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ANALYTICAL INVESTIGATIONS OF DIFFUSION THERMO EFFECTS ON UNSTEADY FREE CONVECTION FLOW PAST AN ACCELERATED VERTICAL PLATE

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ABSTRACT

The objective of this study is to investigate diffusion-thermo and radiation effects on unsteady free convection chemically reacting fluid flow past an accelerated infinite plate with variable temperature and mass diffusion under the influence of uniform transverse magnetic field when the magnetic lines of force are fixed relative to the fluid or to the plate. . Two important cases, when the magnetic lines of force are being fixed relative to the fluid (K=0) or to the moving plate (K=1) have been considered. A general exact solution of the dimensionless governing partial differential equations is obtained by Laplace transform technique without any restriction. The effects of various parameters describing the flow transport in the presence of thermal radiation and Dufour effect on the fluid velocity, temperature, concentration, the rate of heat transfer and the rate of mass transfer are concentrated on through graphs.
Keywords: *MHD, Dufour effect, radiation, free convection, exponentially accelerated plate, chemical reaction.*

1. INTRODUCTION

In nature, there exist flows which are caused not only by the temperature differences but also the concentration differences. These mass transfer differences do affect the rate of heat transfer. In industries, many transport processes exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy effect in the presence of thermal radiation. Hence, radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites, combustion and furnace design, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications. If the temperature of surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal radiation and mass diffusion**.**

The Effects of thermal diffusion and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium was studied by Sivaiah *et al.* (2012). Hetnarstki (1975) and Puri and Kythe (1988) reported an algorithm for generating some inverse transforms of exponential form. Narahari and Debnath (2013) investigated unsteady magnetohydrodynamic free convection flow past an accelerated vertical plate with constant heat flux and heat generation or absorption by Laplace Transform method. Prabhakar Reddy (2014) studied effects of thermal diffusion and viscous dissipation on unsteady MHD free convection flow past a vertical porous plate under oscillatory suction velocity with the heat sink. Seth *et al.* (2010) investigated MHD natural convection flow past an impulsively moving vertical plate with ramped wall temperature in the presence of thermal diffusion with heat absorption. The effects of thermal radiation and heat source on an unsteady MHD free convection flow past an infinite vertical plate with thermal diffusion and diffusion thermo was analyzed by Raju *et al.* (2013). Prakash *et al.* (2013) examined the effects of diffusion-thermo and radiation on unsteady MHD free convective flow with variable temperature and mass diffusion. In all the above studies, the stationary vertical plate is considered. Muthucumaraswamy *et al.* (2004) and Rajesh *et al.* (2009) studied radiation and mass transfer effects on exponentially accelerated isothermal vertical plate. Muralidharan and Muthucumaraswamy (2013) analyzed Radiation effects on linearly accelerated isothermal vertical plate with variable mass diffusion in the presence of magnetic field. Rushi Kumar *et al.* (2015) investigated the influence of magnetic field in the presence of thermal radiation and diffusion on a past vertical plate when magnetic lines relative to the fluid or the plate.

An energy flux is generated not only by temperature gradients but by composition gradients as well. Temperature gradients can also create mass fluxes and this is the Soret or thermal-diffusion effect. Generally, the thermo-diffusion and diffusion-thermo effects of smaller order magnitude than the effects prescribed by Fourier's or Fick's laws and are often neglected in heat and mass transfer processes. Due to the importance of thermo-diffusion and diffusion-thermo effects for the fluids with very light molecular weight as well as medium molecular weight, many investigators have studied and reported results for these flows and the contributors such as Eckert and Drake (1972), *Anghel et al.*(1985), Postelnicu (2004) are worth mentioning. Mythreye *et al.* (2015) analyzed the chemical reaction on unsteady MHD convective heat and mass transfer past a semi infinite vertical permeable moving plate with heat absorption. Venkateswarlu *et al.* (2014) analyzed thermal diffusion and radiation effects on unsteady MHD free convection heat and mass transfer flow past a linearly accelerated vertical porous plate with variable temperature and mass diffusion. Alam *et al.* [(2005), (2006), (2006a), and (2006b)] investigated the Dufour and Soret effects on unsteady laminar viscous

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incompressible MHD free convection and mass transfer flow past an impulsively started infinite vertical plate embedded in a porous medium under the influence of transverse applied a magnetic field.

Georgantopoloulos *et al.* (1979) studied the magnetohydrodynamic free convection flow past an impulsively started infinite vertical plate with uniform temperature. Ogulu and Makinde (2009) investigated unsteady hydromagnetic free convection flow of a dissipative and rotating fluid past a vertical plate with constant heat flux. Toki and Tokis (2007) investigated exact solutions for the unsteady free convection flows on a porous plate with time-dependent heating. Swetha *et al.* (2015) studied the effects of thermal radiation and radiation absorption on flow past an impulsively started infinite vertical plate with Newtonian heating and chemical reaction. Several studies were continued on magneto-hydrodynamic free convection flows past a vertical surface under different boundary conditions for various physical situations. Chandran *et al.* (1998) was investigated the effects of magnetic field and buoyancy force on the unsteady free convection laminar flow of an electrically conducting fluid when the flow was generated by uniformly accelerated motion of an infinite inclined plate subjected to constant heat flux. They obtained an exact solution with the help of Laplace transform technique and the numerical results were computed with the approximated error functions appeared in the solution.

However, in the literature we found less attention was paid on unsteady MHD free convection flows past a vertical plate subjected to a variable temperature and uniform mass diffusion with Dufour effect when the magnetic lines being fixed relative to the fluid or to the moving plate even though this situation involves in many engineering applications such as in aeronautics, spacecraft design and analysis of thermal plumes into atmosphere which are responsible for atmospheric pollution. In this paper, it is proposed to study diffusion-thermo and radiation effects on free convection flow past an accelerated infinite vertical plate with variable temperature and mass diffusion discussed in two cases, when the magnetic lines of force are fixed relative to the fluid (*K*=0) or to the plate (*K*=1) in the presence of transverse applied a magnetic field and chemical reaction. The dimensionless governing equations are solved using Laplace transform technique and the solutions are expressed in terms of exponential and complementary error functions.

