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NATURAL CONVECTION ANALYSIS OF A RECTANGULAR SHAPE HEATED BLOCK EMBEDDED IN SQUARE CAVITY

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ABSTRACT

Numerical analysis of heat transfer by convection in a square with a rectangular shape heated block located at the top, center and bottommost has been numerically investigated by applying the principal partial differential equations governing mass, momentum and energy using discontinuous Galerkin weighted procedure for residual finite element with the view of examining the significance of position of rectangular shaped heated block encapsulated within the square cavity. The right wall being adiabatic while the other three walls are maintained at low constant temperature. The heated block is maintained constantly hot. The developed code of COMSOL Multiphysics is employed to perform a parametric study with the aim to determine the influence of Rayleigh number and Nusselt number on isotherm contours and stream functions. The result show that the position of the heated block in the square cavity has significant influence on the heat transfer behaviour and temperature distribution. It is also observed that at specific Rayleigh number, Nusselt number is at maximum around the heated block at the top, middle and bottom position in the cavity considered.

Keywords: *Finite element method, Heat transfer, Natural convection, Rayleigh number, Square cavity.*

1. INTRODUCTION

In several situations of scientific practices, heat generated media are often embedded into cavities that contains fluid using the case of fuel tanks for instance. Some other applications require that heat generated medium be controlled thermally for instance in space heating, electronic equipment, nuclear design and passive cooling (Xiaohui, 2012). Natural convection with embedded heat sources in cavities is gaining more attention due to the importance of regulating temperature gradient in cavity meant for centralized heating or cooling system. The need to analyze the rate of heat transfer by convection of object in cavity is becoming paramount because it is noteworthy to study the temperature profile and rate of heat transfer in thermal design and management of object positioned in cavity.

Consequently, investigation of fluid flow and temperature distributed in a square cavity having a heat generating conducting body within it revealed that local average Nusselt number of vertical walls dramatically changes as a result of heat transfer by convection caused by temperature gradient via the cavity (Jong *et al.* 2007). It has been established that magnetic field is strongly required to reduce heat transferred by convection in cavity especially when the Rayleigh number is increased in cavity containing semi-circular heated body (Bhuiyan *et al.* 2014). Further, from the study conducted in square cavity having a square heated body, it was found that the fluid flow and dimensionless temperature present within the cavity are dependent on the Rayleigh number and magnetic field strength (Jahirul *et al.* 2015). Report of (Roslan *et al.* 2014) showed that for conjugate natural convection of square cavity differentially heated and containing solid, increase in size of the polygon causes increase in the rate of heat transfer until a critical value was reached after which the rate at which heat is being transferred then shrinks in the cavity. A conclusive report of enhanced heat transfer by inserting conductive object which include quadrilateral and rotating cylindrical object at the center of the cavity has been presented (e.g. (Hussain and Hussein, 2011; Fu *et al.* 1994; Das and Reddy, 2006; and Aminossadati and Ghasemi, 2012). Heat transfer in an enclosed domain was studied in the presence of trapezium shape heat sources mounted on an inner square cylinder. In this case, the results of numerical solution revealed a remarkable characteristic on streamlines and isotherms while the number of heat sources were augmented (Nepal *et al.* 2020). Also, (Mahmoodi and Sebdani, 2012) conducted numerical investigation within a square cavity enclosing an insulated square block at its center. Their results showed that a decrease in average Nusselt number of the nanoparticles, the size of the adiabatic square body was increased with increase in Rayleigh number and heat transfer rate. Further, (Sojoudi *et al.* 2015) numerically investigated natural convection inside an attic shaped cavity with temperature gradient at the two inclined walls and occupied with room air temperature. Their report revealed the influence of thermal boundary layer when the average distance between the bottom heat source and cool right wall were reduced. Moreover, from the experimental viewpoint, (Barozzi and Corticelli, 2000) focused on two-dimensional buoyancy force driven in an enclosed cabinet consisting of two vertical heat sources using an accuracy-time finite method. They showed that long term behavior of numerical solution predicted was time-dependent. (Ramiz and Ra'ed, 2020) studied the effects of the position of cooled square object placed at the center of square enclosure. They discovered that when the Rayleigh number and the size of the object in the cavity were varied, the Nusselt number value reached maximum when the object is positioned at the top section of the cavity at a specific body size. However, changes in vertical and horizontal position drastically affect heat distribution by convection in the cavity. Recently,

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(Oyewola *et al.* 2021; and 2022) showed that heat transfer characteristics are significantly influenced within the cavity irrespective of the objects.

