Frontiers in Heat and Mass Transfer (FHMT), 20, 30 (2023)
DOI: 10.5098/hmt.20.30

Global Digital Central
ISSN: 2151-8629

Frontiers in Heat and Mass Transfer

Available at www.ThermalFluidsCentral.org

HEAT TRANSFER ANALYSIS OF MHD CASSON FLUID FLOW BETWEEN TWO POROUS PLATES WITH DIFFERENT PERMEABILITY

V.S. Sampath Kumar, N.P. Pai†, B. Devaki

Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal, Karnataka, 576104, India.

ABSTRACT

In the present study, we consider Casson fluid flow between two porous plates with permeability criteria in the presence of heat transfer and magnetic effect. A proper set of similarity transformations simplify the Navier-Stokes equations to non-linear ODEs with boundary conditions. The homotopy perturbation method is an efficient and stable method which is used to get solutions. Further, the results obtained are compared with the solution computed through an effective and efficient finite difference approach. The purpose of this analysis is to study the four different cases arise viz: suction, injection, mixed suction and mixed injection in this problem, along with magnetic effect and heat transfer characteristics. The results obtained are used to analyse the fluid velocity and temperature profiles, and the skin friction at the upper and lower plate by using both methods are displayed in the form of figures and tables. The semi-analytical solution obtained is in good agreement with the numerical solution.

Keywords: *Navier-Stokes equations, Incompressible flow, Non-linear differential equations, Homotopy Perturbation method, Finite Difference Method.*

1. INTRODUCTION

The study of MHD flow and heat transfer through a porous medium has received attention due to its application in technological and engineering problems, including nuclear reactors, the oil industry, metallurgical processes, plasma studies, etc. Permeability is an important parameter used to characterize porous media. Flow and heat transfer in natural and manufactured porous materials have various applications in biochemical, industrial, and geophysical science. Darcy initiated the work in the nineteenth century. Berman (1953) investigated the laminar flow in a channel with porous walls. Amanifard *et al.* (2007) analysed the heat transfer in porous media. The study made by Khaled and Vafai (2003) showed the importance of porous media in modeling flow and heat transfer. Many authors have contributed to the study of porous media [Rees and Pop (2000); Badruddin *et al.* (2020); Pai *et al.* (2021)].

The problem of heat transfer in MHD flow between parallel plates was studied by Alpher (1961). Later Chandrasekhara and Rudraiah (1971) found a solution for MHD laminar flow between porous disks. Chaoyang and Chuanjing (1989) studied the problem of boundary layer flow and heat transfer of non-Newtonian fluids in porous media. Terril and Shrestha (1965) explored the flow through parallel and uniformly porous walls with different permeability. Bujurke *et al.* (1998) found the series solution for the same problem. Various authors have also contributed in the analysis of Casson fluid flow [Mukhopadhyay *et al.* (2013); Ghiasi and Saleh (2019); Kranthi Kumar *et al.* (2021); Dhange *et al.* (2022); Muhammad *et al.* (2023)]. Elkouh (1967) analysed the flow between two parallel porous disks for small suction-injection. Wilson and Schryer (1978) found a numerical solution for the flow between a stationary and revolving disc with suction at one disc for sufficiently large Reynolds numbers. Several other researchers have contributed significantly to the problems of hydrodynamic/ magneto-hydrodynamic and heat transfer analysis of fluid flows. In the present study we have investigated the problem of Casson fluid flow between two porous plates with different permeability with magnetic and heat transfer characteristics.

All these problems discussed in the above literature are highly nonlinear and difficult to solve. The homotopy perturbation method (HPM) is a powerful mathematical tool to solve various nonlinear problems. He (1999) introduced this method in the nineteenth century. The application of this method to solve heat transfer was studied by Ganji (2006). Besides its mathematical importance links to other branches of mathematics and it is widely used in all ramifications of modern sciences. This method does not need a small parameter or linearization, the solution procedure is straightforward, and only a few iterations lead to highly accurate solution which are valid for the whole solution in the domain. Further, the solution obtained through HPM is also validated by a stable, accurate and one of most efficient algorithms of finite difference method.

2. MATHEMATICAL FORMULATION

The steady incompressible MHD Casson fluid flow along a two-dimensional channel with uniformly porous walls with different velocities at the walls

† *Corresponding author. Email: nithyanand.pai@manipal.edu*

Frontiers in Heat and Mass Transfer (FHMT), 20, 30 (2023)
DOI: 10.5098/hmt.20.30 Global Digital Central
ISSN: 2151-8629

is considered. Let x and y coordinate axes are taken parallel and perpendicular to the channel walls, respectively (Fig.1). Let u and v are velocity components in x and y directions, respectively. At the wall $y = 0$, the

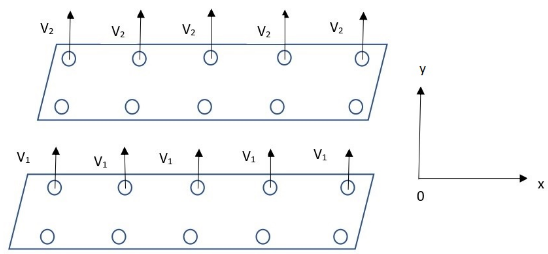


Fig. 1 Schematic diagram of the flow

velocity components $v = V_1$ and $v = V_2$ at the wall $y = h$, where h is the channel width. The Reynolds number $R_1 = (V_1 h)/\mu$ is defined for the cases $|V_1| \geq |V_2|$ and $R_2 = (V_2 h)/\mu$ for $|V_2| \geq |V_1|$ where μ is the viscosity. By assuming the flow to be steady, laminar and incompressible, and by taking the velocity components to be $u = u(x, y)$ and $v = v(x, y)$, the continuity, momentum, and energy equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\big(1+\frac{1}{\gamma}\big)\big(2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x \partial y}\big)-\frac{\sigma}{\rho}B_0^2 u \quad (2)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\big(1 + \frac{1}{\gamma}\big)\big(\frac{\partial^2 v}{\partial x^2} + 2\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x \partial y}\big) \quad (3)$$

and

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\Big[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\Big]. \quad (4)$$

Here ρ is the fluid density, p is the pressure , ν is the kinematic viscosity, γ is the Casson parameter, k is the thermal conductivity and C_p is the specific heat. We introduce a non-dimensional variable $\eta = \frac{y}{h}$. Then the equations (1-4) become

$$\frac{\partial u}{\partial x} + \frac{1}{h}\frac{\partial v}{\partial \eta} = 0 \quad (5)$$

$$u\frac{\partial u}{\partial x}+\frac{v}{h}\frac{\partial u}{\partial \eta} = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\big(1+\frac{1}{\gamma}\big)\big(2\frac{\partial^2 u}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 u}{\partial \eta^2}+\frac{1}{h}\frac{\partial^2 v}{\partial x \partial \eta}\big)-\frac{\sigma}{\rho}B_0^2 u \quad (6)$$

$$u\frac{\partial v}{\partial x} + \frac{v}{h}\frac{\partial v}{\partial \eta} = -\frac{1}{h\rho}\frac{\partial p}{\partial \eta}+\nu\big(1+\frac{1}{\gamma}\big)\big(\frac{\partial^2 v}{\partial x^2}+\frac{2}{h^2}\frac{\partial^2 v}{\partial \eta^2}+\frac{1}{h}\frac{\partial^2 u}{\partial x \partial \eta}\big) \quad (7)$$

and

$$u\frac{\partial T}{\partial x} + \frac{v}{h}\frac{\partial T}{\partial \eta} = \frac{k}{\rho C_p}\Big[\frac{\partial^2 T}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 T}{\partial \eta^2}\Big]. \quad (8)$$

The flow of fluid is through a two-dimensional channel having fluid sucked or injected with constant velocities V_1 and V_2 through its porous walls at $y = 0$ and $y = h$, respectively. The boundary conditions to be satisfied by the flow are

$$u(x, 0) = 0, u(x, h) = 0 \quad (9)$$

$$v(x, 0) = V_1, v(x, h) = V_2 \quad (10)$$

$$T = \begin{cases} T_1 & \text{for} \quad \eta = 0 \\ T_2 & \text{for} \quad \eta = 1. \end{cases}$$

For suction or injection flow, the problems to be solved for the case $|V_2| \geq |V_1|$ can be reduced to the case $|V_1| \leq |V_2|$. But for mixed flow, the case $|V_1| \geq |V_2|$ and $|V_2| \geq |V_1|$ are two different problems and need to be solved separately. The temperature of the lower and upper plates are T_1 and T_2 respectively. It is assumed that for this two-dimensional incompressible flow, there exists a stream function of the form

$$\xi(x, \eta) = \big[\frac{hU(0)}{\alpha_2} - V_2 x\big] f(\eta) \quad (11)$$

where $\alpha_2 = 1 - \frac{V_1}{V_2}$, for the case $|V_1| \geq |V_2|$.
The expressions for the velocity components are

$$u(x, h) = \big[\frac{V(0)}{\alpha_2} - \frac{V_2 x}{h}\big] f'(\eta) \quad (12)$$

$$v(\eta) = V_2 f(\eta). \quad (13)$$

Similarly, for the case $|V_2| \geq |V_1|$ there is a stream function of the form

$$\xi(x, \eta) = \big[\frac{hU(0)}{\alpha_1} - V_1 x\big] f(\eta) \quad (14)$$

where $\alpha_1 = \frac{V_2}{V_1} - 1$.

The above choice of stream function and velocity components reduce (6) and (7) to scalar equation.

$$\big(1 + \frac{1}{\gamma}\big) f''' + R_2(f'^2 - f f'') - M^2 f' = K_2 \quad (15)$$

where K_2 is a constant and f is a function of η.
The boundary conditions are

$$f(0) = 1 - \alpha_2, f'(0) = 0 \quad (16)$$

$$f(1) = 1, f'(1) = 0 \quad (17)$$

where $\alpha_2 = 1 - \frac{V_1}{V_2}$.
Similarly, for the case $|V_1| \geq |V_2|$, the reduced equation is

$$\big(1 + \frac{1}{\gamma}\big) f''' + R_1(f'^2 - f f'') - M^2 f' = K_1. \quad (18)$$

The boundary conditions are

$$f(0) = 1, f'(0) = 0 \quad (19)$$

$$f(1) = 1 + \alpha_1, f'(1) = 0 \quad (20)$$

where $\alpha_1 = \frac{V_2}{V_1} - 1$.

