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NUMERICAL INVESTIGATION OF NATURAL CONVECTION HEAT TRANSFER IN A PARALLELOGRAMIC ENCLOSURE HAVING AN INNER CIRCULAR CYLINDER USING LIQUID NANOFLUID

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ABSTRACT

Fluid flow and natural convection heat transfer in a parallelogram enclosure with an inner circular cylinder using Cu-water nanofluid are studied numerically. Dimensionless Navier-Stokes and energy equations are solved numerically using finite element method based two-dimensional flow and steady-state conditions. This study evaluates the effect of different concentrations of Cu-water nanofluids (0% to 6%) with different Rayleigh numbers $10^3 \leq Ra \leq 10^6$ under isotherm wall temperatures. The effects of geometrical parameters of the parallelogram enclosure (inclination angle in range of $0 \leq \alpha \leq 30$ and location of inner circular cylinder $-0.2 \leq H \leq +0.2$ on the flow field and heat transfer are examined. The results are presented in terms of streamlines, isotherms, local and average Nusselt number. It is found that the inclination angle has a significant effect on flow pattern and heat transfer and the inclination angle of 30° at a vertical location H=-0.2 gives better fluid flow strength. Moreover, the maximum heat transfer enhancement is obtained when the circular cylinder moves vertically downward up to H=-0.1 and the inclination angle is 30°. The results also indicate that as the Rayleigh number, nanofluid concentration increase, the rate of heat transfer will increase.

Keywords: *Natural convection, parallelogrammic enclosure, cylinder, inclination angle.*

1. INTRODUCTION

Heat transfer by means of the free convective flow within enclosures have practical relevance in many applications in thermal engineering sciences such as heating and cooling nuclear reactors, heat exchangers, lubrication technologies, ventilation of the rooms with radiators, cooling of containers and cooling of electronic devices. Natural convection has a significant influence and an important role in the process of the energy transport for the proper design of enclosures to achieve higher rates of heat transfer. The internal convection fluid flow can be classified into two types depending upon on the conditions of applied thermal boundary which they are: heat from the sidewalls of the enclosures hand heated or cooled from the top and the bottom walls of the enclosures Bejan, (1984). Warrington and Powe, (1985) investigated experimentally the natural convective heat and fluid flow between different shapes of inner body like spherical, cylindrical and cubical located concentrically inside cubical enclosure at law Reynolds number. There are many experimental and numerical studies in this regards like Filis and Poulikakos, (1986); November and Nansteel, (1987); Valencia and Frederick, (1989); Hasnaoui et al., (1992); Evren-Selamet et al., (1992). Ganzarolli and Milanez, (1995) investigated numerically the influence of the aspect ratio on the fluid flow and thermal field using a stream function–vorticity formulation the two-dimensional natural convection in a rectangular enclosure heated from one side and cooled from the ceiling. The numerical results indicated that the impact of aspect ratio is stronger when the enclosure is tall and the Rayleigh number is high. Aydin et al., (1999) studied numerically using the stream function–vorticity formulation the aspect ratio impact of the rectangular enclosure heated from bottom walls, the top wall is adiabatic and the both vertical sides conserved at isothermal cold temperature filled with air. Roychowdhury et al., (2002) investigated by using collocated, non-orthogonal grid based finite volume scheme for simulating the two dimensional natural convective heat transfer in a square enclosure within a heated cylinder. Peng et al., (2003) presented using the Taylor-series expansion and least-squares-based lattice Boltzmann method to simulate numerically the natural convection in a concentric annulus between a square outer cylinder and a circular inner cylinder. Moreover, De and Dalal, (2006) investigated numerically the influence of aspect ratio for the shape of the rectangular enclosure. It was showed that the aspect ratio has significant influence on the patterns of fluid flow and thermal field in the enclosure. For constant Rayleigh number as the enclosure's aspect ratio goes up, the heat transfer rate by convection falling down.

Various researchers like Moukalled and Acharya, (1996), Oh et al., (1997), Ha et al., (1999), Asan, (2000), Shu and Zhu, (2002), Humaira Tasnim et al., (2002), Lee et al., (2004), De and Dalal, (2006), Angeli et al., (2008) and Xu et al., (2010) studied the influence of inner body on the natural convective flow inside an enclosure. They concluded that the inner body's size and shape is strongly influencing on natural convection flow. Lee and Ha, (2005) studied numerically the impact of an inner cylinder having the square shape on the natural convection in a cavity under different values of Rayleigh numbers. They reported that the existence of the internal object affects fluid flow and heat transfer characteristics in the cavities.

Moreover, with respect to the effect of the position of an internal object on natural convection within enclosures has been illustrated by Humaira Tasnim et al., (2002); Shu et al., (2001); Ding et al., (2005);

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Kim et al., (2008); Lee et al., (2010); Kang et al., (2013b); Park et al., (2013); Karimi et al., (2014). Their work is by changing the position of of the internal object in the vertical direction like De and Dalal, (2006); Shu et al., (2001); Ding et al., (2005); Kim et al., (2008) as well as horizontally Hussain and Hussein, (2010); Lee et al., (2010); Kang et al., (2013a); Park et al., (2013); Karimi et al., (2014) within the enclosures. Their observation indicated that the position and the Rayleigh number paly as an important parameter on the behavior of fluid flow.

Various researchers examined the effect of the thermal conditions, aspect ratios and the medium of the heat transfer which summarized recently by Das et al., (2017b)

The natural convective in a rhombus cavity with an internal cylinder having the shape of circle had been studied numerically by changing both the position of the cylinder as well as the Rayleigh number Choi et al., (2014). Mun et al., (2017) investigated using immersed boundary theory natural convection phenomenon induced by a difference of the temperature resulted in the hot four inner cylinders located inside an enclosure. The simulations presented under various Rayleigh numbers and distances between each cylinder. The natural convection in a uniformly hot circular cylinder located inside enclosure of the square shape filled with air under different cylinder vertical locations studied numerically in Hussain and Hussein, (2010). It was obtained that high Rayleigh numbers have considerable effect on the flow distribution. Boulahia, Wakif, Chamkha, et al., (2017) used finite volume scheme to solve the Navier-Stokes equations of natural and mixed convection heat transfer of copper-water nanofluid inside a square enclosure containing heating and cooling circular cylinders. It is obtained that the heat transfer improves when the pair of circular bodies changing their direction from the horizontal into the vertical.

Influence of different shapes of inner body like rectangular, triangular and trapezoidal located inside a cubical cavity on the natural convection heat transfer had been studied in Gibanov and Sheremet, (2018). The results indicated that the trapezoidal shape give better heat removal in a comparison with the other types. Boulahia, Wakif, et al., (2017a) demonstrated numerically the buoyancy driven flow within a square enclosure with inner hot cylinder located at the center of the enclosure. They used copper-water nanofluid and solve the problem using finite volume method. They obtained that increasing of the nanofluid volume fraction and the Rayleigh number enhances the heat transfer rate.

Engineering application regarding the natural convection needs solving combined complex problems for the heat transfer and fluid dynamics which depend strongly upon numerous parameters which they are the geometry of the enclosure and the properties of the working fluid.

The changing the parameters of geometric form is a suitable method to increase the thermal performance. So that many researchers have investigated to enhance the heat transfer in the parallelogramic enclosure with the various cross sectional shapes using conventional working fluids and varied methods Chung and Trefethen, (1982); Seki et al., (1983); Hyun and Choi, (1990); Aldridge and Yao, (2001); Baïri, (2008); Saleh et al., (2011). Systematic and comprehensive numerical results of the fluid flow and thermal patterns evolutions are presented for tilt angles 0°, ±15°, ±30°, ±45°, ±60°; and different values of Rayleigh numbers 10^5, 10^6, and 10^7. Prandtl number and of the aspect ratio effects are investigated.