Despite numerous studies on natural convection from the previous literature, it appears that study is sparse on the influence of the location of heated block rectangular shape in a square cavity. It should be noted that the problem of this kind have significant application. For example, the application can be observed from natural convection within vertically-positioned or horizontally-placed radiator. This is due to its significance in the regulation of air temperature in an enclosed integrated cooling and heating system. It is therefore necessary to study the heat distribution as well as transfer in management of thermal fluid and system design. The main motive of the present study is to examine the various effect of the position of rectangular shape heated block placed at the top, center and bottom of a square cavity. The fluid flow configuration, temperature distribution and rate at which heat is transferred by convection naturally will be effective in determining the influence of heat distribution and Rayleigh number on the flow profile.

2. NUMERICAL MODEL AND EQUATIONS

The schematic configuration of a heated block rectangular shape in square cavity is shown in Figure 1. The length of the square is defined by L. Right wall is considered adiabatic while the other three sides of the cavity wall is maintained at a constant cold temperature (Tc). The rectangular shape block with width (W) at all sides maintained at hot temperature and positioned at three different cases: at the top, center and bottom of the cavity.

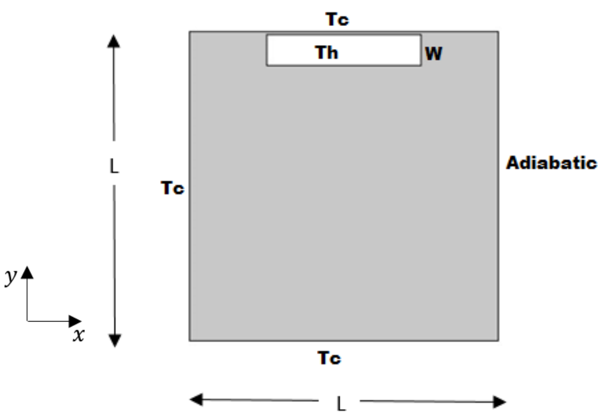

Fig. 1: Schematic cavity configuration

Natural convection inside a square cavity containing embedded rectangular shape heated block is considered. Principal equations for conservation of mass, momentum and energy are stated below for a two-dimensional, laminar, steady, and incompressible flow with Boussinesq approximation (COMSOL cyclopedia, 2022) for connecting the temperature field to fluid field.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\vartheta\left\{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right\} \quad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\vartheta\left\{\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right\}+\beta g(T_h-T_c) \quad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\propto\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

The following non-dimensional quantities are introduced

$$\Psi=\frac{\psi}{\propto},\ \mathrm{X}=\frac{x}{\delta},\quad \Omega=\frac{\omega\delta^2}{\propto},\ \mathrm{Y}=\frac{y}{\delta},\ \mathrm{U}=\frac{u\delta}{\propto},$$

$$\mathrm{V}=\frac{v\delta}{\propto},\quad \theta=\frac{T-T_c}{T_h-T_c}\quad \mathrm{Pr}=\frac{\vartheta}{\propto}$$

$$\mathrm{Ra}=\frac{\beta g(T_h-T_c)\delta^3 Pr}{\vartheta^2},\qquad \mathrm{Nu}=\frac{hl}{k} \quad (5)$$

Equations (1)-(4) in dimensionless form is obtained as follows

$$U\frac{\partial\Omega}{\partial X}+V\frac{\partial\Omega}{\partial Y}=Pr\left(\frac{\partial^2\Omega}{\partial X^2}+\frac{\partial^2\Omega}{\partial Y^2}\right)+Ra.Pr\frac{\partial\theta}{\partial X} \quad (6)$$

