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MIXED BIOCONVECTION FLOW OF A NANOFLUID CONTAINING GYROTACTIC MICROORGANISMS PAST A VERTICAL SLENDER CYLINDER

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ABSTRACT

in this paper, the steady mixed bioconvection flow of a nanofluid containing gyrotactic microorganisms past a vertical slender cylinder is studied. The passively controlled nanofluid model is applied to approximate this nano-bioconvection flow problem, which is believed to be physically more realistic than previously commonly used actively controlled nanofluid models. Using a suitable transformation, the nonlinear system of partial differential equations is converted into non-similar equations. These resulting equations are solved numerically using an accurate implicit finite-difference method. The present numerical results are compared with available data and are found in an excellent agreement. The skin friction coefficient, local Nusselt number, and the local density of the motile microorganism profiles are examined subject to various parameters of interest, namely Richardson number, thermophoresis parameter, Brownian motion parameter, bio-convection Lewis number, bio-convection Rayleigh number, and bio-convection Péclet number for various values of surface transverse curvature parameter. The results indicate that the skin friction coefficient, local Nusselt number, and the local density of the motile microorganisms enhance with a decrease in either of the bioconvection Péclet number or the thermophoresis parameter and with an increase in either of the Brownian motion parameter, bioconvection Lewis number or the Richardson number. Increasing and decreasing the buoyancy ratio parameter and bioconvection Rayleigh number respectively, lead to increase in the local skin friction coefficient and the local rate of heat transfer and reduction in the local density of motile microorganism. This type of study finds application in engineering, geothermal and industrial fields such as the design of microbial fuel cell and bio-convection nano-technological devices.

Keywords: Mixed bio convection, nanofluid, gyrotactic microorganisms, vertical slender cylinder.

1. INTRODUCTION

The study of nanofluids have been active field of research as it plays a crucial role in diverse applications, such as medical engineering, which are continually expanding. Nanofluids are engineered colloids comprising a base fluid (e.g., air, water) and nanoparticles which range in diameter between 1 and 100 nm. Nanofluids typically employ metal or metal oxide nanoparticles, such as copper and alumina, and the base fluid is usually a conductive fluid, such as water or ethylene glycol. The term ''nanofluid'' was firstly used by Choi (1995) to describe the pure fluids with suspended nano particles. Grosan and Pop (2011) have studied the problem of steady axisymmetric mixed convection boundary layer flow of nanofluid past a thin vertical cylinder. Gorla *et al.* (2011) have investigated the boundary layer flow of a nanofluid along a circular cylinder in a stagnant free stream. Tham *et al.* (2012) have analyzed the problem of mixed convection flow of a nanofluid along a horizontal circular cylinder in a stream flowing vertically upwards. Chamkha *et al.* (2013a) have analyzed the unsteady free convection boundary-layer flow of a nanofluid from a vertical cylinder. The heat transfer of a nanofluid over a vertical slender cylinder was considered by Nadeem *et al.* (2012). The problem of mixed convection boundary-layer flow of nanofluid over a vertical circular cylinder embedded in a porous medium is studied by Rohni *et al (2013).* Rajesh *et al.* (2014) have examined the unsteady natural convection flow of a nanofluid and heat transfer from a moving vertical cylinder. EL-Kabeir *et al.* (2014) have discussed theoretically the non-Darcy free natural convection flow of nanofluid with heat transfer from a vertical cylinder saturated a porous medium. Dinarv *et al.* (2015) have reported the mixed convection boundary layer flow of a nanofluid over a vertical circular cylinder with prescribed external flow.

On the other side, the bio convection is a attractive phenomenon of fluid mechanics is a demeanor -forming motion noted in shallow suspensions of cells which swim upward (against gravity). Also it is phenomenon in aquatic environments, affecting the spatial distribution of motile micro-organisms and promoting mixing within the fluid. Platt (1961) coined the term ''bioconvection'' to characterize the phenomenon of manner formation in shallow suspensions of motile micro-organisms at uniform temperature, on a par with those found in convection experiments. Plesset and Winet (1974) employed a quantitative description of bioconvection patterns given in terms of Rayleigh-Taylor instability. Childress et al. (1975) and Levandowsky et al. (1975) determined the first self-consistent theorem for the onset of bioconvection. They have incorporated purely upward swimming cells in a suspension with stress free and rigid upper boundary. Plesset et al. (1976) have explained the configuration of the upper layer "granular nature" of Tetrahymena pyriformis suspensions. A deterministic model for gyrotactic bioconvection using a constant diffusivity was first analyzed in layers of infinite and finite depth by Pedley et al. (1988) and