The boundary conditions (16) and (19) imply the suction case α_2 and α_1, must lie in the range $1 \leq \alpha_2 \leq 2$ and $-1 \geq \alpha_1 \geq -2$, whereas for the mixed cases they are in the range $0 \leq \alpha_2 \leq 1$ and $0 \geq \alpha_1 \geq -1$. For energy equation we use the transformation

$$\theta = \frac{T - T_1}{T_2 - T_1} \quad (21)$$

After transformation, the energy equation become

$$\theta''(\eta) + Pr R_2 f(\eta)\theta'(\eta) = 0 \quad (22)$$

with

$$\theta(0) = 0, \theta(1) = 1. \quad (23)$$

Differential equations of the type (15) and (18) are usually solved by direct integration which frequently involves more than one integration process, because of the two-point nature of the boundary conditions. Moreover, to confirm the validity of numerical results they are to be solved using other possible available methods. Thus, the use of a series solution provides an effective approach.

3. METHOD OF SOLUTION

We adopt two methods to solve the problem.
Method-I: Homotopy Perturbation Method:
HPM for the system of non-linear differential equations:
To describe the HPM solution for the system of non-linear differential equations, we consider

$$D_1[f(\eta)] - f_1(\eta) = 0 \tag{24}$$

$$D_2[\theta(\eta)] - f_2(\eta) = 0 \tag{25}$$

Where D_1, D_2 denote the operators, $f(\eta), \theta(\eta)$ are unknown functions, η denote the independent variable and f_1, f_2 are known functions. D_1, D_2 can be written as

$$D_1 = L_1 + N_1$$
$$D_2 = L_2 + N_2$$

where L_1 and L_2 are simple linear part, N_1 and N_2 are remaining part of the equation (24) and (25) respectively. The proper choice of L_1, L_2, N_1 and N_2 form the governing equations one can get the homotopy equations for equation(24) and equation(25) as follows

$$\begin{aligned} H_1(\phi_1(\eta, q; q)) = (1-q)[L_1(\phi_1, q) - L_1(v_0(\eta))] \\ +q[D_1(\phi_1, q) - f_1(\eta)] = 0 \end{aligned} \tag{26}$$

$$\begin{aligned} H_2(\phi_2(\eta, q; q)) = (1-q)[L_2(\phi_2, q) - L_2(v_0(\eta))] \\ +q[D_2(\phi_2, q) - f_2(\eta)] = 0 \end{aligned} \tag{27}$$

where $v_0(\eta)$ is the initial guess to the equations (24) and (25).
We assume the solution of equation (26) and equation (27) as follows

$$\phi_1(\eta, q) = \sum_{n=0}^{\infty} q^n f_n(\eta) \tag{28}$$

$$\phi_2(\eta, q) = \sum_{n=0}^{\infty} q^n \theta_n(\eta) \tag{29}$$

The solution to the considered problem is equation (28) and equation (29) at $q = 1$.

For mixed suction:

$$f_0 = 1 - \alpha_2(-1+\eta)^2(1+2\eta) \tag{30}$$

$$\begin{aligned} f_1 = \frac{-1}{140(1+\gamma)}\alpha_2(-1+\eta)^2\eta^2(7M^2(-1+2\eta) \\ +2R_2(35+\alpha_2(-19+5\eta-6\eta^2+4\eta^3)))\gamma \end{aligned} \tag{31}$$

$$\begin{aligned} f_2 = \frac{-1}{646800(1+\gamma)^2}\alpha_2(-1+\eta)^2\eta^2(77M^4(3+4\eta \\ -30\eta^2+20\eta^3)+154M^2R_2(-1-4\eta+4\eta^2) \\ +\alpha_2(10+99\eta-117\eta^2+48\eta^3-25\eta^4+10\eta^5) \\ +R_2^2(32340(-1+2\eta)+770\alpha_2(66-112\eta \\ +49\eta^2-42\eta^3+21\eta^4)+\alpha_2^2(-17719+28940\eta \\ -23731\eta^2+23390\eta^3-13265\eta^4+504\eta^5 \\ +1568\eta^6-448\eta^7)))\gamma^2 \end{aligned} \tag{32}$$

$$\theta_0 = \eta \tag{33}$$

$$\theta_1 = \frac{1}{20}PrR_2(-1+\eta)\eta(-10+\alpha_2(7-3\eta-3\eta^2+2\eta^3)) \tag{34}$$

$$\begin{aligned} \theta_2 = \frac{1}{25200(1+\gamma)}PrR_2(-1+\eta)\eta(PrR_2(1+\gamma) \\ (2100(-1+2\eta)-210\alpha_2(-16+35\eta-5\eta^2 \\ -20\eta^3+10\eta^4)+\alpha_2^2(-1252+3158\eta \\ -1042\eta^2-3247\eta^3+2675\eta^4+575\eta^5 \\ -1225\eta^6+350\eta^7))+\alpha_2\gamma(3M^2(-1 \\ -\eta-\eta^2+34\eta^3-50\eta^4+20\eta^5) \\ +2R_2(105(1+\eta+\eta^2-4\eta^3+2\eta^4) \\ +\alpha_2(-53-53\eta-53\eta^2+232\eta^3 \\ -155\eta^4+55\eta^5-35\eta^6+10\eta^7)))) \end{aligned} \tag{35}$$

For mixed injection:

$$f_0 = 1 + \alpha_1(3-2\eta)\eta^2 \tag{36}$$

$$\begin{aligned} f_1 = \frac{-1}{140(1+\gamma)}(-1+\eta)^2\eta^2(7M^2(-1+2\eta) \\ +2R_1(35+\alpha_1(16+5\eta-6\eta^2+4\eta^3)))\gamma \end{aligned} \tag{37}$$

$$\begin{aligned} f_2 = \frac{-1}{646800(1+\gamma)^2}\alpha_1(-1+\eta)^2\eta^2(77M^4(3+4\eta \\ -30\eta^2+20\eta^3)+154M^2R_1(35(-1 \\ -4\eta+4\eta^2)+\alpha_1(-25-41\eta+23\eta^2+48\eta^3 \\ -25\eta^4+10\eta^5))+R_1^2(32340(-1+2\eta) \\ +770\alpha_1(-18+56\eta+49\eta^2-42\eta^3 \\ +21\eta^4)+\alpha_1^2(761+7380\eta+13999\eta^2-8950\eta^3 \\ +2905\eta^4+504\eta^5+1568\eta^6-448\eta^7)))\gamma^2 \end{aligned} \tag{38}$$

$$\theta_0 = \eta \tag{39}$$

$$\theta_1 = \frac{1}{20}PrR_1\eta(-10(-1+\eta)+\alpha_1(3-5\eta^3+2\eta^4)) \tag{40}$$

$$\begin{aligned} \theta_2 = \frac{1}{25200(1+\gamma)}PrR_1\eta(PrR_1(1+\gamma)(-1+\eta) \\ 2100(-1+2\eta)-210\alpha_1(4-5\eta-5\eta^2-20\eta^3 \\ +10\eta^4+\alpha_1^2(8+8\eta+8\eta^2+953\eta^3+575\eta^4 \\ +575\eta^5-1225\eta^6+350\eta^7))+\alpha_1\gamma(3M^2(1 \\ -35\eta^3+84\eta^4-70\eta^5+20\eta^6)+2R_1(105(-1 \\ +5\eta^3-6\eta^4+2\eta^5)+\alpha_1(-52+240\eta^3 \\ -243\eta^4+90\eta^6-45\eta^7+10\eta^8)))) \end{aligned} \tag{41}$$

Method-II: Finite Difference Method:
Finite Difference Method (FDM) is one of the oldest methods used to solve differential equations that are difficult or impossible to solve analytically. The finite difference method is applied directly to the differential form of the governing equations. The principle is to employ a Taylor series expansion for the discretization of the derivatives of the flow variables.

4. RESULTS AND DISCUSSION

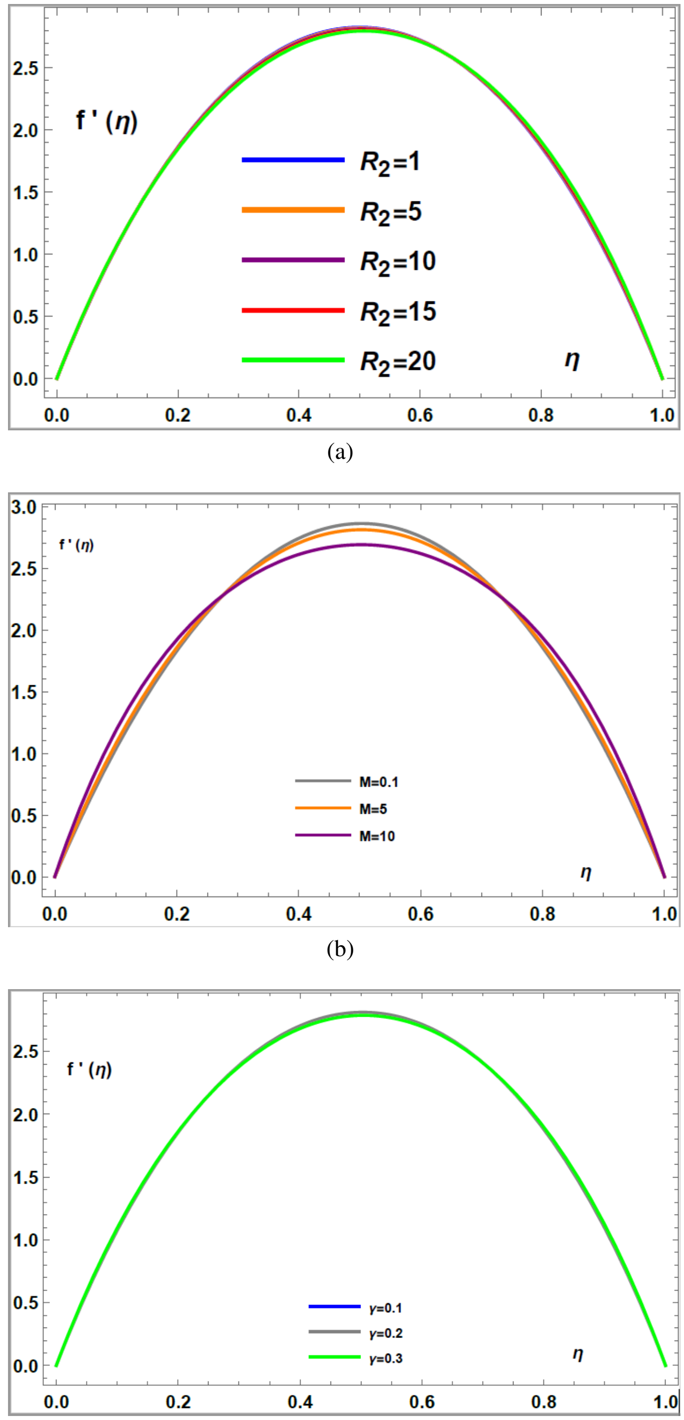
The result obtained for problems of Casson fluid flow between two plates with different permeability with magnetic effect is discussed in this section. The solution obtained through HPM and finite difference scheme used to compute velocity for different values of α_2, γ, M, R_2 and R_1, and results are displayed in Fig.2 - Fig.5 for four different cases.Figures in 2 show velocity profile ($f'(\eta)$) obtained for fixed values of α_2, γ, M and R_2 for the case of suction. It clearly indicates that the nature of flow is similar for different values taken out in Fig.2c (Velocity profile for $\alpha_2 = 1.916, \gamma = 0.2, R_2 = 10$), we find point of inflection at $\eta = 0.3$ and $\eta = 0.7$, when $M = 0.1, 5, 10$.

