



# THERMAL TOPOLOGY OPTIMIZATION DESIGN OF SPINDLE STRUCTURE WITH A HYBRID CELLULAR AUTOMATON METHOD

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## ABSTRACT

A hybrid cellular automaton model combined with a finite element method for thermal topology optimization of spindle structure is developed. The higher order 8-node element and von Neumann strategy are employed for the finite element and the cellular element, respectively. The local sensitivity filtering algorithm and the weight approach are applied. The four validating studies of two-dimensional structure for thermal topology optimization are carried out. The structure evolution and thermal distribution evolution of thermal topology optimization are investigated. The results show the developed hybrid method is more efficient for thermal topology optimization. Meanwhile, the thermal topology optimization for spindle structure can save structural volume under the same condition.

**Keywords:** Hybrid Cellular automaton Model, Spindle Structure, Thermal Topology Optimization

## 1. INTRODUCTION

During high-precision machining processes, thermal error control plays an important role because the thermal error that seriously restricts product quality of high-precision manufacturing industry is about 40% to 70% of the total machining error (Deng *et al.* (2018)). As the required machining precision increases, error compensations become effective methods of thermal error control to improve the machining accuracy of computer numerical control (CNC) machine tools (Yang *et al.* (1999), Pahk *et al.* (2002), Wu *et al.* (2008)), particularly the ultra-precision high-speed machines. For the past few decades, error compensations for the thermal error have been implemented for CNC machine tools by many researchers with numerical and experimental methods, especially for its core structure named as the spindle system.

Mori *et al.* (2009) derived an approach based on finite element method and Taguchi method for a headstock structure optimization design of numerical control lathe to minimize the thermal displacement of the spindle center position. Kimman *et al.* (2010) studied a miniature milling spindle with active magnetic bearings using experiments based on negative stiffness compensation. The results demonstrated that the spindle with a modal controller and a cross-feedback controller could reach to a maximum speed of 150000 rpm with high precision. A machining state monitoring method in high spindle speed system considering active thermal deformation compensation function and a remote operation function was presented by Mitsuishi *et al.* (2001). Li *et al.* (2009) investigated the thermal characteristics of the CNC machine spindle and reduced the thermal error of the spindle system by two approaches of rational arrangement of the heat-dissipating rib plates and parametric optimization design of finite element model. Liang *et al.* (2014) carried out a novel expert hydro/aero-static spindle design system taking thermal error into account. It demonstrated the design system enabled the structure of machine tools to be designed efficiently

with a higher precision. Xia *et al.* (2015) investigated numerically fractal tree-like channels network heat sink in spindle's cooling sleeve by a three-dimensional thermal and hydrodynamic model with thermal effects of conjugate heat transfer in the channel walls.

As one of error compensations, topology optimization can be fundamentally implemented to improve thermal performance of CNC machine tools during the product design stage. Topological optimization is a design method that optimizes the structure of the design area according to the given external load and boundary conditions on the premise of satisfying the constraints of stress, balance and displacement. It is necessary to improve the thermal characteristics of the spindle (such as temperature field, thermal deformation, thermal stress, etc.) through the thermal design theory and technology in the design stage, so as to reduce the thermal error of the machine tool, improve the machining accuracy, reduce the experimental research and prototype production cost. Many efficient and accurate topology optimizations were presented by the numerical models based on cellular automaton algorithm (Sanaei *et al.* (2011), Zakhama *et al.* (2009), Du *et al.* (2013)). Due to the significant challenges of the nonlinearity and numerical instability of cellular automaton algorithm, hybrid cellular automaton method (HCAM) coupling cellular automaton algorithm and finite element model was proposed by Tovar *et al.* (2004, 2005). It was confirmed that HCAM had a fast convergence performance and high calculating accuracy (2006, 2007). Duddeck *et al.* (2016) investigated topology optimization for carshworthiness of thin-walled structures under axial impact using HCAM. Penninger *et al.* (2010) founded the HCAM convergence of topology optimization was affected by parameters of the algorithm.

As the core structure of the CNC machine tools, the spindle system is critical to the cutting speed and machining accuracy, which is one of the most important factors restricting the accuracy. Therefore, it is necessary to reduce the thermal error of the spindle system at the design

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stage. In present study, thermal topology optimization design of spindle structure with HCAM is presented. The verifiable investigations of the developed HCAM are carried out for two-dimensional structures. Furthermore, the optimized analysis of the spindle system is obtained based on the HCAM.

