $\tau=\frac{\varepsilon(\rho c)_p}{(\rho c)_f}$, u and v are the Darcian velocity components along the x and y directions, T and C are the temperature and concentration, respectively, n is the power law index (n < 1, n = 1 and n >1, respectively, Pseudoplastic, Newtonian and Dilatant fluids). The parameters ρ_f, μ and β, respectively, are the density, viscosity and thermal expansion of the fluid while ρ_p is the density of the particle, g is the acceleration due to gravity, the effective heat capacity is $(\rho c)_f$ and effective thermal conductivity of the porous medium is k, D_B is the Brownian diffusion coefficient and D_T is thermophoretic diffusion coefficient.

The boundary conditions necessary to complete the problem formulation are written as

$$y=0:\quad k\frac{\partial T}{\partial y}=\rho_\infty\left[L+c_s(T_m-T_0)\right]v\,,\ T=T_m\,,\ C=C_w \tag{5}$$

$$y\to\infty:\quad u\to U_\infty\,,\ T\to T_\infty\,,\ C\to C_\infty \tag{6}$$

where L and c_s are latent heat of the solid and the specific heat capacity of the solid phase, respectively and U_∞ is the uniform free stream velocity.

The boundary condition (5) at the interface states that the temperature of the plate is equal to the melting temperature of the material saturating the porous matrix and the other condition means that the heat conducted to the melting surface is equal to the sum of heat of melting and the sensible heat required to raise the temperature of the solid, T_0 to its melting temperature T_m. It is important to note that eq. (5) is consistent with a coordinate system fixed to the melting surface, so that the interior of the solid appears to move towards the (stationary) melting surface with constant velocity equal to the melting velocity $v(x,0)$. In the present formulation, transient effects in the solid have been neglected. This assumption is valid as long as the melting solid is large compared to its thermal boundary layer thickness [see Epstein and Cho (1976)].

The continuity equation is automatically satisfied by defining a stream function $\psi(x,y)$ such that $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$. We introduce the following similarity transformation

$$\eta=\frac{y}{x}\chi^{-1}Pe_x^{1/2}\,,\ \psi(\eta)=\alpha\chi^{-1}Pe_x^{1/2}f(\eta)\,,\ \theta(\eta)=\frac{T-T_m}{T_\infty-T_m}\,,$$

$$\phi(\eta)=\frac{C-C_w}{C_\infty-C_w}\,,$$

where

$$\chi^{-1}=1+\sqrt{\frac{Ra_x}{Pe_x}}\,,\ Pe_x=\frac{U_\infty x}{\alpha}\ \text{and}\ Ra_x=\frac{x}{\alpha}\left[\frac{(1-C_\infty)\rho_{f_\infty}Kg\beta(T_\infty-T_m)}{\mu}\right]^{1/n}$$

The above transformation reduces the system of partial differential equations into the following system of non-linear ordinary differential equations:

$$n\,f'^{\,n-1}f''=-\,(1-\chi)^{2n}(\theta'-N_r\phi') \tag{7}$$

$$\theta''+\frac{1}{2}f\theta'+N_b\theta'\phi'+N_t\theta'^2=0 \tag{8}$$

$$\phi''+\frac{Le}{2}f\phi'+\frac{N_t}{N_b}\theta''=0 \tag{9}$$

The boundary conditions are

$$\eta=0:\quad f+2M\theta'=0\,,\ \theta=0\,,\ \phi=0 \tag{10}$$

$$\eta\to\infty:\ f'\to\chi^2\,,\theta\to1\,,\phi\to1 \tag{11}$$

In the above equations $M=\frac{c_f(T_\infty-T_m)}{L+c_s(T_m-T_0)}$ is the melting parameter, $Le=\alpha/D$ is the Lewis number, $N_r=\frac{(\rho_p-\rho_{f_\infty})(C_w-C_\infty)}{\rho_{f_\infty}\beta(T_m-T_\infty)(1-C_\infty)}$ is the buoyancy ratio, $N_t=\frac{\varepsilon(\rho c)_p D_T(T_m-T_\infty)}{(\rho c)_f\,\alpha T_\infty}$ is thermophoresis parameter and $N_b=\frac{\varepsilon(\rho c)_p D_B(C_w-C_\infty)}{(\rho c)_f\,\alpha}$ is Brownian motion parameter.

We note that $\chi=0$ corresponds to pure free convection and $\chi=1$ corresponds to pure forced convection processes.