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Hill et al. (1981). This model was further extended in a completely self-consistent fashion by Pedley and Kessler (1992). Bees (1996) modelled bioconvection using a probability distribution function for the cell swimming direction in a stochastic formulation of gyrotaxis. in all these works, it was found that the gyrotactic instability mechanism depends on the absolute cell concentration, unlike the overturning instability which depends on the gradient of the cell concentration. Bioconvection patterns observed in suspensions of Chlamydomonas sp. due to combination of gravitaxis and gyrotaxis (see Pedley and Kessler (1992), Hill and Pedley (1989).The first analysis of bioconvection in a realistic geometry using a model that deals with random swimming in a rational manner was presented by Bees and Hill (1998). Ghorai and Hill (1999-2000) investigated the structure and stability of two-dimensional gyrotactic plumes in tall, narrow chambers with either stress-free sidewalls or periodic sidewalls using a conservative finite-difference scheme. Also they (2002) have studied gyrotactic bioconvection in an axisymmetric chamber. Czirok et al. (2000) investigated the properties of the patterns near the onset of the instability and later during its evolution into a fully nonlinear convection regime. Lewis (2003) showed that gyrotactic algae in a homogenous and isotropic turbulent flow field maintain their bias and only perturb the effective value of diffusivity of cell concentration due to turbulence. Literature survey reveals that this is the first study of bioconvection under shear, and first quantitative study of bioconvection in horizontal tubes. The problem of bioconvection of gyrotactic microorganisms in nanofluids was first considered in (2004-2005). Bioconvection (1998-2011) in modestly diluted cell suspensions is described by equations for concentrations of bacteria and oxygen are coupled with the incompressible Navier-Stokes equations and the continuity equation. MHD laminar boundary layer flow with heat and mass transfer of an electrically conducting water-based nanofluid containing gyrotactic microorganisms along a convectively heated stretching sheet is investigated numerically by Khan and Makinde (2014). Prasad et al. (2016) studied mixed convective magnetohydrodynamic flow from slender cylinder with chemical reaction. Waqas et al. (2017) investigated the effects of thermophoresis and Brownian motion on convective flow from slandering surface with nonlinear thermal radiation. Mallikarjuna et al. (2018) studied convective flow of 3D Casson nanofluid containing gyrotactic microorganisms from slandering surface with Cattaneo – Christov heat flux

This paper analyzes the steady mixed bioconvection flow of a nanofluid containing gyrotactic microorganisms past a vertical slender cylinder. The boundary-layer equations governing the flow of a nanofluid containing gyrotactic microorganisms are converted to non-similarity equations which are solved using the implicit, iterative, finite-difference method. Importance of the several parameters involved in the problem are discussed and their effects are displayed through graphs. To the best of our knowledge, such analysis has been overlooked in all of previous publications so that the results are novel and original.

2. MATHEMATICAL FORMULATION

Consider a steady, incompressible, laminar, two-dimensional boundary-layer flow of a nanofluid containing Gyrotactic microorganisms over a permeable vertical slender cylinder of length L and outer radius r_o ($L >> r_o$). The physical model and coordinate system are shown in Fig. 1. The gravitational acceleration, g, acts in the downward direction. The velocity at a distance remote from the cylinder is given by u_∞ and the cylinder surface is maintained at a constant temperature T_w and constant density of the motile microorganism n_w , respectively, while far from the surface of the cylinder, the temperature, the nanoparticle volume fraction and the concentration of microorganisms are given as T_∞ , C_∞ and n_∞, respectively . According to the model by Kuznetsov and Nield (2013). Since no nanoparticle flux on the surface is distributed so that the passively controlled model can be applied. . It is assumed that the nanoparticles suspended in the base fluid are stable and have no effect on the swimming direction and velocity of the micro-organisms. in addition, the nanofluid is assumed to be dilute so that the bioconvection instability due to the increase of the suspensions viscosity can be avoided. On the other hand, to satisfy the physical requirement for survival of micro-organisms, the base fluid has to be water.

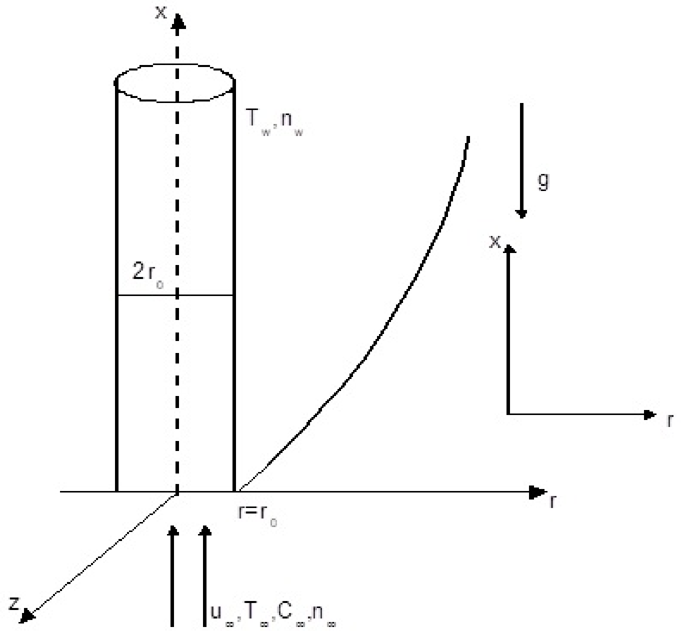

Fig. 1 Physical model and coordinate system

With those assumptions, employing the Oberbeck-Boussinesq and the standard boundary layer approximations, and making use of the above assumptions, the governing equations embodying the conservation of the total mass, momentum, thermal energy, nanoparticle volume fraction, and micro-organisms can be written as; (Chang [38] and Aydin and Kaya [39])