2. MATHEMATICAL ANALYSIS

Diffusion-thermo and radiation effects on unsteady MHD free convection of flow of a viscous incompressible, electrically conducting, radiating fluid past an impulsively started infinite plate with variable temperature and uniform mass diffusion in the presence of transverse applied magnetic field when the magnetic lines of force are fixed relative to the fluid or to the plate. Two important cases, when the magnetic lines of force are being fixed relative to the fluid (*K*=0) or to the moving plate (*K*=1) have been considered. The x'-axis is taken along the plate in vertically upward direction and y'-axis is taken normal to it in the direction of applied transverse magnetic field. Initially, it is assumed that the plate and surrounding fluid are at the same temperature and concentration in a stationary condition for all the points in the entire flow region $y' \geq 0$. At the time $t' \geq 0$, the plate is given an impulsive motion with constant velocity $\mathrm{u} = \mathrm{u}_0 \exp(\mathrm{a}_0'\mathrm{t}')$. At the same time, the plate temperature is raised linearly with time t and the concentration levels near the plate are raised to C'_w. A magnetic field of uniform strength B_0 is assumed to be applied normal to the flow. For free convection flow, it is also assumed that,

- The induced magnetic field is assumed to be negligible as the magnetic Reynolds number of the flow is taken to be very small.
- The viscous dissipation is neglected in the energy equation.
- The effects of variation in density (ρ) with temperature and species concentration are considered only in the body force term, in accordance with usual Boussinesq approximation.
- The fluid considered here is gray, absorbing / emitting radiation but a non-scattering medium.
- Since the flow of the fluid is assumed to be in the direction of x' axis, so the physical quantities are functions of the space co-ordinate y' and t' only.

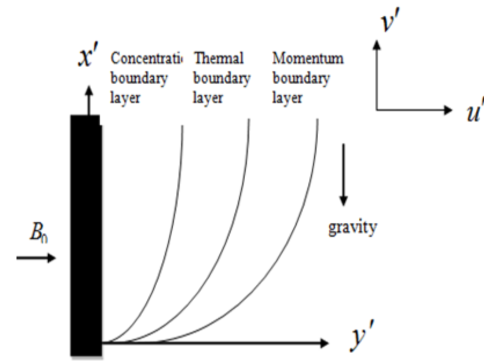

Fig. 1 Physical model

Then by usual Bossiness's approximation, the flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty) + g\beta^*(C'-C'_\infty) + v\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho} \tag{1}$$

$$\rho c_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial t'^2} - \frac{\partial q_r}{\partial y} + \frac{D_m K_T \rho}{C_s}\frac{\partial^2 C'}{\partial y'^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - k'(C'-C'_\infty) \tag{3}$$

If the magnetic field fixed relative to the plate, the momentum equation (1) is replaced by (see [Raptis and Singh (1983), Tokis (1985), Cramer and Pai (1973)])

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty) + g\beta^*(C'-C'_\infty) + v\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}\left[u' - u_0 f(t')\right] \tag{4}$$

Note that the velocity $u_0 f(t')$ of magnetic field B_0 in Eq. (4) appears because of the magnetic lines of force are fixed relative to the plate, which is accelerates with velocity $u_0 f(t')$. Equation (1) and (4) can be combined as

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty) + g\beta^*(C'-C'_\infty) + v\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}\left[u' - K u_0 f(t')\right] \tag{5}$$

where $K = \begin{cases} 0 \text{ if } \mathrm{B}_0 \text{ is fixed relative to the fluid} \\ 1 \text{ if } \mathrm{B}_0 \text{ is fixed relative to the plate} \end{cases}$

For an exponentially accelerated plate, $f(t') = \exp(a_0' t')$, where a_0' is dimensional accelerating parameter.
With the following initial and boundary conditions

$$\begin{aligned} & t' \leq 0: u' = 0,\ T' = T'_\infty, \quad C' = C'_\infty, \text{ for all } \ y' \\ & t' > 0: u' = u_0 \exp(a_0' t'),\ T' = T'_\infty + (T'_w - T'_\infty)At',\ C' = C'_w \text{ at } y' = 0 \\ & \qquad u' \to 0, \qquad T' \to T'_\infty, \qquad C' \to C'_\infty, \qquad \text{as } y' \to \infty \end{aligned} \tag{6}$$

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where $A=\frac{u_0^2}{v}$ the local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'}=-4a^*\sigma\left(T_\infty'^4-T'^4\right) \tag{7}$$

It is assumed that the temperature differences within the flow are sufficiently small and that T'^4 may be expressed as a linear function of the temperature. This is obtained by expanding T'^4 in a Taylor series about T_∞' and neglecting the higher order terms, thus, we get

$$T'^4\cong 4T_\infty'^3T'-3T_\infty'^4 \tag{8}$$

From Esq. (5) and (6), Eq. (2) reduces to

$$\rho c_p\frac{\partial T'}{\partial t'}=k\frac{\partial^2T'}{\partial y'^2}+16a^*\sigma T_\infty'^3\left(T_\infty'-T'\right)+\frac{D_mK_T\rho}{C_s}\frac{\partial^2C'}{\partial y'^2} \tag{9}$$

On introducing the following non- dimensional quantities

$$u=\frac{u'}{u_0},\ t=\frac{t'u_0^2}{v},\ a_0=\frac{va_0'}{u_0^2}\ \ y=\frac{y'u_0}{v},\ \theta=\frac{T'-T_\infty'}{T_w'-T_\infty'},\ C=\frac{C'-C_\infty'}{C_w'-C_\infty'},$$
$$Gr=\frac{g\beta v\left(T_w'-T_\infty'\right)}{u_0^3},\ Gm=\frac{g\beta^*v\left(C_w'-C_\infty'\right)}{u_0^3},\ \Pr=\frac{\mu C_\rho}{k},\ Sc=\frac{v}{D},$$
$$M=\frac{\sigma B_0^2v}{\rho u_0^2},\ R=\frac{16a^*v^2\sigma T_\infty'^3}{ku_0^2},\ k=\frac{vk'}{u_0^2},\ Du=\frac{D_mK_T\left(C_w'-C_\infty'\right)}{c_sc_pv\left(T_w'-T_\infty'\right)} \tag{10}$$

We get the following governing equations which are dimensionless.

$$\frac{\partial u}{\partial t}=\frac{\partial^2u}{\partial y^2}+Gr\theta+GmC-M\left(u-K\exp(a_0t)\right) \tag{11}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}-\frac{R}{\Pr}\theta+Du\frac{\partial^2C}{\partial y^2} \tag{12}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2C}{\partial y^2}-kC \tag{13}$$

The initial and boundary conditions in dimensionless form are as follows:

$$t\le 0: u=0,\quad \theta=0,\quad C=0\quad \text{for all } y$$
$$t>0: u=\exp(a_0t),\quad \theta=t,\quad C=1\quad \text{at } y=0$$
$$u\to 0,\quad \theta\to 0,\quad C\to 0\quad as\quad y\to\infty \tag{14}$$

The appeared physical parameters are defined in the nomenclature.

