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MECHANICALLY DRIVEN OSCILLATING FLOW COOLING LOOPS-A REVIEW

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ABSTRACT

The significant increase in the heat dissipation associated with the increased throughput in computing, renewable energy, and electric vehicles has become a limitation to the evolution of these technologies. The needs for more effective thermal-management methods for higher heat fluxes and more uniform temperature have resulted in the development of mechanically driven oscillating flow cooling systems. The objective of this paper is to review the state-of-the-art of mechanically driven oscillating flow loops (MDOFLs) in terms of several aspects such as heat transfer, fluid mechanics, and thermodynamic principles. In each aspect, essential formulas and related sciences from prior studies are presented to show what has been accomplished in the field. Important parameters on the performance of the cooling systems and challenges for future research are also provided.

Keywords: *Oscillating flow, Heat transfer, Renewable Energy, Electronics cooling*

1. INTRODUCTION

Advances in thermal management systems have produced several important cooling solutions which adopt mechanically driven oscillating flow of the working fluid in the heat loops. As special heat pipe systems, the flows in these devices are also referred to as reciprocating flow and are characterized by periodically reversing flow to achieve higher heat transport between the high and low-temperature reservoirs. Typical examples of these systems include pulse tube cryocooler (Fig. 1), where the flow is driven by electromagnetic or thermoacoustic drivers (Hu et al., 2010 ; Ju, Wang, and Zhou, 1998), Sterling engine regenerator (Fig. 2) Tanaka, Yamashita, and Chisaka, (1990) which is applied between the high temperature reservoir and the low temperature reservoir of a Stirling engine, and more recently the reciprocating mechanism driven heat loop (RMDHL) (Fig. 3) for high heat flux electronic cooling or battery pack cooling for electric vehicles, which may be solenoid operated Cao and Gao, (2008) or bellows pump driven Popoola, Bamgbade, and Cao, (2016) through a linear actuator. The advantages offered by these oscillatory flow heat transfer devices include hermitically sealed leak-proof design Popoola, Soleimanikutanaei, and Cao, (2016) improved temperature uniformity across cold plate Popoola, Soleimanikutanaei, and Cao, (2016) heat transfer in both single and two-phase operation for large heat capacity and high heat flux applications Cao and Gao, (2008). The two-phase mode may offer a compact design that eliminates the so-called cavitation problem commonly encountered by the conventional pumps.

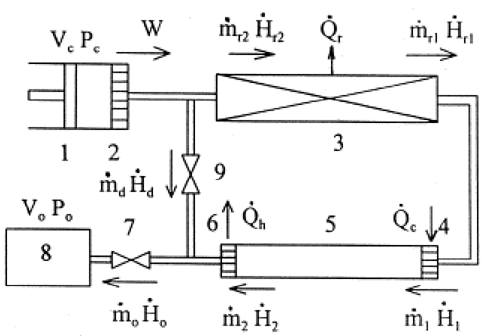

Fig. 1 Schematic of the physical model for the pulse tube refrigerator: 1. compressor; 2. aftercooler; 3. regenerator; 4. cold end heat exchanger; 5. pulse tube; 6. hot end heat exchanger; 7. orifice; 8. reservoir; 9. double-inlet valve Ju, Wang, and Zhou, (1998)

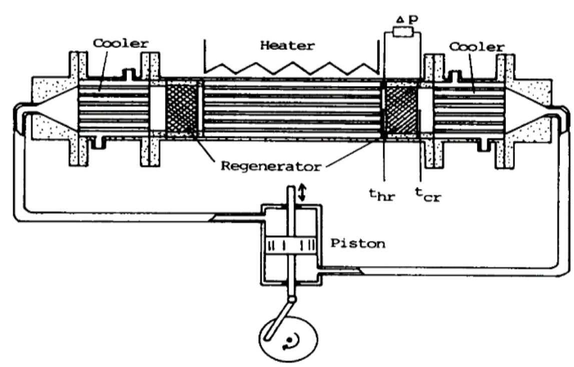

Fig. 2 Physical model of Sterling engine regenerator Tanaka, Yamashita, and Chisaka, (1990)

However, a significant hindrance to the wide adoption of these devices is posed by the fact that analytical studies of these reciprocating flow systems can be challenging. Unlike conventional unidirectional flows, the mathematical description of the oscillatory flow is complicated and not easy to interpret. Particular integral and the general solutions to the homogeneous governing equations for oscillating flow, for the most part, are too complex to provide explicit information (Chen, Luo, and Dai, 2007; Currie, 2003). Depending on the physical description of the problem and the coordinate system adopted, the solution contains

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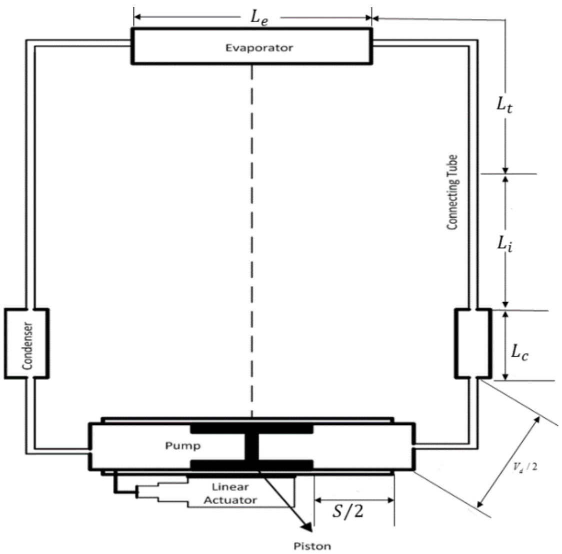

Fig. 3 A linear-actuator operated reciprocating mechanism driven heat loop (RMDHL)

complex numbers, Bessel functions, and hyperbolic trig and exponential terms by Uchida, (1956). In most cases, these terms require additional simplification. For example Eq. (1) below is a solution for the dimensionless temperature profile for single-phase, oscillating, laminar, incompressible flow in a parallel wall channel, with a constant wall temperature Grossman and Nachman, (2006). It is expected that Eq. (1) will take an even more complex form when additional layers of multi-phase and turbulence are included. In some instances, the results from the simplifications provided by different researches are not in agreement.

$$\frac{T-T_L}{T_H-T_L}=\frac{L-x}{L}+\left(\frac{a^4\Delta P}{\rho\nu^2L^2}\right)\frac{\sqrt{Pr}}{4V^4(Pr-1)(\sinh^2V+\cos^2V)}\times$$
$$\left\{\begin{aligned}&\left[\begin{aligned}&\frac{\left[\sin V\cos V\sinh V_1\cos V_1-\sinh V\cosh V\cosh V_1\sin V_1\right]s(V_1\eta)}{\left(\sinh^2V+\sin^2V_1\right)}\\&-\frac{\left[\sin V\cos V\cosh V_1\sin V_1+\sinh V\cosh V\sinh V_1\cos V_1\right]c(V_1\eta)}{\left(\sinh^2V+\sin^2V_1\right)}\\&+\sqrt{Pr}\left[c(V\eta)c(V)+s(V\eta)s(V)\right]-\frac{(Pr-1)}{\sqrt{Pr}}\left(\sinh^2V+\cos^2V\right)\end{aligned}\right]\cos\omega t\\&+\left[\begin{aligned}&\frac{\left[\sin V\cos V\cosh V_1\sin V_1+\sinh V\cosh V\sinh V_1\cos V_1\right]s(V_1\eta)}{\left(\sinh^2V+\sin^2V_1\right)}\\&-\frac{\left[\sin V\cos V\sinh V_1\cos V_1-\sinh V\cosh V\cosh V_1\sin V_1\right]c(V_1\eta)}{\left(\sinh^2V+\sin^2V_1\right)}\\&+\sqrt{Pr}\left[s(V\eta)c(V)-c(V\eta)s(V)\right]\end{aligned}\right]\sin\omega t\end{aligned}\right\} \tag{1}$$

with $c(z)=cosh(z)cos(z)$; $s(z)=sinh(z)sin(z)$; $\eta=y/a$ is the dimensionless lateral coordinate, and $V=\sqrt{V_a/8}$

Compared to research work on conventional flow, research into the oscillating flow so far is limited. It was observed in many literatures that correlations for heat transfer under oscillating flow conditions are usually based on correlations derived from unidirectional flow assumptions Kuosa et al., (2012). Some classical laminar and turbulent steady unidirectional correlations, which are used to approximate the convective heat transfer coefficient in Sterling engines, were presented in Xiao et al., (2014). These methods may prove to be somewhat inappropriate and insufficient as they do not capture certain critic phenomena like phase lag in heat transfer and the influence of the oscillation frequency on the heat transfer (Walther, Kühl, and Schulz, 2000; Popoola and Cao, 2016). Specific dissimilarities between oscillating flow and conventional flow have been itemized and the shortfalls in the correlations have been highlighted Allan, (2014). While some of these correlations provide a very close approximation, the shortfall in many other predictions is as wide as 25 %.