The magnetic field influence in enclosure filled with porous medium and nanofluid on the convection onset using the Buongiorno's model the behavior of heat transfer and fluid flow is investigated by Wakif et al., (2017); Wakif, Boulahia, and Sehaqui, (2018); Wakif, Boulahia, Mishra, et al., (2018).

Moreover, the effect of the structure of the physico-chemical of the fluid has been taken a lot of interest in many studies recently. The capability of a fluid medium to transmit a high quantity of heat through a small difference in temperature enhances the energy conversion efficiency and improves the design and performance of heat exchangers. Heat transfer through natural convection is depending strongly on the fluid thermal conductivity. Most of conventional fluids like air, water and oil have law thermal conductivity which is a limitation in the enhancing the heat transfer rate. For this reason nanotechnology may be considered as a new type of the working fluids which enhance the heat transfer Sheikholeslami, (2018b); Sheikholeslami et al., (2018); Sheikholeslami, (2018a)

In this way to enhance the fluid thermal conductivity, solid nanoparticles in the fluid medium suspension is playing as an effective strategy in improving the heat transfer rate because the thermal conductivity of solids is greater than that of fluids in an order of magnitude. The new achieved structure is by inserting solid particles of nanometer in sizes into the fluid in order to improve its effective thermal conductivity. The obtained mixture is named nanofluid.

Studies on natural convection in differentially heated enclosures filled with nanofluids are very limited compare with those filled with pure fluid. Garoosi and Hoseininejad (2016) Studied numerically of heat transfer by natural as well as mixed convection between differentially heated cylinders located inside an adiabatic cavity filled with nanofluid.

Boulahia et al., (2017b) simulate the natural convection heat transfer in a wavy walled enclosure filled with copper-water nanofluid. The numerical results indicate that heat transfer can be augment by reduces the amplitude of the wavy surface, increasing undulations number, Rayleigh number and nanoparticle volume fraction. Aminossadati and Ghasemi, (2011) studied numerically the natural convection of water–CuO nanofluid with volume concentrations up to 4% in a cavity with pairs of two heat source–sink. The results show that regardless of the position of the pairs of source–sink, the Rayleigh number and the solid volume fraction enhance the rate heat transfer. Selimefendigil and Öztop (2014) studied numerically the mixed convection in an enclosure with inner cylinder in the presence of heat generation to illustrate the effect of nanoparticle shapes of SiO_2 on heat transfer rate. They found that cylindrical shapes gives better heat transfer. Öğüt, (2009) investigated the effect of various nanofluid types such as Cu, Ag, Al_2O_3, and TiO_2 on heat transfer enhancement. The results indicate that Cu and Ag enhance the heat transfer rate because of their higher thermal conductivity in a comparison with the other types. The effect of magnetics field on natural convection around body located inside enclosure filled with nanofluid has been reported in Sheikholeslami, Gorji-Bandpay, et al., (2012); Sheikholeslami et al., (2016); Sheikholeslami, Gorji-Bandpy, et al., (2012). Kumar et al. (2010) carried out in numerical study of fluid flow and thermal field in cavity filled with nanofluid using single phase thermal dispersion model. It was noted that as the solid volume fraction increases, the average Nusselt number will increase. Aminossadati Aminossadati and Ghasemi, (2009) obtained that the reduction of the highest temperature is provided by Cu among all the nanoparticles (i.e. 42.80) while the lowest reduction in temperature is provided by TiO_2 (i.e. 2.59%). Boulahia et al., (2018) performed a numerical investigation for mixed convection heat transfer in a vented square enclosure with circular cooled obstacle. Three different exit port locations on the enclosure right wall are presented. The numerical results indicate that the bottom location of the exit port is better for heat transfer augmentation. Ali et al.) studied numerically the mixed convection between inner corrugated cylinder located inside rotating circular cylinder filled with nanofluid.

Moreover, the natural convection heat transfer in a parallelogramic cavity open from top and filled with Cu-water nanofluid are examined by Hussein and Mustafa, (2017). They noted that as the Rayleigh number rises, the average value of the Nusselt numbers will go up under diverse values of parallelogramic enclosure inclination angle and vertical locations of cylinder having the circle shape.

However, the case of natural convection heat transfer in a parallelogramic enclosure containing circular cylinder maintained at isothermal hot temperature using CuO-water nanofluid has not been investigated in full-detail. The current study presents the computational investigation of the natural convection heat transfer and fluid flow from

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a heated circular cylinder inserted in a parallelogramic enclosure cooled from the two vertical sides and adiabatically insulated at the top and the bottom walls with different inclination angle $0 \le \alpha \le 30$, vertical location $-0.2 \le H \le +0.2$, and Rayleigh number $10^3 \le Ra \le 10^6$ and, using different Nano fluid concertation(0% to 6%). The finite element techniques will be used in the present study to simulate the natural convection fluid flow and solving the dimensionless governing equations of continuity, energy and momentum of the fluid.

2. MATERIAL AND METHODS

2.1 Model Description

The model configuration of the parallelogram enclosure with inner circular cylinder along with the thermal boundary conditions is presented in Fig.1. The parallelogram enclosure main dimensions like its height and width are denoted W because each side has the same length. The circular cylinder of radius R = 0.2 is located inside a paralelogramic enclosure and the space between them is filled with CuO-water nanofluid. The cylinder is moving vertically upward and downward. The dimensionless vertical location of the cylinder is changes from H = −0.2 to H = +0.2.The cylinder is maintain at isothermal high temperature while the two verticals sidewalls of the paralleogramic enclosure are kept at cooled isothermal temperature. The top and bottom walls are assumed adiabatic. The fluid flow field and the characteristics of the heat transfer are considered under steady state conditions, laminar in nature and incompressible. No slip conditions are applied to each wall of the paralleogramic enclosure as well as the circular cylinder. There is no internal heat generation. The thermophysical properties of the nanofluids are considered constant except in the density in the body force term of Y-Momentum Navier-Stokes equations which is treated based on the Boussinesq approximation.

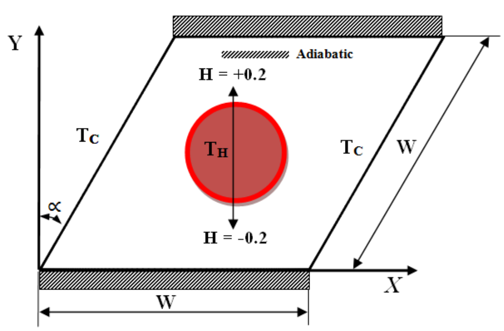

Fig. 1 Schematic Diagram of the parallelogramic enclosure

2.2 Governing Equations

The governing equations of two-dimensional parallelogramic enclosure filled with nanofluid containing circular cylinder can be describe by the governing equations of heat and fluid flow. The dimensionless governing equations can be written in terms of the dimensionless parameters :

$$X = \frac{x}{L}; Y = \frac{y}{L}; U = \frac{uL}{\alpha_f}; V = \frac{vL}{\alpha_f}; P = \frac{pL^2}{\rho_f \alpha_f^{\,2}}; \theta = \frac{T - T_c}{\Lambda T}; \tag{1}$$

$$\alpha_{nf} = \frac{k_{nf}}{\rho_{nf} cp_{nf}}; \Pr = \frac{\nu_f}{\alpha_f}; Ra = \frac{g\,\beta_f\,(\nabla T)L^3}{\alpha_f \nu_f}$$

The steady state continuity, momentum and energy equations for non-dimensional laminar natural convection fluid flow and heat transfer with the Boussinesq approximation in y-direction are as following Mahmoodi and Sebdani, (2012):

