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HEAT TRANSFER INTENSIFICATION IN A 3D CAVITY USING HYBRID CNT-AL$_2$O$_3$ (15-85%) NANOFLUID

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ABSTRACT

In this work, a computational study of convective heat transfer in a hybrid CNT-Al_2O_3/water nanofluid cavity filled. The main considered parameters are the Rayleigh number and nanoparticles volume fraction. Results are presented in terms of flow structure, temperature field, and average Nusselt number. Since CNT and Al_2O_3 have different shapes to models are used to evaluate the effective thermal conductivity. It was found that both increasing Rayleigh number and nanoparticles volume fraction increase the heat transfer intensify the flow and affect the temperature field. Adding nanoparticles enhances the heat transfer due to the enhancement of the effective thermal conductivity. The maximum percentage of heat transfer enhancement occurs at the transition regime (Ra = 10^4) and is equal to 28%.

Keywords: *Natural convection, Numerical simulation, Hybrid nanofluid.*

1. INTRODUCTION

Nanofluids are revolutionary fluids that correspond to a suspension of nanoparticles in a base fluid. Choi (1995) was the first that who innovated such fluids. The characterization of nanofluids showed that they have enhanced thermophysical properties allowing better performances of heat transfer and thus a reduction of the required heat exchange area which is the challenge of engineers and researchers working on heat exchangers, electronic devices cooling and thermal storage system. Various materials were used with nanoparticles. These materials are characterized by different properties. Some of these materials are more expensive than others but have better properties. Thus to compromise between heat transfer enhancement and operating cost, some researchers proposed to use hybrid nanofluids.

Enhancement of heat transfer using nanofluids has been the subject of several studies. Kolsi *et al.* (2016) studied the combined buoyancy-thermocapillary convection and entropy generation in a 3D cavity filled with Al_2O_3 nanofluid. It was found that the increase in nanoparticles volume fraction for all Marangoni number, intensify the flow and increase heat transfer and total entropy generation. Kolsi *et al.* (2017) studied the effective magnetic field in an open cubic cavity filled with CNT-water nanofluid and equipped with an inclined plate. They mentioned that the presence of the magnetic field opposes the enhancement caused by the addition of nanoparticles.

Al-Rashed *et al.* (2018a), studied the mixed convection and entropy generation in a nanofluid filled cubical open cavity with a central isothermal block. It was found that the effect of adding nanoparticles on heat transfer is limited for small size hot block and low Richardson numbers. Other results related to convective heat transfer of nanofluids can be found in (Al-Rashed *et al.* 2018b, Al-Rashed *et al.* 2018c, Rahimi *et al.* 2018a, Rahimi *et al.* 2018b, Rahimi *et al.* 2018c, Al-Rashed *et al.* 2017 and Rahimi *et al.* 2017). Recently some works investigated the use of hybrid nanofluids to enhance heat transfer. Kasaeipoor *et al.* (2017). The heat transfer is directly related to Rayleigh number and solid volume fraction. Kalidasan *et al.* (2017) studied the laminar natural convection of Copper - Titania/Water hybrid nanofluid in an open-ended C-shaped enclosure with an isothermal block. A monotonical heat transfer enhancement occurs with the increase in the percentage of hybrid nanoparticles. Izadi *et al.* (2018) studied numerically the natural convection inside a ⊥ shaped cavity filled with MWCNT-Fe3O4/water hybrid nanofluids. They indicated that heat transfer degrades in respect with the cavity obstruction ratio due to the development of the thermal boundary layer thickness. Kalidasan and Kanna (2017) investigated the natural convection of nanodiamond-cobalt oxide/water hybrid nanofluid in an open square cavity containing diagonally placed heaters and adiabatic square block. The authors mentioned that the strength of the primary vortex depreciated with the increasing percentage of nano-composites and heat transfer is more important on the right compared to the left one. According to the above literature review, it can be noticed that an important number of studies on the enhancement of heat transfer using nanofluid can be found. The works related to the use of hybrid nanofluids are scarce and deal with 2D configurations. The main objective of this computational research is to study the effect of using hybrid CNT-Al_2O_3/water nanofluid on the 3D convective heat transfer inside a differentially heated cubic cavity. In addition to the use of hybrid nanofluids in a 3D geometry the novelty of this work is related to the use of differently shaped-nanoparticles and thus two models are used and combined to evaluate the effective thermal conductivity.