Figure.3 shows velocity calculated for fixed values of α_2, γ, M and different values of R_2 for injection at the wall. It is clear that for injection case also, a similar trend is observed. But in Fig.3c again a points of inflection are found at $\eta = 0.3$ and $\eta = 0.7$, when $M = 0.1, 5, 10$. Figure.4 represent the velocity profiles obtained for the case of mixed suction like the previous two cases. All figures in Fig.4 have points of inflection at $\eta = 0.5$ (for Fig.4a), $\eta = 0.6$ (for Fig.4b) and $\eta = 0.7$ (for Fig.4c). This indicates that in the case of mixed suction, there is a flow pattern changing for different values of α, γ, R_2 and M as shown. For mixed injection case, the velocity profiles in Fig.5 reveal that, entire flow pattern shows a reverse phenomena with point of inflection at $\eta = 0.5$ (Fig.5a), 0.6 (Fig.5b) and $\eta = 0.35$ and 0.8 (Fig.5c).

Figures in Fig.6 show temperature profile of obtained for fixed values of α_2, R_2, Pr, and M for different values of γ. Figures. 6a and 6b are similar in nature, but the results obtained in Fig 6c, have a point of inflection, where a flow pattern changes at $\eta = 0.5$. Figures in Fig.7 show results for injection case, $\theta(\eta)$ is increasing as η increases (Fig.7a , Fig.7b). But in Fig 7c, the temperature profile is parabolic in nature, and the point of inflection is at $\eta = 0.5$. Figures in Fig.8 show the variation in temperature profile for different values of Casson parameter (Fig 8a) and different values of Hartmann number (Fig 8b). There is no much difference in the profiles Fig.8a and Fig.8b. But Fig.8c shows a totally different temperature profile (for mixed suction case). Finally, the figures in Fig.9 (mixed injection case) show a similar trend as that of mixed suction for the values of γ, M, Pr, α_1 and R_1.

The results obtained through HPM for $f''(0)$ and $f''(1)$ for four different cases are tabulated in form of Tables (1-4). The skin friction wall $f''(0)$ and $f''(1)$ for suction case, (for fixed M, γ, and different R_2) is monotonically increasing in magnitude. The same trend is observed for injection. But, in the cases of mixed suction and mixed injection, $f''(0)$ values are decreasing in magnitude whereas $f''(1)$ values are increasing in magnitude for different R_2 and R_1 respectively. The results are compared with numerical solutions obtained through the independent method. They are in good agreement, and the same is shown in Tables (1-4).

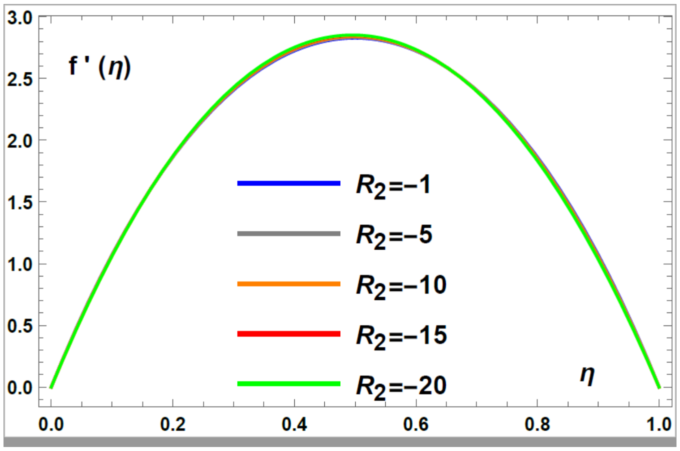
Tables (5-8) show the heat transfer rate at the walls ($\theta'(0), \theta'(1)$). The results are calculated for fixed M, Pr, γ and different R_2 for suction and injection cases. The values of $\theta'(0)$ and $\theta'(1)$ decrease in magnitude in case of suction, whereas they are increasing in case of injection at the walls. But the values of $\theta'(0)$ and $\theta'(1)$ (for fixed M, Pr, γ and for different R_2 and R_1) are increasing in magnitude and decreasing in magnitude respectively. The results are shown in Tables (5-8).Even if M values are high the results obtained are consistent and convergent. For this study we have taken R_1 values in the range $1 \le R_1 \le 20$, so also R_2. Inspite of the moderately large values taken for R_1, R_2 and M the results obtained are accurate and they converge. In all the cases the results obtained through HPM and numerical scheme are in good agreement. Further more the results are obtained in minimum time and storage of the computer.



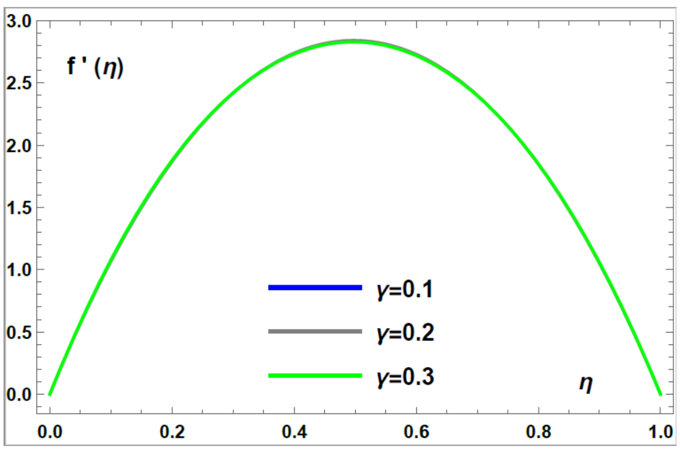
(a) (b) (c)

Fig. 2 Velocity profiles with $\alpha_2 = 1.916$: (a) for different values of R_2 when $\gamma = 0.2$ and $M = 5$, (b) for different values of M when $\gamma = 0.2$ and $R_2 = 10$, (c) for different values of γ when $M = 5$ and $R_2 = 10$.

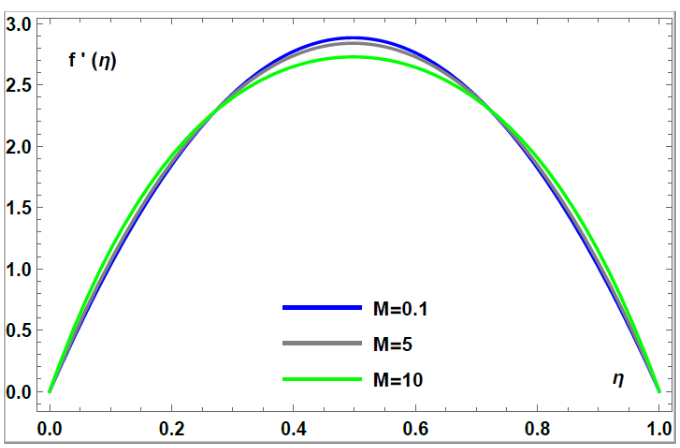
Frontiers in Heat and Mass Transfer (FHMT), 20, 30 (2023)
DOI: 10.5098/hmt.20.30
Global Digital Central
ISSN: 2151-8629



(a)

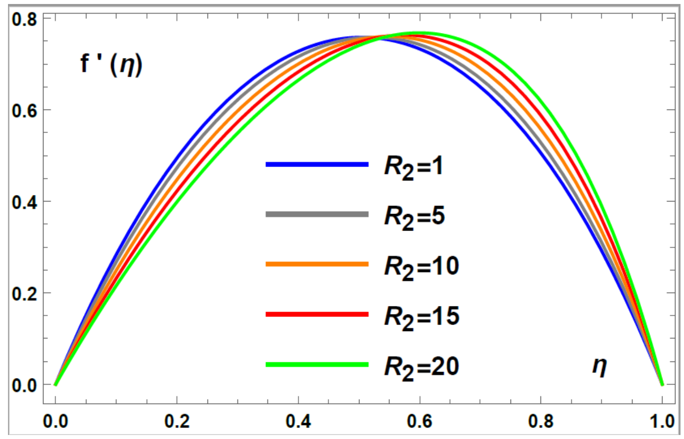


(b)

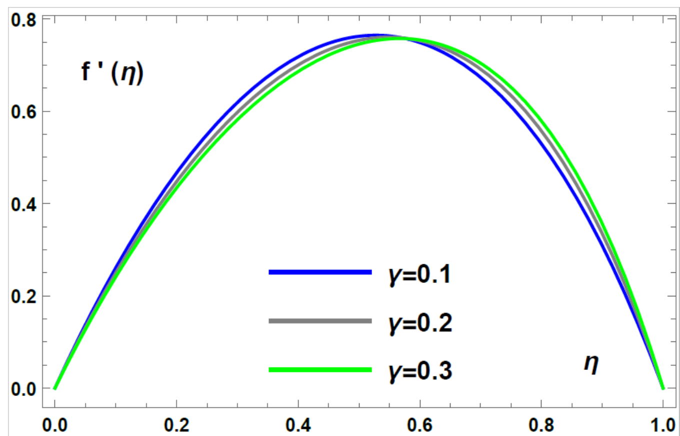


(c)

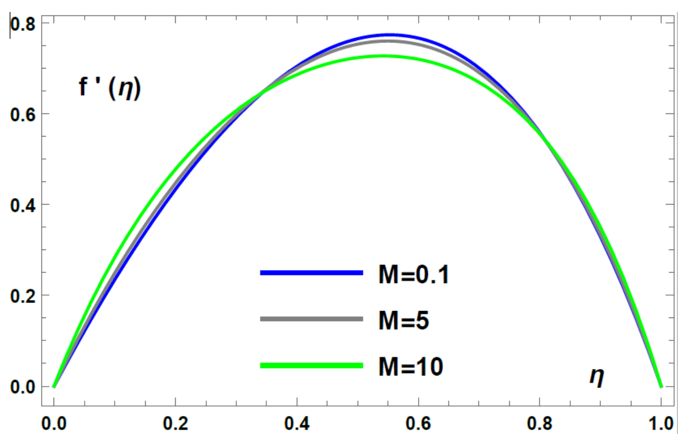
Fig. 3 Velocity profiles with $\alpha_2 = 1.916$: (a) for different values of R_2 when $\gamma = 0.2$ and $M = 5$, (b) for different values of M when $\gamma = 0.2$ and $R_2 = -10$, (c) for different values of γ when $M = 5$ and $R_2 = -10$.