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2} \quad (7)$$

The stream function is defined as the scalar function of space and time with respect to u and v.

$$u=\frac{\partial\psi}{\partial y}, \quad (8)$$

$$v=-\frac{\partial\psi}{\partial x} \quad (9)$$

Boundary Conditions for velocity flow at the walls are stated as follows

Velocity: $u(x,0)=u(x,L)=u(0,y)=u(L,y)=0$

$v(x,0)=v(x,L)=v(0,y)=v(L,y)=0$

The Temperature distribution around the walls is defined thus;

At the left and horizontal walls:

$T(0,y)=T(x,L)=T(x,0)=T_c$

At the right wall: $\frac{\partial T}{\partial x}(L,y)=0$

3. NUMERICAL SOLUTION

In the present work, COMSOL Multiphysics software was employed to solve the nonlinear dimensionless principal equations of the conservation of mass, momentum, and energy in partial differential form. The analyzed mesh and boundary elements for the considered cavity is 8206 and 304 respectively. The coupled system provides a factor of estimated error as 400 while the tolerance converged at 10^{-6}. Further iteration on the non-dimensional parameter and Boussinesq approximation for buoyancy force leads to an efficient solution.

Moreover, the present code was corroborated with the solution benchmark of (Mousa, 2010; and Davis, 1983) who studied insulated horizontal walls square cavity heated at the vertical left wall as well as cooled at the opposite wall. Some results of Average Nusselt numbers are presented in Table 1 for program comparison. It is obvious that there is a good agreement between the present work and those of (Mousa, 2010; and Davis, 1983).

Table 1. Average Nusselt number comparison with other studies

Ra	Present study	(Mousa,2010)	(Davis, 1983)
10^4	2.242	2.245	2.243
10^5	4.516	4.521	4.519
10^6	8.725	8.819	8.799

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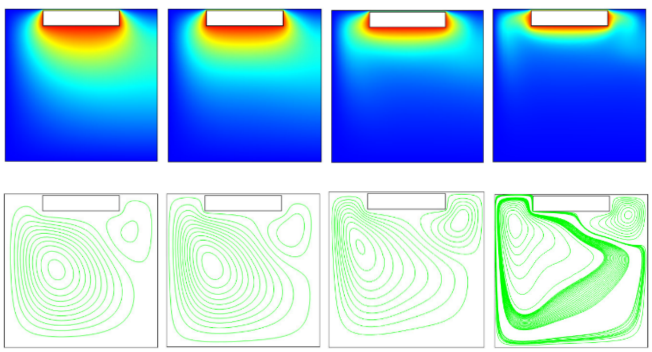
4. RESULTS AND DISCUSSION

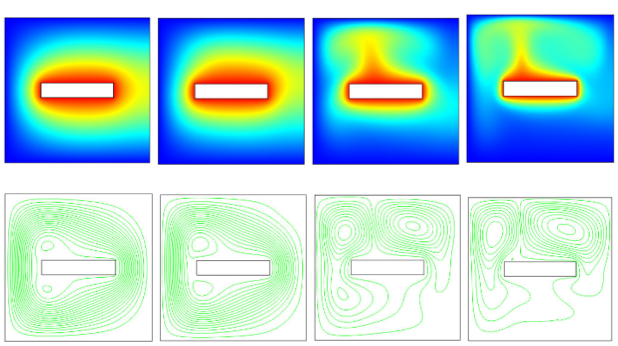
The natural convective process in square cavity with rectangular shape heated block at the top, center and bottom of a square cavity has been numerically investigated. The square cavity is kept at constant cold temperature at the sides except for the right wall which is maintained adiabatic. The results of non-dimensional parameter of Prandtl number at 0.71 and Rayleigh number of 10^3 , 10^4, 10^5 and 10^6 has been obtained. Figure 2 shows that Isotherms plots around the block form eccentric circles. The direction of the heat tends towards the insulated wall and less dense fluid arises as the Rayleigh number increases. As can be observed from the streamlines, about two counter-rotating eddies are formed in the cavity, the larger one being at the bottom of the heated block while the other eddies are just beginning to form at the either side of the rectangular shape heated block. The increase in the Rayleigh number produces more eddies which is a response to increase convectional mode of heat transfer.