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{3}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) + \frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f} Ra \Pr \theta \tag{4}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{5}$$

2.3 Thermophysical Properties of Nanofluid

The thermos-physical properties like the density, heat capacity and coefficient of the thermal expansion, of the nanofluid can be calculated as follow, respectively. Those formulas has been used to calculation of the thermophysical properties of nanofluid in numerical simulation of natural convection in recently published articles Khanafer et al., (2003); Jou and Tzeng, (2006); Oztop and Abu-Nada, (2008); Aminossadati and Ghasemi, (2009); Ghasemi and Aminossadati, (2010); Das et al., (2017a)

$$\rho_{nf} = (1-\phi)\rho_{bf} + \phi\rho_p \tag{6}$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_{bf} + \phi(\rho C_p)_p \tag{7}$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_{bf} + \phi(\rho\beta)_p \tag{8}$$

The thermal conductivity of the nanofluid for nanoparticles can be calculated according to the Maxwell formula

$$k_{nf} = \frac{(k_p + 2k_{bf}) - 2\phi(k_{bf} - k_p)}{(k_{bf} + 2k_p) + \phi(k_{bf} + k_p)} k_{bf} \tag{9}$$

To estimation of the effective dynamic viscosity of the nanofluid the Brinkman model is employed.

$$\mu_{nf} = \frac{\mu_{bf}}{(1-\phi)^{2.5}} \tag{10}$$

Table 1 Thermo-physical properties of base fluid (pure water) and (Cu) nanoparticles Oztop and Abu-Nada, (2008).

Properties	C_p(J/kg k)	ρ (kg/m^3)	k (W/m. k)	β(1/k)	μ (kg/m.s)
Copper (Cu)	385	8933	401	1.67×10^{-5}	-
Pure water	4179	997.1	0.613	21×10^{-5}	0.000372

The motion of the fluid is presented by the stream function Ψ obtained from the two components of velocity U and V. The relationships between velocity components and the stream function may be written as follow:

$$U = \frac{\partial \Psi}{\partial Y}; V = -\frac{\partial \Psi}{\partial X} \tag{11}$$

which is a single formula

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = \frac{\partial \Psi}{\partial Y} + \frac{\partial \Psi}{\partial X} \tag{12}$$

3- NUMERICAL COMPUTATION AND GRID-INDEPENDENCE STUDY

The set of governing and constitutive equations presented in Section (2.2) were solved using the commercial finite element method The huge fulfillment of the finite element technique, wherein it's far almost completely applied, inside the research area of mechanics of solids has encouraged dense research on its software within the area of Computational Fluid Dynamics (CFD). The FEM some clear advantages that recommend it as an effective alternative numerical technique for ocean models. The most essential benefit of FEM is that it

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lets in the usage of unstructured grids with variable resolution. In the present work, CFD code which is finite element method is utilized. We consider laminar fluid flow for equations (3-4) and heat transfer in fluid for equation (5). In order to assure the numerical solutions stability, P2-P1 Lagrange elements and the Galerkin least squares technique are utilized.

The mesh is considered finer near to the enclosure walls and the heated circular cylinder to resolve the hydrodynamic and thermal boundary layer high gradients. Independence study of mesh is done to find out an optimal grid distribution with accurate results and minimal computational time. The grid distribution for the considered problem is shown in Fig. 2 for the full computational domain and near the cylinder surface at Ra = 10^6, φ = 0.06, α = 15º. Five different grid sizes are tested and the convergence in the length-averaged Nusselt number. To validate the numerical code, the Average Nusselt number through a square enclosure with a circular cylinder at different vertical locations are compared with the results of the numerical study done by Kim et al. Kim et al., (2008) as shown in Fig.3. The present surface-averaged Nusselt numbers are in good agreement with the values of (Kim et al., 2008) as in Table 2.

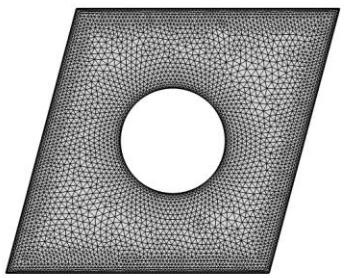
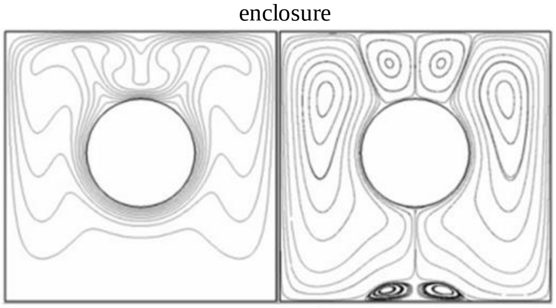
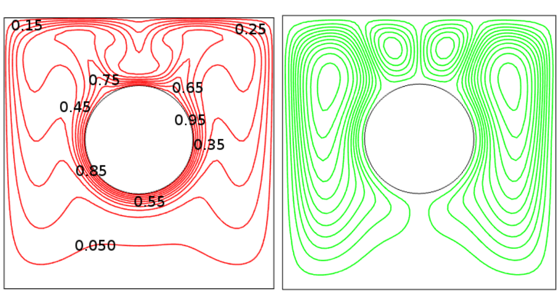


Fig. 2 Schematic diagram the mesh geometry of the parallelogramic enclosure



Kim et al., (2008)



Present Work

Fig. 3 Isothermals and streamlines for four different Rayleigh numbers of (a) 10^3, (b) 10^4, (c) 10^5 and (d) 10^6 (Kim et al., (2008).

Table 2 Validation of average Nusselt number of the present work with Kim et al., (2008)

Ra	Top wall (present study)	Top wall Kim et al., (2008)
	Nu_{ave}	Nu_{ave}
10^3	1.9761	1.99
10^4	2.3522	2.39
10^5	4.8690	4.9
10^6	10.130	10.150

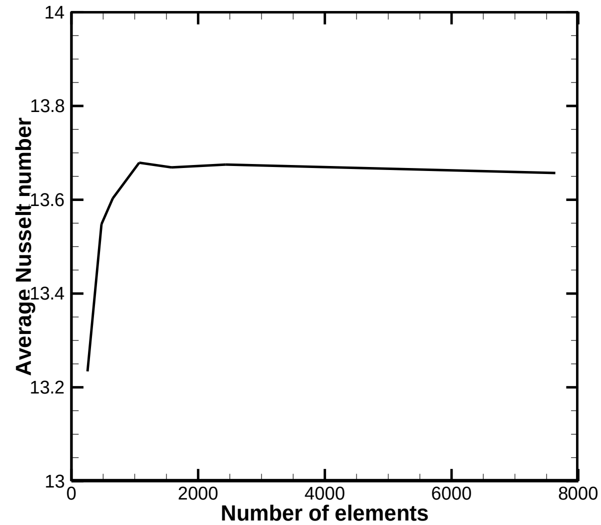


Fig. 4 Mesh independent study in terms of Average Nusselt number along the circular cylinder at Ra = 10^6, α = 0, H = 0

4. RESULTS AND DISCUSSION

The main purpose of the current investigation is to examine systematically the characteristics of flow and heat transfer in parallelogram enclosure with an inner circular cylinder using Cu-water Nano fluid.

4.1 The effect of geometrical parameters

In order to optimize the geometrical parameters of the parallelogram enclosure, different parametric are tested. The effect of inclination angle of the parallelogramic enclosure and vertical locations of the circular cylinder inside the cavity on the fluid flow and thermal field under various Rayleigh numbers are presented in this section.