2. GEOMETRY, MATHEMATICAL MODEL, AND NUMERICAL METHOD

The considered configuration is shown in fig. 1. Two vertical walls of the enclosure are differentially heated and all other walls are adiabatic. The cubic enclosure is filled with a hybrid CNT-Al_2O_3/water nanofluid having $Pr = 6.2$. The thermophysical properties of the pure fluid (water) and CNT particles are shown in Table 1. The flow is

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considered to be three-dimensional, laminar, incompressible, and unsteady. The physical properties of the fluid are assumed as constant except for the density in the buoyancy term by the Boussinesq's approximation.

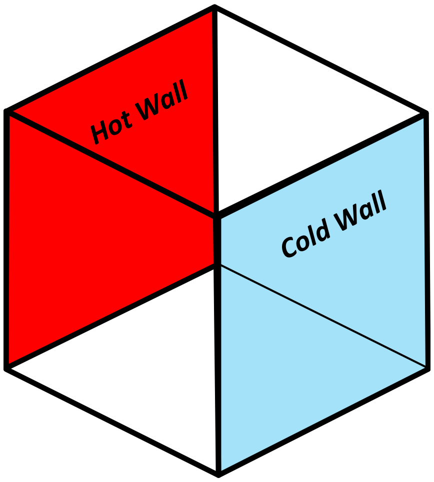

Fig. 1. Considered configuration

Table 1. Thermophysical properties of water and nanoparticles

Physical properties	Water	CNT [3]	Al_2O_3 [2]
C_p *(J/kg.K)*	4179	425	765
ρ *(kg/m³)*	997.1	2600	3970
k (W/m.K)	0.613	6600	40
β (K^{-1})	21.10^{-5}	$1.6.10^{-6}$	$0.85.10^{-5}$

Governing equations were written using 3D vorticity-vector potential formalism. This formalism allows to eliminate the pressure^, which is delicate to treat. The vorticity and vector potential are respectively defined by the following two relations (Kolsi *et al.* (2010)):

$$\vec{\omega}' = \vec{\nabla} \times \vec{V}' \tag{1}$$

and

$$\vec{V}' = \vec{\nabla} \times \vec{\psi}' \tag{2}$$

The system of equations governing the phenomenon is

$$-\vec{\omega} = \nabla^2 \vec{\psi} \tag{3}$$

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{V}.\nabla)\vec{\omega} = (\vec{\omega}.\nabla)\vec{V} + \frac{\nu_{nf}}{\nu_f}\Pr.\nabla^2\vec{\omega} - \frac{\beta_{nf}}{\beta_f} Ra \Pr \nabla \times T\vec{g} \tag{4}$$

$$\frac{\partial T}{\partial t} + \vec{V}.\nabla T = \frac{\alpha_{nf}}{\alpha_f}\nabla^2 T \tag{5}$$

In these equations, the dimensionless *Pr*, *Ra*, and *Ha* numbers are respectively defined as

$$\Pr = \frac{\nu_f}{\alpha_f} \text{ and } Ra = \frac{g.\beta_f.\Delta T.l^3}{\nu_f.\alpha_f} \tag{9}$$

To get the above-mentioned equations (3-7), the different variables: $t', \vec{V}'$, $\vec{\psi}'$ and $\vec{\omega}'$, are put in their dimensionless form using: l^2/α α/l, α l^2/α, respectively, and the dimensionless temperature is:

$$T = (T' - T_c') / (T_h' - T_c') \tag{10}$$

The effective density of the nanofluid is given by Kahveci (2010) as:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s \tag{11}$$

The heat capacitance of a nanofluid is expressed by Kahveci (2010) as

$$\left(\rho C_p\right)_{nf} = (1-\varphi)\left(\rho C_p\right)_f + \varphi\left(\rho C_p\right)_s. \tag{12}$$