There has been significant research work to address the challenges associated with the aspects of the fluid flow and heat transfer in mechanically driven oscillatory flow cooling loops. Some of these works are to provide further insight into the mechanism for improved heat transfer in oscillating flow over conventional flow Li and Yang, (2000). Implications of the so-called annular effect where the maximum axial velocity in some oscillatory flow occurs near the wall was first observed (Richardson and Tyler, 1929; Xiao et al., 2014; Kuosa et al., 2012). More significant work for inventors and designers of thermal management systems among these research efforts is to improve the accuracy of the analytical correlations associated with quantifying system fluid mechanics and thermal variables. Some of these correlations were developed from approximate analytical or numerical solutions of the Navier-Stokes equations and experimental measurements. The majority of the research works into the correlations span several decades from several researchers of different countries and are yet to be harmonized.

The goal of this work is to provide a comprehensive compilation of heat transfer and flow characteristics for oscillating solutions and correlations that have been scattered in literatures. The current study has become necessary as a result of the challenges the authors have experienced during the design process for the RMDHL. In some cases, inaccuracies in results were observed when correlations were wrongly copied by authors in prior literatures or when boundaries for application of the correlations were not properly specified. This study would also serve as a more recent literature review on correlations for oscillatory flow since an excellent review of the subject presented nearly 20 years ago Zhao and Cheng, (1998b).

2. GOVERNING EQUATIONS

A thermal management application may employ an oscillatory flow of the working fluid within a channel, a pipe, or a combination of channels and pipes. Such oscillatory fluid flow driven by a sinusoidal displacer can be regarded as a problem of an incompressible, viscous fluid oscillating within a confined boundary connecting two reservoirs. The two reservoirs include a hot reservoir at T_H and a cold reservoir at T_C. Oscillating flow regimes have been described by Walther, Kühl, and Schulz, (2000) indicating that there are phases in which the flow is turbulent while laminar flow occurs during acceleration after flow reversal. Similar to conventional flows, the governing equations are the continuity equation, the Naiver-Stokes equations, and the energy equations. For ease of analysis, a number of different formulations have been developed based on dimensionless coordinates and parameters. Given that a sinusoidal flow is either prescribed by a sinusoidal mean velocity u_m, Eq. (2), with a maximum velocity $u_{\max}$, Eq. (3), or driven by a sinusoidally varying pressure gradient $\partial p/\partial x$, Eq. (4), in conjunction with possible phase angle ϕ, Eq. (5), the most common formulations of the governing equations are in terms of oscillation angular frequency ω, time t, pipe or channel hydraulic diameter D or width h, stream function Ψ, kinematic viscosity ν, density ρ, and thermal conductivity, k_f. Commonly used continuity, momentum, and energy equations by different researchers are respectively described by Eqs. (6-17) as follows

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$$u_m = u_{\max} \sin\phi \tag{2}$$

$$u_{\max} = \frac{x_{\max}\omega}{2} \tag{3}$$

$$-\frac{\partial p}{\partial x} = p_o \cos\phi \tag{4}$$

$$\omega t = 2\pi(i-1)+\phi \tag{5}$$

i = number of cycles

Continuity:

$$\nabla \cdot \vec{V} = 0 \tag{6}$$

Zhao and Cheng, (1998b)

$$\nabla^2\Psi = -\zeta \tag{7}$$

Iliadis and Anagnostopoulos, (1998)

Momentum (Table 1):

Table 1 Different momentum equations for oscillatory flow.

Equation		Parameters	
$\mathrm{Re}_\omega \frac{\partial \vec{V}}{\partial \tau} + \mathrm{Re}_{\max}\left[\vec{V}.\nabla\vec{V} + \nabla P\right] = \nabla^2\vec{V}$	(8)	$\mathrm{Re}_{\max} = \frac{A_o}{2}\mathrm{Re}_\omega$	Zhao and Cheng, (1998b)
$\frac{\partial V}{\partial \tau} + \frac{A_o}{2}\left[V.\nabla V + \nabla P\right] = \frac{1}{\mathrm{Re}_\omega}\nabla^2\vec{V}$	(9)		
$Str\frac{\partial V}{\partial \tau} + V.\nabla V + \frac{\nabla P}{\rho} = \frac{1}{\mathrm{Re}_\omega}\nu\nabla^2 V$	(10)		Simon and Seume, (1988)
$\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{u_i u_j}\right) + \frac{\partial \overline{p}}{\partial x_i} = \frac{1}{\mathrm{Re}_\tau}\frac{\partial^2 \overline{u_i}}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + P_g\delta_{i1}$	(11)		Wang and Lu, (2004)
$\frac{\partial \zeta}{\partial t} + \frac{\partial \Psi}{\partial y}\frac{\partial \zeta}{\partial x} - \frac{\partial \Psi}{\partial x}\frac{\partial \zeta}{\partial y} = \nu\nabla^2\zeta$	(12)		Iliadis and Anagnostopoulos, (1998)
$\frac{\partial \zeta}{\partial t} + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(\zeta\frac{\partial \Psi}{\partial \theta}\right)\right) - \frac{\partial}{\partial \theta}\left(\zeta\frac{\partial \Psi}{\partial r}\right) = \nu\nabla^2\zeta$	(13)		(Zhang, Dalton, and Brown, 1993; Justesen, 1991)
$g(\xi)\frac{\partial \zeta}{\partial t} + \left(\frac{\partial}{\partial \xi}\left(\zeta\frac{\partial \Psi}{\partial \eta}\right)\right) - \frac{\partial}{\partial \eta}\left(\zeta\frac{\partial \Psi}{\partial \xi}\right) = \frac{2}{\mathrm{Re}}\nabla^2\zeta$	(14)		

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Energy (Table 2):

Table 2 Different Energy equations for oscillatory flow.

Equation		Parameters	
$\frac{\partial \overline{T}}{\partial t}+\frac{\partial}{\partial x_j}\left(\overline{Tu_j}\right)=\frac{1}{\mathrm{Re}_\tau \mathrm{Pr}}\frac{\partial^2 \overline{T}}{\partial x_j \partial x_j}-\frac{\partial q_j}{\partial x_j}$	(15)		Wang and Lu, (2004)
$\rho c_p\left[2\mathrm{Re}_\omega \frac{\partial T}{\partial t}+\mathrm{Re}_{\max} u.\nabla T\right]=2Ec\,\mathrm{Re}_\omega \frac{\partial^2 P}{\partial t}-Ec\,\mathrm{Re}_\omega u.\nabla P+\frac{\nabla.\left(k\nabla T\right)}{\mathrm{Pr}}+Ec\,\phi$	(16)	$Ec=\frac{u_{\max}^2}{c_p\left(T_H-T_L\right)}$	Simon and Seume, (1988)
$\frac{\partial \theta}{\partial \tau}+\frac{A_o}{2}\left(\vec{V}.\nabla\right)\theta=\frac{1}{\mathrm{Re}_\omega \mathrm{Pr}}\nabla^2\theta$	(17)	$\theta=\frac{T-T_C}{T_H-T_C}$ $\theta=\frac{k_f T}{q_w D}$	Zhao and Cheng, (1998b)