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0 \tag{1}$$

$$\rho_{f\infty}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+ \left[(1-\phi_\infty)g\rho_{f\infty}\beta(T-T_\infty)-(\rho_p-\rho_{f\infty})g(\phi-\phi_\infty)-(\rho_{m\infty}-\rho_f)g\gamma(n-n_\infty)\right] \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\nu}{\Pr}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\tau\left[D_B\frac{\partial C}{\partial r}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial r}\right)^2\right] \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial r}=D_B\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)+\frac{D_T}{T_\infty}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{4}$$

$$u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial r}+\frac{bW_C}{C_\infty}\frac{1}{r}\left[\frac{\partial}{\partial r}n\left(r\frac{\partial C}{\partial r}\right)\right]=D_n\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) \tag{5}$$

The corresponding boundary conditions for this problem can be written as:

$$r = r_0, u = 0, v = 0, T = T_w,\ D_B\frac{\partial C}{\partial r}+\frac{D_B}{T_\infty}\frac{\partial T}{\partial r}=0\ \ n = n_w \tag{6a}$$

$$r\to\infty, u\to u_\infty, T\to T_\infty, C\to C_\infty, n\to n_\infty , \tag{6b}$$

where u and v are the velocity components in the x- and r-direction, respectively. T and C are the temperature and species concentration, respectively. α , β , $\rho_{f\infty}$, $\rho_{m\infty}$, υ and μ are the thermal diffusivity of the base fluid, the volume expansion coefficient of the fluid, the density of the base fluid, the microorganisms density, the kinematic viscosity and the viscosity of the fluid, respectively. D_B , D_T and D_n are the Brownian diffusion coefficient, the thermophoretic diffusion

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coefficient of the microorganisms and the diffusivity of microorganisms, respectively. σ , γ , w_c and $\tau = (\rho c)_p / (\rho c)_f$ are the motile parameter , the average volume of a microorganism, the maximum cell swimming speed and Ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, respectively.
in order to get non-dimensionalized equations, we introduce the following transformations

$$\xi = \frac{4}{r_0}\left(\frac{\nu x}{u_\infty}\right)^{1/2}, \quad \eta = \left[\frac{r^2 - r_0^2}{4r_0}\right]\left(\frac{u_\infty}{\nu x}\right)^{1/2}, \quad \psi = r_0(\nu u_\infty x)^{1/2} f(\xi,\eta),$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \chi = \frac{n - n_\infty}{n_w - n_\infty}, \quad \phi = \frac{C - C_\infty}{C_\infty}, \quad u = \frac{1}{r}\frac{\partial \psi}{\partial r}, v = -\frac{1}{r}\frac{\partial \psi}{\partial x}. \quad (7)$$

Where ψ is the stream function satisfying the continuity equation Eq. (1), $f(\eta)$ is the dimensionless stream function, n is the concentration of the microorganisms, $\chi(\eta)$ is the Dimensionless density of motile microorganisms, η is Dimensionless co-ordinate , $\theta(\eta)$, $\phi(\eta)$ are the dimensionless temperature and concentration of the fluid in the boundary layer region respectively,
Substituting eqns. (7) into eqns (1) through (5) yields:

$$(1+\xi\eta)f''' + \xi f'' + ff'' + Ri\xi^2(\theta - Nr\phi - Rb\chi) = \xi\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right) \quad (8)$$

$$\frac{1}{\Pr}(1+\xi\eta)\theta'' + \frac{\xi}{\Pr}\theta' + f\theta' + Nb\theta'\phi' + Nt\theta'^2 = \xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right) \quad (9)$$

$$\frac{1}{Le}(1+\xi\eta)\phi'' + \frac{\xi}{Le}\phi' + f\phi' + \frac{1}{Le}\frac{Nt}{Nb}\left[(1+\xi\eta)\theta'' + \xi\theta'\right] = \xi\left(f'\frac{\partial \phi}{\partial \xi} - \phi'\frac{\partial f}{\partial \xi}\right) \quad (10)$$

$$(1+\xi\eta)\chi'' + \xi\chi' + Lbf\chi' - Pe\left[\chi'\phi' + \left((1+\xi\eta)\phi'' + \xi\phi'\right)(\chi + \sigma)\right]$$
$$= Lb\xi\left(f'\frac{\partial \chi}{\partial \xi} - \chi'\frac{\partial f}{\partial \xi}\right) \quad (11)$$

where a prime indicates differentiation with respect to η and the parameters,

$$Ri = \frac{Gr}{Re}, Re = \frac{u_\infty r_0}{\nu}, Gr = \frac{(1 - C_\infty)g\beta(T_w - T_\infty)}{2\nu^2},$$

$$Nr = \frac{(\rho_p - \rho_{f\infty})C_\infty}{(1 - C_\infty)\rho_{f\infty}\beta(T_w - T_\infty)}, \quad Le = \nu / D_B, \quad \Pr = \nu/\alpha, Pe = \frac{bW_C}{D_n}$$