3. METHOD OF SOLUTION

The dimensionless governing equations from (11) to (13), subject to the boundary conditions (14) are solved by usual Laplace transform technique and the solutions are expressed in terms of exponential and complementary error functions.

$$C(y,t)=\frac{1}{2}\left[\begin{array}{l}\exp(y\sqrt{kSc})erfc(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{kt})\\+\exp(-y\sqrt{kSc})erfc(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{kt})\end{array}\right] \tag{15}$$

$$\theta(y,t)=\left[\begin{array}{l}\left(\frac{t}{2}+\frac{y\Pr}{4\sqrt{R}}\right)\exp\left(y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\frac{Rt}{\Pr}}\right)\\+\left(\frac{t}{2}-\frac{y\Pr}{4\sqrt{R}}\right)\exp\left(-y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\frac{Rt}{\Pr}}\right)\end{array}\right]$$
$$-\frac{a_4}{2}\left[\exp\left(y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{Rt}\right)+\exp\left(-y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{Rt}\right)\right]$$
$$+\left(\frac{a_4}{2}-\frac{a_3}{2}\right)\exp(-a_2t)\left[\begin{array}{l}\exp\left(y\sqrt{R-a_2\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{-a_2t+\frac{Rt}{\Pr}}\right)\\+\exp\left(-y\sqrt{R-a_2\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{-a_2t+\frac{Rt}{\Pr}}\right)\end{array}\right]$$
$$+\frac{a_4}{2}\left[\exp\left(y\sqrt{kSc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{kt}\right)+\exp\left(-y\sqrt{kSc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{kt}\right)\right]$$
$$+\left(\frac{-a_4}{2}+\frac{a_3}{2}\right)\exp(-a_2t)\left[\begin{array}{l}\exp\left(y\sqrt{(k-a_2)Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(k-a_2)t}\right)\\+\exp\left(-y\sqrt{(k-a_2)Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(k-a_2)t}\right)\end{array}\right] \tag{16}$$

$$u(y,t)=\frac{A_0}{2}\exp(a_0t)\left[\begin{array}{l}\exp\left(y\sqrt{M+a_0}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M+a_0)t}\right)\\+\exp\left(-y\sqrt{M+a_0}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M+a_0)t}\right)\end{array}\right]$$
$$+\frac{A_1}{2}\left[\exp\left(y\sqrt{M}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)+\exp\left(-y\sqrt{M}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)\right]$$
$$+A_2\left[\begin{array}{l}\left(\frac{t}{2}+\frac{y}{4\sqrt{M}}\right)\exp\left(y\sqrt{M}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\\+\left(\frac{t}{2}-\frac{y}{4\sqrt{M}}\right)\exp\left(-y\sqrt{M}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)\end{array}\right]$$
$$+\frac{A_3}{2}\exp(-a_6t)\left[\begin{array}{l}\exp\left(y\sqrt{M-a_6}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M-a_6)t}\right)\\+\exp\left(-y\sqrt{M-a_6}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M-a_6)t}\right)\end{array}\right]$$
$$+\frac{A_4}{2}\exp(-a_2t)\left[\begin{array}{l}\exp\left(y\sqrt{M-a_2}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M-a_2)t}\right)\\+\exp\left(-y\sqrt{M-a_2}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M-a_2)t}\right)\end{array}\right]$$
$$+\frac{A_5}{2}\exp(-a_8t)\left[\begin{array}{l}\exp\left(y\sqrt{M-a_8}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M-a_8)t}\right)\\+\exp\left(-y\sqrt{M-a_8}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M-a_8)t}\right)\end{array}\right]$$
$$+A_6\exp(-Mt)erfc\left(\frac{y}{2\sqrt{t}}\right)$$
$$+\frac{A_7}{2}\left[\exp\left(y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\frac{Rt}{\Pr}}\right)+\exp\left(-y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\frac{Rt}{\Pr}}\right)\right]$$
$$-A_2\left[\begin{array}{l}\left(\frac{t}{2}+\frac{y\Pr}{4\sqrt{R}}\right)\exp\left(y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\frac{Rt}{\Pr}}\right)\\+\left(\frac{t}{2}-\frac{y\Pr}{4\sqrt{R}}\right)\exp\left(-y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\frac{Rt}{\Pr}}\right)\end{array}\right]$$
$$+\frac{A_8}{2}\exp(-a_6t)\left[\begin{array}{l}\exp\left(y\sqrt{R-a_6\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{-a_6t+\frac{Rt}{\Pr}}\right)\\+\exp\left(-y\sqrt{R-a_6\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{-a_6t+\frac{Rt}{\Pr}}\right)\end{array}\right]$$
$$+\frac{A_9}{2}\exp(-a_2t)\left[\begin{array}{l}\exp\left(y\sqrt{R-a_2\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{-a_2t+\frac{Rt}{\Pr}}\right)\\+\exp\left(-y\sqrt{R-a_2\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{-a_2t+\frac{Rt}{\Pr}}\right)\end{array}\right]$$

$$+\frac{A_{10}}{2}\left[\exp\left(y\sqrt{kSc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{kt}\right)+\exp\left(-y\sqrt{kSc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{kt}\right)\right]$$

$$+\frac{A_{11}}{2}\exp(-a_2t)\left[\begin{array}{l}\exp\left(y\sqrt{(k-a_2)Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(k-a_2)t}\right)\\+\exp\left(-y\sqrt{(k-a_2)Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(k-a_2)t}\right)\end{array}\right]$$

$$+\frac{A_{12}}{2}\exp(-a_8t)\left[\begin{array}{l}\exp\left(y\sqrt{(k-a_8)Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(k-a_8)t}\right)\\+\exp\left(-y\sqrt{(k-a_8)Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(k-a_8)t}\right)\end{array}\right]$$

$$+A_6\exp(a_0t)-A_6\exp(-Mt) \quad (17)$$

where

$a_1=\frac{-Du\Pr kSc}{Sc-\Pr},\ a_2=\frac{kSc-R}{Sc-\Pr},\ a_3=\frac{-Du\Pr Sc}{Sc-\Pr},\ a_4=\frac{a_1}{a_2},\ a_5=\frac{Gr}{\Pr-1},$