The various dimensionless parameters in the above equations are defined as $\mathrm{Re}_\omega=\omega D^2/\nu$, $\mathrm{Re}_{\max}=u_{\max}D^2/\nu$, $Str=\omega d/u_{\max}$, $\tau=\omega t$, $U=u/u_{\max}$, and $P=p/\rho u_{\max}^2$. Eqs. (6, 8, and 16) in terms of the $\mathrm{Re}_{\max}$ and Re_ω are the most common form of the non-dimensional governing equations. However, it has been shown that $\mathrm{Re}_{\max}$ and Re_ω plays somewhat similar roles Zhao and Cheng, (1998a). Due to the fact that the amplitude and frequency of oscillation are independent of each other and the governing equations presented in Eq. 9 and Eq. 17 are in terms of A_o and Re_ω, additional variables had been adopted by other literatures. In the studies where channel turbulence enhancements like baffles, rectangular and cylindrical cross channel block and gutters are adopted to enhance the heat transfer in the oscillating flow, the vorticity formulation in Eqs. (7) are mostly adopted by researchers, in which $u=(1/r)(\partial\Psi/\partial\theta)$ and $\upsilon=-\partial\Psi/\partial r$. These formulations remove pressure as an independent variable and allows coordinate transformations that can concentrate on mesh spacing and large mesh gradients. Eq. (14) is a non-dimensional form of Eq. (13) using ξ and η as transformation variables. Eq. (16) provides for characterizing heat dissipation using the Eckert number (Ec). Additional details on how the governing equations are reduced to the various forms are provided in (Zhao and Cheng, 1998b; Simon and Seume, 1988; Wang and Lu 2004; Iliadis and Anagnostopoulos 1998; and Zhang, Dalton, and Brown 1993)

3. FLUID DYNAMICS

3.1 Velocity Profile

Over time, Fourier, Laplace, Green and a number of other mathematical methods have been adopted for solving the governing equations. The chronologic development and the state of the art for analytical solutions of the oscillating internal flow have been presented Blythman et al., (2016). The earliest direct solution for the velocity profile of an oscillatory flow pipe was provided by Richard and Tyler (1929) and is given as follows:

$$u(r)=\frac{p_o}{i\omega\rho}\left[1-\frac{J_o(\lambda r)}{J_o(\lambda R)}\right] \quad (18) \qquad \lambda=\frac{i\omega\rho}{\mu}$$

$$Q_o=\frac{\pi R^2 p_o}{i\omega\rho}\left[1-\frac{2J_o(\lambda R)}{\lambda R J_o(\lambda R)}\right] \quad (19)$$

where J_o is the Bessel function of the first kind and is of zero-order Abramowitz and Stegun, (1972).

For real application, Eq. (18) and (19) need to be decomposed to yield the real part explicitly. Significant effort has been undertaken to visualize the flow and thereby verify the solution using various experiments (Harris, Peev, and Wilkinson, 1969; Walther et al., 1998; Simon and Seume, 1988). In Simon and Seume, (1988) plots of velocity profiles for laminar, incompressible, fully developed, and oscillating flow for eight dimensionless frequencies have been provided. Fig 4 shows the variation of velocity profiles in laminar oscillating pipe flow with time during one cycle Walther et al., (1998).

It is obvious from the figure that this flow is characterized by volatility and acute dependence on the oscillating nature of the flow. This can be attributed to viscous dissipation that occurs as a result of the change in the flow direction, and the oscillating flow transitions between different degrees of turbulence and laminar flows in a single cycle Iguchi, Ohmi, & Maegawa, (1982).

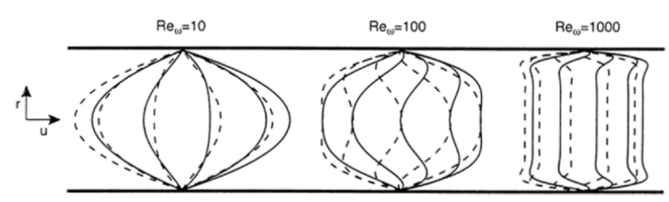

Fig. 4 Variation of velocity profiles in laminar oscillating pipe flow with time during one cycle. Fluid accelerated to the right and left shown respectively by the solid and dashed lines Walther et al., (1998)

For a fully developed laminar reciprocating flow in a circular pipe, an exact solution for the axial velocity profile has been presented (Uchida, 1964; Zhao and Cheng, 1998b):

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$$u = \left(\frac{kD^2}{4\alpha^2 \nu} \left[B\cos\omega t + (1-A)\sin\omega t \right] \right) \tag{20}$$

$Re_{\omega} \geq 16$

$$u_{max} = \frac{kD^2\sigma}{32\nu} \tag{21}$$

Where

$$A = \frac{ber(\alpha)bei(2\alpha R) - bei(\alpha)ber(2\alpha R)}{ber^2(\alpha) + bei^2(\alpha)} \tag{22}$$

$$B = \frac{ber(\alpha)ber(2\alpha R) - bei(\alpha)bei(2\alpha R)}{ber^2(\alpha) + bei^2(\alpha)} \tag{23}$$

$$\phi = \frac{\pi}{2}\left(\omega t - \Lambda\right) \tag{24}$$

$$\Lambda = \tan^{-1}\left(\frac{\alpha - 2C_1}{2C_2} \right) \tag{25}$$

$$E = \left[1 - \left(\frac{r}{R} \right)^2 \right] \sqrt{\frac{Re_{\omega}}{8}} \tag{26}$$

$$\sigma = \frac{8}{\alpha^3}\sqrt{\left(\alpha - 2C_1\right)^2 + 4C_2^2} \tag{27}$$

$$C_1 = \frac{ber(\alpha)bei'(\alpha) - bei(\alpha)ber'(\alpha)}{ber^2(\alpha) + bei^2(\alpha)} \tag{28}$$

$$C_2 = \frac{ber(\alpha)bei'(\alpha) + bei(\alpha)ber'(\alpha)}{ber^2(\alpha) + bei^2(\alpha)} \tag{29}$$

$R = r/D$ is the dimensional radial coordinate,
ber( )and bei( ) are kelvin functions,

$$ber'(x) = \frac{d(ber(x))}{d(x)} \quad \text{and} \quad bei'(x) = \frac{d(bei'(x))}{d(x)}$$

(Richardson and Tyler, 1929; Abramowitz and Irene. Stegun, 1972)

For reciprocating flow in a rectangular duct, the governing equation for the flow between two parallel planes, when there is a pressure gradient varying harmonically with time, is given by Landau and Lifshitz (1987):

$$\frac{\partial V}{\partial t} = ae^{i\omega t} + \nu \frac{\partial^2 V}{\partial y^2} \tag{30}$$

Subject to boundary conditions $v = 0$ for y = ± h. If the *x-z* plane is half-way between the two planes, the solution can be expressed by the following equations:

$$V = \frac{ia}{\omega} e^{i\omega t} \left(1 - \frac{\cos ky}{\cos \frac{kH}{2}} \right) \tag{31}$$

$$k = \frac{\alpha\left(i+1\right)}{H\sqrt{2}} \tag{32}$$

$$i\omega = \nu k^2 \tag{33}$$

Eq. (20) to Eq. (33) are given in terms of the Womersley number (α) defined in Eq. (34), which is related to the thickness of the 'Stokes layer' δ defined in Eq. (35). δ and α play a very important role in defining the velocity profile due to their relationship to ω that is a critical determinant of the generation and dissipation of the viscous forces in the flow.

$$\alpha = \frac{D}{2}\sqrt{\frac{\omega}{\nu}} \tag{34}$$

$$\delta = \sqrt{\frac{2\nu}{\omega}} = \frac{D\sqrt{2}}{\alpha} \tag{35}$$

Additional explicit decompositions of the real portion of the velocity profile are presented in Table 3. Eq. (36) and Eq. (40) are the velocity profiles for a two-dimensional oscillatory flow in a semi-infinite plane above a rigid wall as presented by Ramos et al. (2004). The solutions are based on the assumption that the velocity has no variation in the perpendicular direction to the wall. For oscillating pressure-driven flow, Eq. (37) is the mean flow over a cross section of a channel based on Eq. (31). Eq. (38) and Eq. (39) are limiting cases of Eq. (37) with simplifications based on the width of the channel. Similarly, Eq. (41) and Eq. (42) are limiting cases of Eq. (20) with simplifications based on the oscillating frequency of the flow. Eq. (44) is the average velocity for turbulent oscillating pipe flow. Eq. (43) is a solution for the pressure driven oscillating pipe flow between two parallel plates derived using the Green theorem decomposition. A three-dimensional solution of the governing equation of oscillating flow in the r- θ -x coordinates is presented by Xiao et al. (2014), which was obtained using Maple soft

Table 3 Different velocity equations for oscillatory flow.

