



COMPUTATIONAL INVESTIGATION OF DOUBLE-DIFFUSIVE MIXED CONVECTIVE FLOW IN AN ENCLOSED SQUARE CAVITY WITH SORET EFFECT

C. G. Mohan, A. Satheesh*

School of Mechanical Engineering, VIT University, Vellore, Tamilnadu, 632014, India

ABSTRACT

In this study, a two-dimensional steady state double-diffusive mixed convective flow in a square cavity with Soret effect is presented. The numerical investigation is considered with two different conditions, (a) top and bottom walls move with same velocity (U_0) towards right and (b) top wall moves towards right and bottom wall moves towards left with the same velocity (U_0). The left and right walls remain stationary. The top and bottom walls are adiabatic; the left wall is maintained at high temperature and concentration. The right wall is maintained at low temperature and concentration. Governing equations are solved by finite volume method using SIMPLE algorithm in association with QUICK scheme. The Prandtl number ($0.71 \leq Pr \leq 10$) is varied with different Richardson numbers ($0.01 \leq Ri \leq 100$) to analyze the streamline, temperature and concentration contours with and without Soret effect. Results are investigated using Nusselt and Sherwood numbers. The results are also validated with the existing literature.

Keywords: Numerical simulation, Double diffusion, Mixed convection flow, Soret number, Finite Volume Method.

1. INTRODUCTION

The combined effect of temperature and concentration gradients along with some externally applied forces in which heat and mass transfer occur simultaneously is known as double-diffusive mixed convection. Double-diffusive mixed convection in cavities is an area extensively investigated because of the multitude of engineering applications they are often encountered in. The aforementioned areas of research include nuclear reactors, solar collectors, cooling of electronic systems, chemical processing equipment, etc. Double-diffusive convection is also referred to as buoyancy driven flow induced by both temperature and concentration gradients. Bergman (1989) studied the influence of Soret-induced solutal buoyancy forces on the hydrodynamics and heat transfer rates of an initially uniform concentration fluid contained in an enclosure associated with natural convection. The effect of Soret phenomenon may or may not be important in natural convection. However, the Soret effect must be included in the analysis of the water-based binary fluid. Reima et al. (1993) carried out numerical studies for mixed convection in a driven cavity with a stable vertical temperature gradient. Flow is generated by the top horizontal boundary wall, which slides in its own plane at constant speed. A stabilizing externally-applied vertical temperature gradient is imposed on the system boundaries: the top wall is maintained at a higher temperature than the bottom wall. It was observed that at $Ri < 1.0$, in the bulk of the interior, fluids are well mixed, and temperature variations are small. For $Ri > 1.0$, the majority of the fluid interior is stagnant.

Mansour et al. (2004) worked on the numerical study of Soret effect on multiple steady state solutions induced by double diffusive convection in a square porous cavity. They considered the Darcy model and presented numerical results for $Ra = 100$, $Le = 0.1$ and Soret number

varying from -31.4 to 40. They found that the Soret number had a strong effect on the multiplicity of solutions. The bi-cellular flow disappeared in the absence of Soret effect and reappeared when the Soret effect is introduced but only when it was negative. Al-Amiri et al. (2007) have investigated steady mixed convection in a square lid-driven cavity under the combined buoyancy effects of thermal and mass diffusion. The investigation is carried out for different Ri , Le and Buoyancy ratio (N). The positive N enhances both heat and mass transfer rates whereas negative N deteriorates these effects. Gaikwad et al. (2007) studied the onset of double diffusive convection in a two component couple stress fluid layer with Soret and Dufour effect using both linear and non-linear stability analysis. It is concluded from results that, if Du is taken into account, there are enhancement in the stability of the couple stress fluid system in both stationary as well oscillatory modes. Also, the positive and negative Soret number are significant and stabilizes the couple stresses in the fluid system differently for stationary and oscillatory modes.

Abidi et al. (2008) numerically investigated double diffusive convection in a bunch of cubic cavities subjected to temperature and concentration gradients using FVM. The results were obtained for varying buoyancy ratio (N) in the range of $[-2,0]$ with fixed parameters as $Le = 10.0$, $Pr = 10.0$ and $Ra = 10^5$. The findings were, that heat and mass transfer differs with the change in magnitude of the temperature and concentration gradients. These gradients decide the thermal and solutal dominant regions in the cavity. The effect of double-diffusive natural convection of water in a partially heated enclosure with Soret and Dufour effect around the density maximum is numerically analyzed by Nithyadevi et al. (2009). The temperature and concentration between the two walls are varied. They found that the rate of heat and mass transfer increases when the values of thermal Ra increase in all heating locations

* Corresponding Author: satheesh.a@vit.ac.in; egsatheesh@gmail.com

and it is non-linear due to maximum density effect. The influence of Soret and Dufour effect on the natural convection heat and mass transfer near a vertically truncated cone with variable wall temperature and concentration in a fluid-saturated porous medium studied by Cheng (2010). They had concluded that there is an increase in the thickness of concentration boundary layer and decrease in mass transfer rate with increase in Soret parameter.

Ibrahim et al. (2010) studied the effect of chemical reaction on free convection heat and mass transfer over a vertical plate in the presence of yield stress and Soret effect. For comparison with standard results they have kept the value of $Sr < 1.0$ and they found that if there is chemical reaction occurring in the power law fluids, there will be a decrease in local Nu and increase in local Sh . Also, with an increase in the yield strength, local Nu , and local Sh decreases. Kumar et al. (2010) analyzed the double diffusion natural convection from a curved vertical surface immersed in a fluid-saturated semi-infinite porous medium due to Soret and Dufour effects. Finite difference scheme based on the Keller-Box approach in conjunction with blocktridiagonal solver is applied to get a solution to the dimensionless governing equations. The Soret effect is responsible for the increase in heat transfer and decreases in the mass transfer, but the Dufour effect does the vice-versa. The double-diffusive mixed convection in a rectangular enclosure with insulated moving lid is numerically studied by Teamah et al. (2010). They have concluded that the heat and mass transfer is greater when the upper wall is moving towards right than when it is moving towards left in case of positive N . On the other hand, same phenomena are observed in a reverse manner in case of negative N for high values of Ri .

The MHD free convective heat and mass transfer past a moving vertical plate with Soret and Dufour effect in the presence of suction/injection parameters are analyzed by Olanrewaju et al. (2011). The governing equations are solved numerically by shooting iteration technique together with a sixth order Runge–Kutta integration scheme. It is found that the skin friction coefficient and Nu decreases with increasing Du and decreasing Sr . A numerical model to examine the combined effects of Sr and Du on mixed convection MHD heat and mass transfer in a Darcian porous medium in the presence of thermal radiation and Ohmic dissipation developed by Pal et al. (2011). The influence of Schmidt number (Sc), Da , Pr , Sr , Du , magnetic parameter, on the velocity, temperature, and concentration fields are studied. Magyari et al. (2011) studied the double-diffusive natural convection past a vertical plate embedded in a fluid-saturated porous medium in the boundary-layer and Boussinesq approximation, considering that the Soret–Dufour cross-diffusion effects are significant. They found that when the Soret effect is more dominating than Dufour effect the Sherwood number influences the flow with thermosolutal symmetry than Nusselt number. The Dufour effects influence the temperature profile in the thermal boundary layer. The three dimensional numerical analysis of laminar mixed convection and entropy generation in a cubic lid-driven cavity is carried out by Lioua et al. (2011). The results were obtained for different Richardson number ranging from 0.01 to 100. The findings were, for low values of Ri the entropy generation is symmetrical in the cavity whereas it is insignificant for higher Ri .