$a_6=\frac{R-M}{\Pr-1},\ a_7=\frac{Gr}{Sc-1},\ a_8=\frac{kSc-M}{Sc-1},\ a_9=\frac{-Gm}{Sc-1},\ a_{10}=\frac{a_5}{a_6^2},$

$a_{11}=\frac{a_5}{a_6},\ a_{13}=\frac{a_1a_5}{a_2a_6},\ a_{14}=\frac{a_1a_5}{a_6(a_2-a_6)},\ a_{15}=\frac{a_1a_5}{a_2(a_2-a_6)},$

$a_{16}=\frac{a_3a_5}{(a_2-a_6)},\ a_{17}=\frac{a_1a_7}{a_2a_8},\ a_{18}=\frac{a_1a_7}{a_8(a_1-a_8)},\ a_{19}=\frac{a_1a_7}{a_2(a_2-a_8)},$

$a_{20}=\frac{a_3a_7}{(a_2-a_8)},\ a_{21}=\frac{a_9}{a_8},\ a_{22}=\frac{MK}{M+a_0},\ A_0=(1-a_{22}),$

$A_1=(-a_{10}-a_{13}+a_{17}-a_{21}),\ A_2=a_{11},\ A_3=(a_{10}+a_{14}-a_{16}),$

$A_4=(-a_{15}+a_{16}+a_{19}-a_{20}),\ A_5=(-a_{18}+a_{20}+a_{21}),$

$A_6=a_{22},\ A_7=(a_{10}+a_{13}),\ A_8=(-a_{10}-a_{14}+a_{16}),$

$A_9=(a_{15}-a_{16}),\ A_{10}=(-a_{17}+a_{21}),\ A_{11}=(-a_{19}+a_{20}),$

$A_{12}=(a_{18}-a_{20}-a_{21}).$

3.1 Nusselt Number

From temperature field, the Nusselt number which is given in non-dimensional form as follows:

$$Nu=-\left[\frac{\partial\theta}{\partial y}\right]_{y=0} \quad (18)$$

From equations (16) and (18), we get Nusselt number as follows

$$Nu=-\left[t\sqrt{R}\,erf\sqrt{\frac{Rt}{\Pr}}-\sqrt{\frac{t\Pr}{\pi}}\exp\left(-\frac{Rt}{\Pr}\right)-\frac{\Pr}{2\sqrt{R}}erf\sqrt{\frac{Rt}{\Pr}}\right]$$
$$-a_4\left[\left(-\exp(-\frac{Rt}{\Pr})\right)\sqrt{\frac{\Pr}{\pi t}}-\sqrt{R}erf(\sqrt{\frac{Rt}{\Pr}})\right]$$
$$-(a_4-a_3)\exp(-a_2t)\left[\left(-\exp\left(-\frac{Rt}{\Pr}+a_2t\right)\right)\sqrt{\frac{\Pr}{\pi t}}-\sqrt{R-a_2\Pr}erf\left(\sqrt{\frac{Rt}{\Pr}-a_2t}\right)\right]$$
$$+a_4\left[\sqrt{\frac{Sc}{\pi t}}\exp(-kt)+\sqrt{kSc}\,erf\sqrt{kt}\right]$$
$$-(a_3-a_4)\exp(-a_2t)\left[\left(-\exp(-kt+a_2t)\right)\sqrt{\frac{Sc}{\pi t}}-\sqrt{kSc-a_2Sc}erf\left(\sqrt{kt-a_2t}\right)\right] \quad (19)$$

3.2 Sherwood Number

From concentration field, Sherwood number which is given in non – dimensional form as follows:

$$Sh=-\left[\frac{\partial C}{\partial y}\right]_{y=0} \quad (20)$$

From equations (15) and (20) we get Sherwood number as follows:

$$Sh=\left[\exp(-kt)\sqrt{\frac{Sc}{\pi t}}+\sqrt{kSc}\,erf\left(\sqrt{kt}\right)\right] \quad (21)$$

4. RESULTS AND DISCUSSION

In order to get a clear insight of the physical problem, the velocity, temperature, concentration, the rate of heat transfer and the rate of mass transfer have been discussed by assigning numerical values to the parameters like radiation parameter (*R*), magnetic parameter (*M*),exponential accelerated parameter (a_0), Schmidt parameter (*Sc*), Prandtl number (*Pr*), Dufour number (*Du*), thermal Grashof number (*Gr*), mass Grashof number (*Gm*), chemical reaction (*k*) and time (*t*) from Figs. 2 -14.

Concentration profiles for different values of Schmidt number *Sc* and chemical reaction parameter *k*are presented through Fig. 2reveals that the concentration field due to variation in Schmidt number for the gasses hydrogen(*Sc*=0.22), water vapour(*Sc*=0.60), ammonia (*Sc*=0.78), carbon dioxide (*Sc*= 0.96). It is observed that concentration field is arrived regularly for hydrogen and accrues for carbon dioxide in comparison to watervapour. Thus,watervapour can be used for maintaining concentration field and hydrogen can be used for maintaining good concentration field. An increasing in Schmidt number leads to decreases in the concentration boundary layer thickness. From Fig. 3it is seen that the concentration decreases with an increasing value of *k*.

The influence of various flow parameters on the fluid temperature are illustrated in Figs. 4 -7. Fig. 4 depicts that the effects of the Dufour number on the fluid temperature. It can be clearly seen from this figure that the diffusion thermal effects show the significant effect on the fluid temperature. As the values of Dufour number increases, the fluid temperature is also enhanced. The various values of Prandtl number are chosen such that for air (*Pr*=0.71), electrolytic solution (*Pr*=1.0) and water (*Pr*=7.0). Fig.5revealsthat the temperature decreases with the increasing values of Prandtl number. This is due to the fact that an increment in Prandtl number fluid has comparatively low thermal conductivity, which reduces conduction and there by the thermal boundary layer thickness and as a result, temperature decreases. Effect of thermal radiation *R* on the temperature field is illustrated in Fig. 6 illustrates that the radiation parameter restricts the fluid temperature. Therefore, using radiation we can control the fluid temperature. The temperature profiles at different time are shown in Fig. 7 it is observed that the temperature is increasing with time and approach to a steady state temperature as the time arises. The velocity profiles for an exponentially accelerated plate are presented from Figs. 8 -13when the magnetic lines of force are fixed relative to the fluid (*K*=0) or to the plate (*K*=1).The effects of *M*, *Du*, *Gr, Gm*, a_0 and *t* on the velocity field are shown in Figs. 8 -13 in the presence of radiation and chemical reaction respectively when other parameters fixed.