Equations		Parameters	
$u(y,t)=U_\infty\left[\cos(\omega t)-\left(\cos(\omega t)-\left(\frac{\omega}{2\nu}\right)^{1\backslash 2}y\right)\exp^{\left[-\left(\frac{\omega}{2\nu}\right)^{1\backslash 2}y\right]}\right]$	(36)	oscillatory boundary layer flow	Ramos et al., (2004)
$\overline{u(y,t)}=\frac{iP_x}{\omega}e^{-i\omega t}\left(1-\left(\frac{2}{\frac{h(1+i)}{\delta}}\tan\left[\frac{1}{2}\frac{h(1+i)}{\delta}\right]\right)\right)$	(37)		Landau and Lifshitz, (1987)
$\overline{u(y,t)}=P_x e^{-i\omega t}\left(\frac{h^2}{12\nu}\right)$	(38)	$\frac{h}{\delta}\ll 1$	
$\overline{u(y,t)}=\frac{iP_x}{\omega}e^{-i\omega t}$	(39)	$\frac{h}{\delta}\gg 1$	
$U(Y,\tau)=e^{-\eta}\cos(\tau-\eta)$	(40)	$\eta=\frac{y}{\delta}$; $U(Y,\tau)=\frac{u(y,t)}{u_{\max}}$ Flat plate with infinite length	Zhao and Cheng, (1998b)
$U(r,\tau)=\frac{2\cos\tau}{a\sigma}\left[1-\left(\frac{r}{a}\right)^2\right]+\frac{1}{32a\sigma}\mathrm{Re}_\omega\left[\left(\frac{r}{a}\right)^4+4\left(\frac{r}{a}\right)^2-5\right]\sin\tau$	(41)	$\mathrm{Re}_\omega<16$; $U(r,\tau)=\frac{u(r,t)}{u_{\max}}$	
$U(r,\tau)=\frac{32}{a\sigma\,\mathrm{Re}_\omega}\left[\sin\tau-\frac{e^{-E}}{\sqrt{r/a}}\sin(\tau-E)\right]$	(42)	$\mathrm{Re}_\omega>16$ $U(r,\tau)=\frac{u(r,t)}{u_{\max}}$	
$U'(y,z,t)=-\frac{16\nabla P}{\rho\pi^2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{Z}{(2m+1)(2n+1)}\times\frac{\nu\beta\cos(\omega t)+\omega\sin(\omega t)-\nu\beta e^{\nu\beta t}}{\nu^2\beta^2+\omega^2}$	(43)	$Z=\sin\left(\frac{(2m+1)\pi y}{a}\right)\sin\left(\frac{(2n+1)\pi z}{b}\right)$ $\beta=\left(\frac{(2m+1)\pi}{a}\right)^2+\left(\frac{(2n+1)\pi}{b}\right)^2$	Blythman et al., (2016)
$\lvert\bar{u}\rvert=\frac{1}{\kappa}\left(\frac{\mathrm{Re}_R}{\mathrm{Re}_A}\right)^{0.25}\left[0.25\ln(\mathrm{Re}_A)+0.75\ln(\mathrm{Re}_R)+1.92\right]$	(44)	$0.38\le\kappa\le.47$ κ = von Kármán constant $\mathrm{Re}_A=\frac{2\pi fA^2}{\nu}$ $\mathrm{Re}_R=\frac{2\pi fR^2}{\nu}$	Su, Davidson, and Kulacki, (2012)

3.2 Transition to turbulence

The answer to the question of where the transition from laminar to turbulent flow would occur in oscillating flow is a complex one as there are varying approaches for flow description. When one looks at the classical flow description for oscillating flow, some describe the flow in terms of Reynolds number based on linear velocity (Re), Reynolds number based on angular velocity (Re_ω), Stokes number, dimensionless oscillation amplitude of fluid (A_o), Wormersley number (α), depth of penetration (δ), Strouhal number (St), Valensi number (Va), or at times critical number that is a combination of any of the listed numbers herein.

$\mathrm{Re}_c=305\left[\frac{D}{\delta_\nu}\right]^{1/7}$	(45)	Ohmi and IguchI, (1982)
$A_o\sqrt{\mathrm{Re}_\omega}>761$	(46)	Zhao and Cheng, (1996a)

However, there is a significant lack of agreement in the definition of the entrance length, transition values from laminar to turbulent flow, and

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definition of the transition flow Simon and Seume, (1988). Additionally, oscillating flow is similar but fundamentally different from pulsating flow; however, they are used interchangeably in literatures. When oscillation is superimposed on a non-vanished steady velocity, the flow is called pulsating flow that may be different from the oscillating flow herein. The most popular nomenclature for describing the transition to turbulence in oscillating flow is the critical value, $\beta_{cri} = A_o\sqrt{\mathrm{Re}_\omega}$. The mechanism of transition to turbulence and the interplay of the transition have been presented by a number of authors. Detailed experimental investigation as well as a table of the different transition points recorded by the various investigations is provided by (Akhavan, Kamm, and Shapiro, 1991; Ohmi and Iguchi, 1982; Zhao and Cheng, 1996a, 1998b) and more recently by Xiao et al., (2014). Eq. (45) and Eq. (46) are two widely adopted conditions for the transition to turbulence.

4. MASS OF VAPORIZATION (m_V)

For an oscillating flow typical of the RMDHL, power is required to move the working fluid around the loop for overcoming friction. In the case of phase change, additional power is required to overcome the inertia and body forces that arise as a result of instantaneous expansion during evaporation at the hot plate and gradual contraction at the condenser.

m_v is required to adequately quantify the power consumption during phase change. The exact characteristics of the thermodynamic cycle of the RMDHL are still unknown because the rate of vapor generation and condensation at the evaporator and condenser is not constant. Twice within the cycle, the rate of pressure change and the velocity become zero. At these points, the simultaneous heating and pressure rise of the fluid in the evaporator becomes a rather complex process. These points in the process are characterized by a rate of phase change more rapid than the phase change at any other point within the cycle. These points will be accompanied by isentropic pumping of the working fluid due to the phase change as well as the superheating of the saturated vapor generated. Even though the exact nature of the mass vaporization and condensation have not been measured, these points have been captured Popoola, Bamgbade, and Cao, (2016) and significant efforts have been made to quantify m_v West, (1982) as shown in Eq.(47) and Eq. (48) for a two-phase, two-component Stirling engine with controlled evaporation. Fig. 5 shows the expected pattern of the relative phasing of cylinder volume, the mass of vapor, and evaporation condensation rate West, (1982) in an oscillating flow.

$$m_v(t) = \frac{P_m V_e}{R_v T_e}\frac{\sqrt{k_c^2-1}}{2}\frac{1+\cos(\omega t)}{k_c+\cos(\omega t)} \quad (47)$$

$$k_c = 1 + 2\frac{V_{cd}}{V_c}$$ West, (1982)

$$m_v(\max) = \frac{P_m V_e}{R_v T_e}\frac{k_c-\cos(\omega t)}{\sqrt{k_c^2-1}} \quad (48)$$

where, V_{cd} = unswept volume of the cold end of the engine, V_c = volume stroke of the (cold) compression piston, T_e = temperature of the evaporator and expansion cylinder, P_m = mean pressure of working fluid and R_v = gas constant for the vapor component of the working fluid.

5. PRESSURE DROP

For typical regenerative heat transfer systems, understanding the pressure drop within the system is required to accurately predict the thermodynamic equilibrium, uniformity of temperature through the loop, and phase change. Some of the analytical correlations for the prediction of the pressure drop in a reciprocating loop are given in Table 4, Eq. (49) to Eq. (52). Eq. (49) presents a differential equation for approximating the non-dimensional pressure drop $p/\rho u_{\max}^2$ in a fully developed laminar oscillating pipe. In addition, the pressure drop in turbulent oscillating flow was also solved using a phase averaging technique Walther, Kühl, and Schulz, (2000). Fig 6 compares the solution of Eq. (49) with this "phase-averaging" method for values of Re_{max} = 30000 and R_ω = 800. Eq. (50) is a solution for the instantaneous value of the pressure