(a) Ra=10^3 (b) Ra=10^4 (c)Ra=10^5 (d) Ra=10^6

Fig. 2: Isotherms (up) and Streamlines (down) of heated block located at the top of the cavity for (a) Ra=10^3 (b) Ra=10^4 (c) Ra=10^5 (d)Ra=10^6

It is apparent in Figure 3 that the dimensionless temperature distribution around the hot block at the center generated symmetry isotherms and directed towards the right insulated wall of the cavity at a low Rayleigh number. This is an indication of a weak fluid flow in the cavity. However, a more stratified temperature distribution occurs in the cavity when the Rayleigh number increases, the heat therefore move towards the top wall, a symbol of strong effect of convection heat transfer.

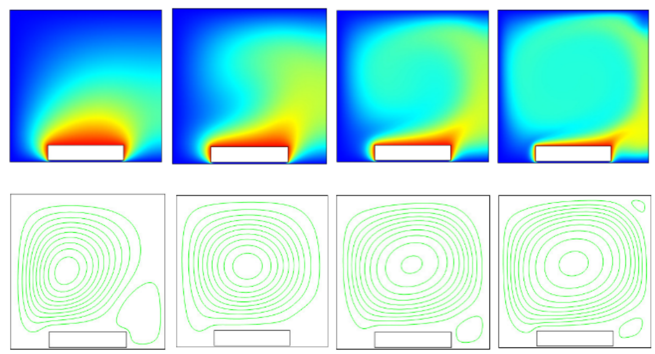


(a) Ra=10^3 (b) Ra=10^4 (c)Ra=10^5 (d) Ra=10^6

Fig. 3: Isotherms (up) and Streamlines (down) of heated block located at the center of the cavity for (a) Ra=10^3 (b) Ra=10^4 (c) Ra=10^5(d)Ra=10^6

The stream function with the rectangular block at the center presents a buoyancy driven flow in the cavity forms around the hot block, more complex circulating flow are formed at the upper section of the cavity due to the effect of buoyancy force. With increase in Ra, the stream function shows two vortices at the top of the block due to increase in the fluid flow velocity at the top section of the cavity.

Figure 4 represents the temperature dispersion around the block at the bottom of the cavity, the isotherms plot shows a conduction dominant at Ra=10^3 due to a low temperature gradient within the cavity. Buoyancy force becomes stronger when the Ra increases and less dense fluid quickly rises to the insulated wall before turning back to the other parts of the walls. With increased buoyant force observed at Ra=10^6 a heat transfer by convection dominated the cavity. As for Streamline formation, velocity of the fluid flow becomes greater only when the Ra increases in the cavity, the formation of extra vortex at the right edges of the insulated wall of the cavity at Ra=10^6 was observed. Hence, velocity of fluid flow in cavity gain more energy when Ra is increased.



(a) Ra=10^3 (b) Ra=10^4 (c)Ra=10^5 (d) Ra=10^6

Fig. 4: Isotherms (up) and Streamlines (down) of heated block located at the bottom of the cavity for(a) Ra=10^3 (b Ra=10^4 (c)Ra=10^5(d) Ra=10^6

The Nusselt number deviation against the vertical wall at x=0.75m in the square cavity is shown in Figures 5, 6 and 7 for rectangular shape heated block at the top, center and bottommost of the cavity respectively. The deviation in temperature is measured along the left cold wall to determine the rate of heat transfer. It is observed that Nusselt number rises with Rayleigh number at the either side of the rectangular block.