4.1.1 The effect of inclination angles

Figs.5a-c illustrate the local Nusselt number behavior for the heated circular cylinder arc length at inclination angles [α=0, α=15 and α=30], respectively. It is noted that the local Nusselt number increases till the mid length of the circle for all cases. After the mid length, a reverse behavior of local Nusselt number is observed. Figs.6 illustrates the isotherms (left) and streamlines (right) for different inclination angle. The symmetrical property is obtained when the inclination angle is zero [i.e., enclosure is square], around the hot circular cylinder. But, as soon as the angle of the inclination increases from α=15 to α=30, various shapes of vortices for the stream function and isotherms can be observed. The rotating vortices fills almost the entire cavity appears around the cylinder. Finally, the effect of inclination angle at Ra = 10^6, H = 0 on the heat transfer enhancement is illustrated in Fig.7. Increasing inclination angle of the parallelogramic enclosure will improve the average Nusselt number of the hot circular cylinder which is as an indicator of enhancing the heat transfer rate. Thus, the inclination angle of α = 30 will be considered in the next sections to optimize the best geometrical parameters.

4.1.2 The effect of location of inner circular cylinder

The influence of vertical location for circular hot cylinder on heat transfer and fluid flow are discussed, at nanofluid concentration = 0.06, inclination angle α=30.

Fig.8 illustrate the local Nusselt numbers field around the circumference the hot cylinder for various vertical location at various Ra and α=30. It can be seen that as the circular cylinder is moving downwards vertically, the local Nusselt number is greater than their corresponding when the cylinder is moving upwards. The physical reason behind this is because of the fluid flow circulation which is presented in terms of maximum stream function increases when the cylinder is moving in the vertical direction downwards as illustrated in Fig.9.

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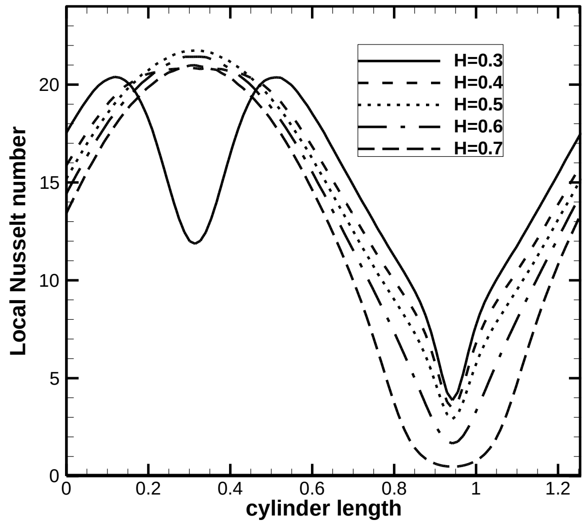
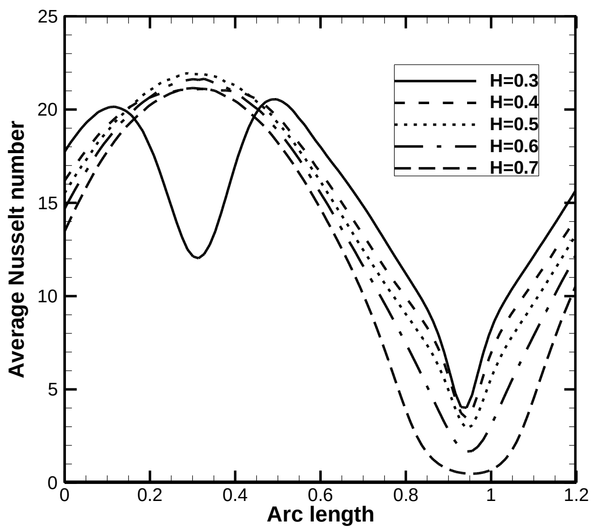
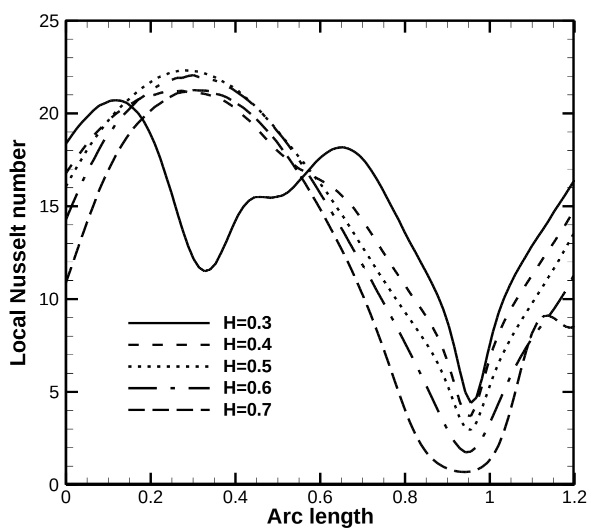


α=0 and Ra=10^6



α=15 and Ra=10^6



α=30 and Ra=10^6

Fig. 5 Local Nusselt number along the hot cylinder for various cylinder vertical location and different of inclination angles [α=0, α=15 and α=30], at Ra=10^6.

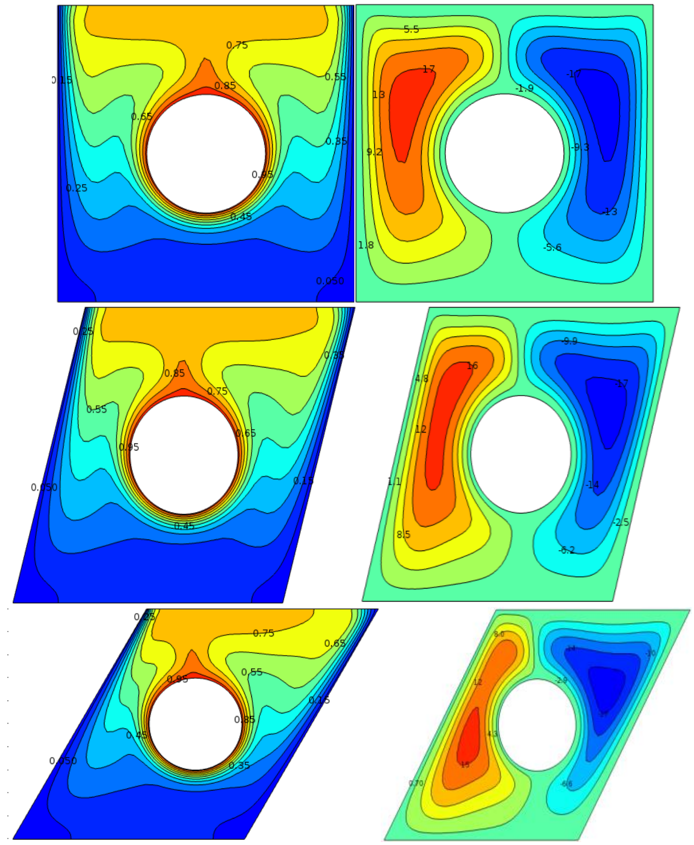


Fig. 6 Variations of isotherms and streamlines for different inclination angles at H =0 at a) α = 0 , at Ra=10^6 , b) α = 15 , at Ra=10^6 c) α = 30 , at Ra=10^6