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The local Nusselt number Nu_x and Sherwood number Sh_x are physical properties. The non-dimensional heat and mass transfer coefficients are defined as

$$Nu_x / \chi^{-1} Pe_x^{1/2} = \theta'(0) \tag{12}$$

$$Sh_x / \chi^{-1} Pe_x^{1/2} = \phi'(0) . \tag{13}$$

3. Results and discussion

The resulting ordinary differential equations (7)-(9) along with the boundary conditions (10)-(11) are solved using Matlab BVP solver bvp4c, which is a finite difference code that implements the 3-stage Lobatto IIIa formula. The integration length η_∞ varies with the parameter values and it has been suitably chosen each time such that the boundary conditions at the outer edge of the boundary layer are satisfied. The results obtained here are accurate up to the 4^{th} decimal place. At first, the accuracy of the numerical solution was tasted by accurately producing the findings of Cheng and Minkowycz (1977) for the case of a Newtonian fluid without melting *(* $M = 0, n = 1$ *)*, then the findings of Chen and Chen (1988) for the case of natural convection of (i) a Pseudoplastic fluids without melting, (ii) a Newtonian fluids without melting, (iii) a Dilatant fluids without melting, and (iv) a Newtonian fluid with melting and they are shown in the Table (1). Also, the present results for the Nusselt and Sherwood numbers are compared with those obtained by Kairi and Murthy (2012) in the absence of Brownian motion and thermophoresis parameter and they are found to be good agreement, but the details are not shown here for brevity. The following values are considered for the parameters: $0.5 \le n \le 1.5$, $0 \le \chi \le 1$, $0 \le M \le 1$, $0.1 \le N_r \le 0.5$, $0.0 \le N_b \le 1.0$, $0.1 \le N_t \le 1$ and $0.1 \le Le \le 5$. It was reported that in case of non-Newtonian fluids, heat and mass transfer coefficients decreased with increases in the melting parameter (Kairi and Murthy 2012). The effect of these parameters in power law nanofluids on the velocity, temperature heat and transfer coefficient is presented through some selected values of these parameters. Their combined effects on the heat and mass transfer coefficients are discussed.

Table 1. *Comparison of values of* $-\theta'(0)$ *for free convection along a vertical plate in Newtonian and non-Newtonian fluid saturated porous medium in the absence of Brownian motion and thermophoresis parameter with* $\chi = 0$, $N_r = 0$ and $Le = 0$.

S.No .	Fluid type with/ without melting effects	Cheng and Minkowycz (1977)	Chen and Chen(1988)	Present
(i)	$M = 0, n = 0.5$	-----------	0.3768	0.37682
(ii)	$M = 0, n = 1$	0.4440	0.4437	0.44370
(iii)	$M = 0, n = 1.5$	-----------	0.4752	0.47525
(iv)	$M = 1, n = 1$	-----------	0.2910	0.29103

Figures 2 and 3 show that the variations of non-dimensional temperature θ and concentration ϕ against the similarity variable η for different values of thermophoresis parameter N_t and melting parameter M with fixed values of N_r, N_b, Le and χ. It is observed that as increasing in the thermopohoresis parameter, the temperature boundary layer thickness increases in the presence of melting parameter and it is also noted that with influence of thermopnoresis parameter an increment in the values of melting parameter the temperature profile decreases for both pseudoplastic and dilatant fluids. Further, the concentration decreases with increase of melting and thermophoresis parameter for both pseudoplastic and dilatant fluids.

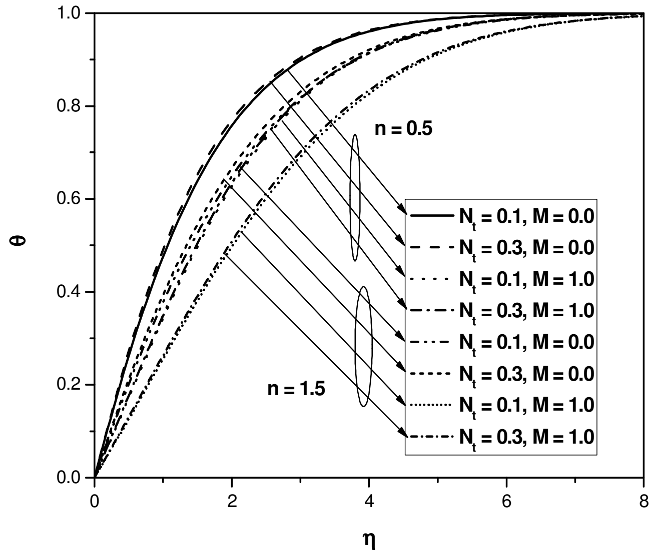

Fig. 2 Variation of temperature profiles for $n = 0.5$ and $n = 1.5$ with the similarity variable η varying N_t and M with fixed $\chi = 0.5$, $N_r = 1$, $N_b = 0.5$ and $Le = 1$.

