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STEADY MHD FLOW OVER A YAWED CYLINDER WITH MASS TRANSFER

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ABSTRACT

This paper examines the steady magnetohydrodynamic (MHD) flow of water over a yawed cylinder with variable fluid properties and non-uniform mass transfer. The impact of viscous dissipation is taken into consideration. The velocity and temperature fields are governed by coupled nonlinear partial differential equations together with boundary constraints. These governing equations are converted to dimensionless form with suitable non-similar transformations and then solved using an implicit finite difference method and the quasi-linearization technique. The results indicate that the yaw angle enhancement declines the skin friction coefficient in the axial direction and the heat transfer coefficient. It is also ascertained that the separation can be delayed by enhancing the MHD effect, the suction parameter with slot movement in the downstream direction.

Keywords: *Boundary layer flow, Magnetohydrodynamic, Non-uniform mass transfer, Non-similar, Variable viscosity*

1. INTRODUCTION

Magnetohydrodynamic heat and mass transfer have numerous applications in magnetohydrodynamic electrical power generation, boundary layer control, MHD generators, MHD pumps, petroleum industries, plasma studies and many others. Many methods have been evolved to control the boundary layer's behavior, and out of that, the enhancement of the MHD principle plays a crucial part in altering the boundary layer's structure and thus influencing flow in the intended direction. The mutual interaction of electromagnetic field and fluid velocity characterize the MHD boundary layer flow. Aldoss *et al.* (1996) have investigated analytically and numerically the effect of MHD mixed convection flow over a horizontal circular cylinder. El-Amin (2003) has studied the effect of MHD forced convection over a non-isothermal horizontal cylinder embedded in a fluid-saturated porous medium. Nagaraju *et al.* (2019); Kumar *et al.* (2021) analyzed the MHD flow and heat transfer with geometries of the circular horizontal pipe and porous disks, respectively.

The geometry considered here is an infinite yawed circular cylinder due to its extensive engineering design applications, yet only a handful of investigations have been made so far. Several authors have published experimental results (King, 1977; Ramberg, 1983; Thakur *et al.*, 2004; Mityakov *et al.*, 2017) and numerical results (Marshall, 2003) of flow past a yawed cylinder. Recently, Patil *et al.* (2020) have published a work on mixed convection flow past a yawed cylinder. However, no investigation is found on MHD flow past a yawed cylinder so far.

Boundary layer is non-similar in nature. This non-similarity occurs due to the body's curvature or the velocity profiles at the edge or due to the surface mass transfer or perhaps an amalgamation of all the above-mentioned factors. A brief review on obtaining non-similar solutions and the references of apposite works are found by Dewey and Gross (1967). Non-similar solutions for a compressible laminar boundary-layer flow had been obtained by Davies and Walker (1977) using the finite difference method, and their behavior was presented near the point of separation. Venkatachala and Nath (1980) have brought non-similar solutions using the finite difference method for a boundary layer flow of steady laminar incompressible two-dimensional and axisymmetric porous bodies with pressure gradient. Roy (2001) shows non-similar solutions for a compressible flow over the yawed cylinder using the implicit finite difference method along with the quasi-linearization technique. Subhashini and Samuel (2016) obtained a non-similar solution of steady compressible flow over a thin cylinder.

Fundamental physical properties of fluid often change notably with temperature; hence it is essential to acknowledge such properties as a function of temperature. Several investigators have studied the impact of variable fluid properties for steady laminar flows over different heated surfaces (Chin *et al.*, 2007; Eisenhuth and Hoffman, 1981; Pantokratoras, 2005; Mohammad, 2020).

Surface mass transfer has a substantial impact in preventing or slowing down the boundary layer separation. The influence of non-uniform mass transfer on water flow past various bodies has been studied by Saikrishnan and Roy (2003a,b), and it is discovered that non-uniform suction delays the point of separation. Tashtoush *et al.* (2000) have investigated the mass transfer's effect of non-Newtonian fluid on a power-law stretched surface. Researchers Ponnaiah (2012); Revathi *et al.* (2014); Roy and Saikrishnan (2004) have solved steady/unsteady forced convection flow problem over a yawed cylinder taking non-uniform slot suc-

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tion/injection into consideration. Ganapathirao *et al.* (2019); Jenifer *et al.* (2021) analyzed the effects of non-uniform slot suction/injection on MHD mixed convective flow past a vertical wedge and sphere, respectively.

The current study is concentrated on the effects of MHD flow over a yawed cylinder with non-uniform mass transfer and temperature-dependent viscosity and Prandtl number. The fluid considered here is water due to its extreme practical applications.