(a)



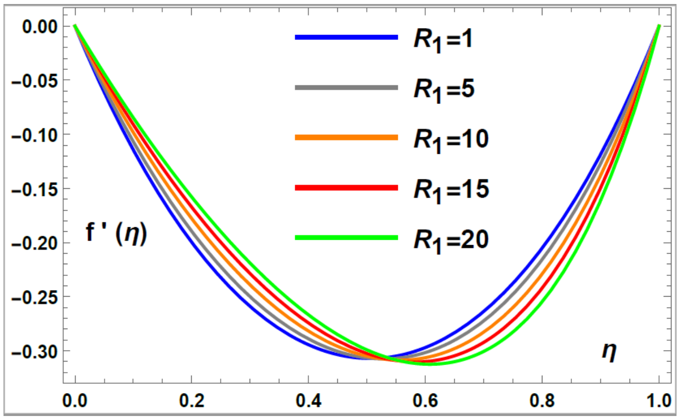
(b)



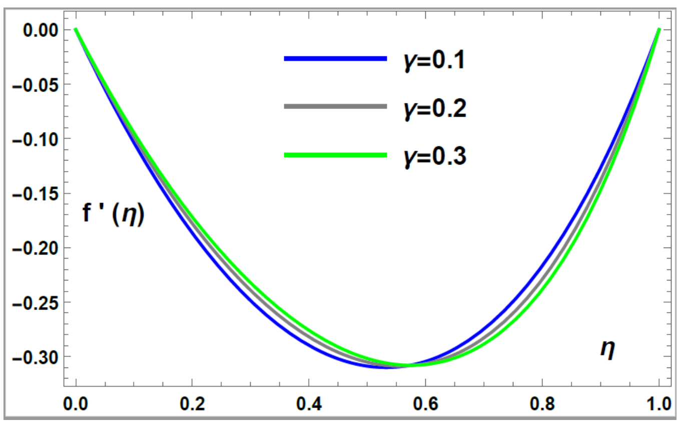
(c)

Fig. 4 Velocity profiles with $\alpha_2 = 0.51425$: (a) for different values of R_2 when $\gamma = 0.2$ and $M = 5$, (b) for different values of M when $\gamma = 0.2$ and $R_2 = 10$, (c) for different values of γ when $M = 5$ and $R_2 = 10$.

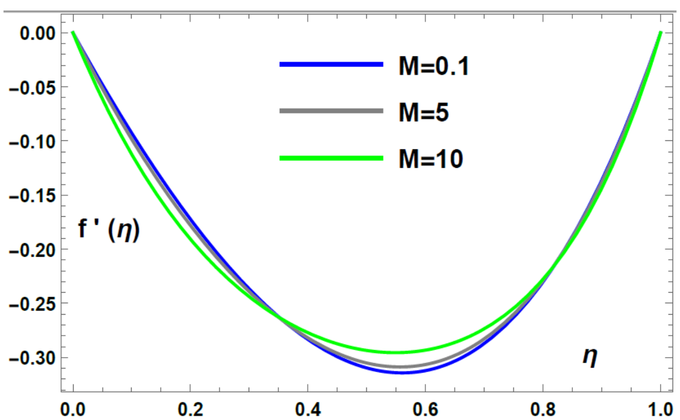
Frontiers in Heat and Mass Transfer (FHMT), 20, 30 (2023)
DOI: 10.5098/hmt.20.30
Global Digital Central
ISSN: 2151-8629



(a)

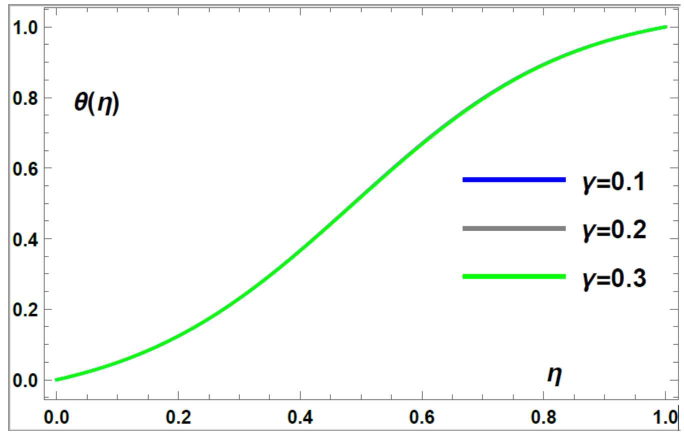


(b)

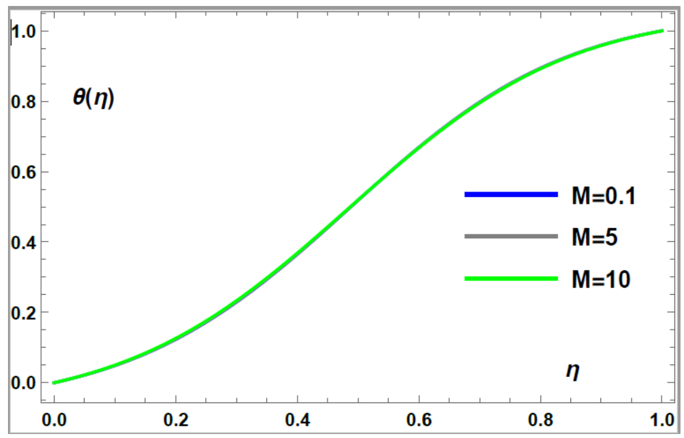


(c)

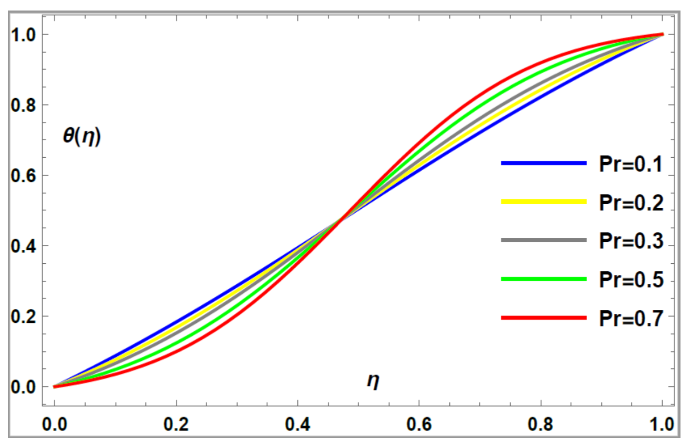
Fig. 5 Velocity profiles with $\alpha_1 = -0.20820$: (a) for different values of R_1 when $\gamma = 0.2$ and $M = 5$, (b) for different values of M when $\gamma = 0.2$ and $R_1 = 10$, (c) for different values of γ when $M = 5$ and $R_1 = 10$.



(a)



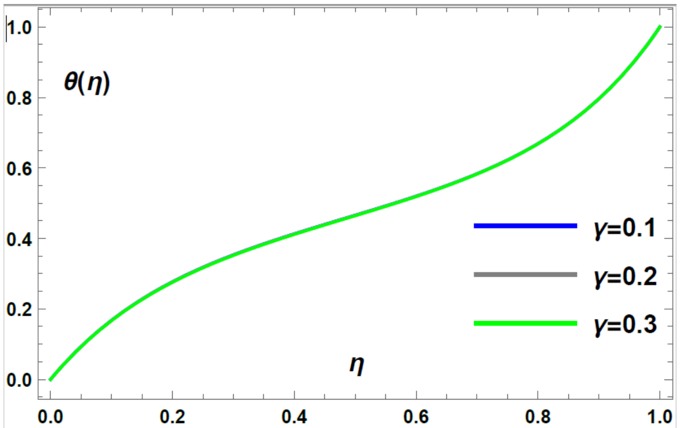
(b)



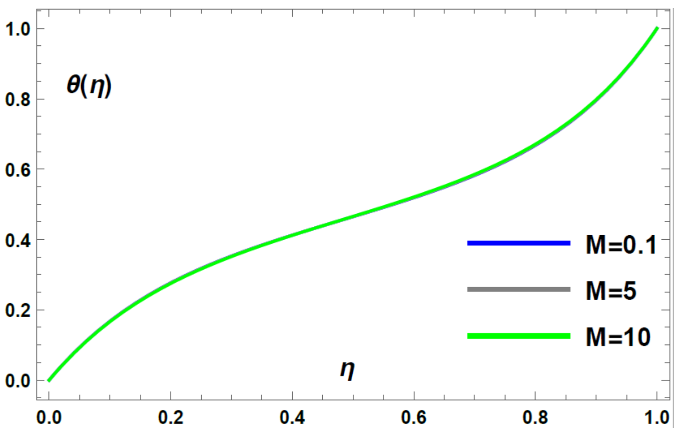
(c)

Fig. 6 Temperature profiles with $\alpha_2 = 1.916$: (a) for different values of γ when $R_2 = 10$, $Pr = 0.5$ and $M = 5$, (b) for different values of M when $R_2 = 10$, $Pr = 0.5$ and $\gamma = 0.2$, (c) for different values of Pr when $R_2 = 10$, $\gamma = 0.2$ and $M = 5$.