The effective thermal conductivity of CNT-nanofluid is approximated by Xue (2005) as:

$$\frac{k_{nf}}{k_f} = \frac{1-\phi+2\varphi\dfrac{k_s}{k_s-k_f}\ln\left(\dfrac{k_s+k_f}{2k_f}\right)}{1-\varphi+2\varphi\dfrac{k_f}{k_s-k_f}\ln\left(\dfrac{k_s+k_f}{2k_f}\right)}. \tag{13}$$

The effective thermal conductivity of the Al_2O_3 nanofluid is approximated by the Maxwell–Garnetts model (1904) as:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\varphi\left(k_f - k_s\right)}{k_s + 2k_f + \varphi\left(k_f - k_s\right)}. \tag{14}$$

The effective dynamic viscosity of a nanofluid is given by the Brinkman model as

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}} \tag{15}$$

The properties of the hybrid CNT-Al_2O_3/water nanofluid are calculated as follows:

$$F_{hnf} = F_{CNT}.y_{CNT} + F_{Al_2O_3}.(1-y_{CNT}) \tag{16}$$

Where F is any thermophysical property and y_{CNT} is the mass fraction of CNT.

Boundary Conditions

The boundary conditions for the present problem are given as follows:

Temperature:

$T = 0$ at $x = 1$, and $T = 1$ at $x = 0$.

$\frac{\partial T}{\partial n} = 0$ on the other walls (i.e., adiabatic).

Vorticity:

$\omega_x = 0$, $\omega_y = -\frac{\partial V_z}{\partial x}$, $\omega_z = \frac{\partial V_y}{\partial x}$ at $x = 0$ and 1.

$\omega_x = \frac{\partial V_z}{\partial y}$, $\omega_y = 0$, $\omega_z = -\frac{\partial V_x}{\partial y}$ at $y = 0$ and 1.

$\omega_x = -\frac{\partial V_y}{\partial z}$, $\omega_y = \frac{\partial V_x}{\partial z}$, $\omega_z = 0$ at $z = 0$ and 1.

Vector potential:

$\frac{\partial \psi_x}{\partial x} = \psi_y = \psi_z = 0$ at $x = 0$ and 1.

$\psi_x = \frac{\partial \psi_y}{\partial y} = \psi_z = 0$ at $y = 0$ and 1.

$\psi_x = \psi_y = \frac{\partial \psi_z}{\partial z} = 0$ at $z = 0$ and 1.

Velocity:

$V_x = V_y = V_z = 0$ on all walls.

The local Nusselt number (Nu) is defined as follows

$$Nu = \left(\frac{k_{nf}}{k_f}\right)\left.\frac{\partial T}{\partial x}\right|_{x=0,1} \tag{17}$$

The average Nusselt number on the hot wall (Nu_{av}) is expressed by:

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$$Nu_{av} = \int_0^1 \int_0^1 Nu.dy.dz \qquad (18)$$

The normalized average Nusselt number is defined as:

$$Nu_{av}^{*} = \frac{Nu_{av}}{Nu_{av}^{0}} \qquad (19)$$

Where Nu_{av}^{0} is the average Nusselt number four $\varphi = 0$

3. CONVERGENCE, GRID TESTING, AND CODE VALIDATION

An extensive mesh testing procedure was conducted to guarantee a grid independent solution. The grid independence test has been performed on the cubical enclosure with Pr = 6.2, Ra = 10^5, φ = 0.05. The tests were conducted for the spatial meshes of 61^3, 71^3, 81^3 and 91^3. The average Nusselt number on the hot wall is selected as a sensitive parameter. The results of the analysis were presented in Table 2. The incremental increase in the percentage of Nu_{av} for the grid 71^3 to 81^3 is only 0.107 %. Hence considering the computational economy and accuracy, the spatial mesh size of 71^3 and a time-step of 10^{-3} have opted for the present study and all results are presented for a dimensionless time equal to 2.