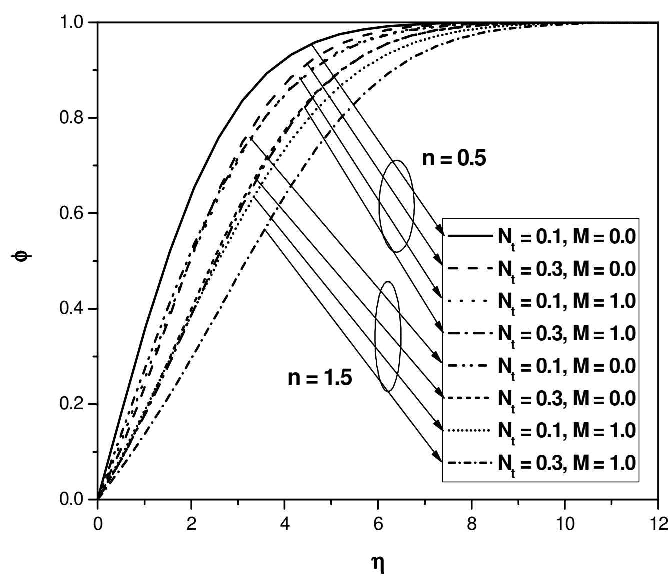

Fig. 3 Variation of concentration profiles for $n = 0.5$ and $n = 1.5$ with the similarity variable η varying N_t and M with fixed $\chi = 0.5$, $N_r = 1$, $N_b = 0.5$ and $Le = 1$.

The variations of the non-dimensional temperature θ and concentration distribution ϕ against the similarity variable η for different values of N_b and M are shown in Figs. 4 and 5, respectively. It is noted that for both pseudoplastic and dilatants fluids, the temperature and concentration boundary layer thicknesses increase as the value of N_b rises in the presence of melting parameter and the temperature and concentration decreases with increase of melting parameter with influence of Brownian motion.

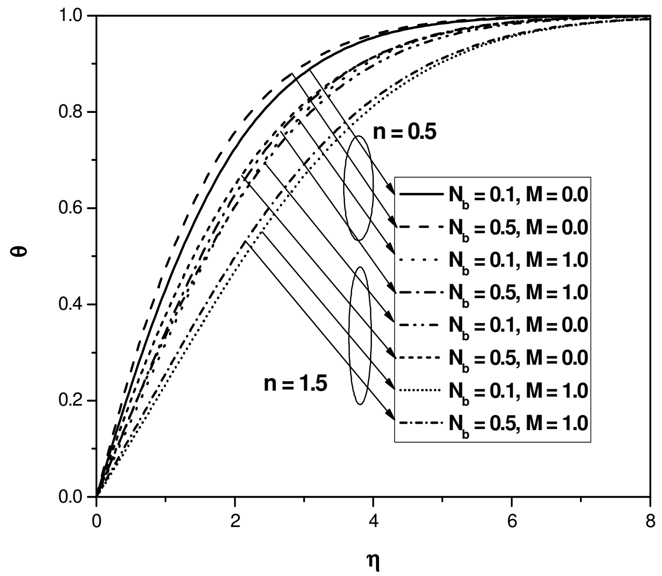

Fig. 4 Variation of temperature profiles for $n = 0.5$ and $n = 1.5$ with the similarity variable η varying N_b and M with fixed $\chi = 0.5$, $N_r = 1$, $N_t = 0.3$ and $Le = 1$.