2. MATHEMATICAL FORMULATION

The flow model of the steady laminar MHD past a yawed infinite cylinder of radius R, set in a free stream with the oncoming free stream velocity U_∞ (see Fig. 1). A constant magnetic field B_0 is applied in the normal direction to the cylinder's surface. The surface of a yawed cylinder and the free stream are at constant temperatures of T_w, T_∞, respectively.
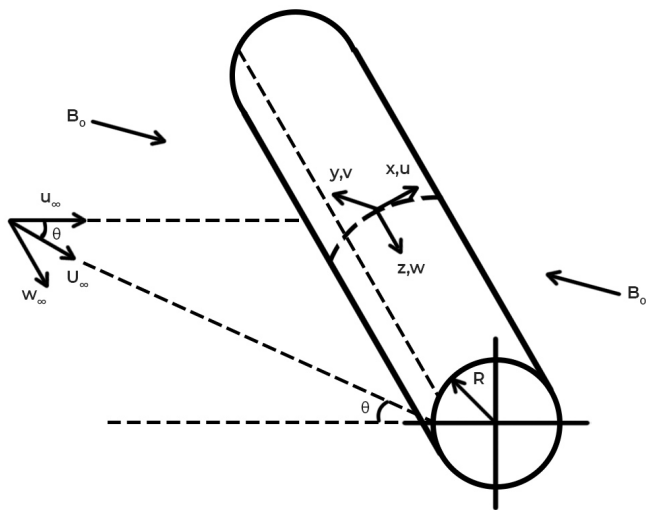

Fig. 1 Flow model

The variation in temperature between the sphere's surface and the free stream is taken in the range $0^oC - 40^oC$. The viscosity (μ) and thermal conductivity (k) show a significant variation with temperature, and so does the Prandtl number (Pr). So, both μ and Pr can be expressed as inverse linear functions of T as given by Eswara and Nath (1994).

$$\mu = \frac{1}{(a+bT)} \quad \text{and} \quad Pr = \frac{1}{(c+dT)} \quad (1)$$

where

$$a = 53.41, \quad b = 2.43, \quad c = 0.068, \quad d = 0.004 \quad (2)$$

Within this temperature limit considered, fluid density (ρ) and specific heat (c_p) vary with the temperature only up to a maximum of 1%. This minute variation allows us to take ρ and c_p as constants.
The governing equations are:

$$(ru)_x + (rv)_y = 0 \quad (3)$$

$$uu_x + vu_y = u_e(u_e)_x + \frac{1}{\rho}(\mu u_y)_y - \frac{\sigma B_0^2}{\rho}(u - u_e) \quad (4)$$

$$uw_x + vw_y = \frac{1}{\rho}(\mu w_y)_y - \frac{\sigma B_0^2}{\rho}w \quad (5)$$

$$uT_x + vT_y = \frac{1}{\rho}\left(\frac{\mu}{Pr}T_y\right)_y + \frac{\mu}{\rho c_p}(u_y^2 + w_y^2) + \frac{\sigma B_0^2}{\rho c_p}(u^2 - u_e u) \quad (6)$$

Boundary conditions:

$$u(x,0) = 0, \quad u(x,\infty) = u_e(x), \quad v(x,0) = v_w(x),$$

$$w(x,0) = 0, \quad w(x,\infty) = w_e(x), \quad T(x,0) = T_w, \quad T(x,\infty) = T_\infty \quad (7)$$

The transformations to non-dimensionalize the Eqs. (3) - (6) are,

$$\psi(x,y) = Ru_\infty\left(\frac{2\xi}{Re}\right)^{1/2}f(\xi,\eta), \qquad u = \psi_y, \qquad v = -\psi_x,$$

$$\xi = \int_0^x \frac{u_e}{u_\infty}d\left(\frac{x}{R}\right), \qquad \eta = \left(\frac{u_e}{u_\infty}\right)\left(\frac{Re}{2\xi}\right)^{1/2}\left(\frac{y}{R}\right),$$

$$G = \frac{T - T_\infty}{T_w - T_\infty}, \qquad u_\infty = U_\infty\cos\theta, \qquad Re = \frac{u_\infty R}{\nu} \quad (8)$$

The above transformations satisfy Eq. (3) identically the non-dimensional form of the Eqs. (4), (5) and (6) are given below.

$$(NF_\eta)_\eta + fF_\eta + \beta(1 - F^2) + MP(1 - F) = 2\xi(FF_\xi - f_\xi F_\eta) \quad (9)$$

$$(NS_\eta)_\eta + fS_\eta - MPS = 2\xi(FS_\xi - f_\xi S_\eta) \quad (10)$$

$$\left(\frac{1}{Pr}NG_\eta\right)_\eta + fG_\eta + NEc\left[\left(\frac{u_e}{u_\infty}\right)^2\cos^2\theta F_\eta^2 + \sin^2\theta S_\eta^2\right]$$

$$+ PEcM\left(\frac{u_e}{u_\infty}\right)^2\cos^2\theta F(F - 1) = 2\xi(FG_\xi - f_\xi G_\eta) \quad (11)$$

with the boundary conditions

$$F(\xi,0) = 0, \quad F(\xi,\infty) = 1$$
$$S(\xi,0) = 0, \quad S(\xi,\infty) = 1$$
$$G(\xi,0) = 1, \quad G(\xi,\infty) = 0 \quad (12)$$

where

$$N = \frac{\mu}{\mu_\infty} = \frac{a + bT_\infty}{a + bT} = \frac{1}{1 + E_1 G}$$

$$E_1 = \frac{b\Delta T_w}{a + bT_\infty}$$

$$Pr = \frac{1}{c + dT} = \frac{1}{E_2 + E_3 G},$$

$$E_2 = c + dT_\infty, \qquad E_3 = d\Delta T_w,$$

$$\Delta T_w = T_w - T_\infty, \qquad \frac{u}{u_e} = f_\eta = F,$$

$$Ec = \frac{U_\infty^2}{c_p(T_w - T_\infty)}, \qquad \beta(\xi) = \frac{2\xi}{u_e}\frac{du_e}{d\xi},$$