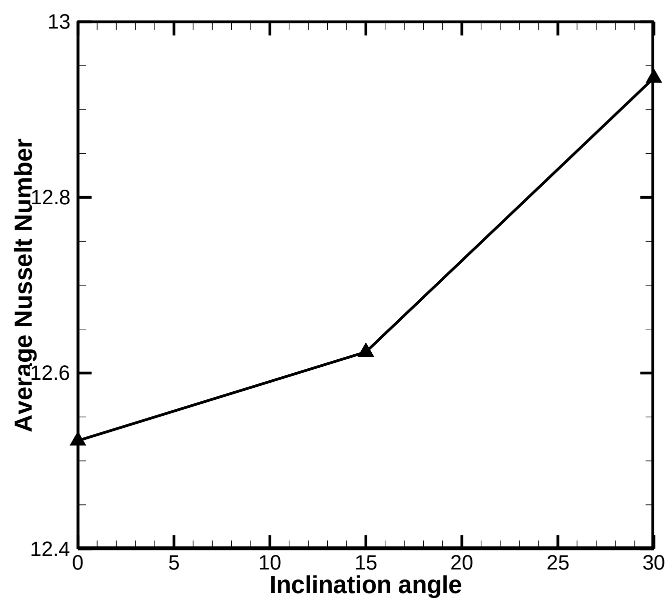


Fig. 7 The effect of inclination angle on the average Nusselt number at Ra = 10^6, H = 0

For the case in which the cylinder is at the center of the paralelogramic enclosure at low Rayleigh number (i.e., [Ra = 10^4 and H = 0]), the strength of the fluid flow circulation will rise from ψ = 0.86398 into ψ = 1.0978 as the inclination angle will rise from α = 0 to α = 30. While at [Ra = 10^6 and H = 0], the strength of the circulation decreases from ψ = 15.893 into ψ = 14.718. Now for Ra = 10^4, when the cylinder transfer in the vertical positive direction, the size vortices start to increases in the region below the internal hot cylinder.

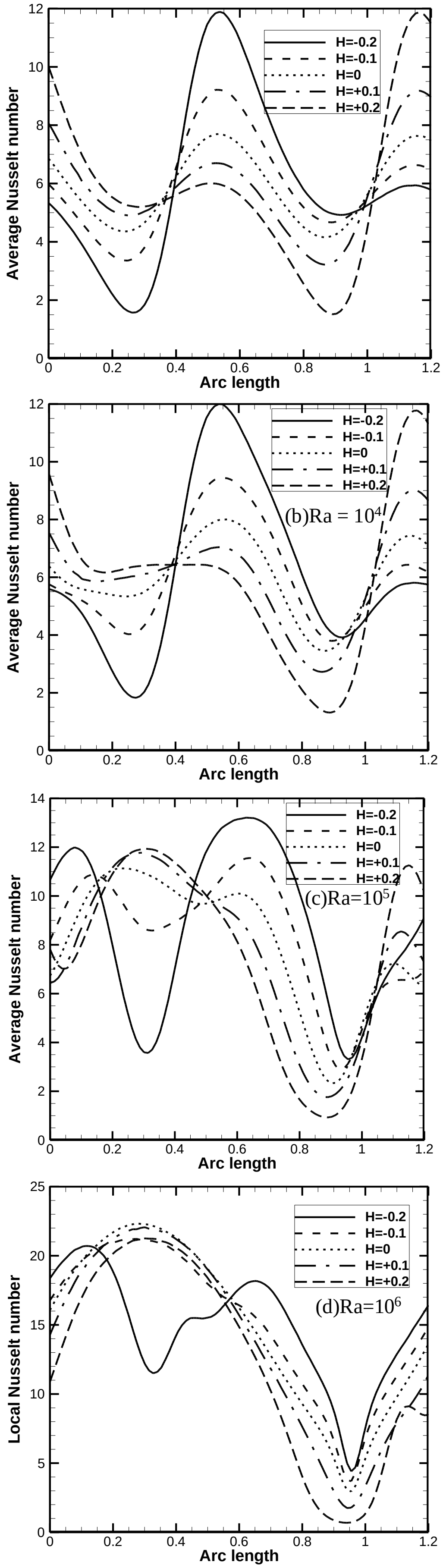


Fig. 8 Local Nusselt number along the hot cylinder for various cylinder vertical location for various Rayleigh number at α=30.

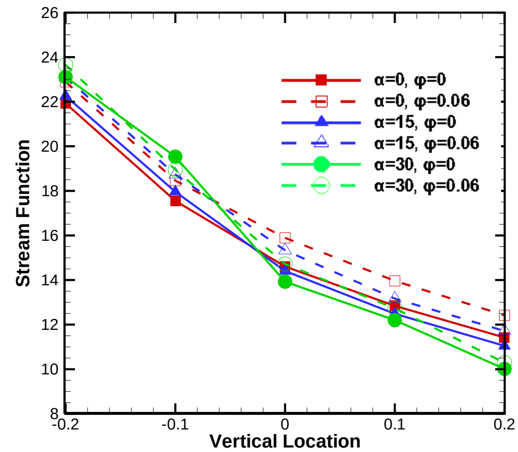


Fig. 9 Stream function versus the vertical location of the circular cylinder under various inclination angles and nanofluid concentrations

The physical reason behind that is due to the space between the cylinder and the upper wall of the enclosure will diminish which leads to push the vortices to spread below the inner cylinder. When α=0 and as the cylinder moves in the positive direction with an increasing in the vertical locations from [H =+0.1] to [H = +0.2], the strength of the fluid flow circulation will improve from Ψ = 0.91188 to Ψ = 0.98574. The same observation is obtained when the inclination angle α=15. For example the stream function increases from ψ=1.0792 into ψ=1.1103. While the behavior is inversed completely for α=30. It is obtained that the stream function is decreases from Ψ = 1.1538 into Ψ = 1.1332 as the vertical location increases from [H =+0.1] to [H = +0.2]. The maximum strength is obtained at H=+0.1 for α=30 while the minimum stream function for the fluid flow filed is obtained for H=+0.1 at α=0. At [Ra=10^6] and for various inclination angles [i.e., α=0, α=15 and α=30], it is obtained that the flow strength is reduced for all inclination angles when the circular cylinder move upward [H=+0.1] to [+H=0.2]. In this case, the maximum flow circulation strength can be obtained for α=0 at H=+0.1 which is ψ=13.964. While the minimum is for α=30 at H=+0.2 which is ψ=10.369. It is important to examine that the symmetric property of shapes vortices is seen for α=0, while it is gradually diminished as the inclination angles increases to α=15 and α=30.

From the other hand, for Ra = 10^4, when the cylinder moves vertically downward, the vortices size start to increases in the region above the inner hot cylinder. The physical reason behind that is due to the space between the cylinder and the lower wall of the enclosure will decrease leading to push the vortices to extend above the inner cylinder. It may be noted that the angle of inclination has significant effect on the shapes of the flow circulations. For example, when α=0, the shapes of streamlines are identical. While increasing inclination angle to α=15 and α=30 makes the shapes unsymmetrical. Now for Ra=10^4 when the circular cylinder moving downward vertically [i.e. H=-0.1 and H=-0.2], the strength of the flow circulations increases from Ψ = 1.1953 to Ψ = 1.5882 when the cylinder move from H=-0.1 to H=-0.2, respectively at α=0. The same observation is obtained for α=15 while the behavior is reverse for α=30 where the strength of the flow circulation decreases from Ψ = 1.0043 to ψ=0.65035 when circular cylinder move vertically from H=-0.1 to H=-0.2. The maximum strength of the flow is obtained at α=0 , H=-0.2 which it is Ψ = 1.5882 and the minimum is at α=30 , H=-0.2 which is Ψ = 0.65035. At Ra=10^6, the strength of the flow circulation increases for all the inclination angles [i.e., α=0, α=15 and α=30], when the cylinder moves vertically downward from H=-0.1 to H=-0.2. The maximum strength of the flow is obtained for α=30 , H=+0.2 which they are Ψ = 23.658 and the minimum is at α=30 , H=0 which is Ψ = 14.718. With respect to the effect of inclination angle and vertical location on isotherms contours is less than the effect of Rayleigh number, so that the next section will discuss the effect of Rayleigh number on heat transfer and fluid flow.