Table 2. Grid sensitivity analysis for: Pr = 6.2, Ra = 10^5 and φ = 0.05

Grid size	Nu_{ova}	Percentage increase	Incremental increase
60^3	5.5213	-	-
71^3	5.6838	2.943147	-
81^3	5.68989	3.053448	0.107147
91^3	5.6945	3.136942	0.081021

The solution is considered acceptable when the following convergence criterion is satisfied for each period of time:

$$\sum_{i}^{1,2,3} \frac{\max\left|\psi_i^n - \psi_i^{n-1}\right|}{\max\left|\psi_i^n\right|} + \max\left|T_i^n - T_i^{n-1}\right| \le 10^{-5} \qquad (17)$$

The numerical code used in the present work was validated by comparing it to the results obtained by Oztop and Abu-Nada (2008) (Fig. 2) who considered the natural convection in a differentially heated cavity filled with Cu-nanofluid. It is noteworthy that these authors considered spherical nanoparticles, thus similarly the Maxwell–Garnetts model has been used to evaluate the effective thermal conductivity. The comparison showed a good concordance between the results.

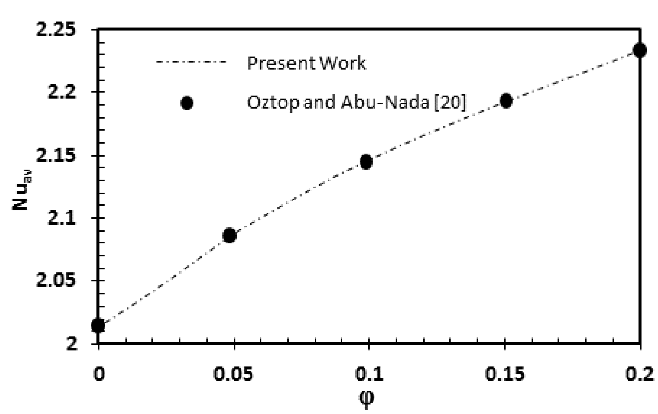


Fig. 2. Nusselt number versus φ for Ra=10^4; comparison with results of Oztop and Abu-Nada (2008).

4. RESULTS AND DISCUSSION

A computational study is performed to solve equations of buoyancy-induced flow in a 3D differentially heated cavity. The working fluid is the hybrid CNT-Al_2O_3/water nanofluid. The flow structure, temperature field, and heat transfer are analyzed according to different nanoparticle volume fractions and different Rayleigh numbers.

Figure 3 presents some particles trajectories for $\varphi = 0.05$ and different Rayleigh numbers. For Ra = 10^3 and 10^4, the flow is characterized by one central vortex. The flow is convergent from the back and front walls to central plan (z = 0.5). For higher Ra, vortexes number becomes 2 for Ra = 10^5 and 3 for Ra = 10^6. The apparition of the multi-vortexes structure is due to the intensification of the flow and the viscous effects.

For a better understanding of the flow structure, the velocity vector projections at z = 0.5 plan are plotted on Fig.4 for different Rayleigh numbers and different nanoparticles volume fractions. Within each of these findings,it is identied that in opposition with the 2D case the streamlines aren't closed. As mentioned above the flow structure is multi-vortexes and the increase of the concentration of nanoparticles causes the increase of the distance between the centers of the vortexes indicating an intensification of the flow.