$$P = 3\xi\left(\frac{u_\infty}{u_e}\right)^2, \qquad M = \frac{2}{3}\frac{\sigma B_0^2 R}{\rho u_\infty}$$

$$\nu_\infty = \frac{\mu_\infty}{\rho}$$

$$f = \int_0^\eta Fd\eta + f_w$$

where

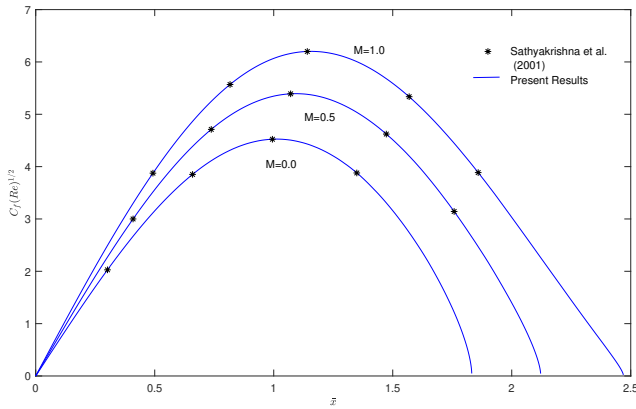$$f_w = -(2\xi)^{-1/2}(Re)^{1/2}\int_0^x \frac{v_w(x)}{u_\infty}d\left(\frac{x}{R}\right) \quad (13)$$

The free stream velocity components in chordwise and spanwise directions are given by

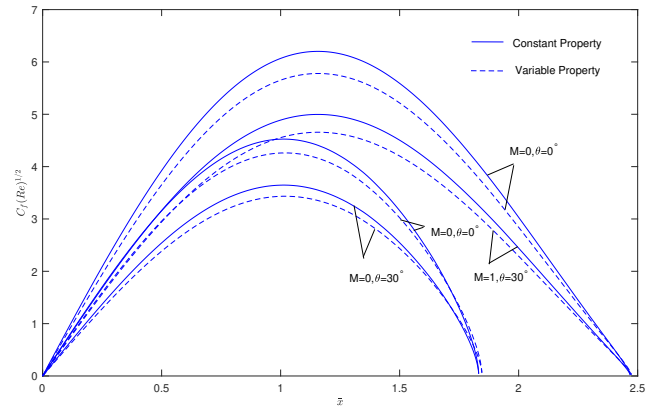$$u_e = 2u_\infty\sin\bar{x}, \quad u_\infty = U_\infty\cos\theta$$

$$w_e(\bar{x}) = w_\infty = U_\infty\sin\theta, \quad \bar{x} = \frac{x}{R} \quad (14)$$

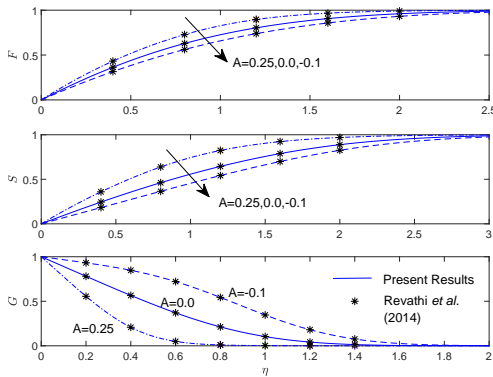$\xi, \beta(\xi)$ and $P(\xi)$ can be written as expressions in $\bar{x}$ as follows.

$$\xi = 2P_1(\bar{x}), \quad \beta = \frac{2\cos\bar{x}}{P_2}, \quad P = \frac{3}{2P_2}$$
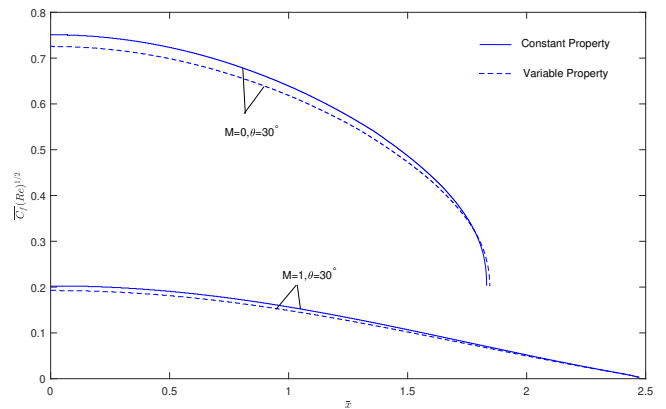
Fig. 2 Comparison of the effect of MHD parameter on the skin friction coefficient for constant fluid properties with those of Sathyakrishna *et al.* (2001) where $T_\infty = 18.7^oC, \Delta T_w = 10.0^oC, \theta = 0°$

Fig. 3 Comparison of the effect of mass transfer parameter on the velocity and temperature profiles for variable fluid properties with those of Revathi *et al.* (2014) where $M = 0.0, \theta = 30°$

Fig. 4 Effect of the constant and variable properties on the skin friction coefficient $(C_f(Re)^{1/2})$

Fig. 5 Effect of the constant and variable properties on the skin friction coefficient $(\overline{C_f}(Re)^{1/2})$