Fig.8 depicts that the velocity various magnetic parameter. It is observed that an increase in magnetic parameter the velocity decreases. It is due to the fact that the application of transverse magnetic field will result a Lorentz force similar to drag force, which tends to resist the fluid flow and thus reducing its velocity and it is also noticed that the momentum boundary layer thickness increases with increasing value of magnetic parameter in case of moving plate. From these Fig. 9 it is noticed that the velocity increases with the increasing values of *Du*. It can be clearly seen from this figure that the diffusion thermal effects show the significant effect on the fluid velocity.The effects of *Gr*, *Gm* and *t* on the velocity field are shown in figures 10 - 12. It is observed that the velocity of the fluid increases with increasing values of *Gr*, *Gm* and *t.* This is because an increase in *Gr* or *Gm* or *t* leads to an increase in the buoyancy force which causes an increase in the fluid velocity. From Fig. 13reveals that the velocity increases with in increasing exponential accelerated parameter.

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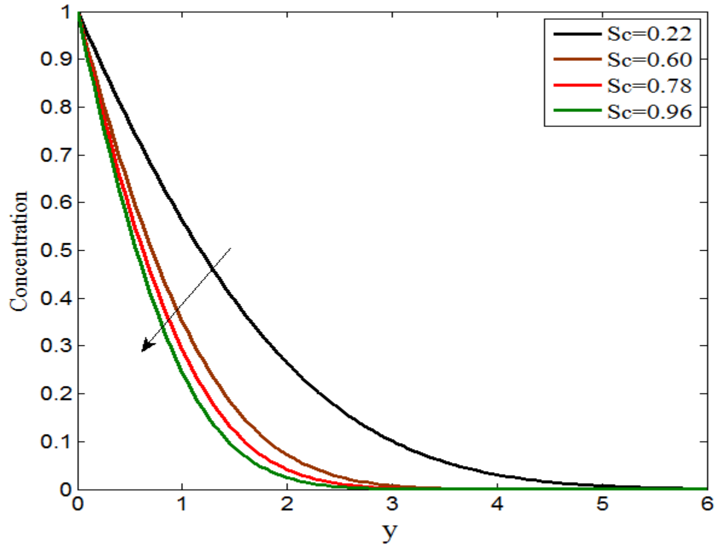

Fig. 2 Concentration profiles for various values of *Sc* when *k* =0.5, *t*=0.4

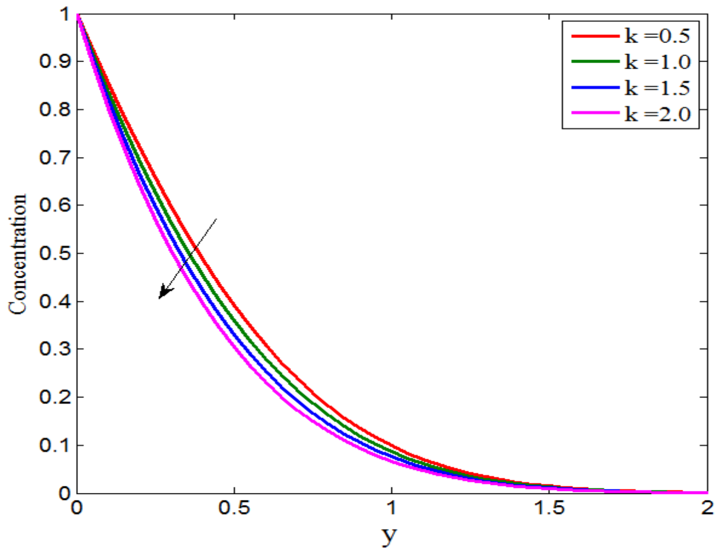

Fig. 3 Concentration profiles for various values of *k* when *Sc* =2.01, *t*=0.4

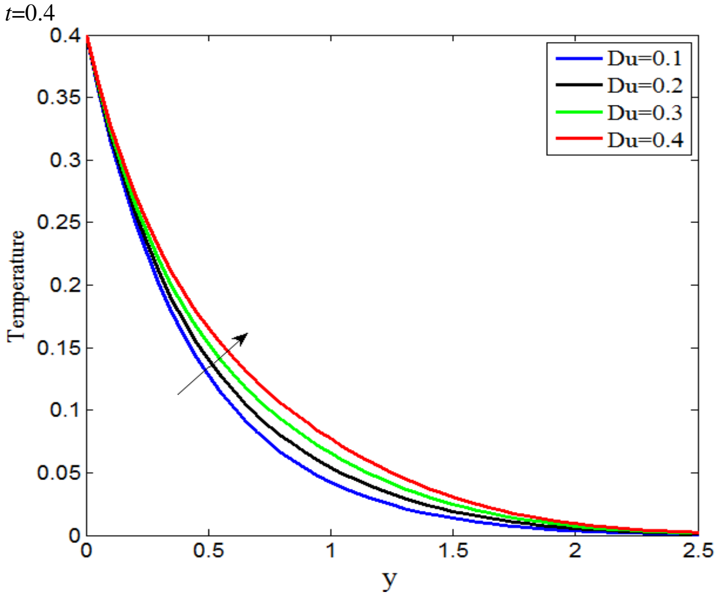

Fig. 4 Temperature profiles for various values of *Du* when *Sc* =2.01, *t*=0.4, *Pr*=0.71, *R*=4, *k*=0.5