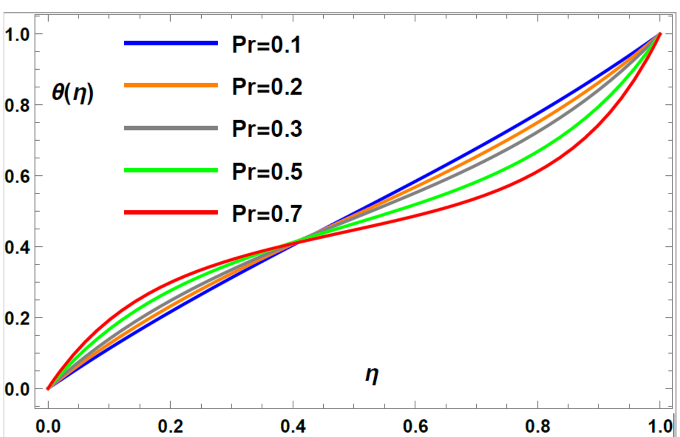
DOI: 10.5098/hmt.20.30
ISSN: 2151-8629



(a)

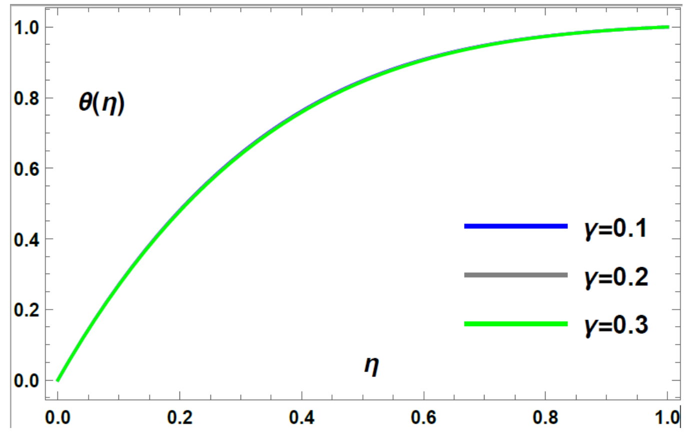


(b)

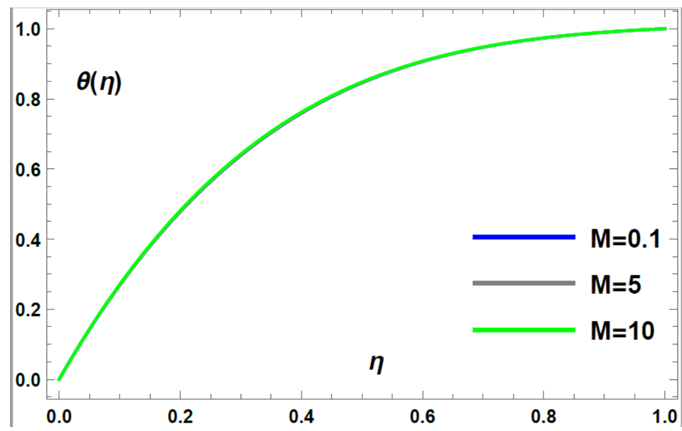


(c)

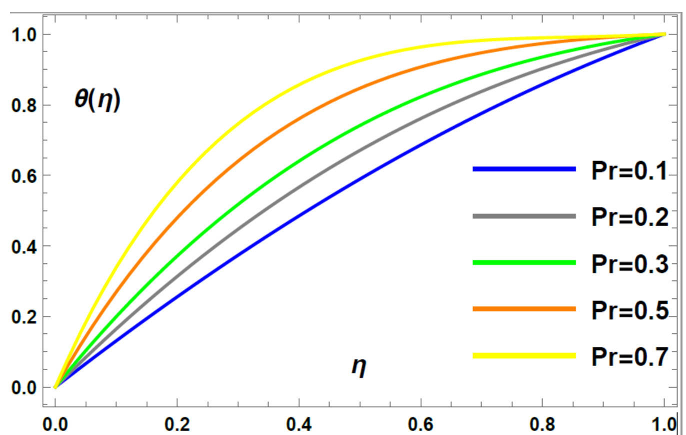
Fig. 7 Temperature profiles with $\alpha_2 = 1.916$: (a) for different values of γ when $R_2 = -10$, $Pr = 0.5$ and $M = 5$, (b) for different values of M when $R_2 = -10$, $Pr = 0.5$ and $\gamma = 0.2$, (c) for different values of Pr when $R_2 = -10$, $\gamma = 0.2$ and $M = 5$.



(a)

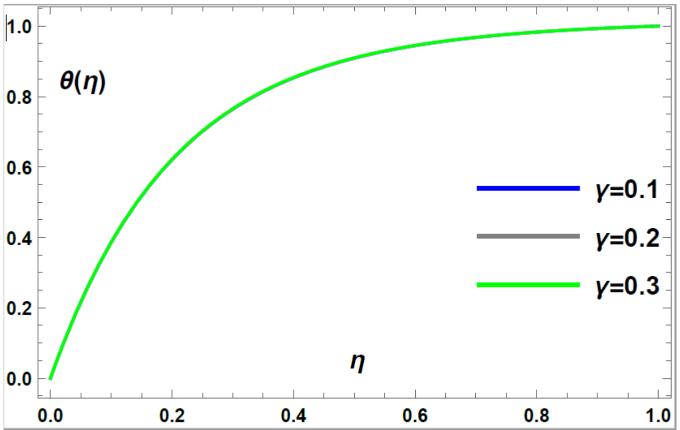


(b)

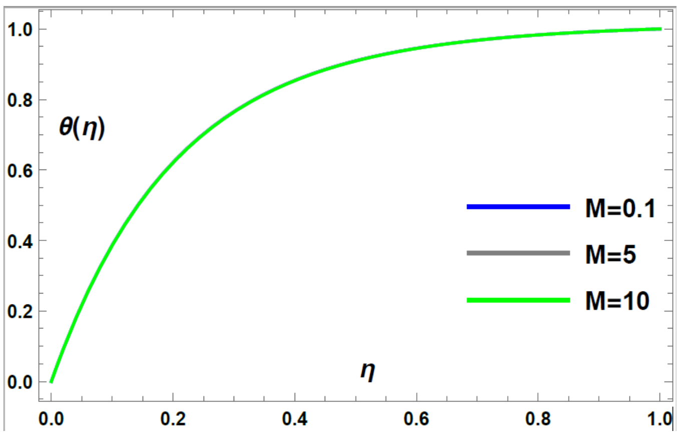


(c)

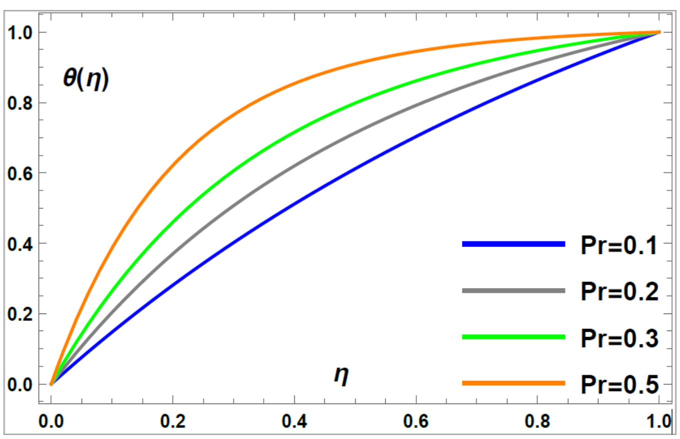
Fig. 8 Temperature profiles with $\alpha_2 = 0.51425$: (a) for different values of γ when $R_2 = 10$, $Pr = 0.5$ and $M = 5$, (b) for different values of M when $R_2 = 10$, $Pr = 0.5$ and $\gamma = 0.2$, (c) for different values of Pr when $R_2 = 10$, $\gamma = 0.2$ and $M = 5$.



(a)



(b)



(c)

Fig. 9 Temperature profiles with $\alpha_1 = -0.20820$: (a) for different values of γ when $R_1 = 10$, $Pr = 0.5$ and $M = 5$, (b) for different values of M when $R_1 = 10$, $Pr = 0.5$ and $\gamma = 0.2$, (c) for different values of Pr when $R_1 = 10$, $\gamma = 0.2$ and $M = 5$.

5. CONCLUSIONS

The current research is focussed on Casson fluid flow between two porous plates with different permeability with special reference to magnetic and heat transfer characteristics. The results for all the parameters in all the four cases obtained through HPM and finite difference scheme. The solution obtained show in the form of tables and figures indicate the following:

- The skin friction at the walls is increasing in the case of suction and injection in magnitudes where as the $f''(0)$ decreasing and $f''(1)$ increasing in case of mixed suction and mixed injections.
- The amount of heat transfer at the walls decreasing in magnitude for suction case and reverse trend is observed for injection. For the case of mixed suction and mixed injection, the values of $\theta'(0)$ increase and $\theta'(1)$ decrease for increase in R_2 and R_1 values respectively.
- The velocity profiles show that, velocity decreases in the case of Casson fluid and in the presence of magnetic effect.
- The temperature profiles also indicate that there is decrease in temperature flow and in the presence of magnetic effects.

NOMENCLATURE

x, y	Cartesian coordinates
u, v	Velocity components in x and y directions
B_0	Magnetic field intensity
C_p	Specific heat at constant pressure
h	Channel width
k	Thermal conductivity
M	Magnetic parameter
R_1, R_2	Reynolds numbers
T	Temperature
θ	Non-dimensional temperature
p	Pressure
Pr	Prandtl number
Greek Symbols	
α	Suction/injection parameter
η	Non-dimensional variable
γ	Casson parameter
μ	Viscosity
ν	Kinematic viscosity
ρ	Fluid density
ξ	Stream function

REFERENCES

Alpher, R.A., 1961, “Heat transfer in magnetohydrodynamic flow between parallel plates,” *International Journal of Heat and Mass Transfer*, **3**, 108–112.
https://doi.org/10.1016/0017-9310(61)90073-4.

Amanifard, N., Borji, M., and Haghi, A., 2007, “Heat transfer in porous media,” *Brazilian Journal of Chemical Engineering*, **24**, 223–232.
https://doi.org/10.1590/S0104-66322007000200007.

Badruddin, I.A., Azeem, Yunus Khan, T., and Ali Baig, M.A., 2020, “Heat Transfer in Porous Media: A Mini Review,” *Materials Today: Proceedings*, **24**, 1318–1321.
https://doi.org/10.1016/j.matpr.2020.04.447.

Berman, A.S., 1953, “Laminar flow in channels with porous walls,” *Journal of Applied Physics*, **24**.
https://10.1063/1.1721476.

Bujurke, N.M., Madalli, V.S., and Mulimani, B.G., 1998, “Long series analysis of laminar flow through parallel and uniformly porous walls of different permeability,” *Comput Methods Appl Mech and Engrg*, **160**.
https://doi.org/10.1016/S0045-7825(97)00281-8.