## 2. PROBLEM STATEMENT AND FORMULATION

### 2.1 Problem Formulation

Fig. 1 shows three-dimension and two-dimension physical model for the spindle structure. The cylindrical spindle with 60mm diameter and 120mm length is subject to the working environment with uniform temperature of 20 °C. As the figure shown, there are 5mm bearings at both ends of the spindle. Due to bearings' friction during the high-speed revolving process, a lot of friction heat conduct into the spindle. Therefore, they are heat sources of the spindle structure. In the two-dimension model of present study, the heat sources are simplified as linear sources with heat flux  $q_i = 5 \text{ W/m}$ .

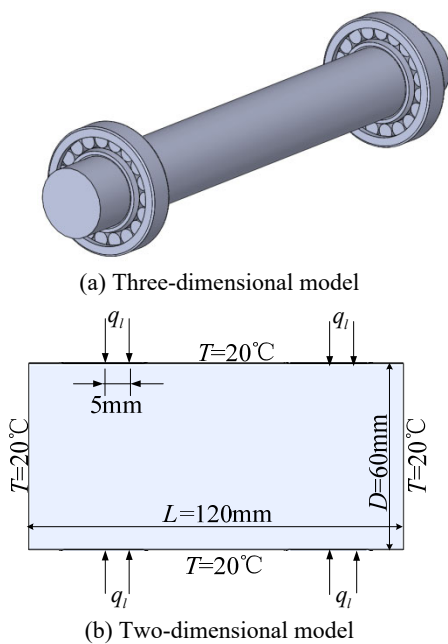


Fig. 1 Physical model of the spindle structure

Due to thermal stress caused by the friction heat, the performance of the spindle at high speed is changed and thermal error emerges. Furthermore, the more friction heat, the more thermal stress and error. It may lead to the unqualified machining products. What is far worse than low precision, the thermal stress can exceed the material's yield strength as the temperature increases. Therefore, it should take thermal effects into consideration for topology optimization of spindle structure. According to the linear thermal stress theory (Hetnarski *et al.* (2009)), the governing equations for the spindle structure can be expressed as follows.

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) \quad (1)$$

Boundary conditions for thermal and mechanical equations can be obtained as,

$$T=293.15 \text{ at } x=0 \text{ or } x=120 \quad (2)$$

$$T=293.15 \text{ at } y=0 \text{ or } y=60 \quad (3)$$

$$q_i = 5 \text{ at } 10 \leq x \leq 15 \text{ and } 0 \leq y \leq 60 \quad (4)$$

$$q_i = 5 \text{ at } 105 \leq x \leq 110 \text{ and } 0 \leq y \leq 60 \quad (5)$$

### 2.2 HCAM Statement

To obtain the global optimum solution of the spindle structure with thermal effects, HCAM coupling cellular automaton algorithm and finite element model is developed for the thermal topology optimization. In present study, the material thermal conductivity  $\lambda_i(t)$  and heat flux  $\Phi_i(t)$  as design variable and state variable of optimization method, respectively. The optimization goal is minimizing the difference between average heat flux  $\bar{\Phi}_i(t)$  and setting point of state variable  $\Phi^*(t)$  based on the design variable. Therefore, the objective function for the HCAM topology optimization problem can be given by,

$$\begin{cases} \min |e_i(t)| = \left| \bar{\Phi}_i(t) - \Phi^*(t) \right| \\ \text{s.t. } 0 < \lambda_{\min} \leq \lambda_i(t) \leq 1, \quad i = 1, 2, \dots, N \end{cases} \quad (6)$$

where  $e_i(t)$  is the difference between average heat flux  $\bar{\Phi}_i(t)$  and setting point of state variable  $\Phi^*(t)$ . The subscript sequence  $i$  represents the cellular number. To avoid singular matrix during the optimization process, the prescribed minimum of the material thermal conductivity should be set. Here, the prescribed minimum  $\lambda_{\min} = 10^{-1}$ .

The average heat flux of each cellular determined by it and its neighboring cellars can be obtained by,

$$\bar{\Phi}_i(t) = \frac{\Phi_i(t) + \sum_{j=1}^{\Delta N} \Phi_j(t)}{\Delta N + 1} \quad (7)$$

where  $\Delta N$  is the number of neighboring cellars.