Wang et al. (2011) analyzed the stability of double diffusive convection for a viscoelastic fluid with Sr in a porous medium using a modified Maxwell-Darcy model. They also concluded that the effect of the Sr irrespective of whether it is positive or negative it does destabilize the system in case of oscillatory mode. The numerical investigation for the unsteady MHD double-diffusive of an electrically conducting fluid past a flat plate embedded in a non-Darcy porous medium considering the Dufour and Soret effects carried out by Al-Odat et al. (2012). It is determined that the temperature contour increases with the decrease in the Soret effect. Whereas, the concentration contour decreases with the increase in Du . Dulal Pal et al. (2013) worked on the thermophoresis particle deposition and Soret–Dufour effects on the convective flow, heat and mass transfer characteristics of an incompressible Newtonian electrically conducting fluid having temperature-dependent viscosity

over a non-isothermal wedge in the presence of thermal radiation. They concluded that skin friction coefficient and local Sh increase with an increase in the thermal radiation and heat generation whereas the opposite is true for the local Nu .

Teamah et al. (2013) numerically analyzed the double-diffusive laminar mixed convection in an inclined cavity. The top lid is considered to move in both directions to introduce the forced convection effect. The top lid and the bottom surface are maintained at a constant temperature with top lid at a higher temperature than the later. The inclination angle, Le and N are varied within different ranges. High Le values improve the mass transfer rate whereas its impact on heat transfer rate is insignificant. The double diffusive convection in a cubic cavity filled with binary mixture is numerically investigated by Maatki et al. (2013) using FVM. Fixed parameters are $Le = 10.0$, $Pr = 10.0$ and $Ra = 10^5$ with varying buoyancy ratio. It is found that for negative buoyancy ratio ($N = -2.0$) the influence of Hartmann number (Ha) on heat and mass transfer is insignificant. Also, near the active walls there is reduction in heat and mass transfer.

Nayak et al. (2014) discussed the influence of Soret and Dufour effects on the mixed convection flow in the hydro-magnetic unsteady flow over an impermeable vertical stretching sheet in a fluid-saturated porous medium in the presence of chemical reaction. The governing partial differential equations are solved numerically by the Runge-Kutta fourth order scheme in association with the shooting method for achieving better accuracy. It is observed that there is an advancement in the velocity profile as the time passes from initial unsteady state to final steady state. Whereas, reverse phenomenon is seen in the case of temperature and concentration. Interestingly this effect enhances when there is the inclusion of Soret and Dufour effect. A model to study thermo-solutal buoyancies with Soret and Dufour effects for double diffusive convection under unsteady state conditions is developed by Wang (2014). The model is discretized by using FVM and obtained the solutions numerically using SIMPLE algorithm along with QUICK scheme. The results showed that there is a change in the flow structure as develops from conduction-dominated to convection-dominated regime with increasing buoyancy ratio or Rayleigh number.

Balla et al. (2015) studied the influence of inclination of the cavity on double diffusive natural convection boundary layer flow inside a square cavity filled with saturated porous media under the effect of Soret and Dufour. They concluded that the reason behind the resistance to fluid flow in the cavity is Dufour effect and there is an increase in concentration value with increase in Dufour parameter. Jha et al. (2015) analytically studied fully developed natural convection heat and mass transfer in a vertical annular non-Darcy porous medium in the presence of Soret effect. It is evident from the results that the buoyancy force is dominating the flow such that velocity decreases with increase in buoyancy ratio evolves from transient into steady-state. A numerical study of the laminar magneto-hydro dynamic (MHD) thermosolutal Marangoni convection along a vertical surface in the presence of Soret and Dufour effects performed by Mahdy et al. (2015). The research concluded that an increase in Hartmann number (Ha) increases both temperature and concentration distributions while decreasing velocity features. Wang et al. (2015) studied a coupling-diffusive model to analyze buoyancy convection with Soret and Dufour effects in a horizontal cavity. The results are obtained for different values of Rayleigh number and buoyancy ratio. It is found that the heat and mass transfer increases with increase in Rayleigh number and buoyancy ratio in thermosolutal diffusion because of coupling-diffusive interaction effect.

Recently, Mohan and Sathesh (2016) studied the fluid behavior in a two sided lid-driven cavity due to double diffusive mixed convection and MHD considering porous media. To get the solution for coupled governing equations FVM is used in association with SIMPLE and QUICK scheme for better accuracy. The finding was, the fluid flow decreases with increase in Hartmann number (Ha). Also, there is significant change in the magnitudes of velocities for high Ha and it also substantially affects thermal and concentration gradients. They came to

a conclusion that MHD on mixed convection is so important that the magnetic field cannot be neglected in mixed convection heat transfer problems. The Soret effect due to mixed convection on unsteady magnetohydrodynamic flow past a semi-infinite vertical permeable moving plate investigated by Raju et al. (2016). The solution to the non-linear equation of system is obtained by using perturbation technique. This study also shows like all other studies that there is an increase in the velocity distribution profile with an increase in Soret effect and in the presence of permeable media. The numerical study of double-diffusive convection in a vertical cavity with Soret and Dufour effects by lattice Boltzmann method on a graphics processing unit is performed by Ren and Chan (2016). In this research paper, they have concluded that the average Nu and Sh were found to increase with higher Rayleigh number (Ra). Wang et al. (2016) studied oscillatory double diffusive convection in a horizontal cavity with Sr and Du effects. The governing equations solved numerically based on SIMPLE algorithm and QUICK scheme. Their study found that the fundamental frequency and fluctuation amplitude increases with Ra or N. With the decrease in aspect ratio, the oscillatory convection.

Motivated by work mentioned above the author's intentions are to study the double diffusive mixed convection in a two sided lid-driven cavity and understand the flow behavior for selected range of essential parameters. It is worth mentioning that previous researchers have given less attention on Soret effect assuming its influence is negligible. Also for solving the governing equations many researchers have used Shooting scheme but it is observed that there are no appreciable results for different contours are obtained. By keeping this in mind, in present study authors used FVM to discuss temperature, concentration and streamline profiles by varying Ri and Pr for two different boundary conditions and also discussed the influence of Soret effect.

2. MATHEMATICAL FORMULATION

A two-dimensional steady, laminar, incompressible combined convective flow, heat, and mass transfer in a two-sided square cavity is studied. The top and bottom walls of the cavity is allowed to move with constant velocity (U_0) and are considered as adiabatic. The left and right vertical walls are stationary and are maintained at uniform but different temperatures and concentrations as shown in Fig. 1. This study considers two cases, wherein the first case the top and bottom walls move towards the right in the second case, the top wall moves towards right and bottom wall moves towards left. The gravity acts downwards.