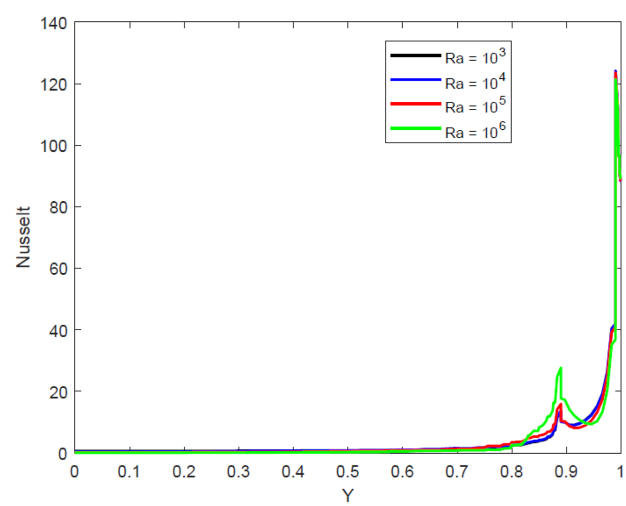


Fig. 5: Deviation of Nusselt number along the vertical section with rectangular block at the Top of the cavity.

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Deviation of Nusselt number to the horizontal wall at a distance of 0.75m in Figure 5 indicates highest value of Nusselt number noticed at the top wall where the hot block is situated and drastically reduces the moment after the block leading down to the bottom of the cavity. A sharp increase at a section close to the top reveals opposite side of the heated block strongly affected by the buoyancy force as a result of the small temperature experienced below the heated block in the cavity.

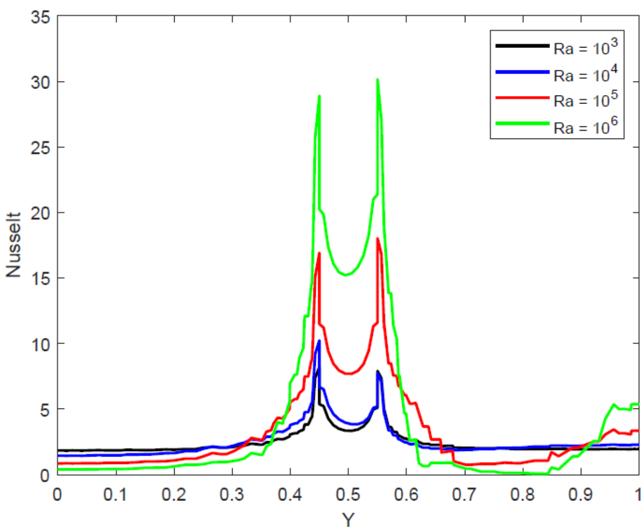

Fig. 6: Deviation of Nusselt along the vertical section with rectangular block at the Middle of the cavity

At the center of the cavity, the heated block in Figure 6 reveals that Nusselt number increases to its highest value at the top edge of the block and reduces along the vertical section of the heated block, it then drastically reduces to zero, the result obtained presents a sharp increase of Nusselt number at the top and bottom wall of the heated block, hence the higher the Rayleigh number, then Nusselt number is augmented and rate at which heat transfer by convection increases.

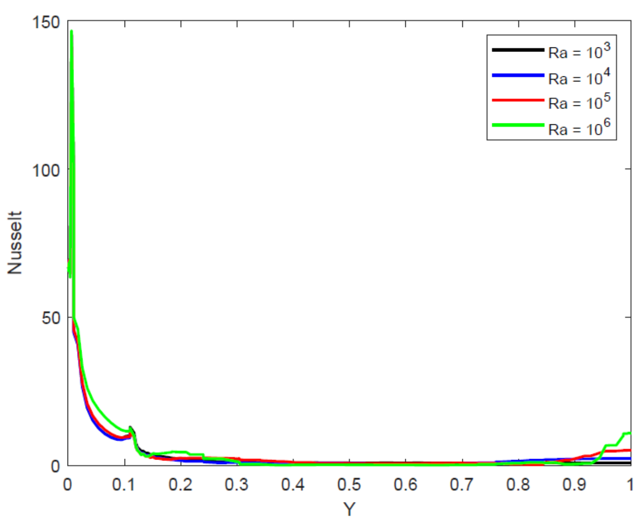

Fig. 7: Deviation of Nusselt number along the vertical section with rectangular block at the Bottom of the cavity