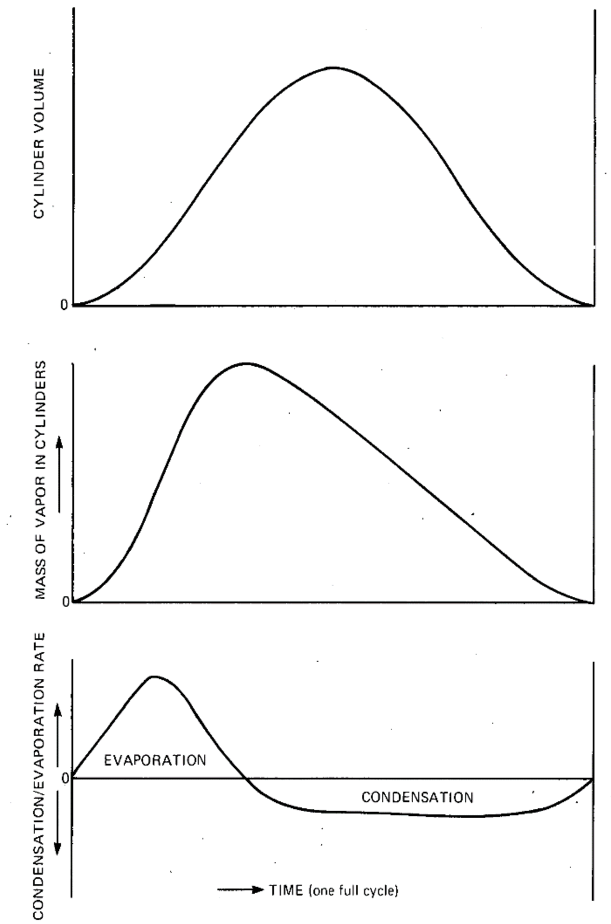

Fig. 5 Relative phasing of cylinder volume, the mass of vapor, and evaporation condensation rate West, (1982).

drop per unit length in the axial direction of the flow (Schwendig 1992; Barreno et al., 2015). Eq. (51) was derived for laminar reciprocating flow under the conditions of and while Eq. (52) is for turbulent reciprocating flow under the conditions of Barreno et al. (2015). For a certain combination of flow parameters, a number of research works have been able to separate the frictional loss components from the pressure drop. The cycle averaged friction coefficient is presented in Table 5, Eq. (53) to Eq. (58) while the cycle averaged friction factor is presented in Eq. (59) and Eq. (60).

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Table 4 Different governing equations for pressure.

Pressure Equations		Parameters	
$-2\frac{\partial\left(p/\rho u_{\max}^2\right)}{\partial\left(x/d_h\right)}=\frac{2}{\lvert s(\eta)\rvert}\frac{\text{Re}_\omega}{\text{Re}_{\max}}\sin\left(\omega t+\frac{\pi}{2}-\beta\right)$	(49)	$\tan(\beta)=\frac{im(s(\eta))}{real(s(\eta))}$; $s(\eta)=\frac{I_2(\eta)}{I_0(\eta)}$; $\eta=\frac{1}{1}\sqrt{i\,\text{Re}_\omega}$	Walther, Kühl, and Schulz, (2000)
$-\left(\frac{\Delta p}{\Delta L}\right)=\frac{32\mu}{D^2}\left(K_{re}(\text{Re}_\omega)+i\frac{\text{Re}_\omega}{32}K_{im}(\text{Re}_\omega)\right)U_o e^{i(\omega t)}$	(50)	$K_{re}(\text{Re}_\omega)=1+.0017\,\text{Re}_\omega^{1.3}-.0004075\,\text{Re}_\omega^{1.5}$; $+.000001642\,\text{Re}_\omega^2$ $K_{im}(\text{Re}_\omega)=1+\frac{1}{3\left(1+0.0030r\,\text{Re}_\omega-0.0000003689\,\text{Re}_\omega^2\right)}$ $\text{Re}_\omega\le 1000$	Schwendig, (1992)
$\left(\frac{\Delta p}{\Delta L}\right)=\frac{\mu U_0}{D^2}\left(\text{Re}_\omega\cos(\omega t)+(32+f_1)\left(\frac{120}{L/D}\right)^{0.01}\sin(\omega t+\phi)\right)$	(51)	$f_i=0.19416\,\text{Re}_\omega-0.000575\,\text{Re}_\omega^{1.75}$; $\beta=\left(A_0\sqrt{\text{Re}_\omega}\right)$ $\phi=\phi_1\frac{1}{2}\tan^{-1}\theta\left(\frac{\text{Re}_\omega}{32+1.67\,\text{Re}_\omega^{0.59}}\right);25\le\beta\le 350$ $\phi=\frac{\phi_1}{650}\left(1000-A_0\sqrt{\text{Re}_\omega}\right);350<\beta\le 1000$ $25\le\text{Re}_\omega\le 600\ ;1\le A_o\le 600;750\le\beta\le 350$	Barreno et al., (2015)
$\left(\frac{\Delta p}{\Delta L}\right)=\frac{\mu U_0}{D^2}\left(\begin{matrix}\text{Re}_\omega\cos(\omega t)+\\ 2\,\text{Re}_{\max}^{0.75}\,0.0791\left(\frac{120}{L/D}\right)^{0.02}\text{f}_2\sin^{1.75}(\omega t+\phi)\end{matrix}\right)$	(52)	$f_i=0.19416\,\text{Re}_\omega-0.000575\,\text{Re}_\omega^{1.75}$; $\beta=\left(A_0\sqrt{\text{Re}_\omega}\right)$ $\phi=\phi_1\frac{1}{2}\tan^{-1}\theta\left(\frac{\text{Re}_\omega}{32+1.67\,\text{Re}_\omega^{0.59}}\right);25\le\beta\le 350$ $\phi=\frac{\phi_1}{650}\left(1000-A_0\sqrt{\text{Re}_\omega}\right);350<\beta\le 1000$ $25\le\text{Re}_\omega\le 600\ ;1\le A_o\le 600;750\le\beta\le 3000$	

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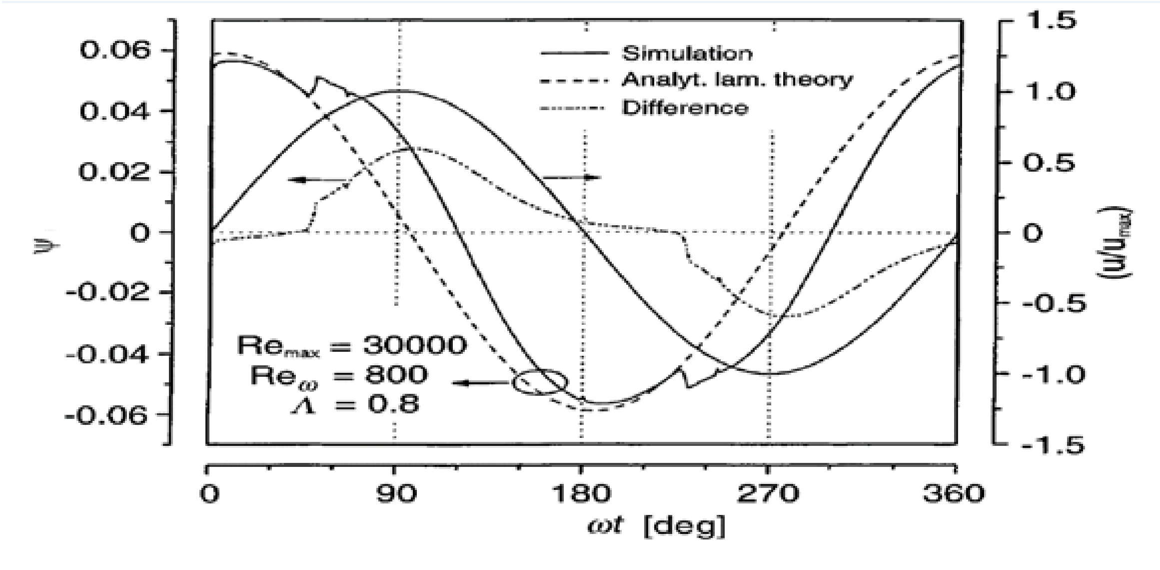


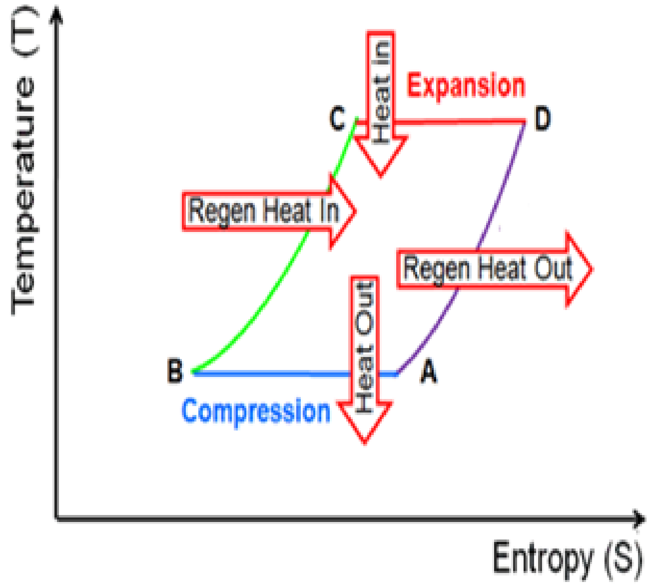
Fig. 6 Pressure drop in turbulent oscillating pipe flow compared to laminar theory Walther, Kühl, and Schulz, (2000)

Table 5 Different frictional coefficient formula.