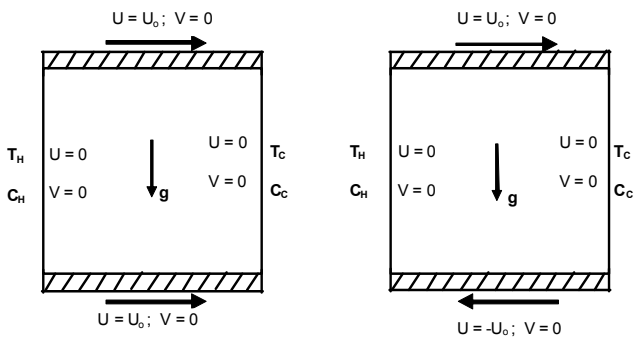


Fig. 1 Schematic diagram of parallel and anti-parallel wall movements of top and bottom walls

The thermophysical properties of the fluid are assumed to be constant, except the density, which is assumed to vary linearly with both temperature and concentration. The Boussinesq approximation is valid and viscous dissipation is assumed to be negligible. Since the Dufour effect is significantly small as compared with the Soret effect, the Dufour effect is neglected in the study (Nithyadevi, 2009).

$$\rho = \rho_0[1 - \beta_T(\theta - \theta_c) - \beta_c(c - c_c)] \quad (1)$$

Where,

$$\beta_T = -\frac{1}{\rho_0} \left(\frac{\partial \rho}{\partial \theta} \right)_{p,c} \quad \beta_c = -\frac{1}{\rho_0} \left(\frac{\partial \rho}{\partial c} \right)_{p,\theta} \quad (2)$$

The parameters β_T and β_c denotes fluid volumetric coefficient of expansion for thermal and concentration, respectively. The governing equations for the flow, heat, and mass transfers in a two-dimensional Cartesian coordinate system are given by the following equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (4)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g[\beta_T(\theta - \theta_c) + \beta_c(c - c_c)] \quad (5)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \alpha \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (6)$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) + \frac{Dk_T}{\theta} \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right). \quad (7)$$

Where, k_T , ν , ρ , D , θ , α , and g are the thermal conductivity, kinematic viscosity, density, mass diffusivity, mean fluid temperature, thermal diffusivity, and acceleration due to gravity, respectively. Making the Eq is convenient. (3) to (7) dimensionless using the following dimensionless variables such as:

$$X = \frac{x}{L}; \quad Y = \frac{y}{L}; \quad U = \frac{u}{U_0}; \quad V = \frac{v}{U_0}; \quad T = \frac{\theta - \theta_c}{\theta_H - \theta_c}; \quad C = \frac{c - c_c}{c_H - c_c}; \quad P = \frac{p}{\rho U_0^2}$$

The previous governing equations result in the following dimensionless equations

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (8)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (9)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ri(T + NC) \quad (10)$$

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{RePr} \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) \quad (11)$$

$$U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{ReSc} \left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) + \frac{Sr}{Re} \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) \quad (12)$$

Where, Reynolds number (Re) = $\frac{U_0 L}{\nu}$, buoyancy ratio number (N) =

$$\frac{\beta_c(c_H - c_c)}{\beta_T(\theta_H - \theta_c)} = \frac{Gr_c}{Gr_T}, \text{ thermal Grashof number}$$

$$(Gr_T) = \frac{g\beta_T(\theta_H - \theta_c)L^3}{\nu^2}, \text{ solutal Grashof number}$$

$$(Gr_c) = \frac{g\beta_c(c_H - c_c)L^3}{\nu^2}, \text{ Richardson number}$$

$$(Ri) = \frac{Gr_T}{Re^2}, \text{ Prandtl number}$$

(Pr) = $\frac{\nu}{\alpha}$ and Soret number

(Sr) = $\frac{(Dk_T(\theta_H - \theta_C))}{(\theta v(c_H - c_C))}$, Schmidt number (Sc) = $\frac{\nu}{D}$

Table 1 The dimensionless boundary conditions of the problem under consideration can be written as

Boundaries	Condition 1	Condition 2
Top wall	$u = U_0 ; v = 0 ;$ $\frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = 0$	$u = U_0 ; v = 0$ $\frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = 0$
Bottom wall	$u = U_0 ; v = 0$ $\frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = 0$	$u = -U_0 ; v = 0$ $\frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = 0$
Left wall	$u = v = 0 ; T = T_H ;$ $C = C_H$	$u = v = 0 ; T = T_H ;$ $C = C_H$
Right wall	$u = v = 0 ; T = T_C ;$ $C = C_C$	$u = v = 0 ; T = T_C ;$ $C = C_C$

The convective heat and mass transfer rates across the cavity are prime important to understand the flow behavior in the enclosure. Therefore, it can be quantified by measuring Nusselt number (Nu) and Sherwood number (Sh). The local and average Nusselt numbers at the hot wall of the cavity are determined by the following equation

$$Nu = -\left(\frac{\partial T}{\partial X}\right)_{x=0} \quad Nu_{avg} = \int_0^1 Nu dY. \quad (13)$$

Similarly, the local Sherwood and the average Sherwood numbers are expressed by

$$Sh = -\left(\frac{\partial C}{\partial X}\right)_{x=0} \quad Sh_{avg} = \int_0^1 Sh dY. \quad (14)$$

3. METHODOLOGY

A Steady state two-dimensional governing equations are solved to obtain the numerical solutions for the given mathematical model by using SIMPLE algorithm (Patankar, 1980). The conservation of mass is enforced by deriving the pressure correction equation from the continuity equation. The staggered grid arrangement is utilized in the Finite Volume Method. Discretization is performed by the second order central difference scheme for diffusion term, and the third-order differer QUICK scheme for convection term in the border and inner nodal points, respectively. A combination of tridiagonal matrix algorithm (TDMA) and step by step procedure is used to solve the set of algebraic equations. The iterations are repeated until the following conditions are satisfied:

$$\frac{\sum \sum_j |\Phi_{i,j}^{n+1} - \Phi_{i,j}^n|}{\sum \sum_j |\Phi_{i,j}^{n+1}|} \leq 10^{-5}.$$

Where, Φ denotes U, V, T and C . The subscript i and j indices denote the grid locations in x and y directions, respectively. It is observed in the present numerical results the decrease of convergence criteria 10^{-5} does not cause any significant change in the final results.

4. RESULTS AND DISCUSSION

The grid dependency test was performed for $Re = 100, Gr_T = 10^4, Pr = 1.0, Le = 1.0$ and $Sr = 1.0$ for the present study, and the results have been evaluated for different grid sizes. For a selected operating condition, the

midplane u velocity is drawn for various grid sizes of $51 \times 51, 81 \times 81, 101 \times 101$ and 121×121 . It is concluded from the test that the grid size of 101×101 produced suitably accurate results as shown in Fig. 2 and there is no variation upon further refinement. Upon comparing the results of the present code with Teamah and Maghlany, (2010) at $Pr = 0.7, N = 1.0$ and $Ri = 1.0$, the contours of streamline, concentration and temperature for a rectangular two-sided lid-driven cavity shows good agreement, thereby validating the legitimacy of our code (as shown in Fig. 3).

The results are obtained for various values of the parameters involved in the problems of four different cases. The parameters which are kept fixed are as the buoyancy ratio ($N = 1.0$), thermal Rayleigh number ($Ra_T = Gr_T \times Pr = 0.7 \times 10^4$), Reynolds number ($Re = 100$) and Lewis number = 1.0. Richardson number is varied from 0.01 to 100 to analyze the importance of natural convection to the forced convection.

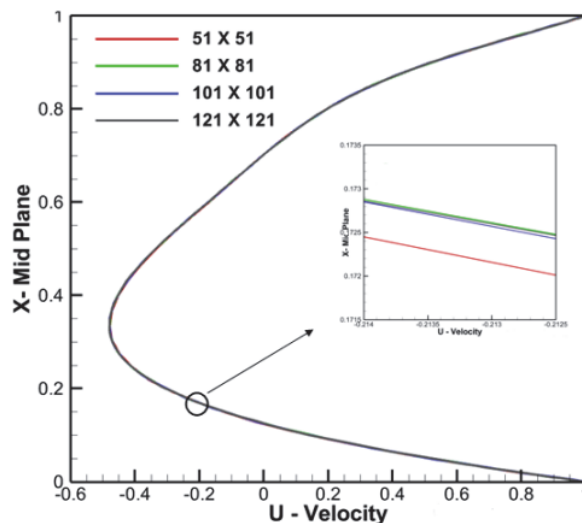


Fig. 2 Grid dependency study of U – velocity at X - mid plane

Fig. 4 shows the temperature, concentration and streamline contours for different Prandtl number at $Ri = 1.0$ without Soret effect when both walls move towards right. It is found that the thermal distribution is increased with increase in Pr which shows that the momentum diffusivity enhances the convective heat transfer rate. It is observed that for $Pr = 0.71$, the variation of temperature and concentration is negligible. With further increase in Pr, the diffusion due to concentration gradient is increased which is very similar to the diffusion due to temperature gradient because Lewis number (Le) is maintained as unity. It is observed from the streamline contours, that two flow circulations are formed near the top and bottom walls. The upper flow circulation is clockwise direction whereas bottom flow circulation is in an anti-clockwise direction. For low Pr, the movement of flow is more near the top wall and keeps on reducing with increase in Pr.