$$Rb = \frac{(\rho_{m\infty} - \rho_f)\gamma(n_w - n_\infty)}{(1 - \phi_\infty)\rho_{f\infty}\beta(T_f - T_\infty)}, Lb = \frac{\nu}{D_n}, , \sigma = \frac{n_\infty}{(n_w - n_\infty)},$$

$$Nt = \frac{\tau D_T(T_w - T_\infty)}{\alpha T_\infty}, Nb = \frac{\tau D_B C_\infty}{\alpha} \quad (12)$$

where Ri is the Richardson number, Re is the Reynolds number, Gr is the Grashof number, Nr is the bouncy-ratio parameter, Le is the Lewis number, Pr is the Prandtl number, Rb is the bioconvection Rayleigh number, Lb is the bioconvection Lewis number, Pe is the bioconvection Peclet number, σ is the Motile parameter, Nt is the thermophoresis number and Nb is the Brownian motion number.
The transformed boundary conditions are:

$$\eta = 0; f' = 0, f + \xi\frac{\partial f}{\partial \xi} = 0, \theta = 1, Nb\phi' + Nt\theta' = 0, \chi = 1, \quad (13a)$$

$$\eta \to \infty; f' = 2, \theta = 0, \phi = 0, \chi = 0 \quad (13b)$$

of special significance, the quantities of practical interest in this study are the skin friction C_f, Nusselt number Nu and density number of the motile microorganisms Nn can be defined in dimensionless form as:

$$2C_f \operatorname{Re}_x^{1/2} = f''(\xi,0) \quad (14)$$

$$2Nu_x \operatorname{Re}_x^{-1/2} = -\theta'(\xi,0) \quad (15)$$

$$2Nn_x \operatorname{Re}_x^{-1/2} = -\chi'(\xi,0) \quad (16)$$

Here, following Kuznetsov and Nield [36], it noted that the fourth boundary condition in Eq. (13a) indicates that the normal flux of nanoparticles is zero at the boundary as thermophoresis is taken into account.

3. NUMERICAL METHOD

The non-similar equations (8) through (11) are nonlinear and possess no analytical solution and must be solved numerically. The efficient, iterative, tri-diagonal, implicit finite-difference method discussed by Blottner (1970) has proven to be adequate for the solution of such equations. The equations are linearized and then descritized using three point central difference quotients with variable step sizes in the η direction and using two-point backward difference formulae in the ξ direction with a constant step size. The resulting equations form a tri-diagonal system of algebraic equations that can be solved by the well-known Thomas algorithm (see Blottner(1970)). The solution process starts at ξ=0 where Eqns. (8) through (11) are solved and then marches forward using the solution at the previous line of constant ξ until it reaches the desired value of ξ. Due to the nonlinearities of the equations, an iterative solution with successive over or under relaxation techniques is required. The convergence criterion required that the maximum absolute error between two successive iterations be 10^{-6}. The computational domain was made of 196 grids in the η direction and 201 grids in the ξ direction. A starting step size of 0.001 in the η direction with an increase of 1.038 times the previous step size and a constant step size in the ξ direction of 0.01 were found to give very accurate results. The maximum value of η (η_∞) which represented the ambient conditions was assumed to be 38. The step sizes employed were arrived at after performing numerical experimentations to assess grid independence and ensure accuracy of the results. The accuracy of the aforementioned numerical method is validated by direct comparisons with the numerical results reported earlier by Chang (2006) and Aydin and Kaya (2011) for the case of forced convection flow about a vertical cylinder (Ri=0) for regular fluid (Rb=Pe=Nb=Nt=0) in the absence of the mass transfer (Nr=0) as given in Table 1. It can be seen from this table that excellent agreement between the results exists. This favorable comparison lends confidence in the numerical results to be reported in the next section.

Table 1. Comparison of values of f″(ξ,0) and -θ′(ξ,0) for various values of ξ with (Ri=0) for regular fluid (Rb=Pe=Nb=Nt=0) in the absence of the mass transfer (Nr=0).

ξ	f″(ξ,0)			-θ′(ξ,0)		
	Chang [38]	Aydin and Kaya [39]	Present results	Chang [38]	Aydin and Kaya [39]	Present results
0.0	1.3280	1.3261	1.329103	0.5852	0.5849	0.5856271
1.0	1.9133	1.9134	1.913843	0.8658	0.8649	0.8657098
2.0	2.3900	2.3902	2.391991	1.0940	1.0938	1.094318
3.0	2.8159	2.8148	2.814610	1.2982	1.2972	1.296471
4.0	3.2187	3.2162	3.217243	1.4925	1.4918	1.489833

4. RESULTS AND DISCUSSIONS

We have computed the solutions for local skin friction coefficient, rate of heat transfer (Nusselt number) and local density of the motile microorganism number as shown graphically in Figs. 2-13. The effects of buoyancy ratio parameter N_r, bioconvection peclet number Pe, Brownian motion number Nb, thermophoresis parameter Nt, bioconvection Rayleigh number Rb, Bioconvection Lewis number Lb and Richardson number Ri.