Fig. 6 Effect of the constant and variable properties on the heat transfer coefficient

where

$$P_1 = 1 - \cos \bar{x}, \quad \text{and} \quad P_2 = 1 + \cos \bar{x} \tag{15}$$

f_w is given by

$$f_w = \begin{cases} 0, & \bar{x} \le \bar{x}_0 \\ A(P_1)^{-1/2}[1 - \cos(\omega^*(\bar{x} - \bar{x}_0))], & \bar{x} \in [\bar{x}_0, \bar{x}_0^*] \\ A(P_1)^{-1/2}[1 - \cos(\omega^*(\bar{x}_0^* - \bar{x}_0))], & \text{otherwise} \end{cases} \tag{16}$$

Also, v_w is considered to be

$$v_w = \begin{cases} -2Au_\infty(Re)^{-1/2}\sin(\omega^*(\bar{x} - \bar{x}_0)), & \text{where} \bar{x} \in [\bar{x}_0, \bar{x}_0^*] \\ 0, \text{otherwise} \end{cases} \tag{17}$$

It is suitable to write the equations in $\bar{x}$ instead of ξ.
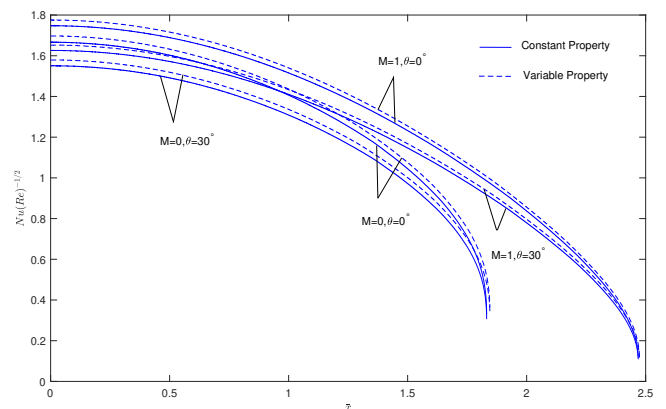$\bar{x}$ and ξ are related by

$$\xi\frac{\partial}{\partial\xi} = Q(\bar{x})\frac{\partial}{\partial\bar{x}} \tag{18}$$
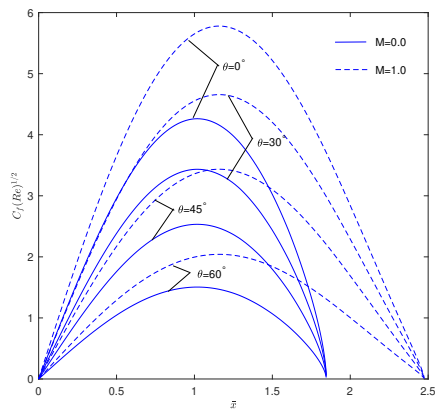
where,

$$Q(\bar{x}) = \tan\frac{\bar{x}}{2} \tag{19}$$

Substituting Eqs. (18) and (19) in the Eqs. (9), (10) and (11), we obtain the dimensionless equations,

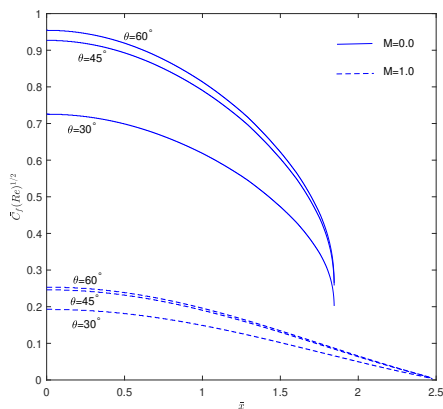$$(NF_\eta)_\eta + fF_\eta + \beta(1-F^2) + MP(1-F) = 2Q(\bar{x})(FF_{\bar{x}} - f_{\bar{x}}F_\eta) \tag{20}$$

$$(NS_\eta)_\eta + fS_\eta - MPS = 2Q(\bar{x})(FS_{\bar{x}} - f_{\bar{x}}S_\eta) \tag{21}$$

Fig. 7 Effect of the MHD parameter M and yaw angle θ on the skin friction coefficient ($C_f(Re)^{1/2}$) for variable viscosity and Prandtl number

Fig. 8 Effect of the MHD parameter M and yaw angle θ on the skin friction coefficient ($\overline{C_f}(Re)^{1/2}$) for variable viscosity and Prandtl number

$$\left(\frac{1}{Pr}NG_\eta\right)_\eta + fG_\eta + NEc\left[\left(\frac{u_e}{u_\infty}\right)^2\cos^2\theta F_\eta^2 + \sin^2\theta S_\eta^2\right]$$

$$+ PEcM\left(\frac{u_e}{u_\infty}\right)^2 F(F-1) = 2Q(\bar{x})(FG_{\bar{x}} - f_{\bar{x}}G_\eta) \quad (22)$$

The boundary conditions become

$$F(\bar{x},0) = 0, \quad F(\bar{x},\infty) = 1$$
$$S(\bar{x},0) = 0, \quad S(\bar{x},\infty) = 1$$
$$G(\bar{x},0) = 1, \quad G(\bar{x},\infty) = 0 \quad (23)$$

The skin friction coefficients in the $x, z-$directions and the heat transfer coefficient can be written as