Fig. 5 shows the Soret effect on temperature, concentration and streamline patterns for different Pr. Diffusion due to the temperature gradient and the diffusion due to concentration gradient is observed significantly even at low Pr. It is found to be increased gradually with increase in Pr though $Le = 1.0$ which shows the major impact on concentration gradient due to Soret effect. The concentration gradient is increased rapidly at $Pr = 5.0$ and higher values of Pr. The shear flow takes place between the fluid layers within the cavity both in clockwise and anticlockwise directions. The negative sign of the contours shows the anticlockwise motion of the concentration gradient. It is also observed that the streamline contours move towards the right wall for higher Pr. It is clearly observed from Figs. 4 and 5 the effect of the Soret number at low Pr is negligible. However, this effect is randomly increased with increase in Pr. This is due to the momentum diffusivity of the fluid plays the major role.

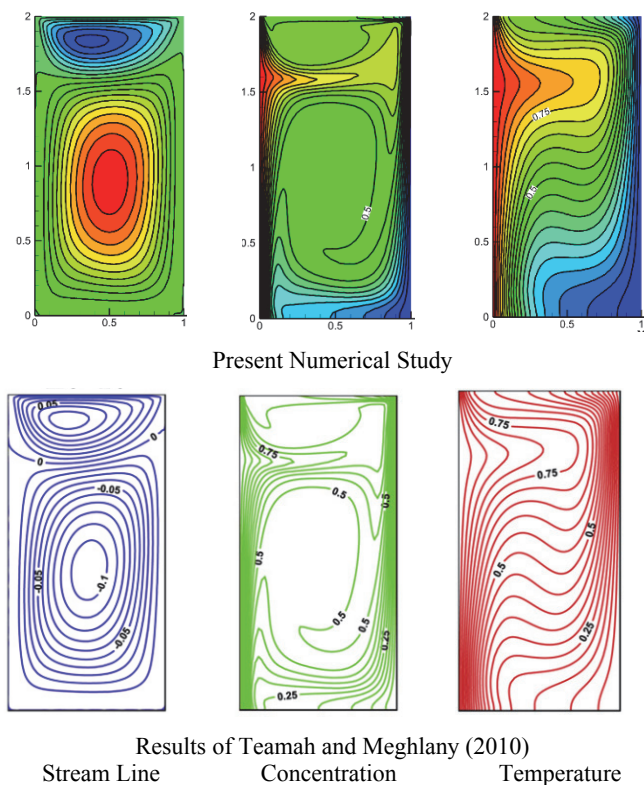


Fig. 3 Comparison of present numerical study with Teamah and Maghlyani (2010) for $Pr = 0.7$, $N = 1.0$, $Ri = 1.0$, $Ra_T = 0.7 \times 10^4$ and top wall moves towards left.

The effect of Pr on temperature, concentration, and streamline contours are presented in Fig. 6 without considering the Soret effect. In this case, both top and bottom walls are moving horizontally at the same velocities but in the opposite direction. Therefore, low temperature and concentration diffusion move towards the bottom left and the high temperature and concentration move towards the top right which makes clockwise flow distribution within the cavity. It is observed from the streamline contours the perturbation takes place at the top left and bottom right corners. With the increase in Pr , the streamline contours change from circular to elliptical shape. This effect is increased due to diffusion of temperature and concentration gradients. Fig. 7 shows the effect of Pr with the presence of Soret effect when both walls are moving in opposite direction. The increase in diffusion rates due to temperature and concentration gradient and streamline contours are uniform up to $Pr = 1.0$ and also the intensity of perturbation is also uniformly increased. Further increase in Pr with Soret effect the temperature and concentration gradients are increased rapidly due to this the elliptical shaped vortex is detached from two neat vortex top and bottom walls. When $Pr = 5.0$, this two vortex is further detached and originated the third vortex at the center of the cavity. It is observed that all three vortices are in the clockwise direction. Due to Soret effect, the temperature and concentration gradients are increased rapidly (as shown in Fig. 7c). When Pr is further increased, these vortices are completely detached, and the magnitude of the middle vortex is increased, but the direction is reversed. Due to this reason the temperature and concentration diffusions are predominant at the center of the cavity.

Fig. 8 shows the midplane U , V , T and C profiles at different Ri with and without Soret effect when both top and bottom walls are moving towards right. In forced convection ($Ri = 0.01$) as shown in Fig. 8(a), the horizontal velocity is gradually decelerated towards the center of the cavity, and Soret effect is negligible. In mixed convection ($Ri = 1.0$), the minimum velocity occurred approximately at $Y = 0.3$ and not at the center of the cavity, because the natural and forced convection equally dominates. Hence, the acceleration due to gravity drags the minimum

velocity at $Y = 0.3$. When $Ri = 1.0$ the influence of Soret effect in horizontal velocity is enhanced up to 5%. In natural convection ($Ri = 100$), two vortices are formed near the top and bottom walls. The maximum U -velocity is reached about double that of the wall velocity. It also observed that the Soret effect further enhances the horizontal velocity approximately 30%. Fig. 8(b) shows the V -velocity at mid Y plane. For low Ri , it is nearly zero, and it is increased marginally up to 0.3 for $Ri = 1.0$. When $Ri = 100$, the V -velocity is increased up to three. For all these conditions, the Soret effect is found to be significant nearly 20% only for natural convection. Figs. 8(c) and 8(d) show the temperature and concentration diffusions. For low Ri , the heat transfer rate is maximum near the moving walls and remains constant except near the center of the cavity. The non-dimensional temperature is further decreased drastically from 0.4 to 0.3 due to the formation of the vortex at the center. When $Ri = 1.0$, the effect of natural convection is increased, and it is observed at the bottom wall the temperature is about 0.55. Due to the movement of the bottom wall, the heat transfer rate is decreased towards the cavity up to $H = 0.3$, and it is increased due to buoyancy effect and movement of the top wall. However, in natural convection ($Ri = 100$) the heat transfer rate is distributed uniformly throughout the cavity. The temperature variation at the X -midplane is observed from 0.22 to 0.8. When the Soret effect is considered for the above conditions, the heat transfer rate is found to be decreased significantly, and the maximum deviation is about 12% for $Ri = 100$. Further, this effect is reduced gradually and negligible for forced convection. The thermal and concentration diffusions are exactly same for the selected range of Ri in the present study without considering Soret effect. This is due to Le is maintained as 1.0. When the Soret effect is considered, the diffusion due to concentration is accelerated from forced to natural convection about 10% to 30% respectively.

Fig. 9 shows the variation of velocities, temperature and concentration at midplane when top and bottom walls are moving in opposite directions. For $Ri = 0.01$ and 1.0, the change in U and V velocities are negligible though the Soret effect is considered and in the natural convection ($Ri = 100$), it is found to be high due to formation of two vortices near the moving walls and the velocities are enhanced by 25% due to Soret effect. It is observed that the temperature and concentration distributions are uniform throughout the cavity for low Ri except near the moving walls because these are rapidly varying due to the wall movement. By considering the Soret effect, the variation in thermal diffusion rate is negligible for low Ri and gradually it is increased to 10%. In the case of concentration diffusion, the Soret effect is significant with an increase in Ri from 0.01 to 100. The maximum variation is found to be 30% for $Ri = 100$.