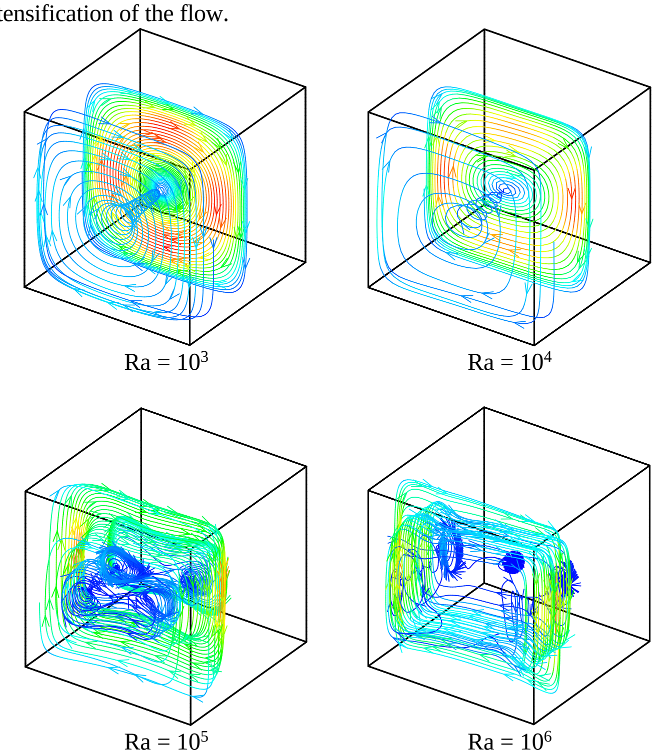
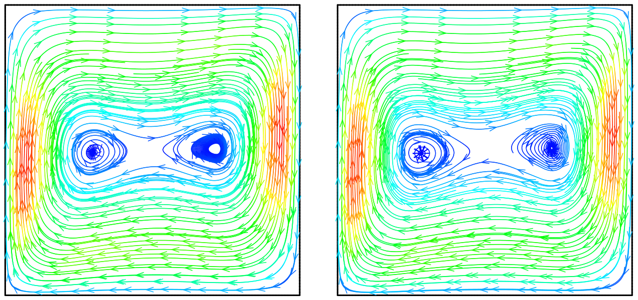


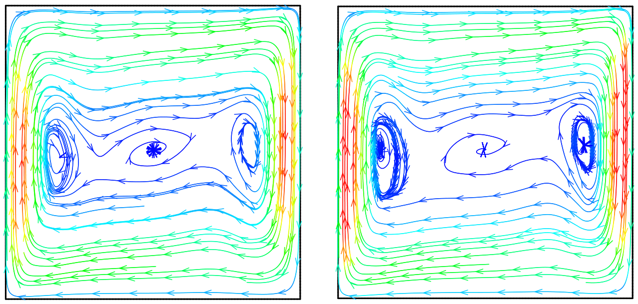
Fig. 3. Some particle trajectories for $\varphi = 0.05$ and different Ra

Figure 5 presents the iso-surfaces of temperature $\varphi = 0.05$ and different Rayleigh numbers. For Ra = 10^3 the iso-surfaces of temperature are vertical and equidistance due to the dominance of the conductive regime. By increasing Ra iso-surfaces of temperature pile up near the bottom of the hot wall and the top of the cold wall. On this level, a vertical stratification occurs in the core of the cavity.

To compare the temperature field with and without adding nanoparticles, isotherms are presented at z = 0.5 plan. Dashed isolines represent $\varphi = 0$ and solid isolines represent $\varphi = 0.05$. Due to the addition of nanoparticles, the vertical stratification becomes more pronounced announcing the enhancement of heat transfer.



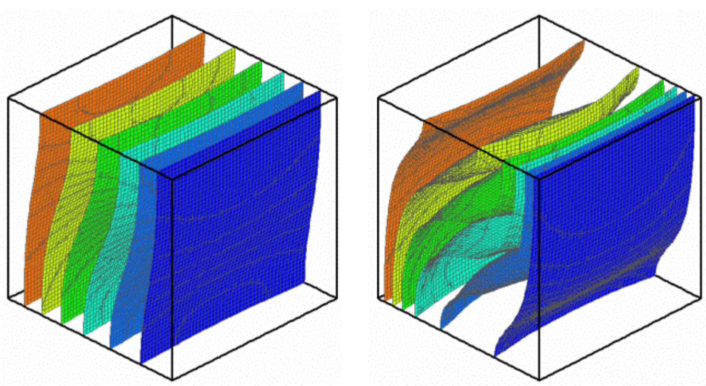
Ra = 10^6, $\varphi = 0$ Ra = 10^6, $\varphi = 0.05$