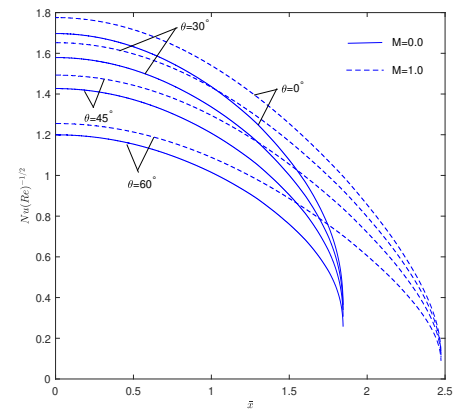$$C_f(Re)^{1/2} = 4P_2P_1^{1/2}(\cos\theta)^{3/2}N_w(F_\eta)_w \quad (24)$$

$$\overline{C_f}(Re)^{1/2} = 2^{3/2}\cos\left(\frac{\bar{x}}{2}\right)\sqrt{\cos\theta}\sin\theta N_w(S_\eta)_w \quad (25)$$

$$Nu(Re)^{-1/2} = -\sqrt{2\cos\theta}\cos\left(\frac{\bar{x}}{2}\right)(G_\eta)_w \quad (26)$$
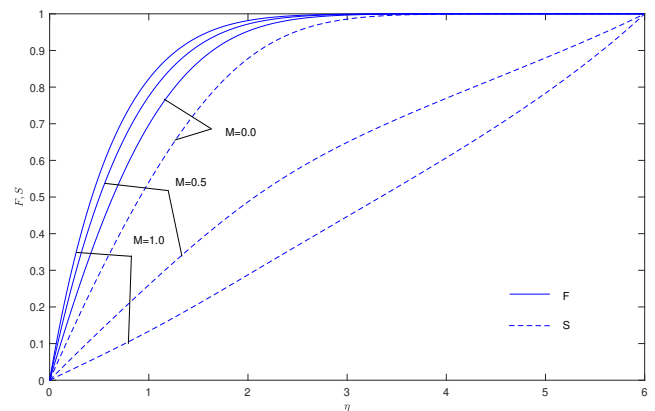
where,

$$C_f = \frac{2[\mu(\frac{\partial u}{\partial y})]_w}{\rho U_\infty^2}, \quad \overline{C_f} = \frac{2[\mu(\frac{\partial w}{\partial y})]_w}{\rho U_\infty^2}$$

$$N_w = \frac{1}{1 + E_1 G_w} = \text{constant}, \quad Nu = \frac{R(\frac{\partial T}{\partial y})_w}{(T_\infty - T_w)}$$

Fig. 9 Effect of the MHD parameter M and yaw angle θ on the heat transfer coefficient for variable viscosity and Prandtl number

Fig. 10 Effect of the MHD parameter M on the velocity profiles F, S for $\theta = 30°$, variable viscosity and Prandtl number

3. RESULTS AND DISCUSSIONS

The quasi-linearization method is utilised to linearise the non-linear PDEs (20), (21) and (22) and the resulting sequence is, in turn, converted into a block tridiagonal system of algebraic equations (Inouye and Tate, 1974). Finally, Varga's algorithm is implemented to solve the block tridiagonal system (Varga, 2000). The convergence of solution is supposed to be achieved when

$$\max\left\{\left|(F_\eta)_w^{(k+1)} - (F_\eta)_w^{(k)}\right|, \left|(S_\eta)_w^{(k+1)} - (S_\eta)_w^{(k)}\right|, \left|(G_\eta)_w^{(k+1)} - (G_\eta)_w^{(k)}\right|\right\} < 10^{-4}$$

The step sizes in η and $\bar{x}$ directions respetively are $\Delta\eta = 10^{-2}$ and $\Delta\bar{x} = 5 \times 10^{-4}$. Here, η_∞ is taken as 6.0.
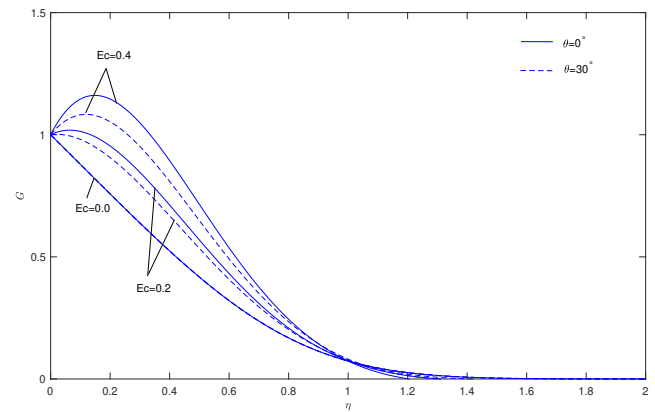
The efficiency of our results are verified by comparing the non-similar solutions obtained for a horizontal cylinder ($\theta = 0°$) with Sathyakrishna *et al.* (2001) for $A = 0, Ec = 0, t^* = 0.0, M = 0, 0.5, 1$ and for a yawed cylinder ($\theta = 30°$) with Revathi *et al.* (2014) for $Ec = 0, t^* = 0.0, M = 0, A = 0.25, 0.0, -0.1$ are presented in Fig. 2 and 3. The results are found to be in excellent agreement.