Figs. 10 (a) and (b) shows the local Nu and Sh at the left wall when both walls move towards right. It is found that for forced convection ($Ri = 0.01$), the local Nu is maximum at the center of the wall and symmetrically decreased towards the top and bottom. There is a sudden increase and followed by a decrease in local Nu at the top and bottom of the wall. The vortex near the top wall moves in a clockwise direction, and the vortex near the bottom wall are in a counter-clockwise direction. Because of this effect, the heat is carried by the fluid at the top and bottom of the wall. For $Ri = 0.01$, the Soret effect does not affect the local Nu . When Ri increases, the local Nu is decreased, and due to bouncy effect, the local Nu is more near the bottom wall than at the top. The Soret effect decreases the local Nu about 5% for $Ri = 1.0$ and 100. The local Sh at the left wall is same as that of local Nu . The local Sh is found to decrease significantly at the center of the left wall when considering Sr . The magnitude of local Sh decreases with increase in Ri . The variation of local Nu and Sh for both walls move in opposite direction is presented in Figs. 11(a) and (b). It is observed that for low Ri the local Nu and Sh is maximum near the bottom of the wall and rapidly decreases towards the top of the wall, further a small variation in local Nu and Sh is identified. This effect is due to the cold with low concentration fluid has initially contacted at the bottom then diffused towards up. For higher values of Ri , the buoyancy effect dominates.

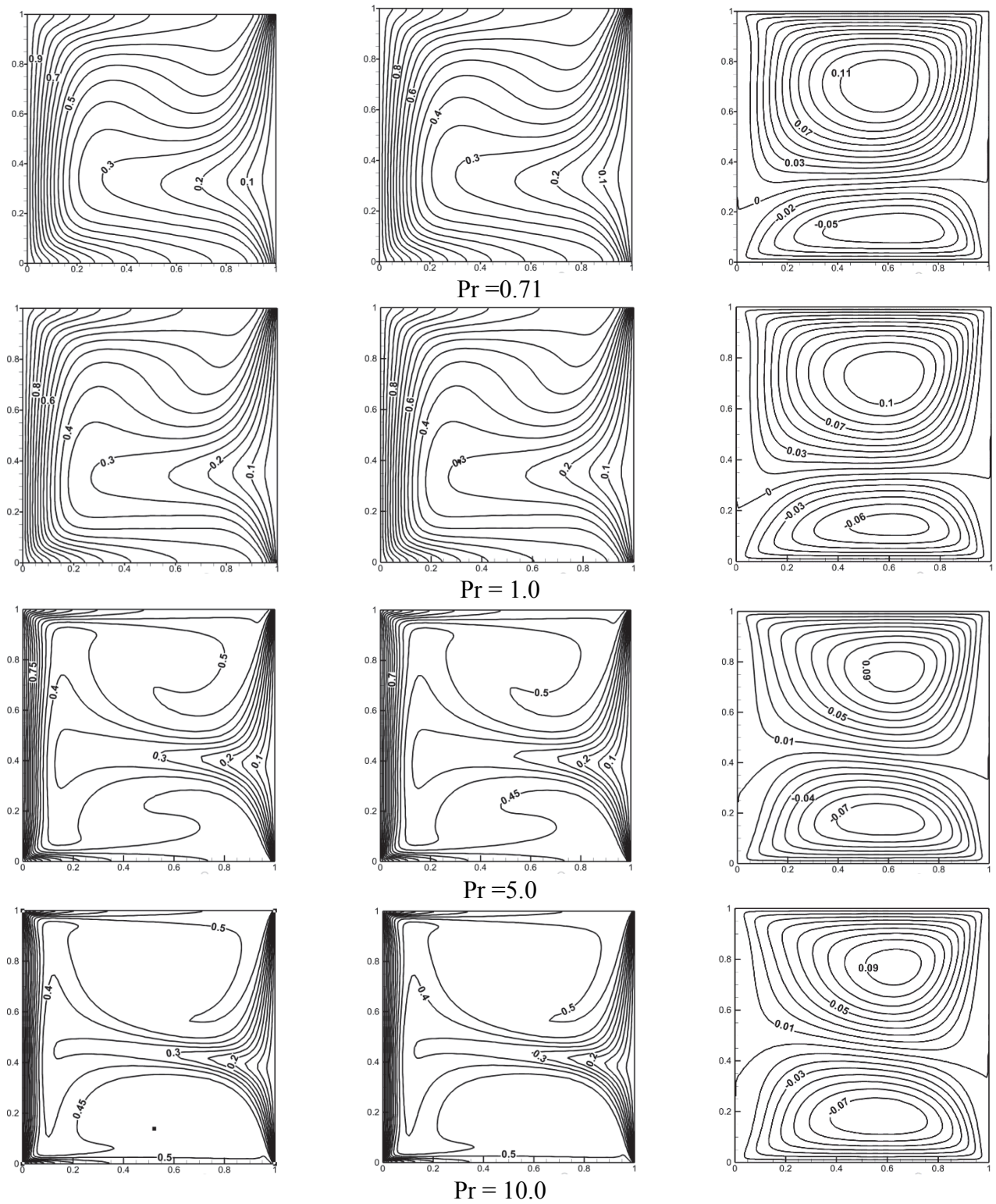


Fig. 4 The effect of Pr on (a) Temperature (b) Concentration (c) Stream line contours at $Ri = 1.0$, $Sr = 0$, top and bottom walls move towards right.