The weight approach for setting point of state variable is applied to accelerate the convergence of the topology optimization as following,

$$\Phi^*(t) = aif \cdot \frac{Q_i}{S_0} \quad (8)$$

where  $aif$  is the weight coefficient  $Q_i$ ,  $S_0$  is the total heat of current iteration and the initial structure area, which can be calculated by

$$Q_i = \sum_{i=1}^N \Phi_i(t) S_i \quad (9)$$

$$S_0 = \sum_{i=1}^N S_i \quad (10)$$

where  $S_i$  is the area of each cellular. In present study, heat flux is obtained by

$$\Phi_i(t) = \lambda_i(t) \frac{T_i^h(t) - T_i^c(t)}{d_i} \quad (11)$$

where  $T_i^h(t)$ ,  $T_i^c(t)$  are the hot surface temperature and cold surface temperature of the  $i$ -th cellular, respectively.  $d_i$  is the distance between the hot surface and the cold surface.

During the topology optimization process, the optimization goal of state variable is obtained based on the control rules of the design variable. For the present HCAM optimization, the heat flux results of Eqs. (1)~(5) are obtained based on the thermal conductivity by the

finite element method, and then the thermal conductivity is renewed as the cellular automaton control rules based on the heat flux. Under preceding cycle calculations, an optimal topology structure is presented while the optimization goal is satisfied. HCAM is a gradient-free optimization algorithm, which does not need sensitivity analysis in the calculation process, uses FEM to calculate and evaluate the field state (strain energy), and successfully solves the problem of global analysis of field state that CA method does not solve. In present study, the control rules of the design variable is expressed by reverse control strategy as,

$$\lambda_i(t) = \begin{cases} 1 & \text{if } \Phi_i(t) \geq \Phi^*(t) \\ 0.75 & \text{if } \Phi^*(t) > \Phi_i(t) \geq \frac{9}{10}\Phi^*(t) \\ 0.5 & \text{if } \frac{9}{10}\Phi^*(t) > \Phi_i(t) \geq \frac{8}{10}\Phi^*(t) \\ 0.25 & \text{if } \frac{8}{10}\Phi^*(t) > \Phi_i(t) \geq \frac{7}{10}\Phi^*(t) \\ 1/\infty & \text{if } \frac{7}{10}\Phi^*(t) > \Phi_i(t) \end{cases} \quad (12)$$

### 2.3 Numerical Solution

During the HCAM iteration process, the heat flux and the material thermal conductivity are renewed alternately by finite element method and cellular Automata algorithm. Considering the convergence of the calculating results and convergence efficiency, the criterion of the total thermal conductivity for the topology structure is established as follow,

$$\frac{|\Delta\lambda_{sum}(t)| + |\Delta\lambda_{sum}(t-1)|}{2\lambda_{sum}(0)} \leq 10^{-3} \quad (13)$$

where  $\Delta\lambda_{sum}(t)$  is total thermal conductivity difference of adjacent iterations by

$$\Delta\lambda_{sum}(t) = \lambda_{sum}(t) - \lambda_{sum}(t-1) \quad (14)$$

where the total thermal conductivity can be obtained by

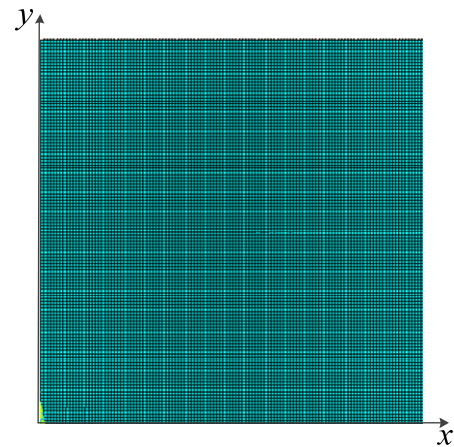
$$\lambda_{sum}(t) = \sum_{i=1}^N \lambda_i(t) \quad (15)$$

So  $\lambda_{sum}(0)$  is the initial total thermal conductivity.

## 3. RESULTS AND DISCUSSIONS

To validate the developed HCAM simulation code for the thermal topology optimization, four test cases for two-dimension testing structure are examined using the computer method under different conditions corresponding to Zuo *et al.* (2005).