Chandrasekhara, B.C., and Rudraiah, N., 1971, “Magnetohydrodynamic laminar flow between porous disks,” *Applied Scientific Research*, **23**, 1573–1987.
https://doi.org/10.1007/BF00413186.

Global Digital Central
ISSN: 2151-8629

Chaoyang, W., and Chuanjing, T., 1989, "Boundary-layer flow and heat transfer of non-Newtonian fluids in porous media," *International Journal of Heat and Fluid Flow*, **10**, 160–165.
https://doi.org/10.1016/0142-727X(89)90011-8.

Dhange, M.Y., Sankad, G.C., and Maharudrappa, I., 2022, "Casson fluid flow due to stretching sheet with magnetic effect and variable thermal conductivity," *Frontiers in Heat and Mass Transfer*, **18**(36).
http://:10.5098/hmt.18.36.

Elkouh, A.F., 1967, "Laminar flow between porous disks," *Journal of the Engineering Mechanics Division*, **93**(4), 31–38.
https://doi.org/10.1061/JMCEA3.0000877.

Ganji, D., 2006, "The application of He's homotopy perturbation method to nonlinear equations arising in heat transfer," *Physics Letters A*, **355**, 337–341.
https://doi.org/10.1016/j.physleta.2006.02.056.

Ghiasi, E.K., and Saleh, R., 2019, "2D flow of Casson fluid with non-uniform heat source/sink and joule heating," *Frontiers in Heat and Mass Transfer*, **12**(4).
https://doi.org/10.5098/hmt.12.4.

He, J.H., 1999, "Homotopy perturbation technique," *Computer Methods in Applied Mechanics and Engineering*, **178**, 257–262.
https://doi.org/10.1016/S0045-7825(99)00018-3.

Khaled, A., and Vafai, K., 2003, "The role of porous media in modeling flow and heat transfer in biological tissues," *International Journal of Heat and Mass Transfer*, **46**, 4989–5003.
https://doi.org/10.1016/S0017-9310(03)00301-6.

Kranthi Kumar, K., Baby Rani, C.H., and Rao, A.V.P., 2021, "MHD Casson fluid flow with an inclined plate in the presence of Hall and aligned magnetic effects," *Frontiers in Heat and Mass Transfer*, **17**(2).
https://dx.doi.org/10.5098/hmt.17.2.

Muhammad, A.M., Faraidun, H., and Hajar, F.I., 2023, "Numerical study of stagnation point flow of Casson fluid over a continuous moving surface," *Frontiers in Heat and Mass Transfer*, **20**(7).
http://dx.org/10.5098/hmt.20.7.

Mukhopadhyay, S., De, P.R., Bhattacharyya, K., and Layek, G., 2013, "Casson fluid flow over an unsteady stretching surface," *Ain Shams Engineering Journal*, **4**(4), 933–938.
https://doi.org/10.1016/j.asej.2013.04.004.

Pai, N.P., Kumar, V.S., and Devaki, B., 2021, "Analysis of MHD flow and heat transfer of laminar flow between porous disks," *Frontiers in Heat and Mass Transfer*, **16**.
http://dx.doi.org/10.5098/hmt.16.3.

Rees, D., and Pop, I., 2000, "Vertical free convection in a porous medium with variable permeability effects," *International Journal of Heat and Mass Transfer*, **43**, 2565–2571.
https://doi.org/10.1016/S0017-9310(99)00316-6.

Terril, R.M., and Shrestha, G.M., 1965, "Laminar flow through parallel and uniformly porous walls of different permeability," *ZAMP*, **16**.
https://doi.org/10.1007/BF01593923.

Wilson, L.O., and Schryer, N.L., 1978, "Flow between a stationary and a rotating disk with suction," *Journal of Fluid Mechanics*, **85**(3), 479–496.
https://doi:10.1017/S0022112078000750.

APPENDIX

Table 1 Suction: Skin friction coefficient for $\alpha_2 = 1.9160$

R_2	γ	$M = 0.5$			
		$f''(0)$		$f''(1)$	
		HPM	FDM	HPM	FDM
1	0.1	11.5074	11.5027	-11.5221	-11.5191
5		11.5354	11.5291	-11.6123	-11.6084
10		11.5703	11.566	-11.7323	-11.72680
20		11.6383	11.6336	-11.9995	-11.9946
1	0.2	11.5169	11.5114	-11.5422	-11.5398
5		11.5684	11.5646	-11.7156	-11.7096
10		11.6314	11.625	-11.9562	-11.95090
20		11.7425	11.7360	-12.5483	-12.5435
1	0.3	11.5250	11.5208	-11.5630	-11.5581
5		11.5963	11.5916	-11.8077	-11.8029
10		11.6811	11.6766	-12.1680	-12.1625
20		11.7997	11.7931	-13.1429	-13.1356
R_2	γ	$M = 1$			
		$f''(0)$		$f''(1)$	
1	0.1	11.5205	11.5155	-11.5353	-11.5306
5		11.5489	11.5459	-11.6258	-11.6199
10		11.5843	11.5792	-11.7461	-11.7408
20		11.6534	11.6464	-12.0140	-12.0103
1	0.2	11.5411	11.5367	-11.5684	-11.5641
5		11.5939	11.5883	-11.7407	-11.7353
10		11.6587	11.6540	-11.9826	-11.9781
20		11.7742	11.7663	-12.5770	-12.5709
1	0.3	11.5587	11.5542	-11.5966	-11.5916
5		11.6325	11.6275	-11.8432	-11.8384
10		11.7210	11.716	-12.2059	-12.2012
20		11.8498	11.8439	-13.1845	-13.1777
R_2	γ	$M = 5$			
		$f''(0)$		$f''(1)$	
1	0.1	11.9344	11.9298	-11.9487	-11.9433
5		11.9742	11.9689	-12.0482	-12.0428
10		12.0246	12.0193	-12.1800	-12.1721
20		12.1268	12.1221	-12.4707	-12.4649
1	0.2	12.2938	12.2882	-12.3194	-12.314
5		12.3845	12.3796	-12.5214	-12.5152
10		12.5017	12.4962	-12.8005	-12.7943
20		12.7440	12.736	-13.4657	-13.4595
1	0.3	12.5938	12.5894	-12.6287	-12.6216
5		12.7398	12.7334	-12.9307	-12.9239
10		12.9323	12.9235	-13.3624	-13.3568
20		13.3369	13.3286	-14.4573	-14.449
R_2	γ	$M = 10$			
		$f''(0)$		$f''(1)$	
1	0.1	13.1522	13.145	-13.1652	-13.1585
5		13.2220	13.2150	-13.2892	13.2811
10		13.3118	13.3054	-13.4518	-13.4465
20		13.4995	13.4927	-13.8044	-13.7968
1	0.2	14.4112	14.4031	-14.4335	-14.4246
5		14.5900	14.5815	-14.7071	-14.6991
10		14.8261	14.8165	-15.0762	-15.0667
20		15.3420	15.3315	-15.9154	-15.9053
1	0.3	15.4065	15.3963	-15.4359	-15.4264
5		15.7039	15.6928	-15.8608	-15.8506
10		16.1047	16.0929	-16.4452	-16.4338
20		17.0097	16.9962	-17.8398	-17.8016

Table 2 Injection: Skin friction coefficient for $\alpha_2 = 1.9160$

R_2	γ	$M = 0.5$			
		$f''(0)$		$f''(1)$	
		HPM	FDM	HPM	FDM
-20	0.1	11.3622	11.3595	-11.1200	-11.1135
-10		11.4305	11.4263	-11.2978	-11.2926
-5		11.4653	11.4608	-11.3957	-11.3910
-1		11.4933	11.4892	-11.4789	-11.4747
-20	0.2	11.2565	11.2524	-10.8718	-10.8678
-10		11.3766	11.3729	-11.1513	-11.1473
-5		11.4397	11.436	-11.3171	-11.3117
-1		11.4911	11.4878	-11.4648	-11.4600
-20	0.3	11.1731	11.1699	-10.6973	-10.6934
-10		11.3320	11.3281	-11.0389	-11.0347
-1		11.4891	11.484	-11.4529	-11.4483
-5		11.4181	11.4142	-11.2539	-11.2484
R_2	γ	$M = 1$			
		$f''(0)$		$f''(1)$	
-20	0.1	11.3736	11.3701	-11.1316	-11.1261
-10		11.4427	11.4384	-11.3101	-11.3063
-5		11.4779	11.4723	-11.4084	-11.4045
-1		11.5063	11.501	-11.4918	-11.4872
-20	0.2	11.2755	11.2718	-10.8912	-10.8869
-10		11.3978	11.3942	-11.1729	-11.1672
-5		11.4622	11.4571	-11.3398	-11.3352
-1		11.5147	11.5099	-11.4884	-11.4848
-20	0.3	11.1974	11.1944	-10.7221	-10.7182
-10		11.3601	11.3554	-11.0676	-11.0633
-1		11.5217	11.5179	-11.4856	-11.4805
-5		11.4485	11.4454	-11.2847	-11.2802
R_2	γ	$M = 5$			
		$f''(0)$		$f''(1)$	
-20	0.1	11.7357	11.7304	-11.4996	-11.4945
-10		11.8280	11.8234	-11.6992	-11.6932
-5		11.8758	11.8707	-11.8084	-11.8039
-1		11.9147	11.9102	-11.9007	-11.8948
-20	0.2	11.8719	11.8655	-11.4999	-11.4963
-10		12.0611	12.0564	-11.8460	-11.8411
-5		12.1637	12.1575	-12.0475	-12.0439
-1		12.2496	12.2444	-12.2249	-12.2179
-20	0.3	11.9570	11.9517	-11.4973	-11.4934
-10		12.2334	12.2282	-11.9561	-11.9514
-5		12.5238	12.3836	-12.4904	-12.2311
-1		12.5238	12.5179	-12.4904	-12.4856
R_2	γ	$M = 10$			
		$f''(0)$		$f''(1)$	
-20	0.1	12.8138	12.8075	-12.5925	-12.5860
-10		12.9691	12.9624	-12.8498	-12.8443
-5		13.0507	13.0433	-12.9887	-12.9824
-1		13.1179	13.1113	-13.1051	-13.0989
-20	0.2	13.6064	13.5992	-13.2614	-13.2548
-10		13.9631	13.9552	-13.7689	-13.7626
-5		14.1591	14.1527	-14.0559	-14.0466
-1		14.3251	14.3177	-14.3034	-14.2958
-20	0.3	14.1296	14.1311	-13.7112	-13.7022
-10		14.6855	14.6773	-14.4372	-14.4284
-5		14.9965	14.9868	-14.8625	-14.8544
-1		15.2651	15.2557	-15.2366	-15.227