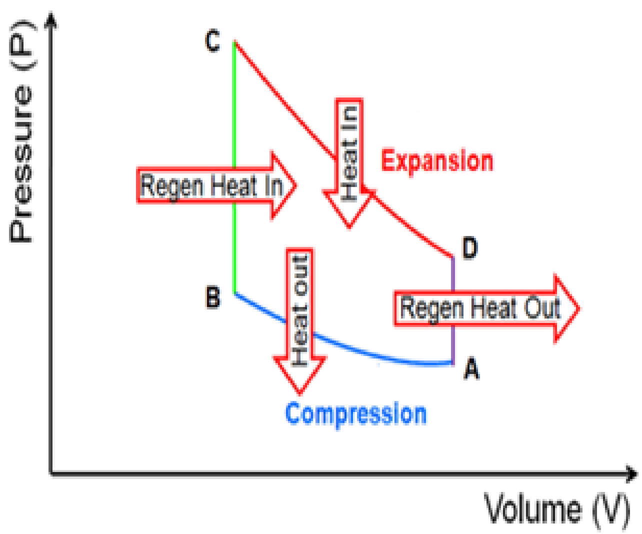
$\left(C_{f,e}\right)_m = \dfrac{3.272192}{A_o(\mathrm{Re}_\omega^{0.548} - 2.03946)}$	(53)	$23 < \mathrm{Re}_\omega < 394$ *and* $0 < A_o < 26.4$	Zhao and Cheng, (1996a)
$\left(C_{f,e}\right)_m = \dfrac{1}{A_o}\left(\dfrac{76.6}{\mathrm{Re}_\omega^{1.2}} + 0.40624\right)$	(54)	$81 < \mathrm{Re}_\omega < 540$ *and* $53.4 < A_o < 113.5$	
$\left(C_{f,e}\right)_m = \dfrac{2\omega^2 h}{\sqrt{2}P_o\alpha\left(c^2+d^2\right)}q$	(55)	$q = e^{\frac{\alpha}{\sqrt{2}}}\left(\left(c+d\right)\sin\left(\omega t - \frac{\alpha}{\sqrt{2}}\right) + \left(c-d\right)\cos\left(\omega t - \frac{\alpha}{\sqrt{2}}\right)\right)$ $- e^{\frac{\alpha}{\sqrt{2}}}\left(\left(c+d\right)\sin\left(\omega t + \frac{\alpha}{\sqrt{2}}\right) + \left(c-d\right)\cos\left(\omega t - \frac{\alpha}{\sqrt{2}}\right)\right)$ $c = e^{\frac{\alpha}{\sqrt{2}}} + e^{-\frac{\alpha}{\sqrt{2}}}$ $d = e^{\frac{\alpha}{\sqrt{2}}} - e^{-\frac{\alpha}{\sqrt{2}}}$	Karagöz and Karagoz, (2002)
$\left(C_{f,e}\right)_m \approx 2St$	(56)	Approximation for all Reynolds number range $St = 0.023\,\mathrm{Re}^{-0.2}\,\mathrm{Pr}^{-0.4}$	Allan, (2014)
$\left(C_{f,e}\right)_m = \dfrac{4}{\sqrt{\left(\mathrm{Re}_\omega\right)_x}}\sin\left(\omega t - \dfrac{\pi}{4}\right)$	(57)	Flow over infinite flat wall	
$\left(\overline{C}_{f,e}\right)_m = \dfrac{3.774}{A_o\left(\mathrm{Re}_\omega^{0.543} - 2.20863\right)}$	(58)	$15 < \mathrm{Re}_\omega < 400$; $8 < A_o < 30$	Zhao and Cheng, (1998a)
$\left(f\right)_m = \dfrac{0.1392}{\mathrm{Re}_\omega^{0.25}}$	(59)	$2800\sqrt{\omega'} < \mathrm{Re}_\omega$ *and* $4 \le \sqrt{\omega'} \le 24$	Ohmi,Iguchi, (1982)
$\left(f\right)_m = \dfrac{0.0791}{\mathrm{Re}_\omega^{0.25}}$	(60)	$\mathrm{Re}_\omega \le 1000$ *and* $3000 \le \mathrm{Re} \le 100,000$	Barreno et al., (2015); Schlunder, (1990)

6. THERMODYNAMIC PRINCIPLES

The discription of the thermodynmics of oscillating flow is closely approximated to the thermodynamic cycle of Sterling engine. This is based on the assumption that the entire system can be modeled as a cylinder with pistons at both ends, with the high-temperature sink located in the midsection and the cold heat sink distributed at both ends Tziranis, (1992). And as with all heat engines, it will have 4 processes, heating



(a)

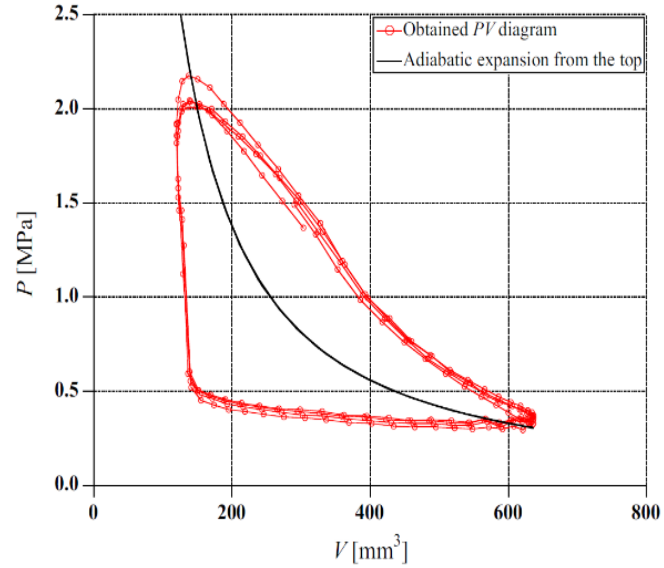


(b)

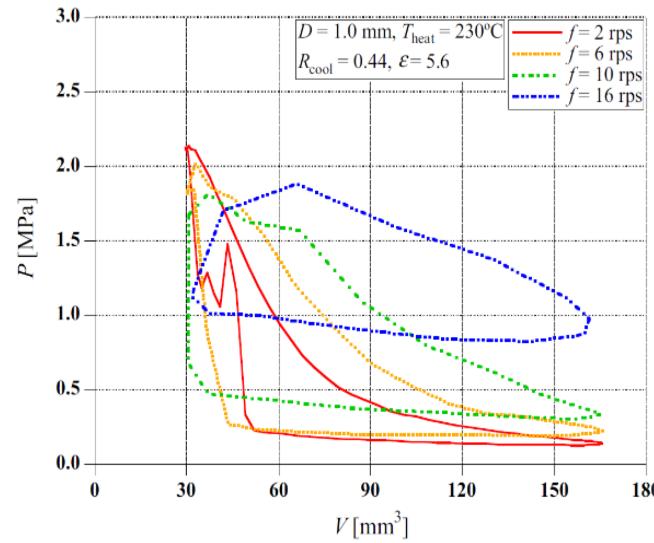
Fig. 7 Thermodynamic cycles for Oscillating cooling (a) *P-V* , (b) *T-S* Woodbank Communications Ltd, (2005; Tziranis, (1992)

accompanied by expansion and cooling associated with compression as well as the regenerative heat in and regenerative heat out.