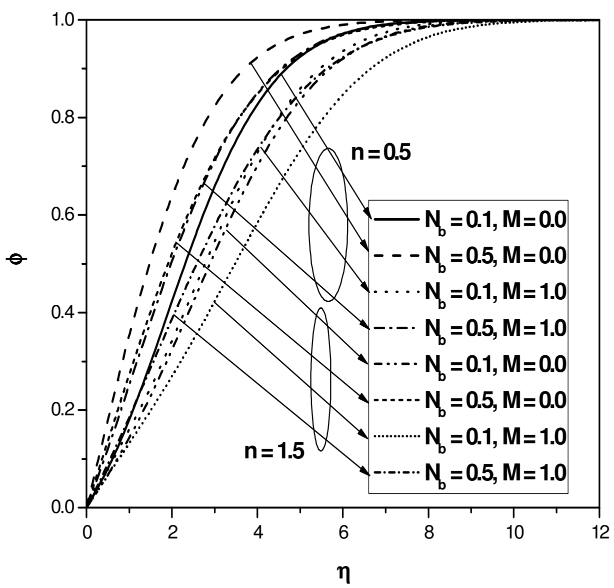

Fig. 5 Variation of concentration profiles for $n = 0.5$ and $n = 1.5$ with the similarity variable η varying N_b and M with fixed $\chi = 0.5$, $N_r = 1$, $N_t = 0.3$ and $Le = 1$.

In figures 6 and 7, the variations of heat and mass transfer coefficients are plotted against χ varying n and N_r with fixed values of other parameters. It is seen that heat and mass transfer coefficients are decreasing with increasing buoyancy ratio, N_r for pseudopastic, Newtonian and dilatants fluids. Similar results were reported by Kairi and Murthy (2012) while investigating the mixed convection heat and mass transfer of non-Newtonian fluids from a vertical surface embedded in porous medium.

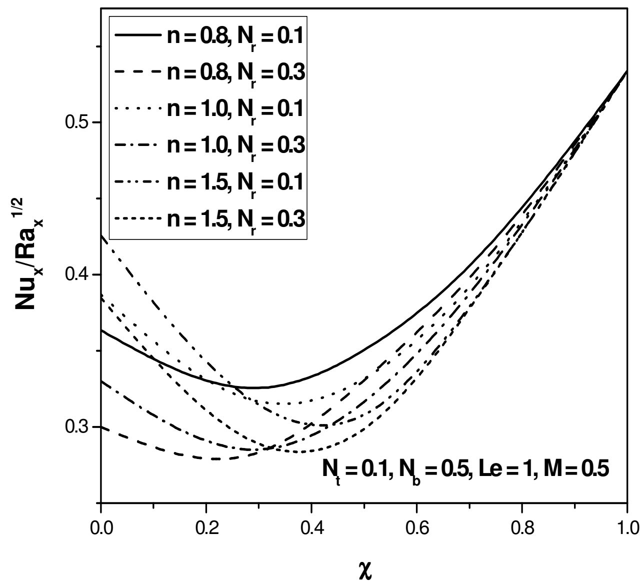

Fig. 6 Variation of heat transfer coefficient against χ varying n and N_r with fixed $Le = 1$, $N_b = 0.5$, $N_t = 0.1$ and $M = 0.5$.

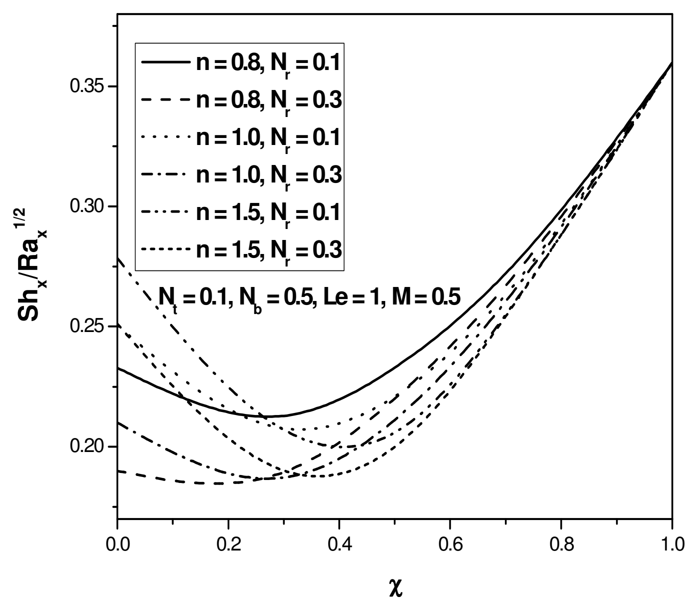

Fig. 7 Variation of mass transfer coefficient against χ varying n and N_r with fixed $Le = 1$, $N_b = 0.5$, $N_t = 0.1$ and $M = 0.5$.