Table 3 Mixed Suction: Skin friction coefficient for $\alpha_2 = 0.51425$

R_2	γ	$M = 0.5$			
		$f''(0)$		$f''(1)$	
		HPM	FDM	HPM	FDM
1	0.1	3.0531	3.05356	-3.1227	-3.1215
5		2.9219	2.91776	-3.2737	-3.27721
10		2.7647	2.76378	-3.4779	-3.47668
20		2.4742	2.47207	-3.9395	-3.93956
1	0.2	3.0264	3.02484	-3.1543	-3.15369
5		2.7914	2.79032	-3.4435	-3.44205
10		2.5215	2.52144	-3.8581	-3.85434
20		2.0650	2.06591	-4.8700	-4.86309
1	0.3	3.0039	3.00736	-3.1813	-3.17914
5		2.6853	2.6834	-3.5966	-3.59507
10		2.3337	2.33154	-4.2189	-4.22095
20		1.7898	1.78966	-5.8105	-5.80573
R_2	γ	$M = 1$			
		$f''(0)$		$f''(1)$	
1	0.1	3.0567	3.0558	-3.1262	-3.12584
5		2.9256	2.92451	-3.2770	-3.27662
10		2.7686	2.76694	-3.4809	-3.48108
20		2.4784	2.47809	-3.9418	-3.93907
1	0.2	3.0329	3.03029	-3.1606	-3.16135
5		2.7985	2.79717	-3.4492	-3.44778
10		2.5291	2.53121	-3.8629	-3.85763
20		2.0734	2.07138	-4.8719	-4.8736
1	0.3	3.0131	3.01110	-3.1899	-3.18927
5		2.6955	2.69517	-3.6040	-3.60063
10		2.3448	2.34365	-4.2242	-4.22298
20		1.8016	1.80075	-5.8093	-5.80455
R_2	γ	$M = 5$			
		$f''(0)$		$f''(1)$	
1	0.1	3.1684	3.16621	-3.2355	-3.23532
5		3.0425	3.04187	-3.3817	-3.38019
10		2.8914	2.89062	-3.5787	-3.57724
20		2.6113	2.61034	-4.0214	-4.01819
1	0.2	3.2368	3.23746	-3.3569	-3.35381
5		3.01819	3.01673	-3.6292	-3.62665
10		2.7653	2.76372	-4.0152	-4.0145
20		2.3305	2.32974	-4.9455	-4.94249
1	0.3	3.2942	3.29377	-3.4572	-3.45402
5		3.0044	3.00257	-3.83875	-3.83700
10		2.6798	2.67789	-4.3999	-4.39765
20		2.15864	2.15776	-5.8117	-5.80842
R_2	γ	$M = 10$			
		$f''(0)$		$f''(1)$	
1	0.1	3.4966	3.49522	-3.5578	-3.55528
5		3.3834	3.38015	-3.6932	-3.69291
10		3.2471	3.24384	-3.8739	-3.87265
20		2.9917	2.99062	-4.2728	-4.26940
1	0.2	3.8083	3.80564	-3.9130	-3.91071
5		3.6226	3.62103	-4.1532	-4.1514
10		3.4043	3.40254	-4.4848	-4.48216
20		3.0154	3.01431	-5.2579	-5.2546
1	0.3	4.0538	4.0516	-4.1919	-4.18928
5		3.8153	3.81291	-4.5182	-4.51576
10		3.5401	3.53819	-4.9784	-4.97471
20		3.0662	3.06481	-6.0835	-6.0777

Frontiers in Heat and Mass Transfer (FHMT), 20, 30 (2023)
DOI: 10.5098/hmt.20.30

Global Digital Central
ISSN: 2151-8629

Table 4 Mixed Injection: Skin friction coefficient for $\alpha_1 = -0.2082$

R_2	γ	$M = 0.5$			
		$f''(0)$		$f''(1)$	
		HPM	FDM	HPM	FDM
1	0.1	-1.2327	-1.23164	1.2666	1.26671
5		-1.1679	-1.16709	1.3365	1.33622
10		-1.0938	-1.09495	1.4292	1.42739
20		-0.9672	-0.96616	1.6309	1.63055
1	0.2	-1.2192	-1.22039	1.2812	1.27816
5		-1.1061	-1.1048	1.4137	1.41335
10		-0.9868	-0.98567	1.5962	1.59845
20		-0.8121	0.81069	2.0092	2.00848
1	0.3	-1.2078	-1.20773	1.2937	1.29226
5		-1.0578	-1.06145	1.4820	1.47926
10		-0.9108	0.90905	1.7483	1.74996
20		-0.7232	-0.72219	2.3602	2.35944
R_1	γ	$M = 1$			
		$f''(0)$		$f''(1)$	
1	0.1	-1.2341	-1.23133	1.2678	1.26814
5		-1.1694	-1.16702	1.3379	1.33743
10		-1.0954	-1.09424	1.4304	1.43224
20		-0.9689	-0.96858	1.6319	1.63233
1	0.2	-1.2219	-1.22218	1.2837	1.28296
5		-1.1089	-1.10756	1.4159	1.41696
10		-0.9898	-0.98529	1.5979	1.60391
20		-0.8150	-0.81420	2.0099	2.0096
1	0.3	-1.2117	-1.21022	1.2972	1.29666
5		-1.0618	-1.06145	1.4849	1.48326
10		-0.9149	-0.91470	1.7502	1.74904
20		-0.7270	-0.72652	2.3601	2.35536
R_1	γ	$M = 5$			
		$f''(0)$		$f''(1)$	
1	0.1	-1.2794	-1.27833	1.3120	1.31186
5		-1.2165	-1.21740	1.3793	1.37791
10		-1.1443	-1.14281	1.4682	1.4688
20		-1.0197	-1.01824	1.6614	1.66175
1	0.2	-1.3043	-1.3053	1.3626	1.36115
5		-1.1966	-1.19764	1.4859	1.48343
10		-1.0811	-1.08153	1.6549	1.65260
20		-0.9053	-0.90466	2.0371	2.03592
1	0.3	-1.3253	-1.32344	1.40442	1.40353
5		-1.1839	-1.18468	1.5749	1.57229
10		-1.0404	-1.04146	1.8146	1.81064
20		-0.8432	-0.84273	2.3678	2.36826
R_1	γ	$M = 10$			
		$f''(0)$		$f''(1)$	
1	0.1	-1.4122	-1.41046	1.4421	1.44191
5		-1.3539	-1.35454	1.5029	1.50035
10		-1.2861	-1.28548	1.5829	1.58224
20		-1.1664	-1.16504	1.7556	1.75735
1	0.2	-1.5355	-1.53416	1.5863	1.58615
5		-1.4384	-1.4375	1.6914	1.69149
10		-1.3302	-1.32755	1.8335	1.83315
20		-1.1533	-1.15301	2.1517	2.15103
1	0.3	-1.6323	-1.63141	1.6993	1.69586
5		-1.5059	-1.50425	1.8392	1.83893
10		-1.3691	-1.36822	2.0317	2.03043
20		-1.1561	-1.15602	2.4723	2.46896

Table 5 Suction Heat transfer rate for $\alpha_2 = 1.916$

R_2	γ	$M = 0.5, Pr = 0.5$			
		$\theta'(0)$		$\theta'(1)$	
		HPM	FDM	HPM	FDM
1	0.1	0.91724	0.91718	0.89823	0.89820
5		0.63519	0.63595	0.57343	0.57350
10		0.38584	0.38598	0.31483	0.31494
20		0.12744	0.12768	0.08622	0.08636
1	0.2	0.91722	0.91718	0.89826	0.89822
5		0.63558	0.63570	0.57395	0.57406
10		0.38496	0.38515	0.31613	0.31627
20		0.12596	0.12625	0.08809	0.08824
1	0.3	0.91721	0.91717	0.89829	0.89825
5		0.63529	0.635422	0.57442	0.57453
10		0.38409	0.38433	0.31738	0.31754
20		0.12417	0.12454	0.09022	0.09034
R_2	γ	$M = 1, Pr = 0.5$			
		$\theta'(0)$		$\theta'(1)$	
1	0.1	0.91725	0.917189	0.89824	0.89819
5		0.63594	0.63601	0.57346	0.57350
10		0.38588	0.38594	0.31486	0.31491
20		0.12757	0.12765	0.08624	0.08633
1	0.2	0.91724	0.91716	0.89828	0.89821
5		0.63564	0.63565	0.57400	0.57400
10		0.38505	0.38505	0.31618	0.316225
20		0.12604	0.12616	0.08812	0.08821
1	0.3	0.91723	0.91715	0.89831	0.89823
5		0.63537	0.63533	0.57448	0.57448
10		0.38423	0.38419	0.31745	0.31747
20		0.12431	0.12438	0.09023	0.09032
R_2	γ	$M = 5, Pr = 0.5$			
		$\theta'(0)$		$\theta'(1)$	
1	0.1	0.91748	0.91742	0.89847	0.89841
5		0.63689	0.63694	0.57426	0.57432
10		0.38725	0.38736	0.31583	0.31590
20		0.12864	0.12887	0.08684	0.08695
1	0.2	0.91766	0.91760	0.89868	0.89863
5		0.63743	0.63749	0.75741	0.57546
10		0.38774	0.387847	0.31786	0.31795
20		0.12856	0.12880	0.08909	0.08919
1	0.3	0.91781	0.91775	0.89885	0.89880
5		0.63789	0.63795	0.57637	0.57642
10		0.38819	0.38827	0.31962	0.31973
20		0.12845	0.12869	0.09124	0.09133
R_2	γ	$M = 10, Pr = 0.5$			
		$\theta'(0)$		$\theta'(1)$	
1	0.1	0.91814	0.91808	0.89910	0.89905
5		0.63958	0.63964	0.57653	0.57658
10		0.39110	0.3912	0.31862	0.31872
20		0.13193	0.13216	0.08863	0.08871
1	0.2	0.91877	0.91872	0.89974	0.89969
5		0.64207	0.64213	0.57917	0.57923
10		0.39461	0.39470	0.32241	0.32251
20		0.13489	0.13509	0.09192	0.09193
1	0.3	0.91925	0.91919	0.90022	0.90017
5		0.64401	0.64406	0.58116	0.58122
10		0.39743	0.39753	0.32532	0.32542
20		0.13753	0.13766	0.09331	0.09448