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4.2 Effect of Rayleigh number

Figs. 10 - 15 illustrate the isotherms (left) and streamlines (right) for different vertical location of circular cylinder (H), Rayleigh numbers and inclination angle. When the Rayleigh number is low, for example when Ra = 10^4, the circulation is weak as illustrated in Fig. 10, 12 and 14 for parallelogramic enclosure when the angle is 0, 15, 30, respectively. The physical reason behinds that is due to slight effect of the natural convection. Generally, the fluid flow behavior and heat transfer characteristics are presented by two major rotating vortices around the inner hot cylinder. The vortices which represented the currents of the natural convection are generated by temperature and density differences between the hot cylinder body and the cold walls of the parallelogramic enclosure. Regarding the isotherms contours, they seem like horizontal lines for low Rayleigh numbers (Ra = 10^4) because the conduction heat transfer mode is dominated. It is also noted that the thermal boundary layer thickness is very slight for low Rayleigh numbers under various vertical location and inclination angle. The maximum stream function is for parallemogtramic enclosure with α=0 and H=-0.2, Ψ = 1.4 . As Rayleigh numbers jumped into Ra = 10^6, both streamlines and isotherms contours completely changes from the uniform shapes into non-uniform shapes as demonstrated in Figs. 11, 13, 15. In this case, the convection effect becomes very strong and the thermal boundary layer thickness increases. The stream function values increase highly in a comparison with their corresponding for low Rayleigh numbers. It is found that the maximum stream function is Ψ = 24 for α=30 and H=-0.2. In the last of this discussion it is noted that the Rayleigh number plays an important role in the heat transfer enhancement and in the controlling of the shapes of both streamlines and isotherms.

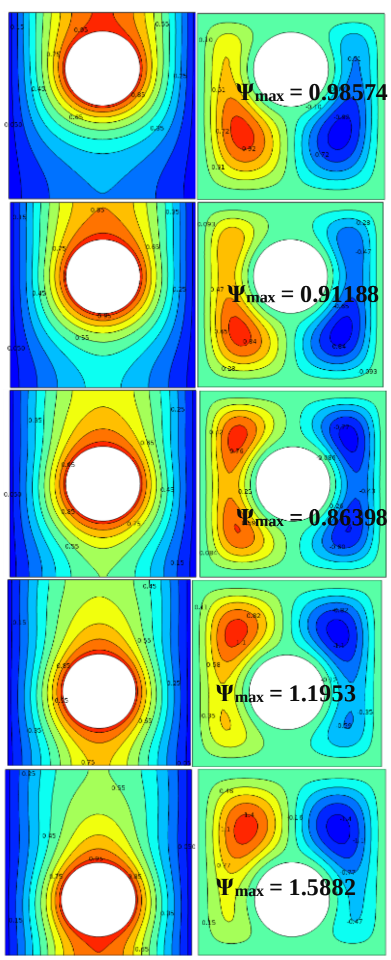

Fig. 10 Variations of isotherms and streamlines for different H = +0.2, +0.1, 0, -0.1, -0.2, respectively at Ra=10^4

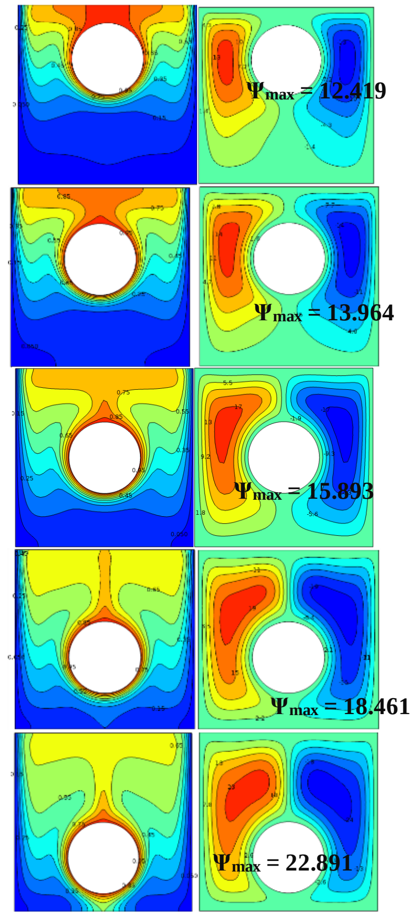

Fig. 11 Variations of isotherms and streamlines for different H = =+0.2,+0.1,0.-0.1,-0.2, at Ra=10^6

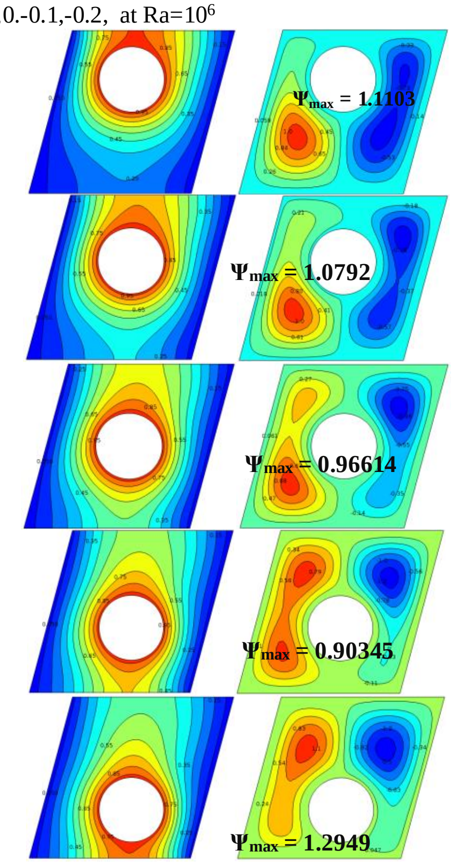

Fig. 12 Variations of isotherms and streamlines for different H =+0.2,+0.1,0.-0.1,-0.2, at Ra=10^4

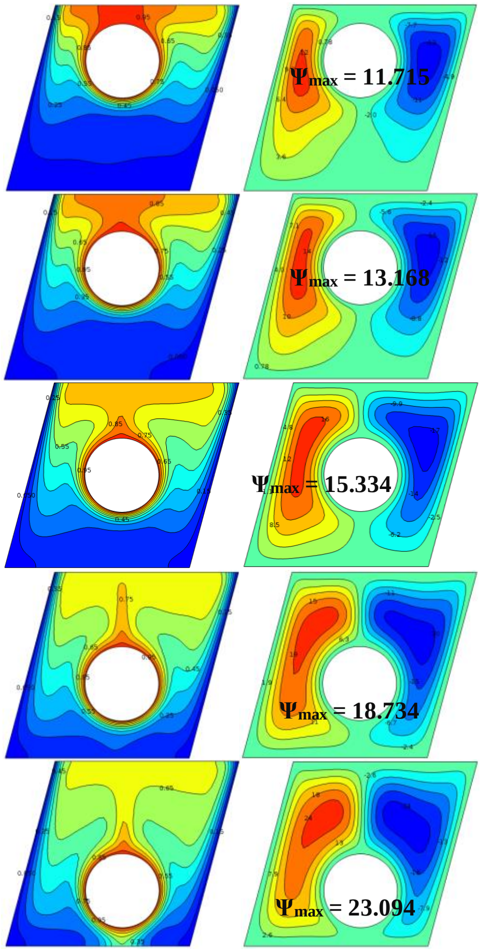

Fig. 13 Variations of isotherms and streamlines for different H = =+0.2,+0.1,0.-0.1,-0.2, at Ra=10^6