Figures 8 and 9 prepared to illustrate the variations of the Nusselt and Sherwood numbers against Le for different values of M and N_t with fixed value of other parameters. It is observed that the with influence of melting parameter, the Nusselt number increases but, Sherwood number decreases with thermophoresis parameter, also it can be noted that the Nusselt and Sherwood number decrease with increase of melting parameter for both pseudoplastic and dilatant fluids. Moreover the thermophoresis effect on both heat and mass transfer rate is prominent for the pseudoplastics than that of dilatant nanofluids.

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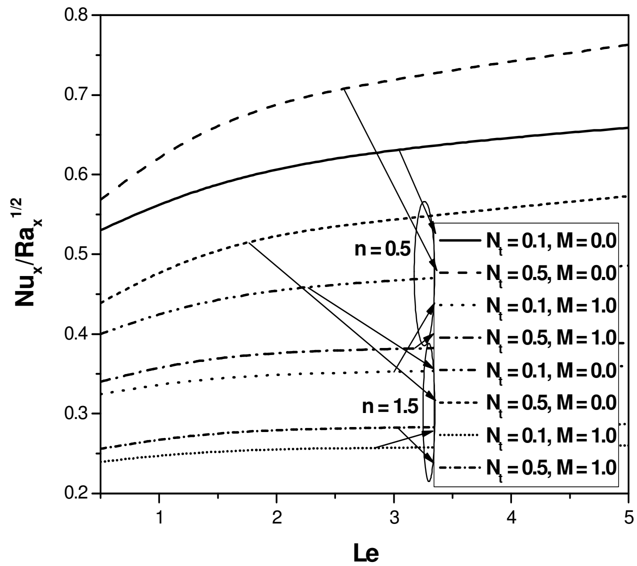

Fig. 8 Variation of heat transfer coefficient against Le varying n, M and N_t with fixed $N_b = 0.5$, $N_r = 0.1$ and $\chi = 0.5$.

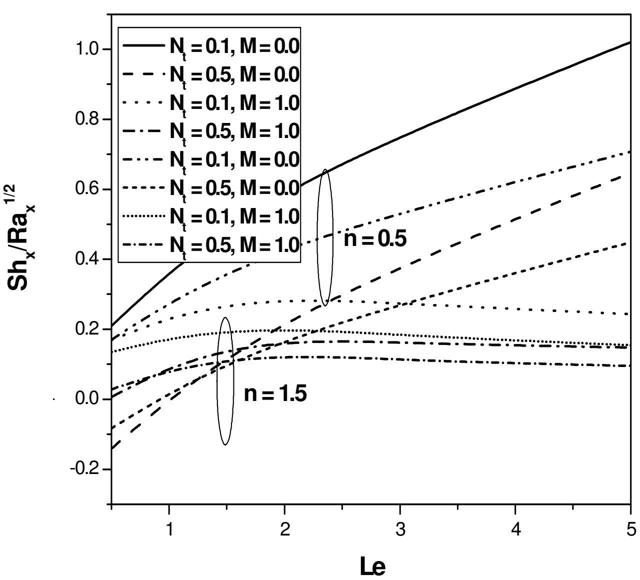

Fig. 9 Variation of mass transfer coefficient against Le varying n, M and N_t with fixed $N_b = 0.5$, $N_r = 0.1$ and $\chi = 0.5$.

Figures 10 and 11 prepared to show that the variation of heat and mass transfer coefficients for pseudoplastic and dilatant fluids against Le for varying M and N_b, and fixed value of other parameters. In the presence of melting parameter, the Nusselt and Sherwood number increases with increase of Brownian motion parameter N_b, and Lewis number Le for all types of non-Newtonian nanofluids. On the other hand, Nusselt and Sherwood number decrease with increase of melting parameter in the presence of thermophoresis for larger values of Le. The influences of the Brownian motion parameter on Nusselt and Sherwood numbers are superior for the pseudoplastic nanofluids with compared to dilatant nanofluids saturated porous medium.