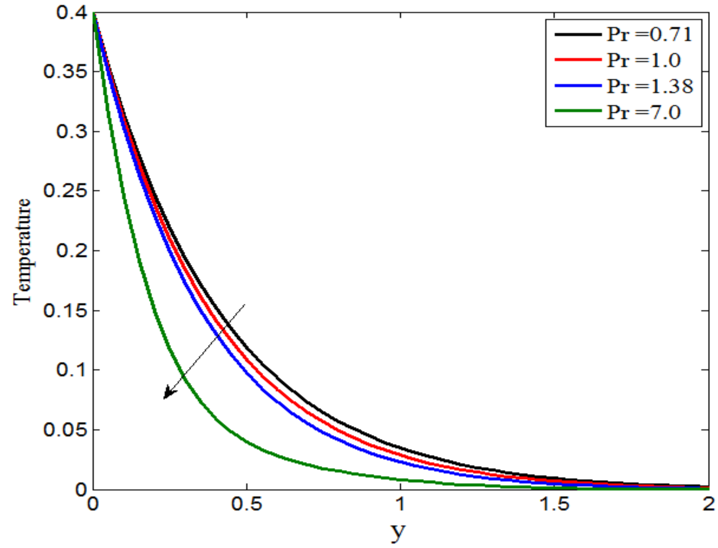

Fig. 5 Temperature profiles for various values of *Pr* when *Sc* =2.01, *t*=0.4, *Du* =0.03, *R*=4, *k*=0.5

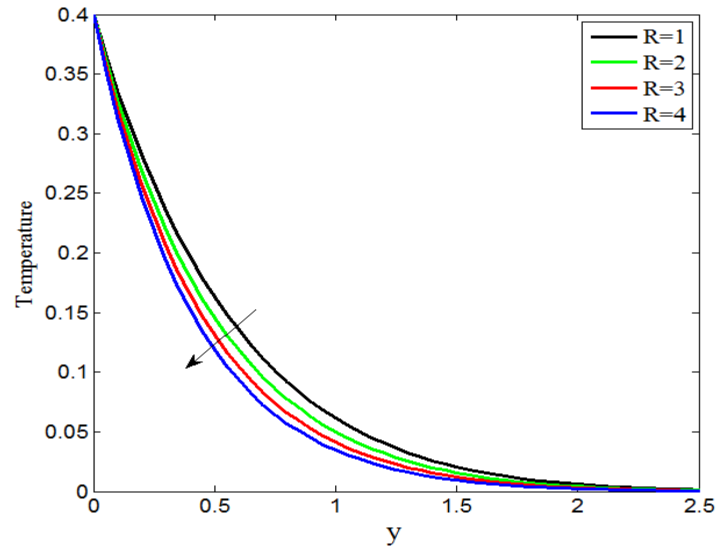

Fig. 6 Temperature profiles for various values of *R* when *Sc* =2.01, *t*=0.4, *Du* =0.03, *Pr*=0.71, *k*=0.5

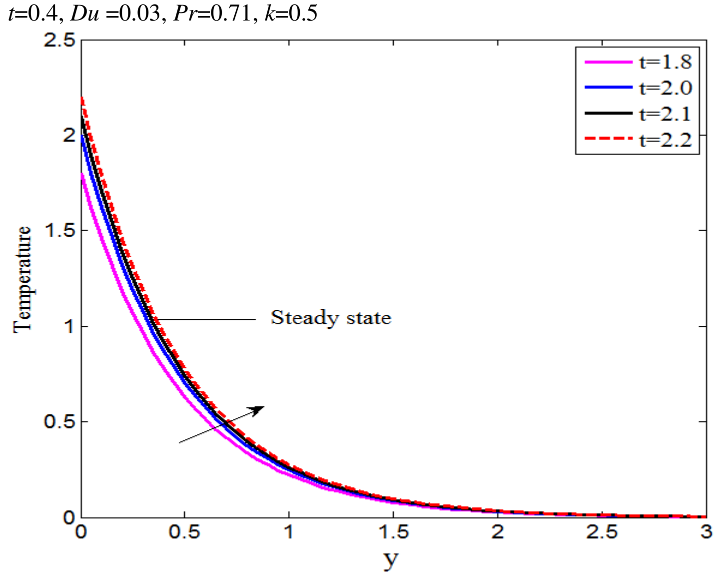

Fig. 7 Temperature profiles for various values of *t* when *Sc* =2.01, *Du* =0.03, *Pr*=0.71, *R*=4, *k*=0.5

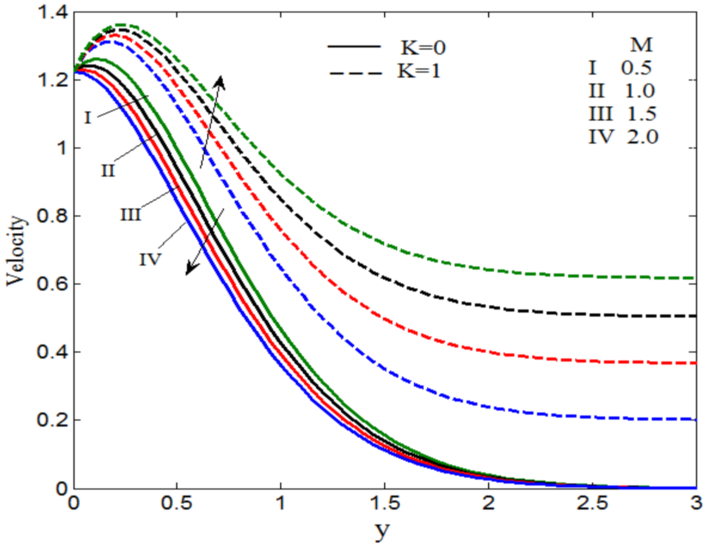

Fig. 8 Velocity profiles for various values of *M* when *Sc* =2.01, *Du* =0.03, *Gr*=10, *Gm*=5 $a_0 = 0.5$, *Pr*=0.71, *R*=4, *k*=0.5, *t*=0.4

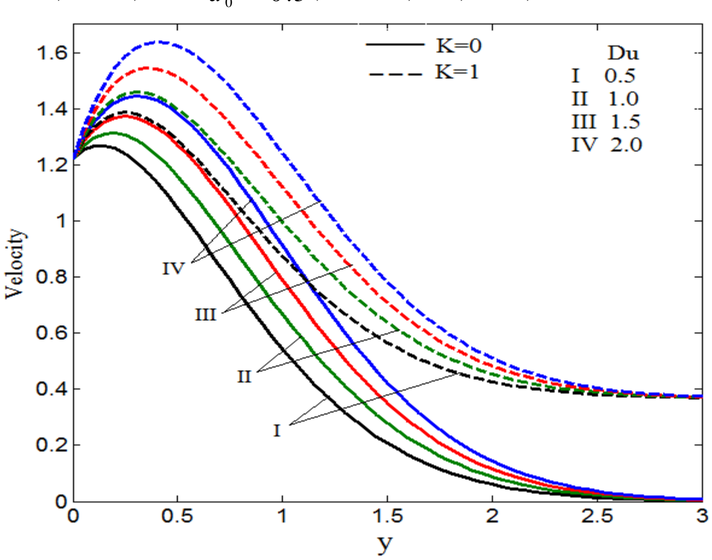

Fig. 9 Velocity profiles for various values of *Du* when *Sc* =2.01, *M* =1, *Gr*=10, *Gm*=5, $a_0 = 0.5$, *Pr*=0.71, *R*=4, *k*=0.5, *t*=0.4