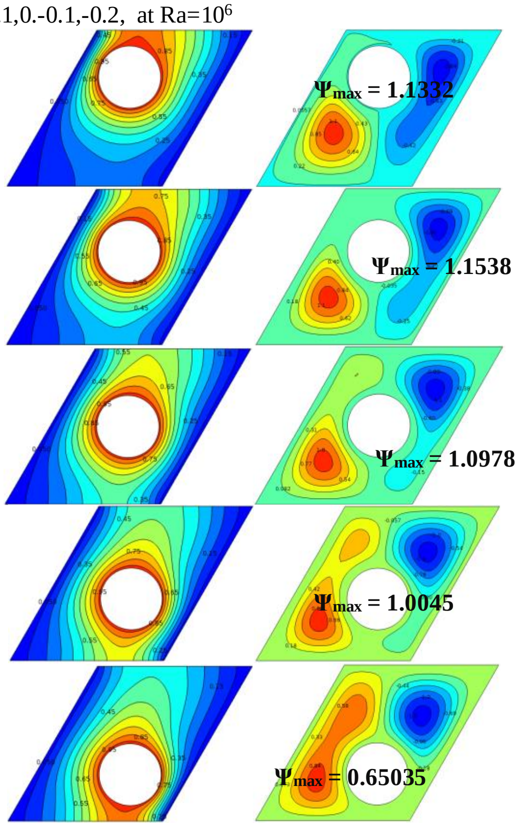

Fig. 14 Variations of isotherms and streamlines for different H = =+0.2,+0.1,0.-0.1,-0.2, at Ra=10^4

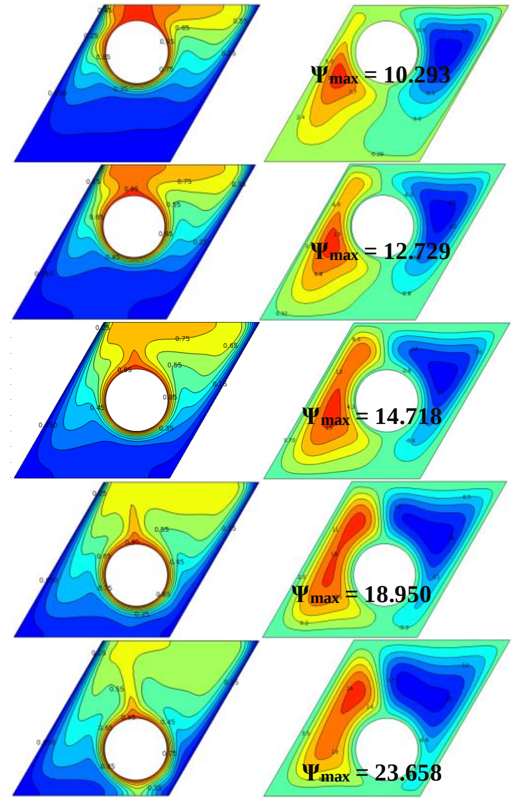

Fig. 15 Variations of isotherms and streamlines for different H = =+0.2,+0.1,0.-0.1,-0.2, at Ra=10^6

It can be noteced from Fig.16 that when the inclination angle is α=30 at H=-0.1, the heat transfer rate is better more than that of α=0 , α=15 for all values of Rayleigh numbers. So that we will consider the paralleogramic enclosure for α=30 when the vertical location is H=0.1 to investigate the effect of nanofluid concentration on the heat transfer enhanecment.

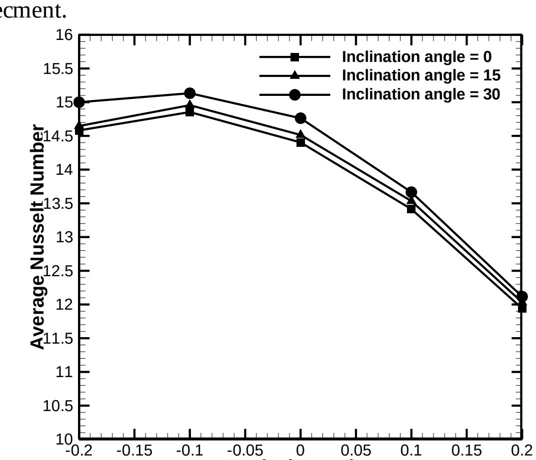

Fig. 16 Average Nusselt number of hot cylinder for various vertical location of cylinder under various inclination angles at Ra=10^6

4.3 Effect of solid volume fraction of nanoparticle

Figs.17-19 demonstrate the streamlines Ψ and isotherms θ for different nanoparticle concentrations when the inclination angle is 0, 15, 30 at Ra = 10^6. The streamlines and isotherms for base fluid (water only) and nanofluid (Cu-water) are presented in two different colors to illustrate the effect of nanofluid on the fluid flow strength and heat transfer rate. The red color is for the base fluid while the green is for the nanofluid. It can be noted that as the nanofluid is used, the fluid flow strength will increase for various inclinations angles. The reason behind this, is that as the thermal energy will increases leading to increase the velocity of

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the fluid and increase the stream function. For example, when the nanofluid volume fraction increases from $\varphi = 0$ into 0.06, the stream function increases from $\Psi = 21.926$ into $\Psi = 22.891$ for paralleogramic enclosure at α=0. The same beahviour is observed for all inclination angles. The nanofluid also increases the local nusselt number for various Rayleigh number as showed in Fig. 20. Fig. 21 depicts the effect of inclination angles and vertical location on the stream function for base fluid (water only) and nanofluid. It may be noted that as the cylinder moves vertically upward, the fluid flow strength will decrease for all inclination angles. On the other hand, solid volume fraction addition will increase the stream function more than that for the base fluid. This is due to increases in the kinematic energy and the thermal transport when the nanoparticle is used. However it is found that when the cylinder is located at a distance H=-0.1 from the center of the parallelogram with nanoparticle will be the optimum in the enhancing of heat transfer. Fig. 21 shows the effect of the solid volume fraction on the heat transfer enhancement in terms of average nusselt number for various inclination angles and Rayleigh number at H=-0.2. As illustrated before, as the Rayleigh number increases, the convection heat transfer will be dominated leading to increase the average nusselt number. It is obtained also, that the effect of nanoparticle will increase the Nusselt number which enhances the heat transfer rate.

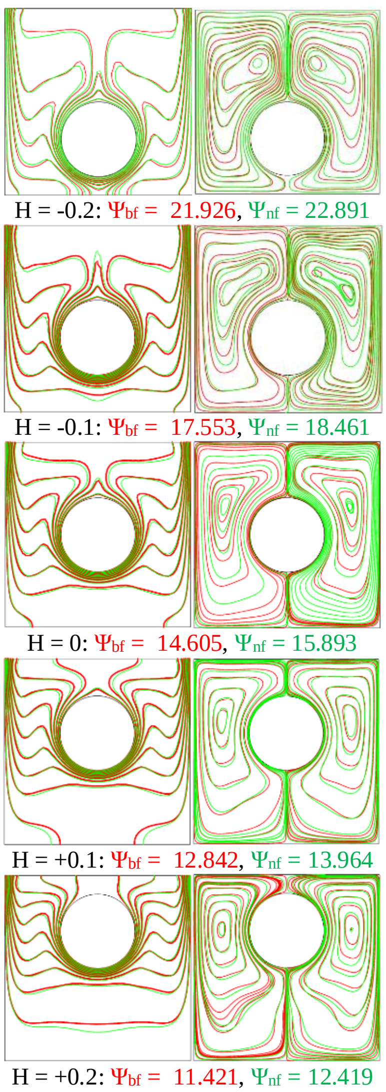

Fig. 17 isotherms (left) and streamlines (right) for paralleogramic enclosure at $\alpha = 0$, different vertical wall locations of circular cylinder at Ra = 10^6.