Fig. 2 shows the physical model and mesh system of two-dimension testing structure. In order for the testing structure to keep its temperature, the optimization goal is the maximum heat flux of the structure. For the four validating cases, both the mesh numbers of finite element and the cellular numbers of cellular automaton are  $120 \times 120$ . Higher order 8-node element is employed for the finite element. The von Neumann strategy of four neighboring cellulars is implemented to renew the thermal conductivity. The local sensitivity filtering algorithm and the weight approach are applied. Table 1 lists boundary conditions and weight coefficients of the four test cases.

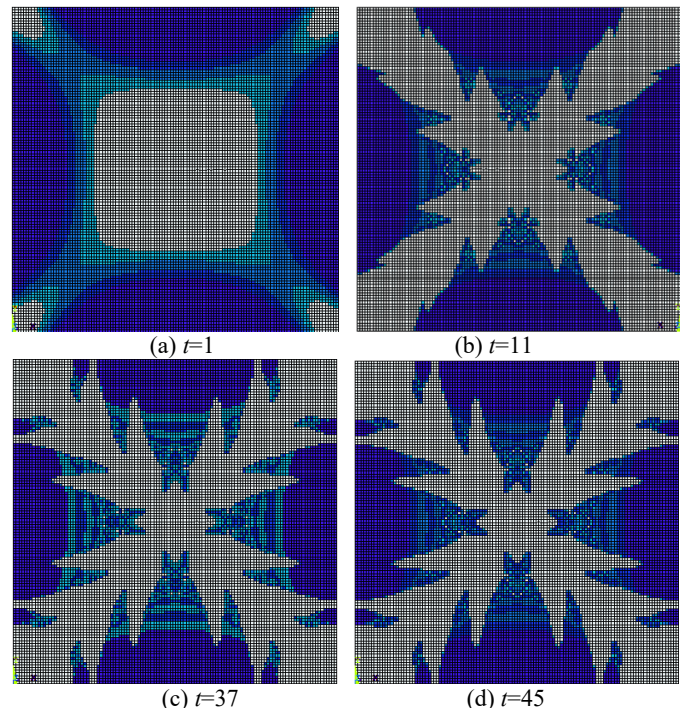


**Fig. 2** Physical model and mesh system of two-dimension testing structure

**Table 1** Boundary conditions and weight coefficients of test cases

Number	Boundary conditions	Weight coefficient
Case 1	Heating uniformly $T=273.15\text{K}$ at the four boundaries	1.1
Case 2	Heating at the center of the structure $T=273.15\text{K}$ at the four boundaries	1.4
Case 3	Heating at the center of the structure $T=273.15\text{K}$ at the middle of the four boundaries	1.1
Case 4	Heating at the center of the structure $T=273.15\text{K}$ at the four corners	1.1

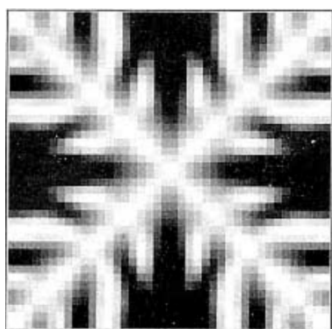
Fig. 3 shows topology optimization process of case 1 with heating uniformly and  $T=273.15\text{K}$  at the four boundaries obtained by HCAM. The white area is the material removing field. It can be seen that it takes 45 iterations for the thermal optimization structure under maximum heat flux by the HCAM simulation. Finally, the X-shape optimization structure forms when it subjects to the optimization goal.



**Fig. 3** Structure evolution of case 1 two-dimension structure by HCAM

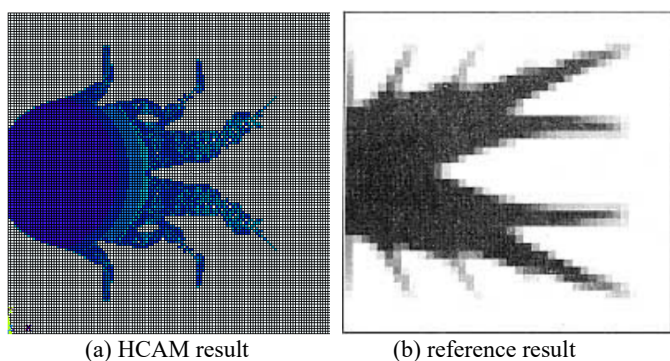


Compared with Fig. 4, the HCAM optimization result of Case 1 shown in Fig. 3d is in good agreement with that obtained by Zuo et al. (2005).