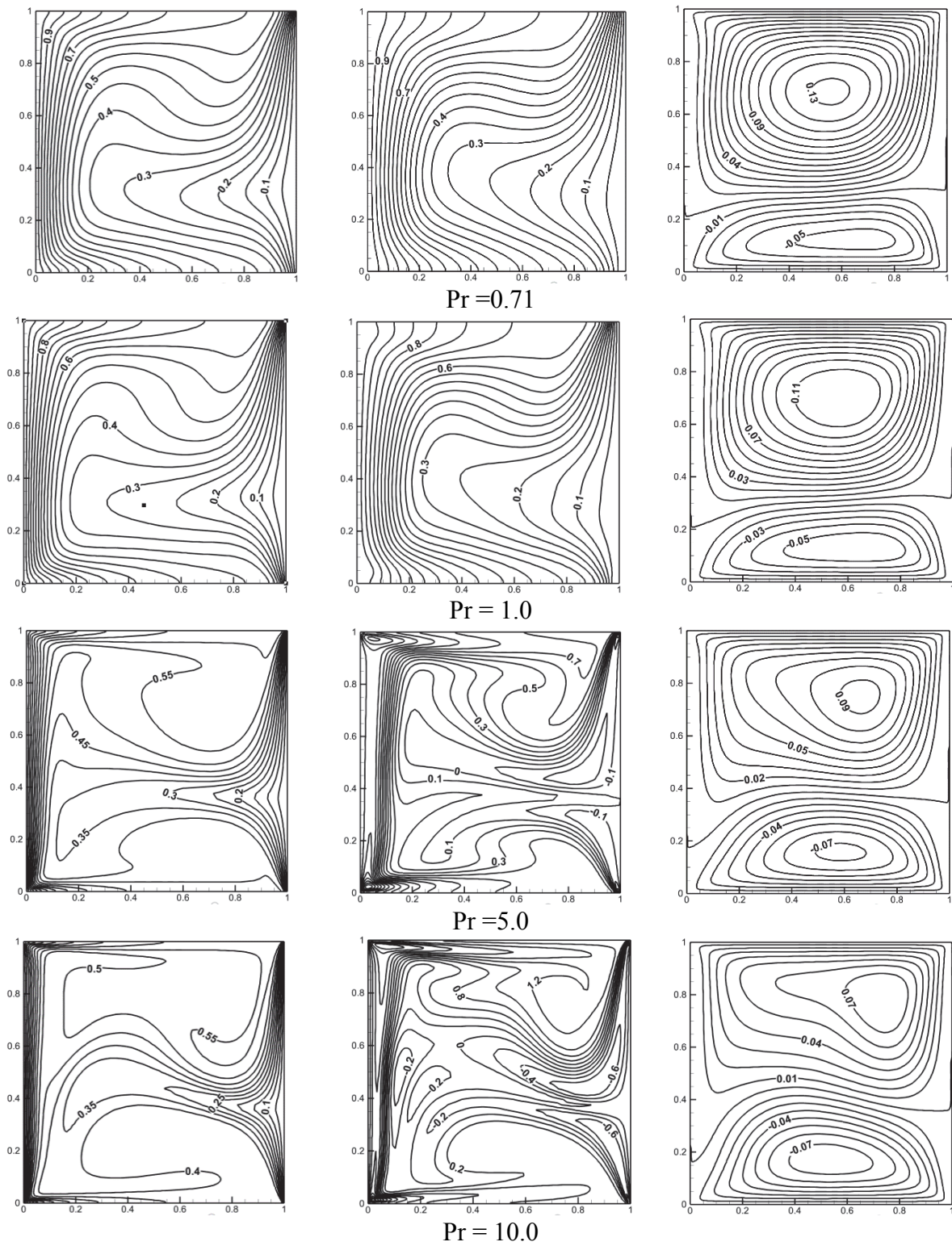


Fig. 5 The effect of Pr on (a) Temperature (b) Concentration (c) Stream line contours at $Ri = 1.0$, $Sr = 1.0$, top and bottom walls move towards right.

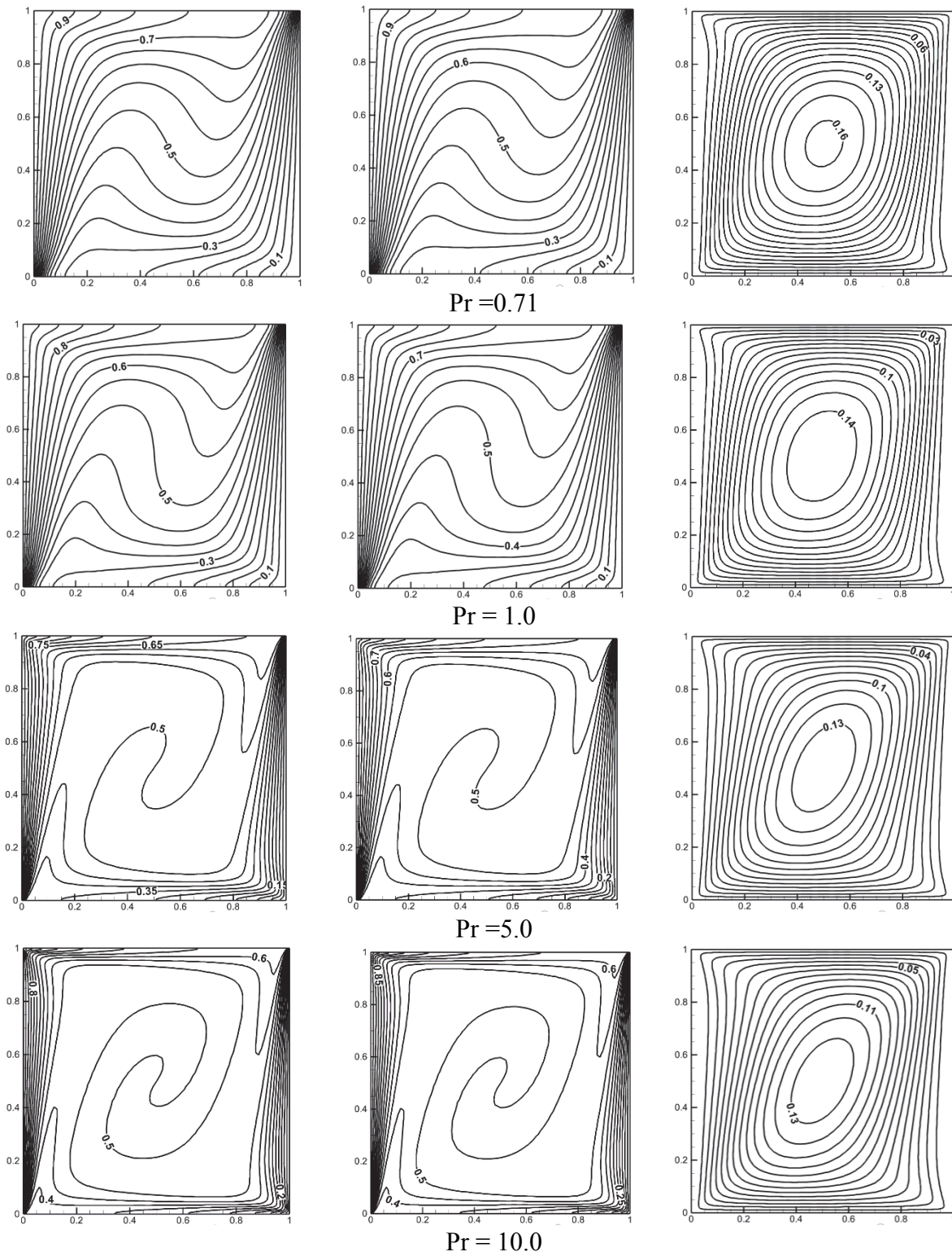


Fig. 6 The effect of Pr on (a) Temperature (b) Concentration (c) Stream line contours at $Ri = 1.0$, $Sr = 0$, top wall moves right and bottom wall moves left.