With the heated block positioned at the bottom of the cavity, deviation of Nusselt number along the vertical section of the square cavity as shown in Figure 7 reveals that peak Nusselt number is noticed at the bottom of the cavity where the rectangular shape heated block is positioned, the Nusselt number thus reduces towards the top of the cavity as a result of reduced velocity of fluid. The Rayleigh number decreases as the Nusselt number decreases along the top section of the square cavity.

Moreover, the values of Nusselt number were compared with the block at the top, center and bottom of the cavity in Figures 8, 9, 10 and 11. It is observed that for a fixed Rayleigh number, Nusselt number is at maximum at the heated block placed at the top of the cavity, also, the increase in Ra has shown a significant upturn on the Nusselt number. The Nusselt number of the heated block at the center of the cavity is the lowest at every specific Rayleigh number but is observed to be augmented as the Rayleigh number increases in the square cavity.

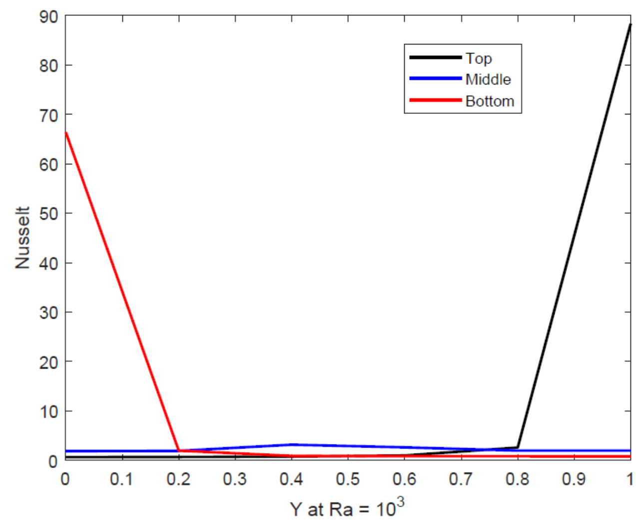

Fig. 8: Deviation of Nusselt number for the three position Top, Center and Bottom at Ra=10^3 within the cavity

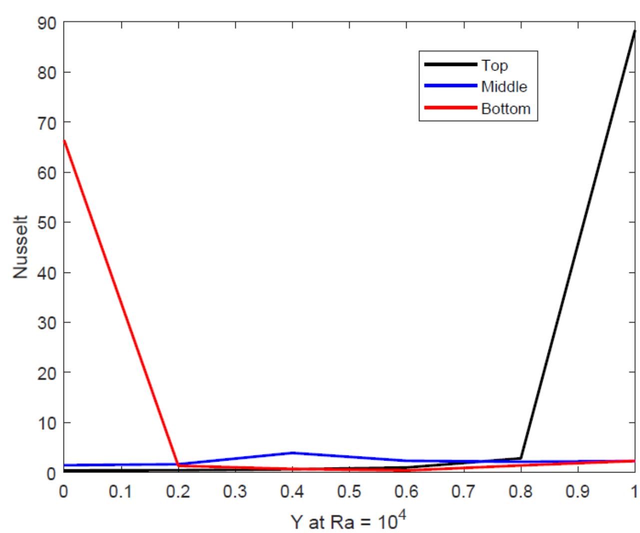

Fig. 9: Deviation of Nusselt number for the three position Top, Center and Bottom at Ra=10^4 within the cavity