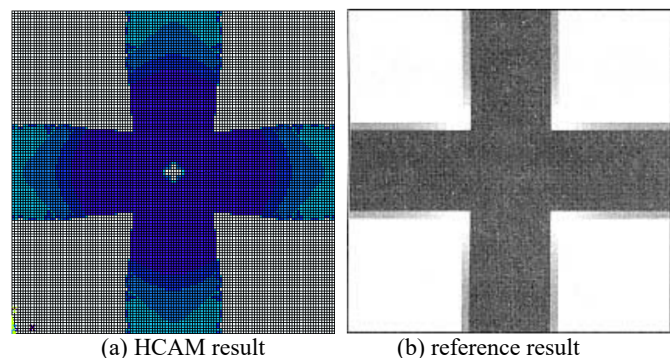


**Fig. 4** Optimization structure of case 1 by reference (Zuo et al. (2005))

Fig. 5~7 shows topology optimization structures of cases 2~4 obtained by HCAM simulation and SIMP method (Zuo et al. (2005)). It can be seen that the results are in good agreement with each other under different conditions. Meanwhile, there is no checkerboard phenomenon for the HCAM results and the structure material of HCAM decreases under equivalent heat flux. Further investigations of heat flux distribution present that the distribution obtained by HCAM is more homogeneous shown in Fig. 8.



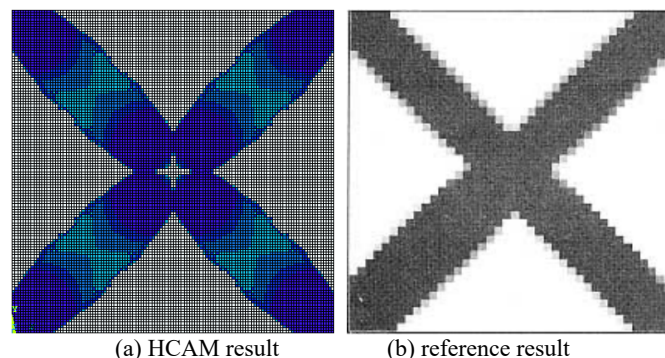
**Fig. 5** Optimization results of case 2 obtained by HCAM and reference (Zuo et al. (2005))



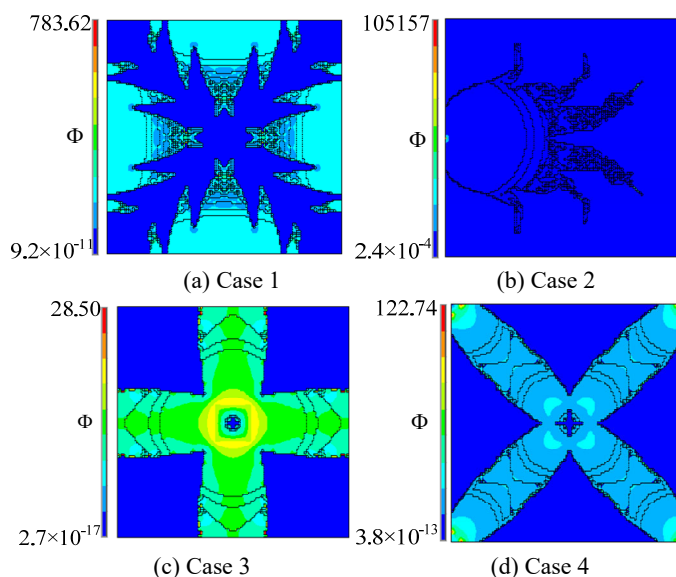
**Fig. 6** Optimization results of case 3 obtained by HCAM and reference (Zuo et al., 2005)

In conclusion, the comparison data show that HCAM can effectively optimize the topological optimization of heat transfer structure, and can optimize and design the topological distribution of heat transfer structure with the best heat dissipation effect, which provides a basis for improving and strengthening the heat dissipation design of spindle structure. Therefore, the developed HCAM code is more efficient for thermal topology optimization.