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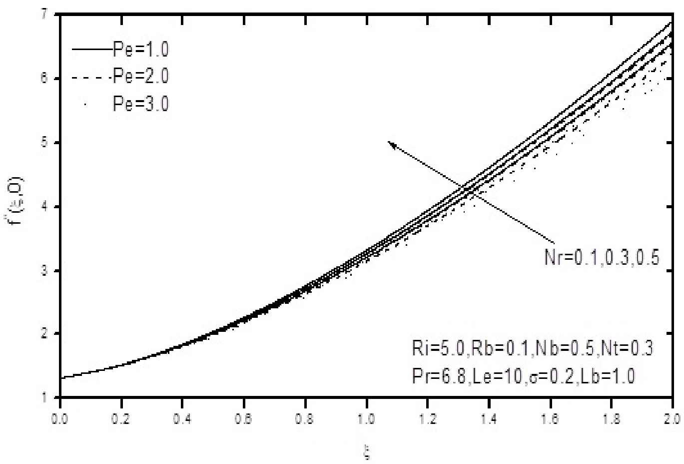

Fig. 2 Effects of the bouncy-ratio parameter Nr and bioconvection Peclet number Pe on the local skin-friction coefficient

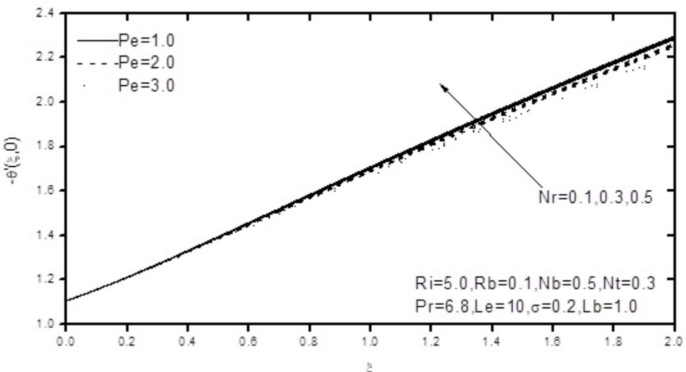

Fig. 3 Effects of the bouncy-ratio parameter Nr and bioconvection Peclet number Pe the local Nusselt number

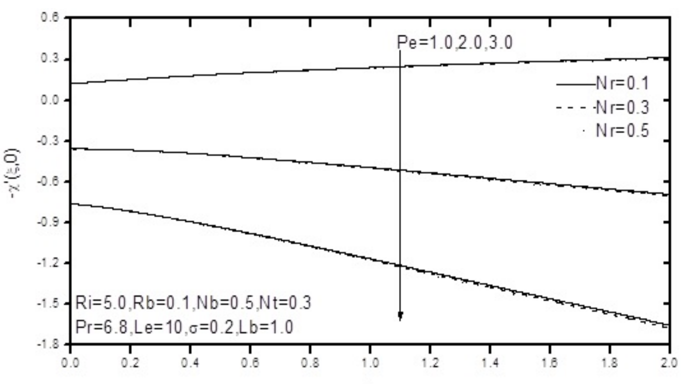

Fig. 4 Effects of the bouncy-ratio parameter Nr and bioconvection Peclet number Pe on the local density of the motile micro-organisms number

The local skin friction coefficient, Nusselt number and local density of the motile microorganism number for different values of the buoyancy ratio parameter Nr (0.1, 0.3 and 0.5) and bio-convection peclet number Pe (1.0, 2.0 and 3.0) with Ri=5.0, Rb=0.1, Nb=0.5, Nt=0.3, Pr=6.8, Le=10, Lb=1.0 and σ = 0.2 are plotted in Figs 3-4. These figures show that an increase in buoyancy ratio parameter serves to strongly enhance local skin friction coefficient and Nusselt number and retards the local density of the motile microorganism at the surface of cylinder. Bioconvection peclet parameter (Pe) arises in dimensionless microorganism density concervation eq. (11), and evidently strongly decreases the local skin friction coefficient and therefore depreciates rate of heat transfer near the cylinder. Similarly, increasing Pe implies reduces in nanoparticle concentration it serves to depreciate the microorganism values. A similar behavior was reported by Kuznetsov and Nield (2013).

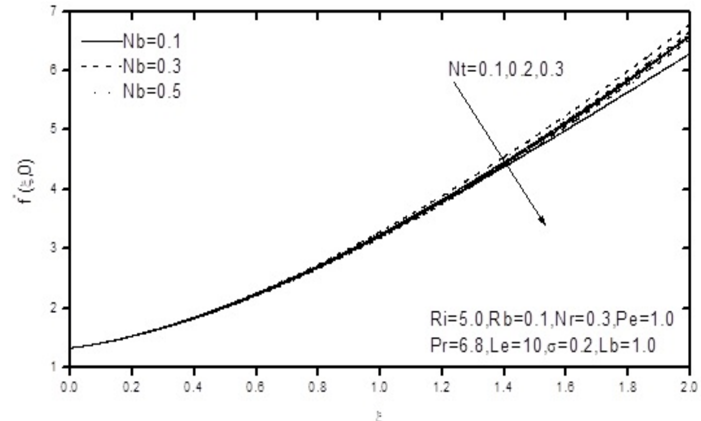

Fig. 5 Effects of Brownian motion number Nb and thermophoresis parameter Nt on the local skin-friction coefficient