The idealized stages are presented in Fig 7. Several aspects of the cycle are yet to be adequately quantified as the cycle has a critical dependence on the oscillating frequency as shown in Figs. 8a and 8b



(a)



(b)

Fig 8 P-V diagram two phase oscillating flow Kanno, Han, and Shikazono, (2017)

7. HEAT TRANSFER

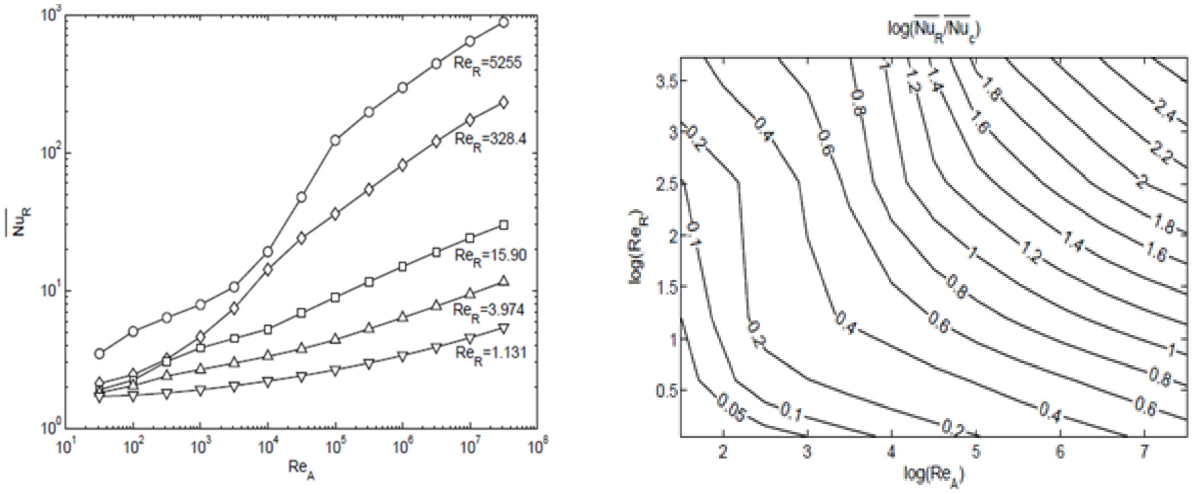
The principle of operation of the RMDHL for cooling application can be theoretically described as heat transfer in a periodically reversing flow between two temperature reservoirs. The two reservoirs (the hot and cold reservoirs) can either be at a constant heat flux or constant temperature. The current thermal management applications developed for the RMDHL are such that the hot reservoir has a constant heat flux while the cold reservoir is approximated to be at a constant temperature.

Though the configuration of the RMDL is simple, the analysis of the thermodynamics of operation, the fluid dynamics of two-phase oscillating flow, and sensible/latent heat transfer are quite complicated Popoola and Cao, (2016). Eq. (61) to Eq. (82) in Table 6 provide the most applied Nu correlations for oscillating pipe flow, some of which include the complex Nusselt number Nu_i proposed by the researchers from MIT. In addition, certain experimental data are plotted in Fig. 9, which would serve as alternative predictions of *Nu* for the analytical equations cited in this paper.

Table 6 Different Nusselt number correlation for oscillatory flow.

Equations		Parameters	
$Nu = \alpha W_0^{0.2} + \frac{\beta(x_{max}/L_h)W_0^2 \Pr}{1+\gamma\left[(x_{max}/L)W_0^2 \Pr\right]^{2/3}}$	(61)	$W_0 = \frac{d_i}{2}\sqrt{\frac{\omega}{\nu}}$ where α , γ and β are constants depending on heating/cooling regions as prescribed by Shin and Nishio, (1998)	Shin and Nishio, (1998)
$Nu = 1.32 \mathrm{Re}_\omega^{0.248} A_o^{0.85}$	(62)	$\Pr \approx 0.7;\ 1000 < \mathrm{Re}_\omega < 4000$ *and* $7.73 < A_o < 12.88$	Akdag and Ozguc, (2009)
$Nu = .0162 \mathrm{Re}_\omega^{0.4} A_o^{0.85}$	(63)	$\Pr \approx 0.7;\ 740 \le \mathrm{Re} \le 4110$ *and* $12 \le \mathrm{Re}_\omega \le 71$	Xiao et al., (2014)
$Nu = .02 \mathrm{Re}_\omega^{0.58} A_o^{0.85}$	(64)	$\Pr \approx 0.7;\ 23 < \mathrm{Re}_\omega < 464$ *and* $8.54 < A_o < 34.9$	Zhao and Cheng, (1996b)
$Nu = 0.00495 \mathrm{Re}_\omega^{0.656} A_o^{0.9}\left[43.74(D/L)^{1.18} + 0.006\right]$	(65)	$\Pr \approx 0.7;\ 10 < \mathrm{Re}_\omega < 400$ *and* $10 < A_o < 35$	Zhao and Cheng, (1995)
$Nu = 1 + \sqrt{\Pr}\left(\frac{D}{1+D}\right)$	(66)	$\mathrm{Re}_\omega \to \infty$	Chen, Luo, and Dai, (2007)
$Nu = 0.548 \mathrm{Re}_\omega^{0.536} A_o^{0.3}$	(67)	Rectangular channel, one-sided heating $43 < \mathrm{Re}_\omega < 684$ *and* $23 < A_o < 600$	(Cooper, Nee, and Yang, 1994; Zhao and Cheng, 1998b)
$Nu = 12.38 \mathrm{Re}_\omega^{0.31} A_o^{0.95}$	(68)	Porous Channel $15 \le \mathrm{Re}_\omega \le 900;\ 3.1 \le A_o \le 4\ \frac{L}{D_h} = 3$	Leong and Jin, (2005)
$Nu = \max\left\{0.000001846.\lvert \mathrm{Re}^* \rvert^{1.236} \times \left(1 + 104.7\left(\frac{L}{d_h}\right)^{0.1}\right); Nu_{\min}\right\}$	(69)	$Nu_{\min} = 11.66 + .00117\left(\mathrm{Re}_\omega^{0.562}\right)\left(\mathrm{Re}_{\max}^{0.812}\right)$ If $\lvert \mathrm{Re} \rvert \le 400\sqrt{\mathrm{Re}_\omega};\ Nu = Nu_{\min}$ Else for $2000 \le \mathrm{Re}_{\max} \le 50000$; $50 \le \mathrm{Re}_\omega \le 1000$; $0.2 \le \Lambda \le 1.0$ $\mathrm{Re}^* = \mathrm{Re}\cos(\phi);\ \phi = 2.948\left(\frac{\mathrm{Re}_\omega}{\mathrm{Re}_{\max}}\right)^{0.682}\ [rad]$	Walther, Kühl, and Schulz, (2000)
$Nu = -0.494 + 0.0777\left(\frac{A_\omega}{1+A_\omega}\right)^2 \mathrm{Re}^{0.7} - 0.00162.\mathrm{Re}^{0.4}\,\mathrm{Re}_\omega^{0.8}$	(70)	$A_\omega = \frac{\chi_{\max}}{L} = \frac{\pi.u_m}{\omega.L} = \frac{d}{L}\frac{\mathrm{Re}}{Re_\omega}$ $7 \le \mathrm{Re} \le 7000$ $7 \le \mathrm{Re}_\omega \le 180;\ 0.06 \le A_\omega \le 2.21$;	Xiaoguo and Cheng, (1993)
$Nu = -0.494 + 0.0777\left(\frac{A_R}{1+A_R}\right)^2 \mathrm{Re}^{0.7} - 0.00162.\mathrm{Re}^{0.4}\left(4\mathrm{Re}_\omega^{0.8}\right)$	(71)	$A_R = \frac{1}{4}\frac{d_h}{L}\frac{\mathrm{Re}_{\max}}{Re_\omega}$; $\mathrm{Re}_{\max} = \frac{u_{\max} d}{\nu}$; $11 \le \mathrm{Re}_{\max} \le 10995$; $\frac{d_h}{L} = 0.0185$; $1.75 \le \mathrm{Re}_\omega \le 45$; $0.2 \le A_R \le 1.13$	Monte et al., (1996)
$Nu = 5.78 + 0.00918 \mathrm{Re}_\omega^{0.969} \Lambda^{-0.367} + 0.178\left\lvert\frac{\mathrm{Re}}{\mathrm{Re}_{\max}}\right\rvert^{0.951} \Lambda^{-0.703} \mathrm{Re}_\omega^{0.526}$	(72)	$\Lambda = \frac{1}{2}\frac{L}{d}\frac{Re_\omega}{\mathrm{Re}_{\max}}$, $\mathrm{Re}_{\max} = \frac{u_{\max} d}{\nu}$ $1000 \le \mathrm{Re}_{\max} \le 400\sqrt{\mathrm{Re}_\omega}$, $50 \le \mathrm{Re}_\omega \le 900$; $0.2 \le \Lambda \le 0.9$	Walther et al., (1998)