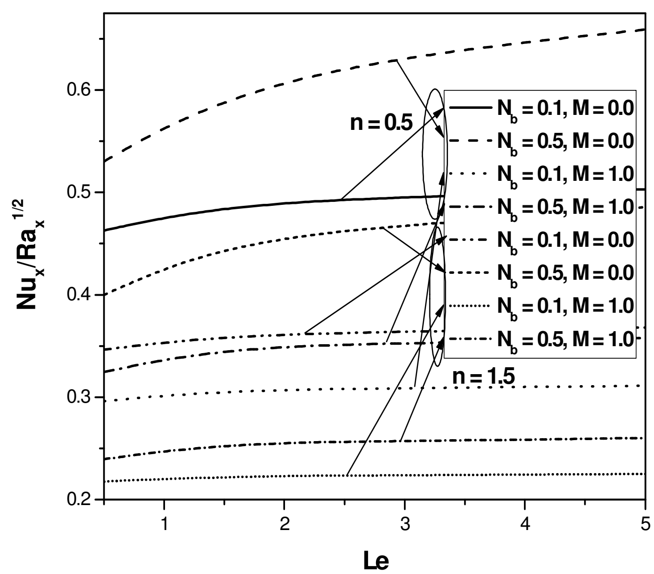

Fig. 10 Variation of heat transfer coefficient against Le varying n, M and N_b with fixed $N_t = 0.3$, $N_r = 0.1$ and $\chi = 0.5$.

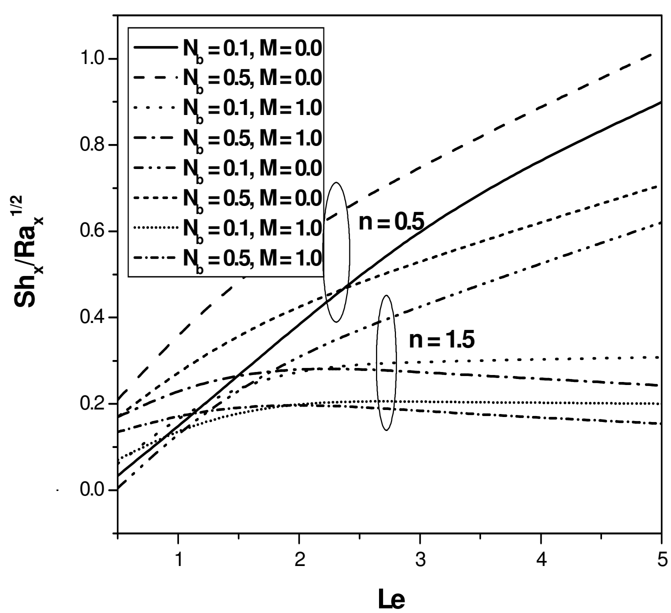

Fig. 11 Variation of mass transfer coefficient against Le varying n, M and N_b with fixed $N_t = 0.3$, $N_r = 0.1$ and $\chi = 0.5$.

4. CONCLUSIONS

In this study the effect of melting on mixed convection heat and mass transfer from a vertical flat plate in a porous medium saturated with non-Newtonian nanofluids is analyzed. It is noted that the temperature and concentration profiles as well as the heat and mass transfer coefficients are significantly affected by melting, thermophoresis and Brownian diffusion. The heat and mass transfer coefficients decrease with increasing M in the whole range of mixed convection for all types of non-Newtonian nanofluids. It is also noted that heat transfer coefficients increases but, mass transfer coefficient decrease with increasing buoyancy ratio N_r in the whole range of mixed convection for all types of non-Newtonian nanofluids. Finally, thermophoresis and Brownian motion effects on Nusselt and Sherwood numbers are prominent in pseudoplastic nanofluids when compared to dilatant nanofluids saturated porous medium.

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NOMENCLATURE

c_f specific heat capacity of the convective fluid
c_s specific heat capacity of the solid phase
C concentration
C_w concentration at the plate
C_∞ concentration at the ambient medium
D_B Brownian diffusion coefficient
D_T thermophoretic diffusion coefficient
f dimensionless stream function
g acceleration due to gravity
k effective thermal conductivity
K permeability of the porous medium
L latent heat of the solid
Le Lewis number
M Melting parameter
n power law index
N_r buoyancy ratio
N_b Brownian motion parameter
N_t thermophoresis parameter
Nu_x local Nusselt number
Pe_x local Peclet number
Ra_x local Rayleigh number
Sh_x local Sherwood number
T temperature
T_m melting temperature
T_0 temperature at the solid phase
T_∞ temperature at the ambient media
u, v average velocity components in x and y directions
U_∞ free stream velocity
x, y coordinates along and perpendicular to the wall

Greek Symbols

α effective thermal diffusivity
β coefficient of thermal expansion
χ mixed convection parameter
ε porosity of the porous medium
ϕ dimensionless concentration
η similarity variable
μ fluid consistency of the inelastic non-Newtonian power – law fluid
θ dimensionless temperature
ρ_f nanofluid density
ρ_p nano-particle mass density
$(\rho c)_p$ heat capacity of nano-particle
$(\rho c)_f$ heat capacity of fluid
ψ dimensionless stream function optical penetration depth

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