Figures 4-6 depict the variations of the skin friction coefficients ($C_f(Re)^{1/2}, \overline{C_f}(Re)^{1/2}$) and the heat transfer coefficient ($Nu(Re)^{-1/2}$) due to constant and variable fluid properties. Irrespective of the MHD parameter M and yaw angle θ, the variable properties diminish $C_f(Re)^{1/2}$ and $\overline{C_f}(Re)^{1/2}$ as compared to the constant fluid properties. Contrastingly, the heat transfer is enhanced by the variable fluid properties.
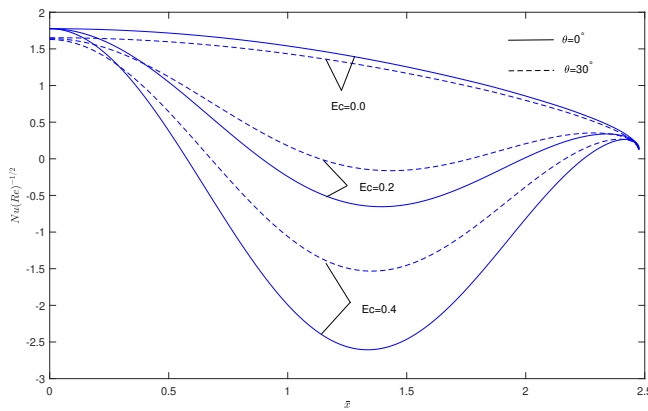
From Figs. 7-9, it is observed that for each M and θ, $C_f(Re)^{1/2}$ enhances from zero, hits its maximum and then declines to zero whereas
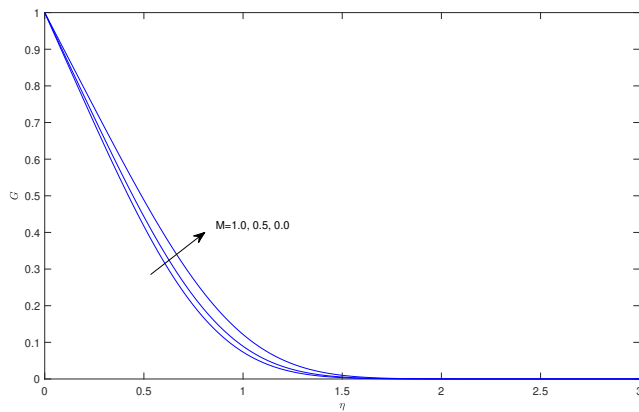
Fig. 11 Effect of the MHD parameter M on the temperature profile G for $\theta = 30°$, variable viscosity and Prandtl number
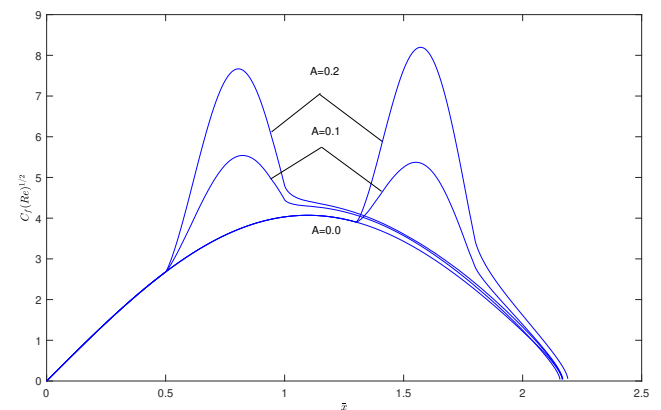

Fig. 12 Effects of the viscous dissipation parameter Ec and yaw angle θ on the heat transfer coefficient for $M = 1$, variable viscosity and Prandtl number

Fig. 13 Effects of the viscous dissipation parameter Ec and yaw angle θ on the temerature profile for $M = 1$, variable viscosity and Prandtl number

Fig. 14 Effects of suction ($A > 0$) and slot location ($\bar{x}_0 = 0.5, 1.3$) on the skin friction coefficient ($C_f(Re)^{1/2}$) for $M = 0.5$, variable viscosity and Prandtl number

Fig. 15 Effects of suction ($A > 0$) and slot location ($\bar{x}_0 = 0.5, 1.3$) on the skin friction coefficient ($\overline{C_f}(Re)^{1/2}$) for $M = 0.5$, variable viscosity and Prandtl number

$\overline{C_f}(Re)^{1/2}$, $Nu(Re)^{-1/2}$ decline monotonically as $\bar{x}$ further increases. With the enhancing of magnetic effect, $C_f(Re)^{1/2}$ and $Nu(Re)^{-1/2}$ increase, but $\overline{C_f}(Re)^{1/2}$ decreases. However, for a constant magnetic effect M, the yaw angle enhancement reverses the above effects. As M changes from 0 to 1, the point of zero skin friction in the $x-$direction moves downstream. This observation reveals that the separation can be slowed down by enhancing the MHD parameter M.

The variations in the velocity profiles in the $x, z-$direction (F, S) and on the temperature profile (G) with respect to M for $\theta = 30°$ are presented in Figs. 10 and 11. F is reduced with the enhancing of the magnetic effect. However, the effect is just the opposite on S and G. This is because the imposed magnetic field generates a supporting force in $x-$direction and an opposing force in $z-$direction. This results in accelerating and decelerating the flow in the respective directions. The impact of M is not remarkable on the temperature profile.