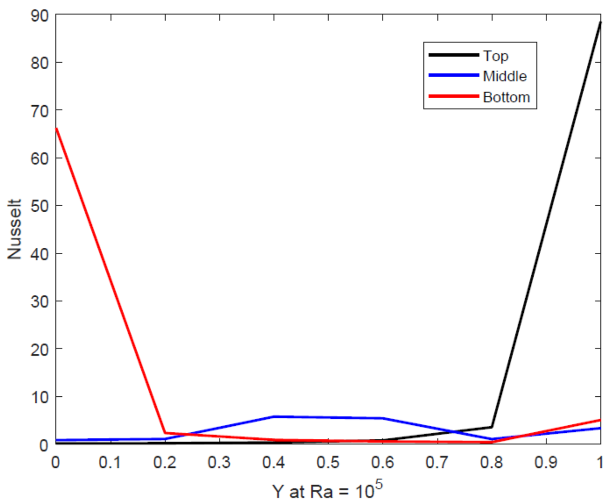

Fig. 10: Deviation of Nusselt number for the three position Top, Center and Bottom at Ra=10^5 within the cavity

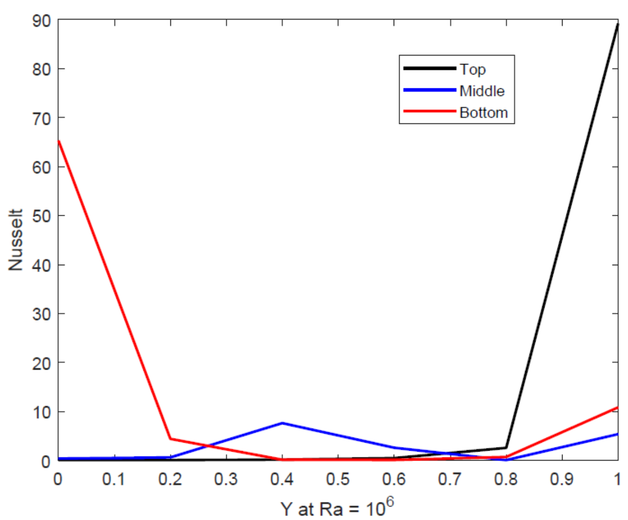

Fig. 11: Deviation of Nusselt number for the three position Top, Center and Bottom at Ra=10^6 within the cavity

5. CONCLUSIONS

A numerical investigation has been conducted to determine the effect of the position of rectangular shape heated block and Rayleigh number on natural convection inside a square cavity. Conclusively, the numerical results can be summarized as follows.

(a.) The dimensionless temperature profiles and fluid flow configurations in the square cavity of rectangular shape heated block at the top, center and bottom of the square cavity are drastically different.

(b.) Nusselt number was strongest around the heated block at the top of the cavity at a particular Rayleigh number

(c.) Nusselt number deviation was experienced at maximum around the position of the heated block and drastically reduced along the rest of the cavity.

(d.) For a specific position of the heated block in the cavity, the velocity of fluid flow rises with an increase in the Rayleigh number within the cavity.

(e.) An optimum buoyancy driven effect exist each period the position of the block has its Rayleigh number increased.

(f.) The position of the heated block in the cavity has a significant effect on the eccentric streamlines and thermal boundary layer with respect to deviation in Rayleigh numbers in the square cavity. Hence the rate of heat transfer was greatly enhanced.

NOMENCLATURE

T_c	Cold temperature [K]
T_h	Hot temperature [K]
g	Gravitational acceleration, m.s^{-2}
W	Breadth of the block [m]
Nu	Local Nusselt number along the heat source
x	Horizontal coordinate
k	Thermal conductivity, W.m^{-1}. K^{-1}
v	Vertical velocity [m/s]
p	Pressure [N/m^2]
u	Horizontal velocity [m/s]
Ra	Rayleigh number
Pr	Prandtl number
T	Temperature [K]
y	Vertical coordinate [m]
Nu	Nusselt number

Greek

ρ	Density [kg/m^3]
Θ	dimensionless temperature
ϑ	kinematic viscosity [m^2.s^{-1}]
Φ	Dimensionless Stream function
Ω	vorticity

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