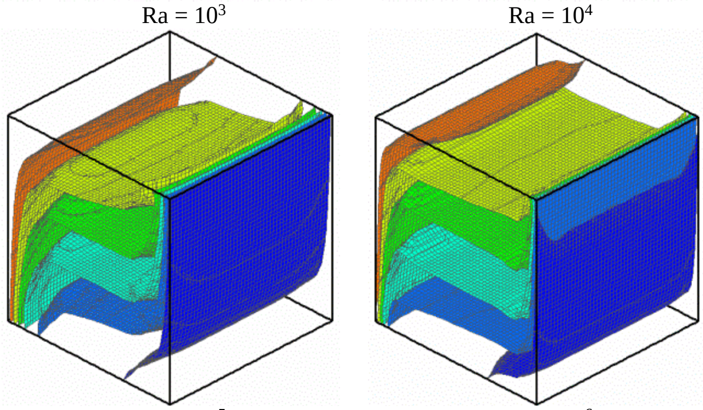
Ra = 10^5, $\varphi = 0$ Ra = 10^5, $\varphi = 0.05$

Fig.4 . Velocity vector projection at z= 0.5 plan

Figure 7 presents the variation of Average Nusselt number versus nanoparticles volume fraction for different Rayleigh numbers. For all Ra values, the variation is quasi-linear and the heat transfer increases with the increase of the oncentration of nanoparticles. The increase of heat transfer is engendred by the enhancement of thermal conductivity that occurs by adding the high conductive nanoparticles. Over and above,that the heat transfer increases by increasing Rayleigh number. This result is obvious and is due to the augmentation of the buoyancy forces.



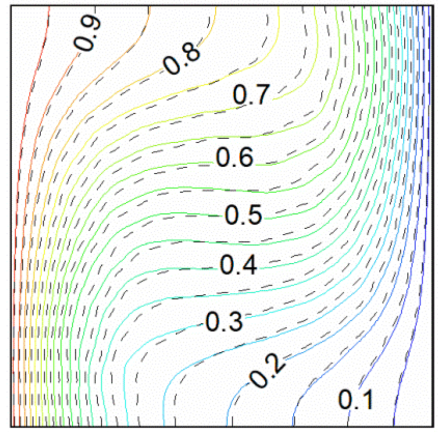
Ra = 10^3 Ra = 10^4



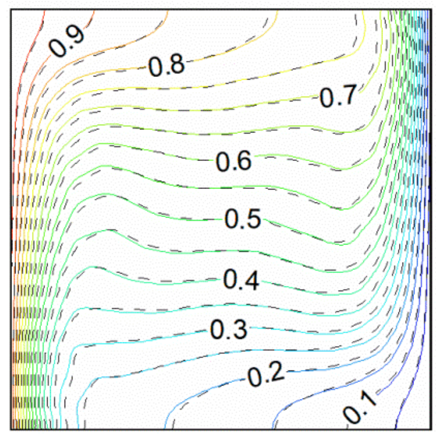
Ra = 10^5 Ra = 10^6

Fig. 5. Iso-surfaces of temperature for $\varphi = 0.05$ and different Ra

The variation of the normalized average Nusselt number versus nanoparticles volume fraction for different Rayleigh numbers is plotted to identify the case where the maximum of heat transfer enhancement occurs (Fig .8). It is found that for all volume fractions the maximum of enhancement is for Ra = 10^4 (transition regime: the passage from the conductive to the convective regime). For example for $\varphi = 0.05$ the percentage of enhancement is about 28%.



Ra = 10^4



Ra = 10^5

Fig. 6. Isotherms at z = 0.5 plan for $\varphi = 0.05$ (solid) and $\varphi = 0$ (dashed)