The influence of viscous dissipation parameter (Ec) and the yaw angle (θ) on $Nu(Re)^{-1/2}$ and G are shown in Figs. 12 and 13. It is noticed that an increment of Ec enhances the temperature of the fluid within the boundary layer. For $Ec > 0$, the fluid's temperature near the wall elevates higher than T_w, owing to viscous dissipation. This results in the temperature profile surpassing 1 near the body's surface, which then declines to zero.

Figure 12 depicts that $Nu(Re)^{-1/2}$ declines with an increase of Ec. The heat transfer parameter becomes negative for $Ec > 0$, showing the
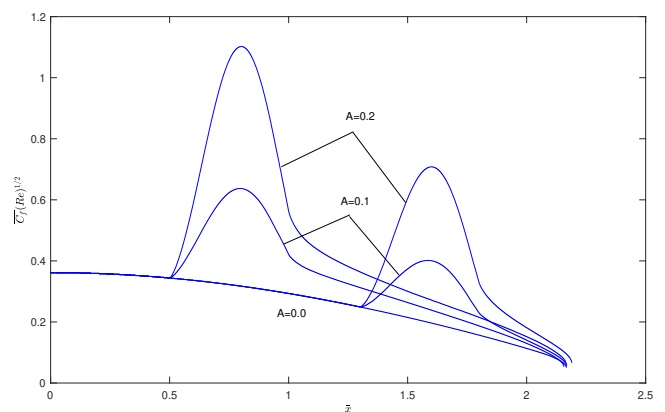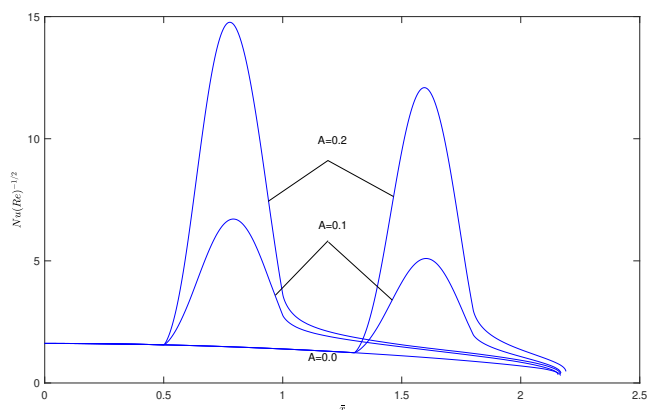
inversion of the heat transfer direction. However, this is not observable for $Ec = 0$. The viscous dissipation parameter has a shallow impact on

Fig. 16 Effects of suction ($A > 0$) and slot location ($\bar{x}_0 = 0.5, 1.3$) on the heat transfer coefficient for $M = 0.5$, variable viscosity and Prandtl number

Fig. 18 Effects of injection ($A < 0$) and slot location ($\bar{x}_0 = 0.5, 1.3$) on the skin friction coefficient ($\overline{C_f}(Re)^{1/2}$) for $M = 0.5$, variable viscosity and Prandtl number

Fig. 17 Effects of injection ($A < 0$) and slot location ($\bar{x}_0 = 0.5, 1.3$) on the skin friction coefficient ($C_f(Re)^{1/2}$) for $M = 0.5$, variable viscosity and Prandtl number
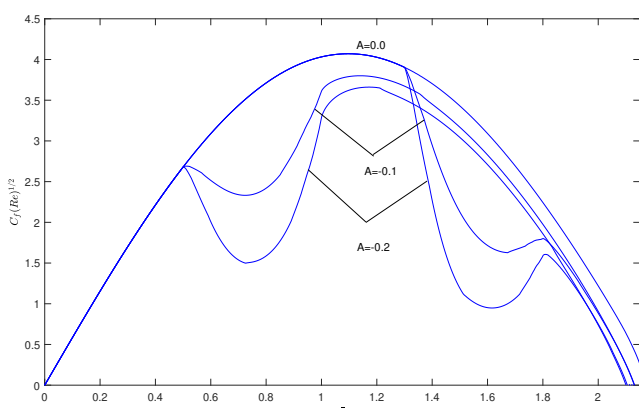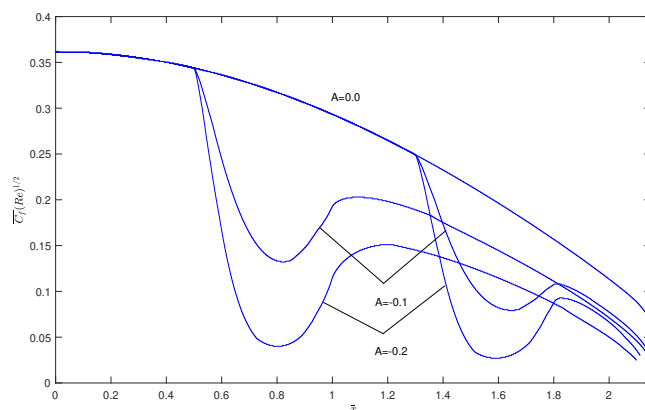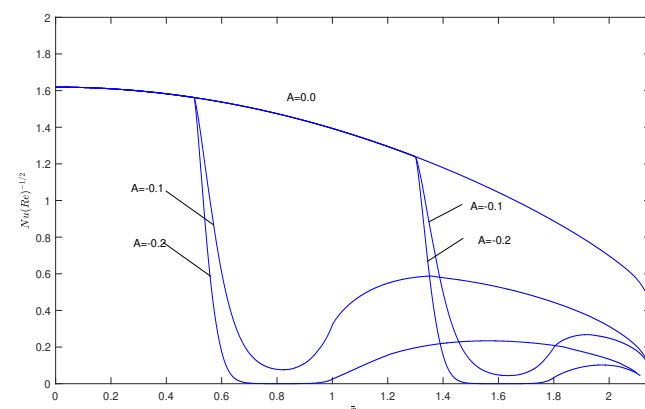