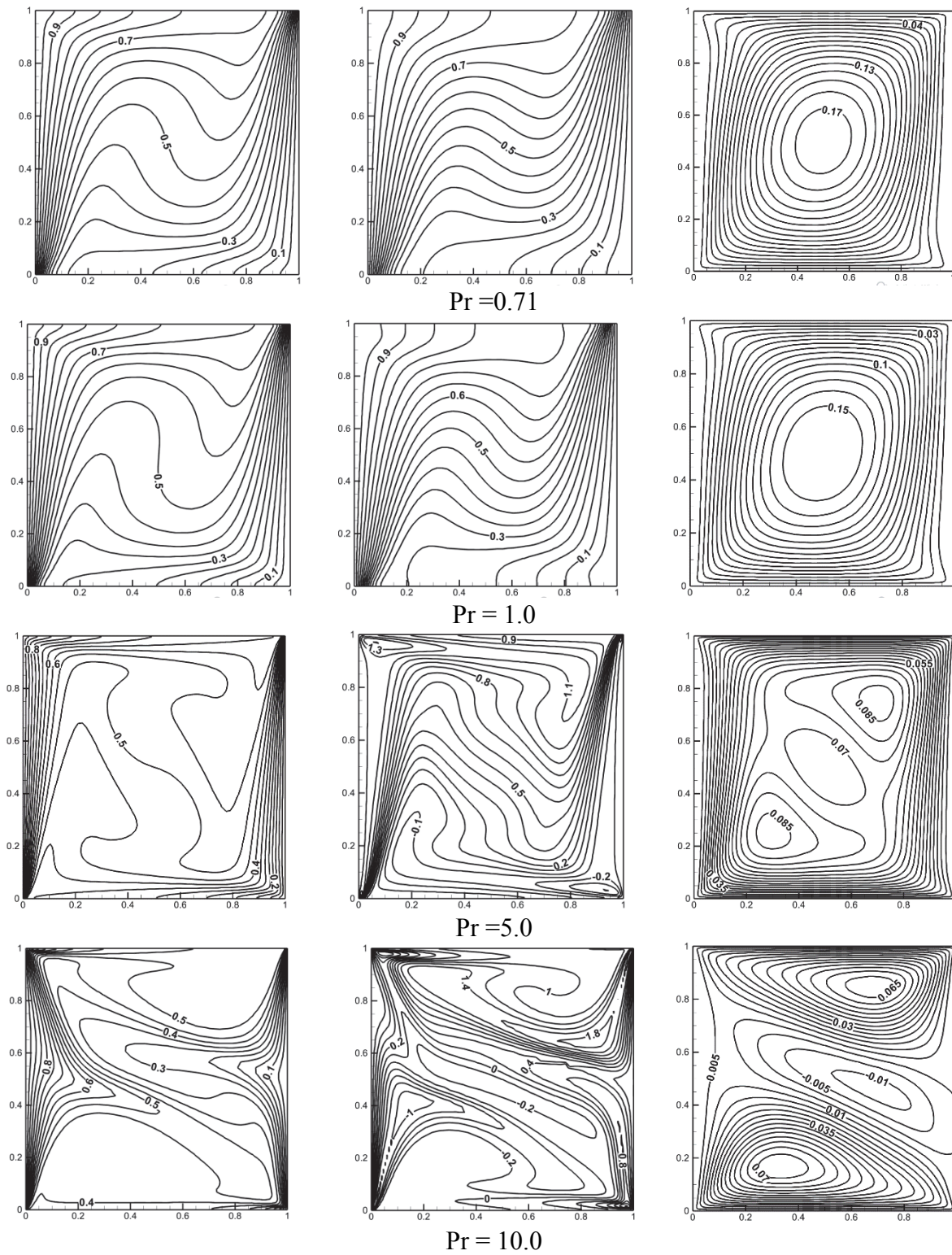


Fig. 7 The effect of Pr on (a) Temperature (b) Concentration (c) Stream line contours at $Ri = 1.0$, $Sr = 1.0$, top wall moves right and bottom wall moves left.

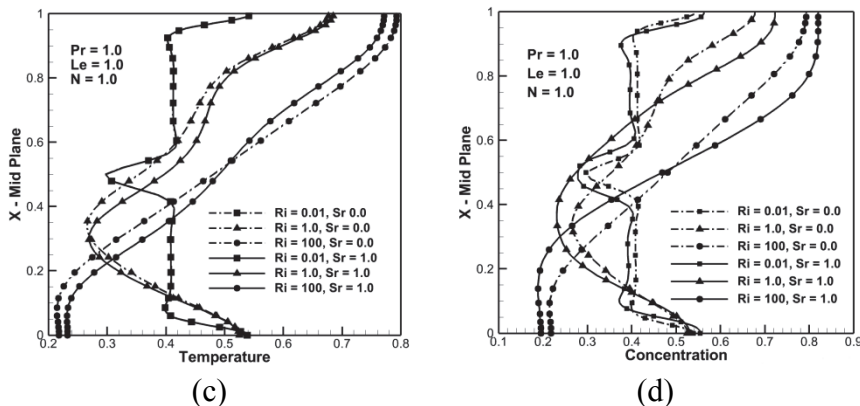
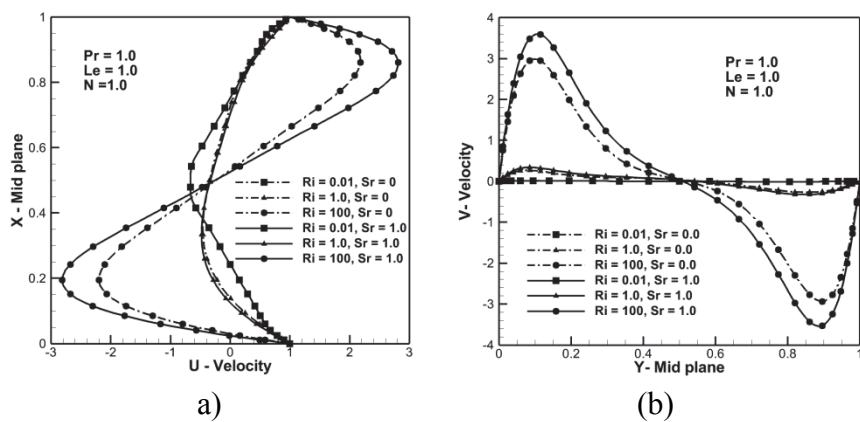


Fig.8 Mid plane (a)U-velocity, (b) V- velocity, (c) Temperature and (d) Concentration profiles for Ri = 0.01, 1.0 and 100 when Top and bottom walls are moving in right direction

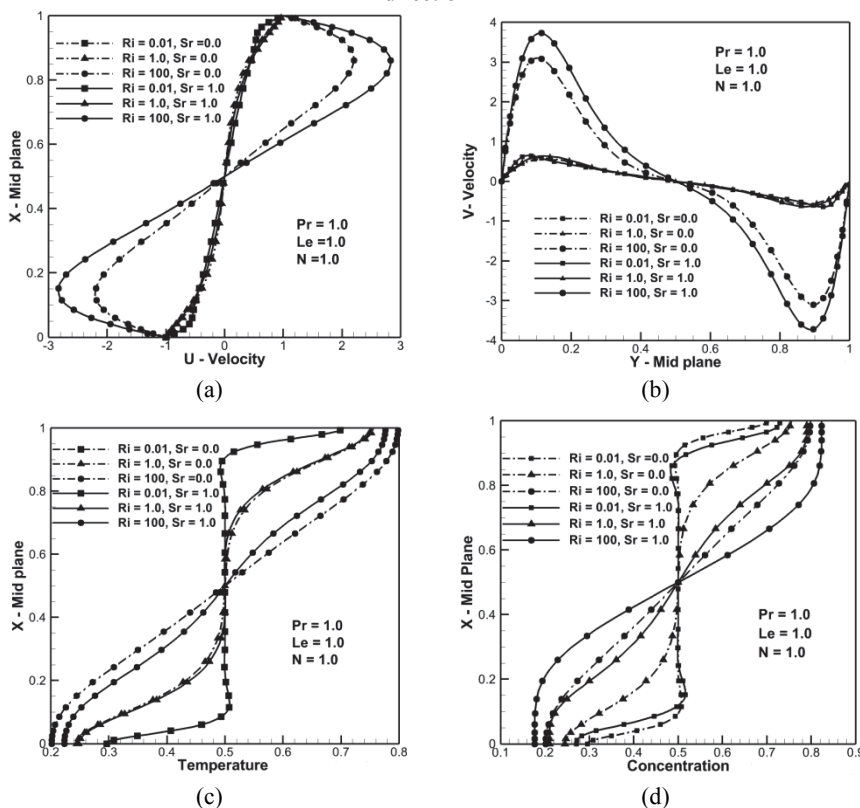


Fig. 9 Mid plane (a)U-velocity, (b) V- velocity, (c) Temperature and (d) Concentration profiles for Ri = 0.01, 1.0 and 100 when top and bottom walls are moving in opposite direction

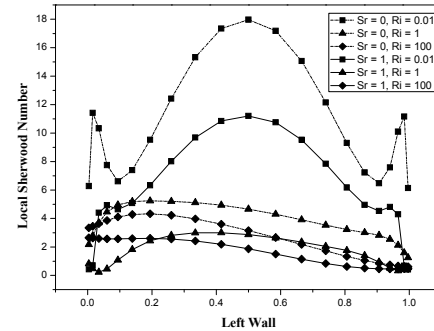
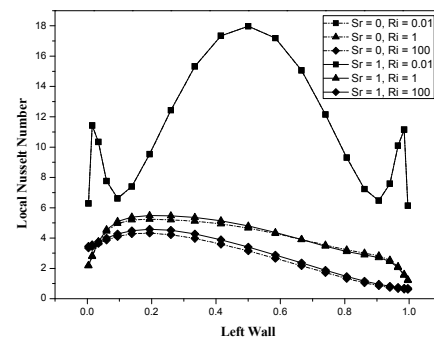


Fig.10 Local Nu and Sh at left wall for both top and bottom walls move towards right.