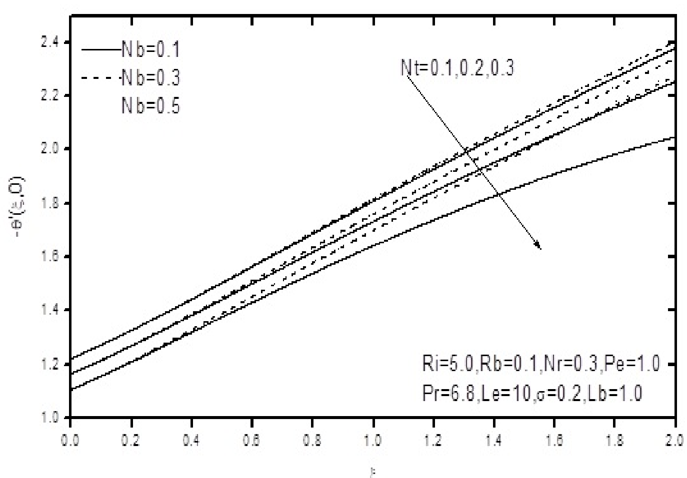

Fig. 6 Effects of the Brownian motion number Nb and thermophoresis parameter Nt on the local Nusselt number

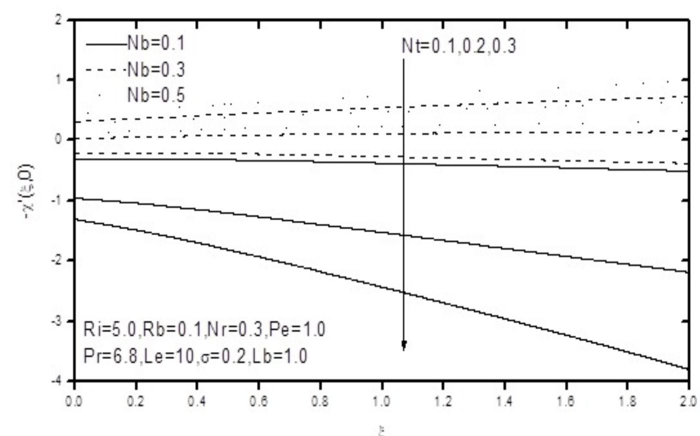

Fig. 7 Effects of Brownian motion number Nb and thermophoresis parameter Nt on the local density of the motile microorganisms number

Variation of local skin friction coefficient, Nusselt number and local density of the motile microorganism number against Brownian motion number Nb (0.1, 0.3 and 0.5) and thermophoresis parameter Nt (0.1, 0.2 and 0.3) with Ri=5.0, Rb=0.1, Nr=0.3, Pe=1.0, Pr=6.8, Le=10, Lb=1.0 and σ = 0.2 are presented in Figs 5-7. As the Brownian motion number Nb increased, the nanoparticle diameter progressively smaller, it leads to increase local skin friction coefficient, and thermal energy to nanofluid and therefore rate of heat transfer values are increased. A similar response has been computed by Anwar Beg *et.al* (2013) for vertical plate. The microorganism density values are also increased with increase in Brownian motion number Nb. Increasing thermophoresis parameter Nt leads to enhance nanoparticle deposition over the vertical slender cylinder. As the result of strong enhance in nanoparticle deposition induces a significant decrease in local skin friction coefficient. The rate of heat transfer and local density of motile microorganism are therefore reduced for larger values of thermophoresis parameter Nt. A similar observation was made by Shaw et.al (2014) for inclined plate.

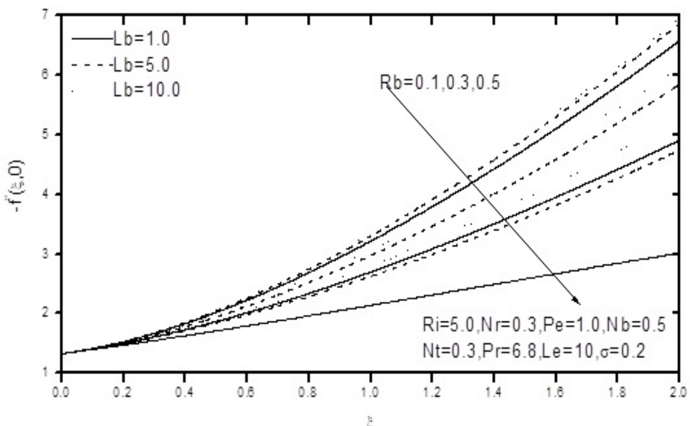

Fig. 8 Effects of bioconvection Rayleigh number *Rb* and bioconvection Lewis number *Lb* on the local skin-friction coefficient