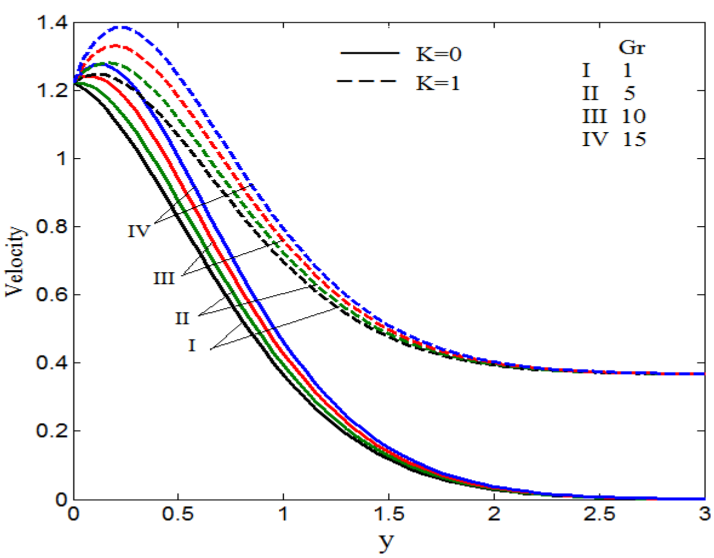

Fig. 10 Velocity profiles for various values of *Gr* when *Sc* =2.01, *Du* =0.03, *M*=1, *Gm*=5 $a_0 = 0.5$, *Pr*=0.71, *R*=4, *k*=0.5, *t*=0.4

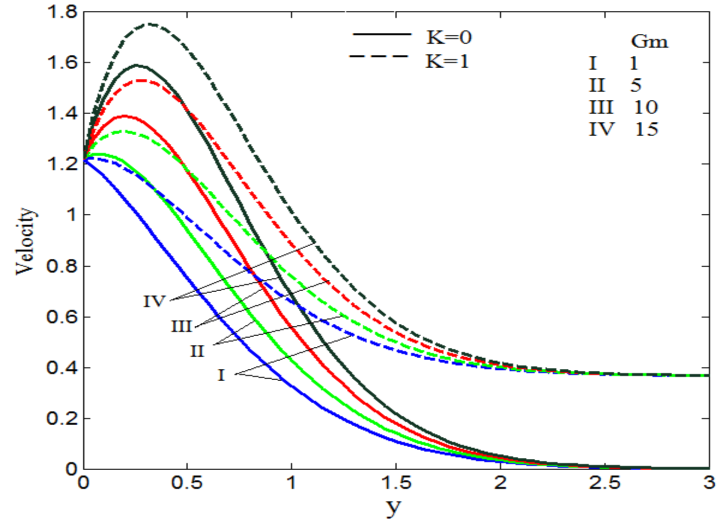

Fig. 11 Velocity profiles for various values of *Gm* when *Sc* =2.01, *Du* =0.03, *Gr*=10, *M*=1 $a_0 = 0.5$, *Pr*=0.71, *R*=4, *k*=0.5, *t*=0.4

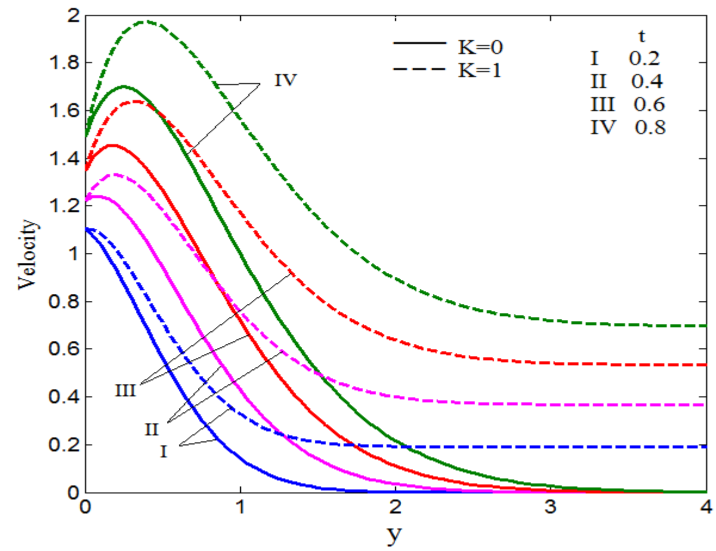

Fig. 12 Velocity profiles for various values of *t* when *Sc* =2.01, *Du* =0.03, *Gr*=10, *Gm*=5 $a_0 = 0.5$, *Pr*=0.71, *R*=4, *k*=0.5, *M*=1

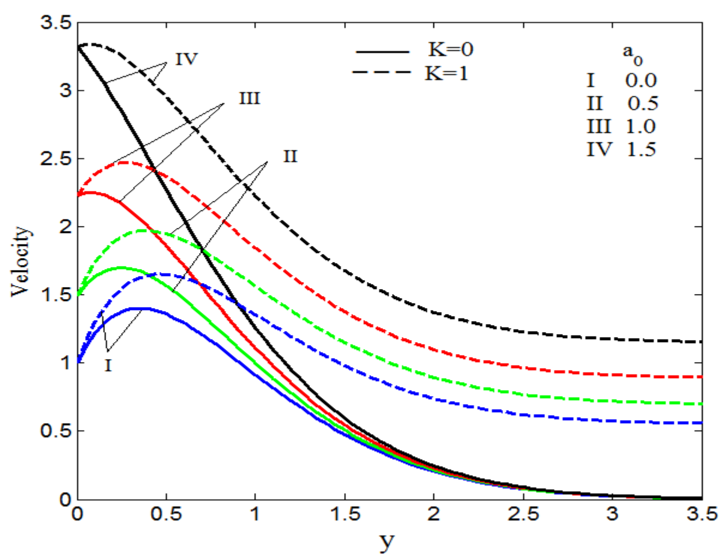

Fig. 13 Velocity profiles for various values of a_0 when *Sc* =2.01, *Du* =0.03, *Gr*=10, *Gm*=5, *Pr*=0.71, *R*=4, *k*=0.5, *M*=1, *t*=0.4