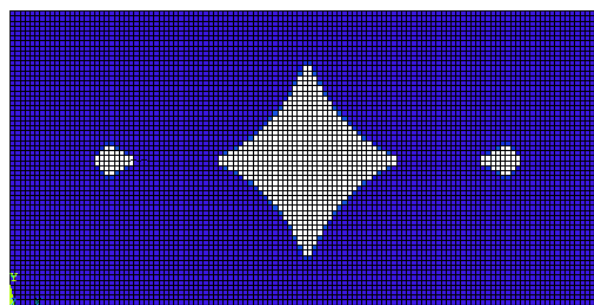
Fig. 9 shows topology optimization spindle structure obtained by HCAM simulation. For the spindle structure, both the mesh numbers of finite element and the cellular numbers of cellular automaton are  $120 \times 60$ . The optimization goal is 30% of the structure's volume under maximum heat flux. The weight coefficient of the cellular automaton algorithm is set as 0.08. It can be seen that there are one large blank area and two small blank areas in the middle of the optimized structure. The two small blank areas are symmetrical about the center of the topology structure. The volume sum of the three areas is about 30% of the optimization goal.



**Fig. 7** Optimization results of case 4 obtained by HCAM and reference (Zuo et al. (2005))

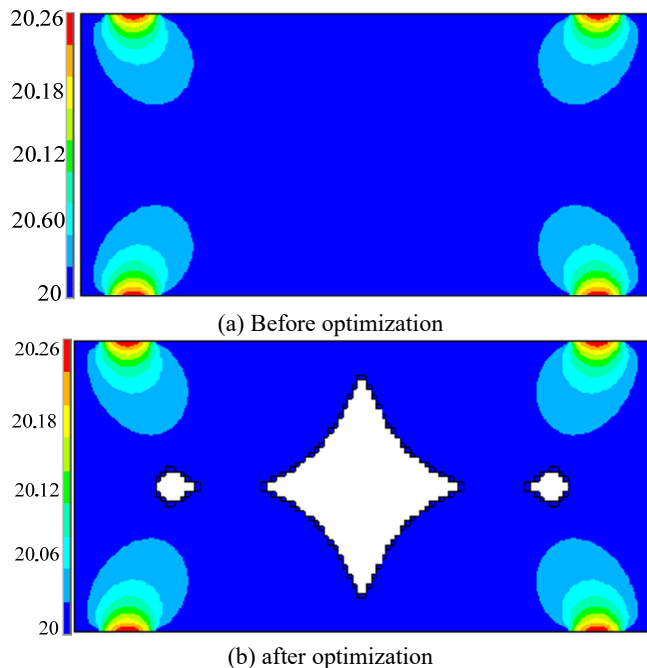


**Fig. 8** Temperature distributions of two-dimensional structures obtained by HCAM



**Fig. 9** Thermal topology optimization of spindle structure by HCAM

Fig. 10 shows temperature distributions of the spindle structure before and after optimization. It can be seen that no change of temperature distributions presents before (shown in Fig. 10a) and after (shown in Fig. 10b) optimization besides the features of decreasing volume. The maximum of the temperatures are both 20.26 °C before and after optimization. Large temperature difference only presents near the four heat sources. It can provide a positive reference for the spindle structure design with the thermal effect.



**Fig. 10** Temperature distributions of spindle structure before and after optimization

#### 4. CONCLUSIONS

A hybrid cellular automaton model combined with finite element method for thermal topology optimization of spindle structure has been developed. The validated studies based on four reference cases showed the developed hybrid model can be efficient to thermal topology optimization. The investigations of structure evolution and thermal evolution of topology optimization have been carried out. It's shown that the developed model can reduce the structure volume under the same condition with maximum heat flux. Furthermore, there is no change for spindle thermal topology optimization before and after optimization besides the features of decreasing volume.

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#### NOMENCLATURE

$c_p$	specific heat ( $\text{kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$ )
$\rho$	density ( $\text{kg}/\text{m}^3$ )
$q$	heat flux ( $\text{J}/(\text{m}^2 \cdot \text{s})$ )
$t$	time (s)
$T$	temperature (K)
$x$	$x$ direction
$y$	$y$ direction

#### Greek symbols

$\lambda$  thermal conductivity ( $\text{W}/(\text{m} \cdot \text{K})$ )

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