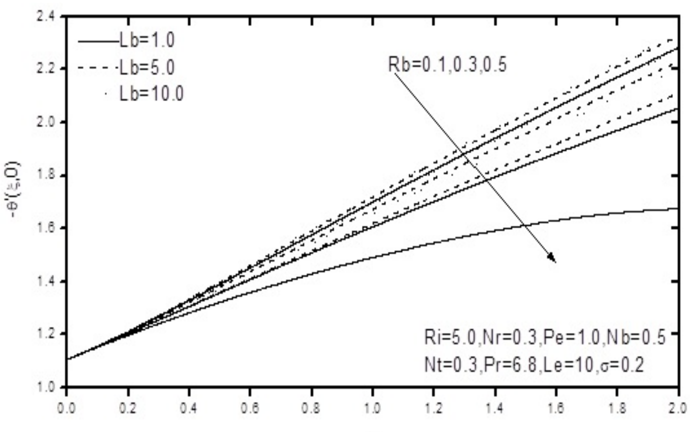

Fig. 9 Effects of bioconvection Rayleigh number *Rb* and bioconvection Lewis number *Lb* on the local Nusselt number

Variation of local skin friction coefficient, Nusselt number and local density of the motile microorganism number against bioconvection Rayleigh number Rb (0.1, 0.3 and 0.5) together with bioconvection Lewis number Lb (1.0, 5.0 and 10.0) are depicted in Figs 8-10 with fixed values of the other parameters. The influence of bioconvection Rayleigh number Rb is more on the momentum equation with the cause of bioconvection plumes, which are characterized by downward motion of the base fluid and oppose the upward motion of the nanofluid by considering the buoyancy effect. Therefore, both local skin friction coefficient and rate of heat transfer are reduced. The contrary response was noticed for local density of motile microorganism values which strongly enhanced with increasing bioconvection Rayleigh number Rb. in fact, the bioconvection Lewis number Lb is a prominent effect in the regime. As increase in Lb leads to enhance local skin friction coefficient which induces to increase the temperature in the nanofluid and nanoparticle concentrations, i.e. strong enhancement in rate of heat transfer. Similarly local density of motile microorganism values are enhanced for larger values of bioconvection Lewis number Lb.

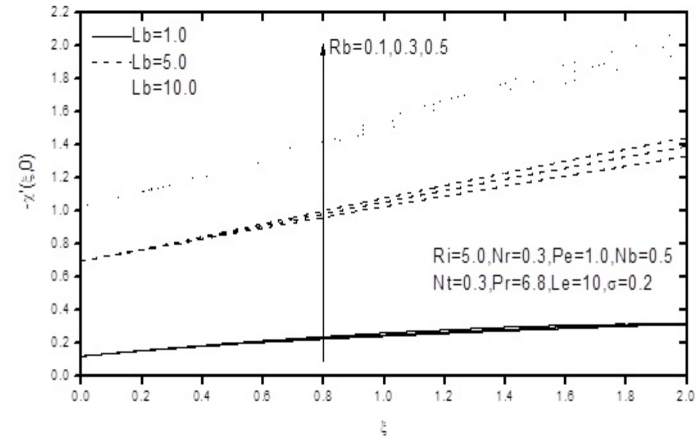

Fig. 10 Effects of the bioconvection Rayleigh number *Rb* and bioconvection Lewis number *Lb* on the local density of the motile microorganisms number

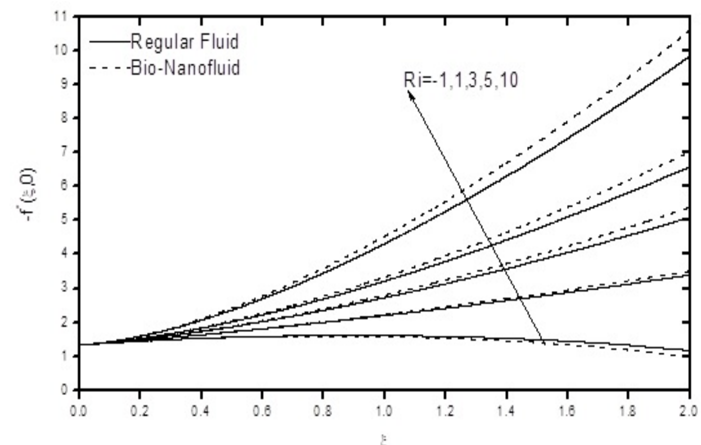

Fig. 11 Effects of the Richardson number Ri on the local skin-friction coefficient with cases of regular fluid and bionanofluid

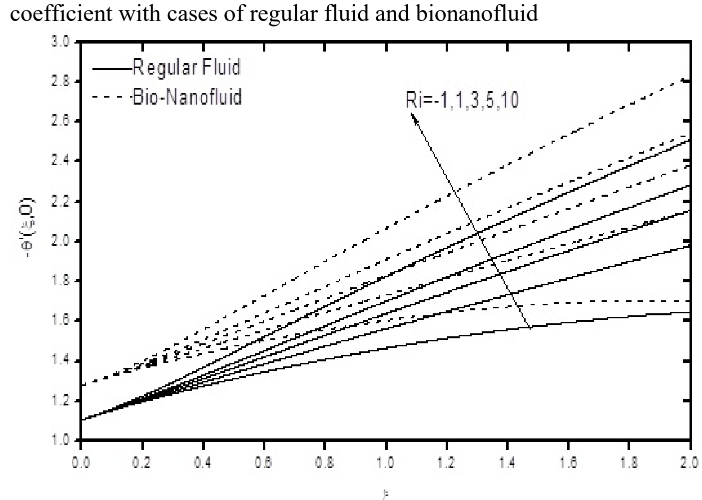

Fig. 12 Effects of the Richardson number Ri on the local Nusselt number with cases of regular fluid and bio-nanofluid