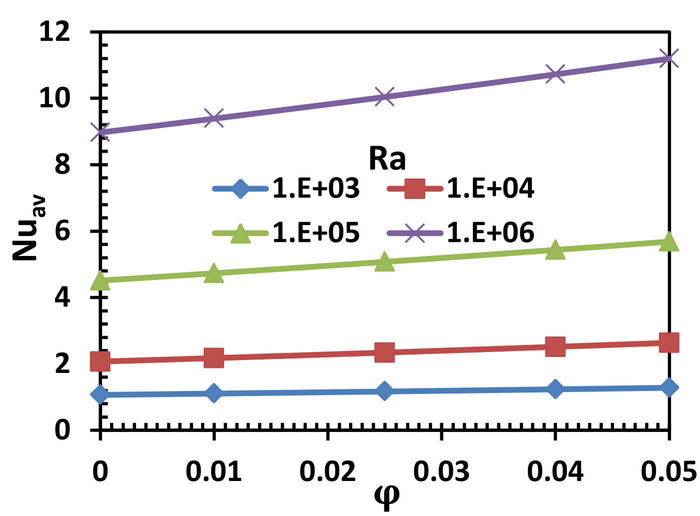


Fig. 7. Variation of Nuav versus $\varphi = 0$ for different Ra

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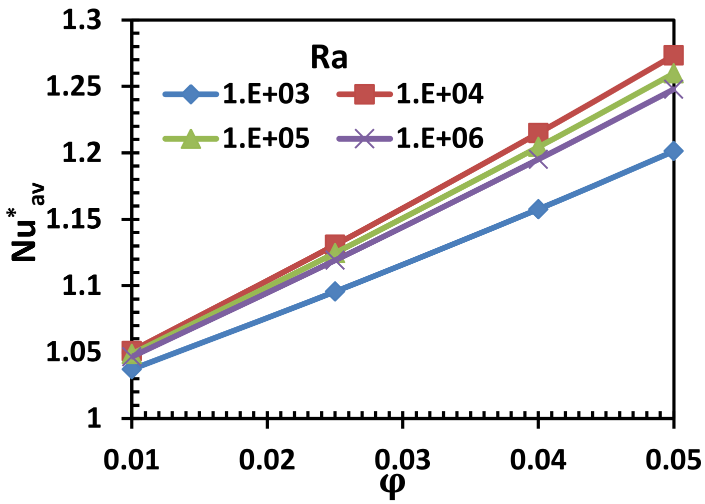

Fig. 8. Variation of Nu_{av}^{*} versus $\varphi = 0$ for different Ra

5. CONCLUSION

This work represents a numerical study of the hybrid nanofluid effect on 3D convective heat transfer. The main results can be summarized like the following

- For high Ra values, there is an apparition of a multi-vortexes structure that is due to the intensification of the flow, this intensification is to more pronounced for higher nanoparticles concentrations.
- For high Ra values a vertical stratification of the iso-surfaces of temperature occurs in the core of the cavity and is more pronounced for higher nanoparticles concentrations.
- For all Ra values, the heat transfer increases with increasing the concentration of nanoparticles.
- the maximum of enhancement percentage due to the adding of nanoparticles occurs for Ra = 10^4

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NOMENCLATURE

C_p	Specific heat at constant pressure (J/kg. K)
g	Gravitational acceleration (m/s^2)
k	Thermal conductivity (W/m.K)
l	Enclosure width and height (m)
Nu	Local Nusselt number
Pr	Prandtl number
Ra	Rayleigh number
t	Dimensionless time ($t'.\alpha / l^2$)
T	Dimensionless temperature [$(T'-T'_c)/(T'_h-T'_c)$]
T'_c	Cold temperature (K)
T'_h	Hot temperature (K)
T_o	Bulk temperature [$T_o = (T'_c + T'_h)/2$] (K)
$\vec{V}$	Dimensionless velocity vector ($\vec{V}'.l/\alpha$)
x, y, z	Dimensionless coordinates (x'/l , y'/l , z'/l)

Greek symbols

α	Thermal diffusivity (m^2 /s)
β	Thermal expansion coefficient (1 / K)
ρ	Density (kg/m^3)
μ	Dynamic viscosity (kg./m.s)
ν	Kinematic viscosity (m^2/s)
φ	Nanoparticle or solid volume fraction
$\vec{\psi}$	Dimensionless vector potential ($\vec{\psi}'/\alpha$)
$\vec{\omega}$	Dimensionless vorticity ($\vec{\omega}'.\alpha / l^2$)
ΔT	Dimensionless temperature difference

Subscripts

av	Average
x, y, z	Cartesian coordinates
fr	Friction
f	Fluid
nf	Nanofluid
S	Solid
th	Thermal
tot	Total

Superscript

'	Dimensional variable

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