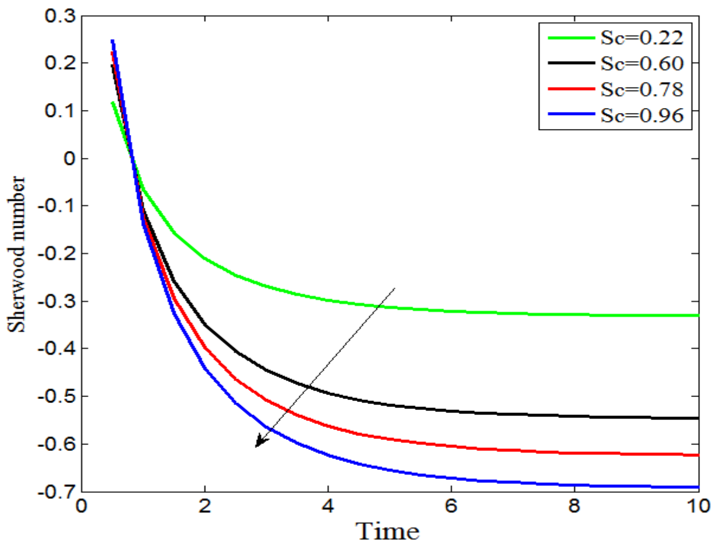

Fig. 14 Sherwood number for various values of *Sc* when *k*=0.5

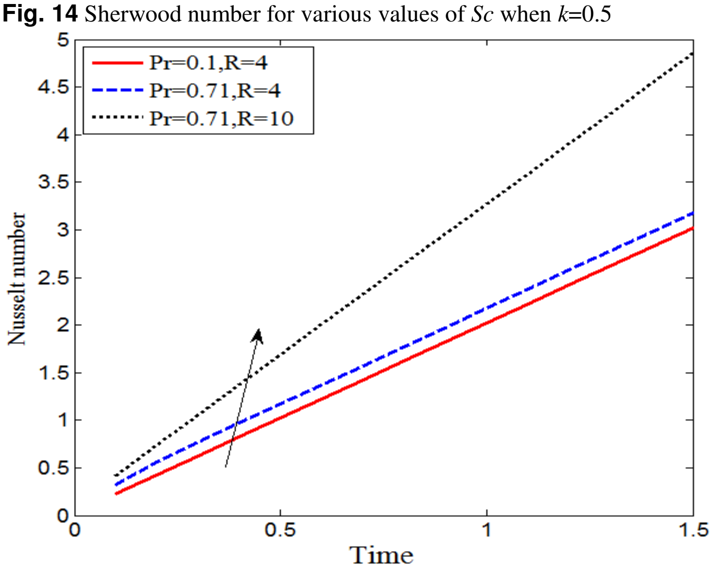

Fig. 15 Nusselt number for various values of *Pr* and *R* when *Sc*=2.01, *k*=0.5, *Du*=0.03

Finally, the rate at which mass transfers and the rate at mass transfers are examined through the figures 14 and 15 respectively. From Fig.14 it is seen that for the fixed values of *k* the Sherwood number *Sh* increases as *Sc* decreases. Figure15 reveals that the rate of heat transfer coefficient *Nu* for different values of Prandtl number *Pr*, radiation parameter *R* and Dufour number *Du* against time *t*. It is observed that for fixed values of *Du* the Nusselt number increases with increasing values of *Pr* or *R*.

5. CONCLUSIONS

A mathematical model has been presented fordiffusion-thermo and radiation effects on unsteady MHD free convection of flow of a viscous, incompressible, electrically conducting fluid past an accelerated infinite plate with variable temperature and uniform mass diffusion in the presence of transverse applied magnetic field.Laplace transform solutions for the non-dimensional momentum, energy and concentration equations subject to boundary conditions have been obtained. The expressions for Nusselt number and Sherwood number at the plate are obtained and solutions are presented graphically for the pertinent governing parameters.From this investigation, the fallowing conclusions arrive.

- The concentration profiles decrease with the increase of *Sc* and *k*.
- The temperature profiles increase with the increasing values of *Du* or *Pr* but reverse trend is obtained for *R*
- The velocity of the fluid decreases with an increasing values of *M*
- The velocity of the fluid increases with an increasing valves of *Du* or *Gr* or *Gm*
- In general, the flow field, temperature, and concentration distribution are affected by the physical parameters.

NOMENCLATURE

a_0 Accelerated parameter
B_0 External magnetic field (A.m^2)
C' Species concentration (kg m^{-3})
C'_w Concentration of the plate (kg m^{-3})
C'_∞ Concentration of the fluid far away from the plate (kg m^{-3})
C Dimensionless concentration (kg m^3)
C_p Specific heat at constant pressure (J kg^{-1}K)
Du Dufour effect
g Acceleration due to gravity (ms^{-2})
Gr Thermal Grashof number
Gm Mass Grashof number
M Magnetic field parameter (Am^{-1})
Nu Nusselt number
$\Pr$ Prandtl number
q_r Radiative heat fluxes in the y-direction
D_m Coefficient of mass diffusivity
R Radiation parameter (cm^2)
Sc Schmidt number
T' Temperature of the fluid near the plate (K)
T'_w Temperature of the plate (K)
T'_∞ Temperature of the fluid far away from the plate (K)
t' Time
t Dimensionless time (Sec)
u' Velocity of the fluid in the y'-direction
u_0 Velocity of the plate
u Dimensionless velocity (ms^{-1})
y' Coordinate axis normal to the plate
y Dimensionless Coordinate axis normal to the plate

Greek symbols

k Thermal conductivity of the fluid (Wm^{-1}K^{-1})
α Thermal diffusivity (Wm^{2}j^{-1})
β Volumetric coefficient of thermal expansion (K^{-1})
β^* Volumetric coefficient of expansion with concentration
μ Coefficient of viscosity (m^2 s^{-1})
ν Kinematic viscosity (m^2 s^{-1})
ρ Density of the fluid (kg m^{-3})
σ Electric conductivity(Sm^{-1})
θ Dimensionless temperature (K)

Subscripts

w Conditions on the wall
∞ Freestream conditions

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