Fig. 19 Effects of injection ($A < 0$) and slot location ($\bar{x}_0 = 0.5, 1.3$) on the heat transfer coefficient for $M = 0.5$, variable viscosity and Prandtl number

the skin friction coefficients, and hence the corresponding figures are not presented here.

The impact of the mass transfer through two different slots $[0.5, 1.0]$ and $[1.3, 1.8]$ on $C_f(Re)^{1/2}$, $\overline{C_f}(Re)^{1/2}$ and $Nu(Re)^{-1/2}$ are shown in Figs. 14-19. For slot suction ($A > 0$), as the slot starts, $C_f(Re)^{1/2}$, $\overline{C_f}(Re)^{1/2}$ and $Nu(Re)^{-1/2}$ increase and hit their maximum before reaching its end. After that $\overline{C_f}(Re)^{1/2}$ and $Nu(Re)^{-1/2}$ decline and reach a non-zero finite value, but $C_f(Re)^{1/2}$ almost vanishes. The injection ($A < 0$) shows the opposite effect. The reason behind this is the function v_w considered in Eq. (17), which describes the effect of mass transfer within a slot. The term $\sin(\omega^*(\bar{x} - \bar{x}_0))$ in v_w increases in the first half of the slot, reaches its highest value of 1 at $(\bar{x}_0 + \bar{x}_0^*)/2$ and decreases to 0 afterwards. Hence, the corresponding changed velocity field influences the temperature field, leading to a change in skin friction and heat transfer coefficients. It is observed that a small increment in the suction parameter $A > 0$ influences the zero skin friction in $x-$direction to move downstream and hence delays the separation. This phenomenon is much noticeable if the location of the slot $[0.5, 1.0]$ is also moved downstream to $[1.3, 1.8]$. However, in the case of injection, increasing the injection parameter $A < 0$ and moving the slot location downstream makes the zero skin friction in $x-$direction to move upstream.

4. CONCLUSIONS

The steady MHD flow problem over a yawed cylinder has been solved numerically, and the following observations are made:

- The separation can be slowed down by enhancing the MHD parameter M.

- With an increase of yaw angle θ, the skin friction coefficient $C_f(Re)^{1/2}$ and the heat transfer coefficient $Nu(Re)^{-1/2}$ declines irrespective of the value of M.

- The enhancement of viscous dissipation effect Ec enhances the temperature (G) of the fluid within the boundary layer while increasing the yaw angle θ diminishes G.

- Overshoot in the temperature profile is observed near the body surface for $Ec > 0$.

- Increasing suction ($A > 0$) and the slot's movement in downstream direction moves the zero skin friction in $x-$direction downstream, whereas the injection's effect is the opposite.

NOMENCLATURE

A	Dimensionless mass transfer parameter
B_0	Magnetic field strength

c_p	Specific heat (kJ/kg $\cdot$ K)
$C_f, \overline{C_f}$	Skin friction coefficient in $x, z-$directions, respectively
Ec	Eckert number (viscous dissipation parameter)
$f_w(x)$	Surface mass transfer distribution
f	Dimensionless stream function
F	Dimensionless velocity component in the $x-$direction
G	Dimensionless temperature
k	Thermal conductivity (W/m $\cdot$ K)
M	MHD parameter
Nu	Nusselt number
N	$\frac{\mu}{\mu_\infty}$ Viscosity ratio
Pr	Prandtl number
Re	Reynolds number
R	Radius of the yawed cylinder
S	Dimensionless velocity component in the $z-$direction
T	Temperature (K)
u, v, w	Dimensional velocity components in $x, y, z-$directions, respectively (m/s)
U	Steady state velocity at the boundary layer's edge
x, y, z	Dimensional meridional, azimuthal and normal distances, respectively (m)
$\bar{x}$	Dimensionless meridional distance
$\bar{x}_0, \bar{x}_0^*$	Ends of slot
Greek Symbols	
$\beta(\xi)$	Pressure gradient
η, ξ	Transformed coordinates
$\Delta\eta, \Delta\bar{x}$	Step sizes in $\eta-, \bar{x}-$ directions, respectively
μ	Dynamic viscosity (Kg/m $\cdot$ s)
ν	Kinematic viscosity (m^2/s)
ρ	Density (Kg/m^3)
σ	Electrical conduction
θ	Yaw angle
ω^*	Slot length parameter
ψ	Dimensional stream function (m^2/s)
Subscripts	
e	Conditions at the edge of the boundary layer
w	Conditions at the surface of the sphere
∞	Conditions in the free stream
$x, z, \bar{x}, \xi, \eta$	Partial derivatives with respect to these variables

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