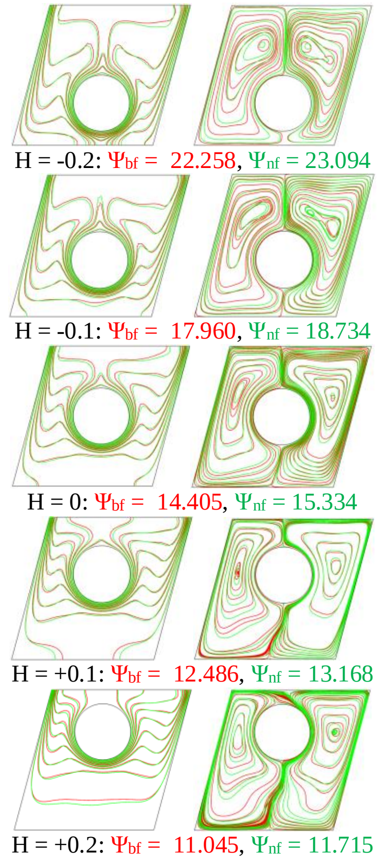

Fig. 18 isotherms (left) and streamlines (right) for paralleogramic enclosure at $\alpha = 15$, different vertical wall locations of circular cylinder at Ra = 10^6.

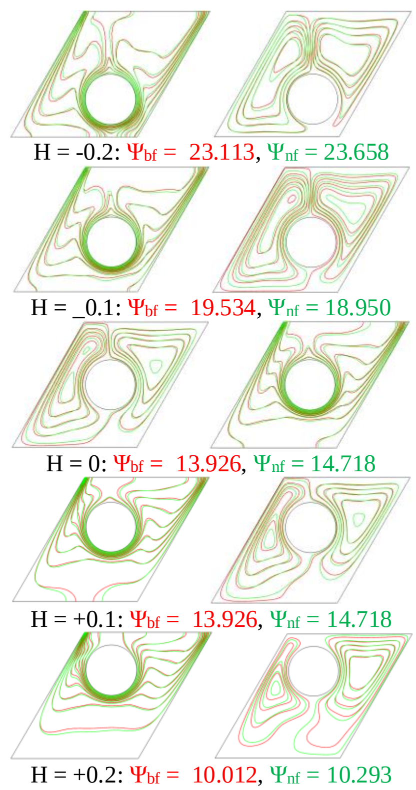

Fig. 19 isotherms (left) and streamlines (right) for paralleogramic enclosure at $\alpha = 30$, different vertical wall locations of circular cylinder at Ra = 10^6.

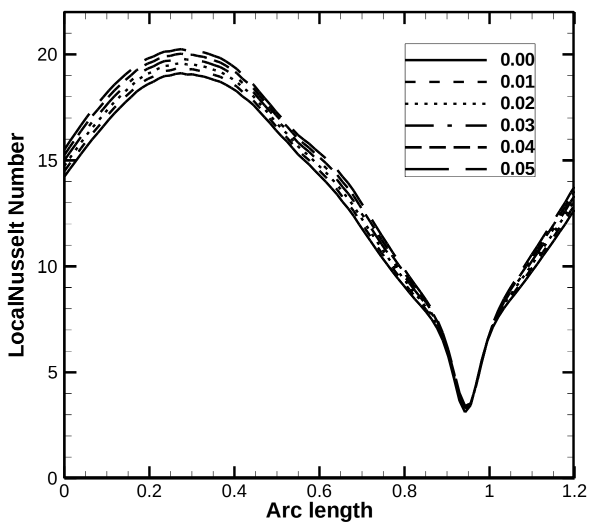

Fig. 20 The effect of nanofluid concentration on the local nusselt number profile along the circular cylinder arc at various Rayleigh numbers.

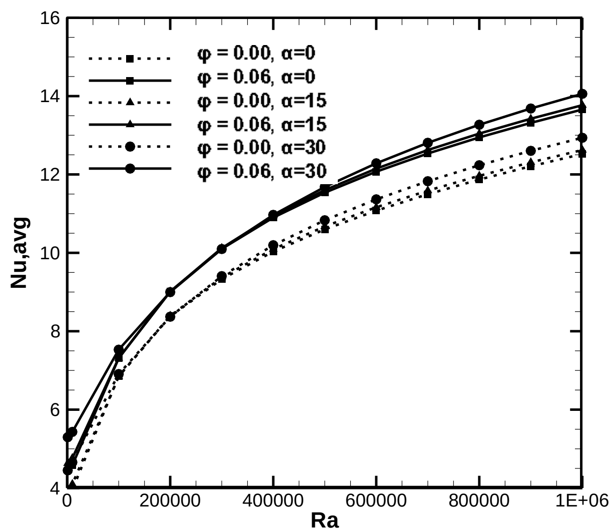

Fig. 21 avegare Nusselt number variation with Rayleigh number for different nanofluid concentration and inclination angle

5. CONCLUSIONS

Numerical investigations to study the fluid flow and heat transfer characteristics in a parallelogramic enclosure containg a hot circular cylinder using nanofluids for natural convection flow were carried out using finite element method. The following findings can be sammarized:

1- As the Rayleigh number increases, both the fluid flow streangth which is presented in terms of maximum stream function and the isotherms line increase dramatically.
2- The heat transfer mechanisem will be transferred from conductive mode into convective at higher values of the Rayleigh number.
3- The inclination angle of the paralleogramic enclsoure influence highly on the fluid flow distribution. For example, when the inclination angle is zero, the vrtices are symmetrical around the hot cylinder. But when the angle of inclination increases, various shapes of vortices and isotherms line are obsreved.
4- In order to increases the heat transfer rate, it is recommended to move the cylinder vertically downwards until it reaches [H=-0.1] and the inclination angle of the paralleogamic enclosure is 30°.
5- The local nusselt number increases as the Rayligh number increases.
6- When the cylinder moves vertically downwards, its Nusselt number will be higher than that when the cylinder moves upwards. When the inlination angle increases, the nusselt number will increases leading to enhance the heat transfer rate.

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NOMENCLATURE

C_p	specific heat (J/kg·K)
g	Gravitational acceleration (m/s^2)
K	thermal conductivity (W/m·K)
W	Length of walls of the cavity
P	Dimensionless pressure
p	Pressure (Pa)
Pr	Prandtl number (ν_f/α_f)
Ra	Rayleigh number
T_C	Temperature of the cold surface (°K)
T_H	Temperature of the hot surface (°K)
Nu	Local Nusselt Number
$\overline{Nu}$	Average Nusselt Number
U	Dimensionless velocity component in x-direction
u	Velocity component in x-direction (m/s)
v	Velocity component in y-direction (m/s)
X	Dimensionless coordinate in horizontal direction
x	Cartesian coordinate in horizontal direction (m)
Y	Dimensionless coordinate in vertical direction
y	Cartesian coordinate in vertical direction (m)

Greek symbols

α	Inclination angle
θ	Dimensionless temperature(T-Tc/ΔT)
Ψ	Dimensional stream function (m^2/s)
ψ	Dimensionless stream function
μ	Dynamic viscosity (kg.s/m)
ν	Kinematic viscosity (μ/ρ)(Pa. s)
φ	Nanoparticle volume fraction (%)
ΔT	Ref. temperature difference
β	Volumetric coefficient of thermal expansion (K^{-1})
ρ	Density (kg/m^3)

Subscripts

c	cold
bf	base fluid
P	nanoparticle
nf	nanofluid

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