$Nu = Nu_{\min} + \left(Nu_{\max} - Nu_{\min}\right)\left\|\sin\left(\omega t\right)\right\|$	(73)	$Nu_{\max} = Nu_G\left(1.1 + \left(\dfrac{10^8\,\mathrm{Re}^{-3.69} + \dfrac{10}{\left(L/D\right)^{1.32}} + 0.09395}{A_0^{6.67/\sqrt{\mathrm{Re}_\omega}}}\right)\right)$ $Nu_G = \dfrac{\left(\left(0.79\ln\mathrm{Re} - 1.64\right)^{-2}/8\right)\mathrm{Re}_{\max}\,\mathrm{Pr}}{1.07 + 12.7\left(f/8\right)^{0.5}\left(\mathrm{Pr}^{2/3} - 1\right)}$; $Nu_{\min} = \left(0.2815 + 0.145\,\mathrm{Re}_\omega^{0.6}\right) \times \left(1 + 0.3734\left(L/D\right)^{0.02} A_0^{0.4}\right)$ $100 \le \mathrm{Re}_\omega \le 600$; $900 \le A_0\sqrt{\mathrm{Re}_\omega} \le 3000$ $1 \le A_0 \le 600$;	Barreno et al., (2015)
$Nu = 0.372\,\mathrm{Re}_\omega^{0.78}$	(74)	vertical annular channel $A_0 = \dfrac{L}{D} = 7.73$; $\mathrm{Pr} = 4.65$; $1000 \le \mathrm{Re}_\omega \le 4000$	Akdag, Ozdemir, and Ozguc, (2008)
$Nu = 0.021\overline{\mathrm{Re}}_{sc}^{0.8}\,\mathrm{Pr}_o^{0.4}\left(\dfrac{T_w}{T_b}\right)^{-0.5}.C'$	(75)	$C' = 0.923 + 0.750\left(\dfrac{T_w}{1000}\right)$ Re_{sc}: Reynolds number (based on Schmidt cycle model Kanzaka and Iwabuchi, (1992)) $\mathrm{Pr} \approx 0.7$; $10 < \mathrm{Re}_\omega < 400$ *and* $10 < A_o < 35$	Kanzaka and Iwabuchi, (1992)
$Nu = 0.0035\,\mathrm{Re}_n^{1.3}\,\mathrm{Pr}_o^{1\backslash 3} + 0.3\left(\dfrac{\mathrm{Re}_o^{2.2}}{\left(\mathrm{Re}_n + 800\right)^{1.25}}\right)^{-0.5}$	(76)	$100 < \mathrm{Re}_n < 1200$; $\mathrm{Re}_n = \dfrac{UD}{\nu}$; $\mathrm{Re}_o = \dfrac{x_{\max}\omega D}{\nu}$	Mackley and Stonestreet, (1995)
$Nu = 0.4Pe + 0.315$	(77)	$5.1 < Pe < 16.6$; $40 < \mathrm{Re}_{\max} < 240$	
$Nu = 0.71Re^{0.38}\,\mathrm{Pr}^{0.4}$	(78)	$61.7 < \mathrm{Re} < 265.9$ U shaped tube	Ni et al., (2015)
$Nu = 0.71Re^{0..47}\,\mathrm{Pr}$	(79)	$448.5 < \mathrm{Re} < 3184.6$	
$Nu = \sqrt{\dfrac{Pe_\omega}{2}}\dfrac{\left[(1+i)\tanh z\right]}{\left[(1+i)\tanh z\right]}$	(80)	$z = (1+i)\sqrt{\dfrac{Pe_\omega}{32}}$; $Pe_\omega = \dfrac{\omega D^2}{\alpha}$	(Grassmyer,1994; Cheng, 1995)
$Nu_r = Nu_i = 0.56(Pe_\omega)^{0.69}$	(81)	$100 \le Pe_\omega \le 10{,}000$ $q_c'' = \dfrac{k}{D_h}\left[Nu_r\left(T_c - T_w\right) + \dfrac{Nu_i}{\omega}\dfrac{dT}{dt}\right]$	Grassmyer, (1994)
$Nu = -\dfrac{2\delta}{kT_w}\left(-I_1\cos t + I_2\sin t\right)$	(82)	$I_1 = \dfrac{\alpha - \gamma_R}{2} - \dfrac{\mathrm{Pr}}{2}\left(\dfrac{-\alpha + \gamma_i\alpha + \left(\alpha + 1 - \gamma_R\right)\left(1 - \mathrm{Pr} + \alpha\right)}{\alpha^2 + \left(1 - \mathrm{Pr} + \alpha\right)^2}\right)$ $I_2 = \dfrac{\alpha - \gamma_R}{2} - \dfrac{\mathrm{Pr}}{2}\left(\dfrac{\left(\alpha + 1 - \gamma_R\right)\alpha + \left(1 - \gamma_i\right)\left(1 - \mathrm{Pr} + \alpha\right)}{\alpha^2 + \left(1 - \mathrm{Pr} + \alpha\right)^2}\right)$ $\gamma_R + \gamma_i = \alpha^4\sqrt{1 + R_\omega^2\,\mathrm{Pr}^2\left(\cos\theta + i\sin\theta\right)}$ $\theta = R_\omega\,\mathrm{Pr}$ Oscillatory flow with sinusoidal wall temperature	Ramos et al., (2004)



(a) (b)

Fig. 9 Contour plot of Nusselt Number Su, Davidson, and Kulacki, (2011) (a) Averaged Nu_r vs. Re_A and (b) Contour plot of ratio of real Nu_r / Nu_i

8. CONCLUSIONS

Mechanically driven cooling loop systems have shown promising results for thermal management of computing, renewable energy, and electric vehicles. This study presents an up-to-date compilation of the research status associated with oscillating flow in different aspects. It shows that the mathematical equations for this kind of system are very complex, and the solutions are substantially different from those of unidirectional flow systems. The study also summarizes the system governing equations in different forms based on dimensionless coordinates and parameters. The detailed research efforts to visualize the flows and their validation by experimental works from different studies have been provided whereas the thermodynamic principle may be applied to the Stirling engine. Comprehensive heat transfer correlations that are fundamental to oscillating-flow systems are assembled, which may be used for system designs and future research in the field.

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NOMENCLATURE

a	width (m)
A_0	dimensionless oscillation amplitude (x_{max} / D)
A	function defined in Equation 22
A_R	function defined in Equation 71
B	function defined in Equation 23
c	function defined in Equation 55
C_1	function defined in Equation 28
C_2	function defined in Equation 29
$C_{f,e}$	cycle-averaged friction coefficient
C'	corrective term expressed in Equation 75
d	function defined in Equation 55
D	width (m)
E_c	Eckert Number
E	function defined in Equation 26
f_i	low Reynolds number function
h	width (m)
I_1	function defined in Equation 82
I_2	function defined in Equation 82
J_o	Bessel function
k	function defined in Equation 32
k_f	thermal conductivity (W/m.k)
k_c	function defined in Equation 47
L	length(m)
m_V	mass of vaporization (kg)
Nu	Nusselt number
$Nu_{\min}$	minimum Nusselt number
$Nu_{\max}$	maximum Nusselt number
Pe	Peclet number
Pe_ω	oscillating flow Peclet number
Δp	pressure difference (N/m^2)
$\Pr$	Prandtl number
q''	heat flux (W/m^2)
q	function defined in Equation 32
Re_ω	Kinetic Reynolds number
Re_{max}	maximum Reynolds number
Re_{sc}	Reynolds number (based on Schmidt cycle model)
Re_n	Reynolds number defined in Equation 76
Str	Strouhal number
St	Staton Number
$\overline{T}$	Resolved temperature (K)
T_w	inner surface temperature of heat exchanger tube (K)

T_L	temperature at cold end (K)
T_H	temperature at hot end (K)
u_m	mean velocity (m/s)
u_{max}	maximum velocity (m/s)
U_0	amplitude of the cross-sectional mean velocity
V_c	volume stroke of the compression piston
W_0	Womersley number ($R(\omega/\nu)^{0.5}$)
x	distance (m)
z	function defined in equation 80

Greek Symbols

α	thermal diffusivity (m/s^2)
α	Womersley number
β	function defined in Equation 43
β	dimensionless parameter indicator of reciprocating flow type $A_0\sqrt{\mathrm{Re}_\omega}$
ρ	density (kg/m^3)
ν	kinematic viscosity (m^2/s)
η	dimensionless lateral coordinates
V_a	Valensi number
Ψ	stream function
ϕ	phase angle
θ	dimensionless temperature
σ	function defined in Equation 27
δ	Stokes layer
ω	oscillation frequency
τ	dimensionless time
Λ	function defined in equation 72

Superscripts

–	axis averaged value

Subscripts

max	maximum value
min	minimum value

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