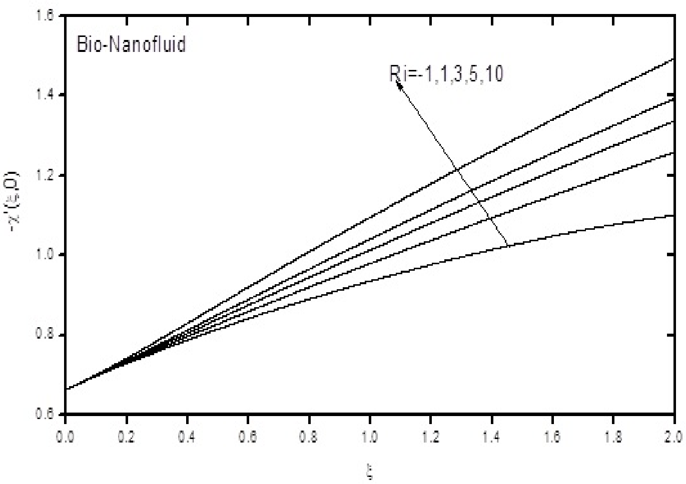

Fig. 13 Effects of the Richardson number Ri on the local density of the motile microorganisms number with bio-nanofluid case

The variation of Richardson number Ri on local skin friction coefficient, local Nusselt number and local density of motile microorganism is depicted in figures (11)-(13). As this parameter influenced in momentum eq. (11) which is coupling with functions of temperature gradient, nanoparticle concentration and microorganism density, all flow variables are significantly enhanced with increase in Ri. The similar results were produced by Orhan and Ahmet (2011). It is also observed from these figures that the results are more pronounced in bio-nanofluid compared to regular fluid.

5. CONCLUSIONS

Bioconvection boundary layer flow of a nanofluid containing gyrotactic microorganisms from a vertical slender cylinder is investigated. The fluid characteristics, momentum, temperature, nanoparticle volume fraction and microorganism conservation boundary layer equations are transformed into non-dimensional equations and then solved numerically using implicit finite difference method. The present numerical results are compared with available data and are found in an excellent agreement. The physical parameters on the skin friction coefficient, local Nusselt number, and the local density of the motile microorganism profiles are examined for different values of surface transverse curvature parameter. The computations have shown that:

- Increasing buoyancy ratio parameter and decreasing bioconvection Rayleigh number induces increases in the local skin friction coefficient and the local rate of heat transfer and a decrease in the local density of motile microorganism.
- Local skin friction coefficient, local rate of heat transfer and the local density of motile microorganisms are depreciates for larger values of bioconvection Peclet number and thermophoresis parameter and lesser values of Brownian motion number.
- Increases in either of the bioconvection Lewis number or the Richardson number produce enhancement in local skin friction coefficient, local rate of heat transfer and local density of motile microorganisms along vertical slender cylinder surface.

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NOMENCLATURE

C	nanoparticle volume fraction
C_f	Skin friction coefficient
C_P	Specific heat at constant pressure
Dn	Diffusivity of the microorganisms
D_B	Brownian diffusion coefficient
D_T	Thermophoretic diffusion coefficient of the microorganisms
f	Dimensionless velocity
Gr	Grashof number
g	Acceleration gravity vector
h	Local heat transfer coefficient
k	Thermal conductivity of fluid
Le	Lewis number
Lb	Bioconvection Lewis number
Nb	Brownian motion number
Nr	Buoyancy ratio parameter
Nt	Thermophoresis number
Nu_x	local Nusselt number
n	Dimensionles s density of motile microorgani sms
Pe	Bioconvection peclet number
Pr	Prandtl number
q	Wall heat flux
Rb	Bioconvection Rayleigh number
Ri	Richardson number
Re	Local Reynolds number
T	Temperature
T_∞	Ambient temperature
T_w	Wall temperature
u	Velocity component in x –direction
v	Velocity component in r –direction
u_∞	Velocity at a distance remote from the cylinder
w_c	Maximum cell swimming speed
x	Horizontal co-ordinate
r	Vertical co-ordinate
r_0	Radius of the vertical cylinder

Greek symbols

α	Thermal diffusivity
β	Thermal expansion coefficient
γ	Average volume of a microorganism
σ	Motile parameter
η	Dimensionless co-ordinate
μ	Dynamic viscosity of the ambient fluid
υ	Kinematic viscosity
θ	Dimensionless temperature
ϕ	Dimensionless nanoparticle volume fraction
ψ	Stream function
χ	Dimensionless density of motile microorganisms
ρ_f	Density of the fluid
$\rho_{f\infty}$	Density of the base fluid
ρ_p	Density of the particles
$\rho_{m\infty}$	Density of the microorganism
$(\rho c)_f$	Heat capacity of the fluid
$(\rho c)_p$	Effective heat capacity of the nanoparticle material
τ	Ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid

Subscripts

w	at the wall
∞	Condition far away from the surface

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