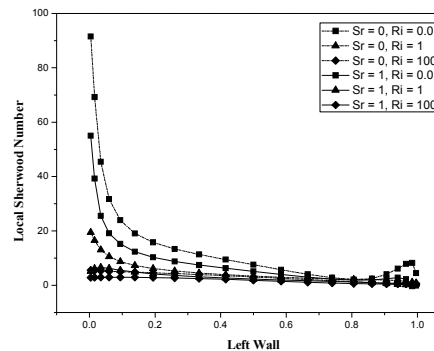
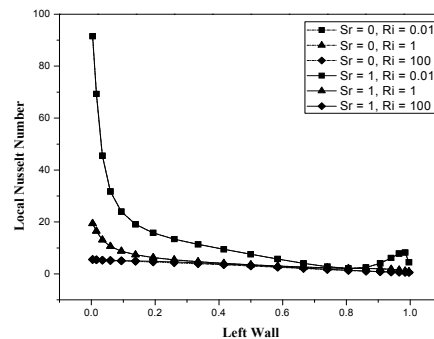


Fig.11 Local Nu and Sh at left wall for top wall moves right and bottom wall moves left.

Therefore, the local Nu and Sh near the lower part of the wall is decreased as compared to $Ri = 0.01$. The effect of Sr is insignificant on local Nu. Irrespective of Ri value, the local Sh is found to be decreased around 40% near the bottom of the wall with considering the Sr effect.

For the selected Ri and Pr, the variation of average Nu and Sh at the left wall with and without Sr is presented in Table 1 when both walls move in the same direction. It is observed that for given Ri, when Pr is increased, the molecular diffusivity increases which enhance the average Nu and due to this reason the influence of Soret effect is negligible. For $Ri = 1.0$, the average Nu is decreased as compared to $Ri = 0.01$ because of buoyancy effect and due to this reason Sr enhances the average Nu. This effect is further increased by increasing Ri. It is found that for the selected Ri and in the absence of Sr, the average Sh increases with Pr. The high concentration fluid diffuses along the direction of top and bottom walls. Hence, the low concentration fluids penetrate at the center towards the high concentration wall. This effect increases with increase in Pr. While incorporating Sr, the average Sh is reduced, and for higher Pr, the concentration gradient is in the opposite direction. The average Sh is further reduced by increasing Ri value. The variation of average Nu and Sh for different Pr with and without Sr at the left wall when both walls move in opposite direction is presented in Table 2. For the selected Ri the average Nu and Sh increases with increase in Pr, but for the given Pr when Ri increases the average Nu and Sh decreases for $Sr = 0$. While Sr is incorporated, the average Nu is further enhanced for these conditions and average Sh is decreased due to retardation of

concentration diffusion and for higher Pr, this effect is reversed in direction.

5. CONCLUSIONS

A two-dimensional steady state double-diffusive mixed convective flow in a square cavity with considering Soret effect is studied. The governing equations are solved by finite volume method with SIMPLE algorithm. Results are presented by solving two different conditions, (a) the top and bottom walls move with same velocity (U_0) towards right and (b) the top wall moves towards right and bottom wall moves towards left with the same velocity (U_0). Results show that irrespective of Soret effect, when both walls are moving in the same direction, two vortices are formed near the top and bottom walls for the selected range of Pr. But when the walls are moving in opposite direction only primary vortex is formed for the given range of Pr in the absence of Soret effect, whereas the primary vortex is disintegrated when Pr is more than 1.0 with Soret effect. The U and V velocities are symmetrically varying with Ri and analogous to thermal diffusion. For low Ri, the effect of thermal diffusion is not influenced by Soret effect whereas for high Ri the thermal diffusion is augmented due to Soret effect. In the case of concentration diffusion, Soret effect plays a vital role, and this effect is accelerated by increasing Ri. For the selected Ri when both walls are moving in opposite direction, the average Nu and Sh increases with increase in Pr, but for the given Pr, the average Nu and Sh decreases with increase in Ri without considering Soret effect.

Table 1 The variation of Average Nu and Sh for different Pr with and without Sr at the left wall (Condition 1: Both wall moves towards right)

	Average Nu						Average Sh					
	Ri = 0.01		Ri = 1.0		Ri = 100		Ri = 0.01		Ri = 1.0		Ri = 100	
	Sr = 0	Sr = 1	Sr = 0	Sr = 1	Sr = 0	Sr = 1	Sr = 0	Sr = 1	Sr = 0	Sr = 1	Sr = 0	Sr = 1
Pr = 0.71	10.720	10.719	3.494	3.542	2.515	2.654	21.439	18.405	6.989	5.696	5.029	4.435
Pr = 1.0	12.267	12.267	4.148	4.226	2.822	3.020	24.535	19.765	8.296	6.259	5.644	4.704
Pr = 5.0	22.094	22.089	8.043	8.534	4.810	5.825	44.187	6.426	16.087	-0.232	9.620	-0.622
Pr = 10.0	27.802	27.749	10.461	11.118	6.077	7.060	55.604	-32.607	20.923	-21.278	12.153	-6.094

Table 2 The variation of Average Nu and Sh for different Pr with and without Sr at the left wall (Condition 2: Both wall moves in opposite direction)

	Average Nu						Average Sh					
	Ri = 0.01		Ri = 1.0		Ri = 100		Ri = 0.01		Ri = 1.0		Ri = 100	
	Sr = 0	Sr = 1	Sr = 0	Sr = 1	Sr = 0	Sr = 1	Sr = 0	Sr = 1	Sr = 0	Sr = 1	Sr = 0	Sr = 1
Pr = 0.71	10.362	10.365	3.916	3.970	2.644	2.775	20.725	17.835	7.832	6.730	5.288	4.640
Pr = 1.0	11.826	12.267	4.473	4.554	2.984	3.168	23.644	19.189	8.947	7.214	5.967	4.933
Pr = 5.0	20.808	20.844	8.098	8.497	5.230	6.169	41.616	5.734	16.196	1.762	10.460	0.718
Pr = 10.0	26.545	26.622	10.316	9.020	6.597	7.646	53.087	-42.160	20.633	-24.849	13.195	-9.392

NOMENCLATURE

c	Concentration (kgm^{-3})
C	Dimensionless Concentration
D	Mass diffusivity
Gr_c	Solutal Grashof number
Gr_T	Thermal Grashof number
g	Gravitational acceleration (ms^{-2})
kt	Thermal conductivity ($\text{Wm}^{-1}\text{K}^{-1}$)
Le	Lewis number
N	Buoyancy ratio
Nu	Nusselt number
P	Dimensionless pressure

Pr	Prandtl number
Re	Reynolds number
Ri	Richardson number
Sr	Soret number
Sh	Sherwood number
T	Dimensionless temperature
U, V	Dimensionless velocity components along x and y axes
X, Y	Dimensionless Cartesian co-ordinates

Greek Symbols

θ	Dimensional temperature (K)
ν	Kinematic viscosity (m^2s^{-1})
ρ	Density (kgm^{-3})
α	Thermal diffusivity (m^2s^{-1})

Subscripts

avg	Average
C	Cold, concentration
f	Fluid
H	Hot
T	Thermal
o	Initial

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