Frontiers in Heat and Mass Transfer (FHMT), 20, 30 (2023)
DOI: 10.5098/hmt.20.30
Global Digital Central
ISSN: 2151-8629

Table 6 Injection Heat transfer rate for $\alpha_2 = 1.916$

R_2	γ	$M = 0.5, Pr = 0.5$			
		$\theta'(0)$		$\theta'(1)$	
		HPM	FDM	HPM	FDM
-1	0.1	1.08784	1.08769	1.11099	1.11083
-5		1.48885	1.48841	1.65611	1.65562
-10		2.0880	2.08695	2.59050	2.58919
-20		3.43320	3.4296	5.33611	5.33172
-1	0.2	1.08781	1.08766	1.11102	1.11087
-5		1.48813	1.48768	1.65719	1.65671
-10		2.08411	2.08305	2.59627	2.59495
-20		3.40873	3.40563	5.36954	5.36458
-1	0.3	1.08774	1.08764	1.11105	1.11089
-5		1.48756	1.48711	1.65806	1.65758
-10		2.08119	2.08014	2.60067	2.59935
-20		3.39210	3.38895	5.39269	5.38777
R_2	γ	$M = 1, Pr = 0.5$			
		$\theta'(0)$		$\theta'(1)$	
-1	0.1	1.08783	1.08768	1.11098	1.11083
-5		1.48881	1.48840	1.65606	1.65553
-10		2.08792	2.08687	2.59037	2.58903
-20		3.43310	3.42985	5.33577	5.33098
-1	0.2	1.08780	1.08765	1.11100	1.11085
-5		1.48806	1.48763	1.65709	1.65659
-10		2.08397	2.08285	2.59601	2.59477
-20		3.40864	3.40554	5.36885	5.36389
-1	0.3	1.08777	1.08762	1.11102	1.11087
-5		1.48746	1.48700	1.65792	1.65744
-10		2.08102	2.08000	2.60031	2.59895
-20		3.39202	3.38883	5.39174	5.38687
R_2	γ	$M = 5, Pr = 0.5$			
		$\theta'(0)$		$\theta'(1)$	
-1	0.1	1.08758	1.08742	1.11072	1.11057
-5		1.48774	1.48701	1.65437	1.65385
-10		2.08514	2.08404	2.58607	2.58480
-20		3.42991	3.42681	5.32474	5.31978
-1	0.2	1.08735	1.08720	1.11053	1.11038
-5		1.48574	1.48530	1.65402	1.65348
-10		2.07956	2.07852	2.58810	2.58676
-20		3.40528	3.40253	5.34813	5.34273
-1	0.3	1.08717	1.08702	1.11038	1.11023
-5		1.48438	1.48395	1.65370	1.65319
-10		2.07554	2.07451	2.58941	2.58806
-20		3.38917	3.38626	5.36333	5.35807
R_2	γ	$M = 10, Pr = 0.5$			
		$\theta'(0)$		$\theta'(1)$	
-1	0.1	1.08686	1.08671	1.10998	1.10982
-5		1.48345	1.48303	1.64958	1.64906
-10		2.07705	2.07605	2.57389	2.57251
-20		3.41987	3.41666	5.29374	5.28883
-1	0.2	1.08616	1.08601	1.10929	1.10914
-5		1.47928	1.47881	1.64592	1.64546
-10		2.06710	2.06611	2.56728	2.56589
-20		3.39316	3.39015	5.29376	5.28861
-1	0.3	1.08564	1.08549	1.10878	1.10863
-5		1.47630	1.47588	1.64320	1.64269
-10		2.06033	2.05925	2.56226	2.56099
-20		3.37686	3.37335	5.29151	5.28727

Table 7 Mixed Suction Heat transfer rate for $\alpha_2 = 0.51425$

R_2	γ	$M = 0.5, Pr = 0.2$			
		$\theta'(0)$		$\theta'(1)$	
		HPM	FDM	HPM	FDM
1	0.1	1.06505	1.06494	0.91811	0.91802
5		1.34460	1.34440	0.64156	0.64155
10		1.73013	1.72986	0.39622	0.39623
20		2.57646	2.57582	0.13839	0.13845
1	0.2	1.0650	1.06489	0.91815	0.91807
5		1.34322	1.34306	0.64248	0.64245
10		1.72426	1.72403	0.39886	0.39884
20		2.55374	2.55327	0.14276	0.14280
1	0.3	1.06496	1.06485	0.91819	0.918111
5		1.34206	1.34189	0.64328	0.64325
10		1.71938	1.71906	0.40113	0.40115
20		2.53616	2.53563	0.14642	0.14652
R_2	γ	$M = 1, Pr = 0.2$			
		$\theta'(0)$		$\theta'(1)$	
1	0.1	1.06505	1.06494	0.91811	0.91803
5		1.34461	1.34444	0.64156	0.64153
10		1.73016	1.72988	0.39622	0.39623
20		2.57656	2.57602	0.13838	0.13842
1	0.2	1.06500	1.06489	0.91816	0.91808
5		1.34325	1.34308	0.64248	0.64245
10		1.72434	1.72416	0.39884	0.39881
20		2.55400	2.55334	0.14272	0.14281
1	0.3	1.06496	1.06485	0.91819	0.91812
5		1.34210	1.34195	0.64327	0.64323
10		1.71951	1.71924	0.40109	0.40110
20		2.53661	2.53605	0.14635	0.14646
R_2	γ	$M = 5, Pr = 0.2$			
		$\theta'(0)$		$\theta'(1)$	
1	0.1	1.06508	1.06497	0.91813	0.91805
5		1.34438	1.34473	0.64157	0.64154
10		1.73106	1.73081	0.39613	0.39612
20		2.57958	2.57903	0.13812	0.13816
1	0.2	1.06506	1.06496	0.91819	0.91810
5		1.34390	1.34374	0.64241	0.64237
10		1.72659	1.72631	0.39838	0.39839
20		2.56176	2.56120	0.14176	0.14181
1	0.3	1.06505	1.06494	0.91823	0.91815
5		1.34315	1.34299	0.64306	0.64303
10		1.72324	1.72296	0.40013	0.40014
20		2.54928	2.54868	0.14456	0.14461
R_2	γ	$M = 10, Pr = 0.2$			
		$\theta'(0)$		$\theta'(1)$	
1	0.1	1.06517	1.06506	0.91819	0.91811
5		1.34565	1.34547	0.64164	0.64162
10		1.73345	1.73315	0.39593	0.39594
20		2.58751	2.58696	0.13749	0.13752
1	0.2	1.06522	1.06511	0.91828	0.91820
5		1.34545	1.34529	0.64233	0.64230
10		1.73176	1.73150	0.39750	0.39751
20		2.57937	2.57880	0.13982	0.13983
1	0.3	1.06526	1.06515	0.91834	0.91826
5		1.34540	1.34523	0.64279	0.64276
10		1.73092	1.73066	0.39852	0.39852
20		2.57519	2.57462	0.14128	0.14126

Table 8 Mixed Injection Heat transfer rate for $\alpha_1 = -0.2082$

R_1	γ	$M = 0.5, Pr = 0.2$			
		$\theta'(0)$		$\theta'(1)$	
		HPM	FDM	HPM	FDM
1	0.1	1.09685	1.09673	0.91686	0.91678
5		1.54615	1.54591	0.63031	0.63027
10		2.24001	2.23944	0.37124	0.37124
20		3.95952	3.95767	0.10778	0.10759
1	0.2	1.09687	1.09675	0.91684	0.91676
5		1.54686	1.54662	0.62988	0.62983
10		2.24302	2.24254	0.37009	0.37005
20		3.96947	3.96768	0.10630	0.10613
1	0.3	1.09689	1.09677	0.91682	0.91674
5		1.54744	1.54715	0.62951	0.62949
10		2.24538	2.24491	0.36915	0.36911
20		3.97616	3.97433	0.10521	0.10508
R_1	γ	$M = 1, Pr = 0.2$			
		$\theta'(0)$		$\theta'(1)$	
1	0.1	1.09685	1.09673	0.91686	0.91677
5		1.54615	1.54591	0.63031	0.63027
10		2.24000	2.23951	0.37125	0.37121
20		3.95947	3.95763	0.10778	0.10759
1	0.2	1.09687	1.09675	0.91684	0.91676
5		1.54685	1.54661	0.62988	0.62983
10		2.24298	2.24259	0.37009	0.37003
20		3.96936	3.96753	0.10632	0.10615
1	0.3	1.09689	1.09677	0.91682	0.91673
5		1.54742	1.54717	0.62951	0.62947
10		2.24531	2.24478	0.36917	0.36915
20		3.97600	3.97413	0.10523	0.10511
R_1	γ	$M = 5, Pr = 0.2$			
		$\theta'(0)$		$\theta'(1)$	
1	0.1	1.09683	1.09671	0.91685	0.91677
5		1.54601	1.54575	0.63032	0.63028
10		2.23955	2.23906	0.37130	0.37127
20		3.95806	3.95624	0.10789	0.10769
1	0.2	1.09684	1.09672	0.91683	0.91674
5		1.54654	1.54627	0.62993	0.62989
10		2.24189	2.24134	0.37031	0.37030
20		3.96614	3.96425	0.10666	0.10646
1	0.3	1.09685	1.09673	0.91681	0.91672
5		1.54692	1.54666	0.62963	0.62960
10		2.24357	2.24301	0.36957	0.36957
20		3.97131	3.96934	0.10587	0.10562
R_1	γ	$M = 10, Pr = 0.2$			
		$\theta'(0)$		$\theta'(1)$	
1	0.1	1.09680	1.09668	0.91683	0.91674
5		1.54565	1.54540	0.63031	0.63027
10		2.23835	2.23785	0.37142	0.37140
20		3.95431	3.95248	0.10817	0.10794
1	0.2	1.09678	1.09666	0.91679	0.91671
5		1.54580	1.54556	0.63000	0.62995
10		2.23935	2.23887	0.37074	0.37071
20		3.95844	3.95648	0.10739	0.10714
1	0.3	1.09676	1.09664	0.91677	0.91668
5		1.54586	1.54562	0.62980	0.62975
10		2.23987	2.23936	0.37031	0.37029
20		3.96053	3.95851	0.10695	0.10666