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Application of Multi-Fractal and Kriging Interpolation Method For the Re-construction of Strata

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Abstract: In order to generate 3-dimensional (3D) strata, a stratum data model of tri-prism based on borehole data is first presented, which is applicable in geological engineering and geotechnical engineering. Due to the local variance and spatial structure among attributes of boreholes, the geological properties at unknown location have to be estimated or interpolated by some special methods. However, classic statistical methods could not resolve such problem which includes selection of sample points, spatial estimation and comparison of bilateral data. A distance-weighted interpolation method for scattered borehole data points is proposed in this paper, which exploits the Kriging method in global geostatistics. On the premise that the distributed statistics characters of the spatial borehole data were obtained at first, accorded to the distributed statistics characters, the spatial interpolation was presenting successively. Moreover, multi-fractal is presented to depict the local singularity which should be ignored by sliding weighted average algorithm such as Kriging method. Case study shows that the generation of 3D strata by proposed interpolation methods could give a good result in global and local occasion.

Keywords: borehole, stratum data model, scattered points interpolation, Kriging method, multi-fractal.

1 Introduction

With the development of economy, more scholars turn their attention to the development and utilization of underground space; 3D geospatial modeling has become a research hotspot [Wang and Zhu (2011)].

3D geological modeling data sources mainly come from the geological document, geological section map and borehole data. Borehole is the most direct way of obtaining origin data, which has been paid more attention by many scholars [Zhu (1998); Yonezawa, Nemoto and Masumoto (2002); Liu and Gong (2001); Zhu,

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Zheng and Wu (2003)], through the acquisition of different stratum boundary of borehole histogram, and the establishment of corresponding topological relations, the surface model, solid model and mixed model of 3D strata would be re-constructed.

Engineering geological borehole could be thought as data source of regionalized variable, which has two remarkable characteristics: the random and structural distribution. First of all, the regionalized variable has local, random and anomaly characteristics. Secondly, the regionalized variable possesses some structural characteristics. Therefore, to obtain spatial statistical characteristics of borehole data, it has great application and profound prospects in simulation of regional engineering geology.

Kriging method calculates the spatial interpolation weight by introducing a semi-variogram function of distance variable, because the semi-variogram could describe the spatial structure characteristics and the random distribution characteristics [Franke (1982); Cressie (1991); Christakos (1992)].

Kriging method is a spatial interpolation method based on semi-variogram. In essence, it is a moving weighted average method which would inevitably cause to data smoothing, especially for the local singularity geological conditions. The Multi-fractal theory could overcome the defaults of Kriging method [Cheng (1994)], it depicted the local singularity rule based on the measurement range and unit.

2 Stratum modeling based on borehole data

To establish 3D stratum of Borehole data has 3 steps: (1) borehole data pre-processing, as shown in Fig.1; (2) creating the surface reference triangulation; (3) generating 3D strata.

Dragging the surface reference triangulation from top to bottom, the results are the tri-prisms of regional 3D strata, as shown in Fig.2.

As mentioned above, according to the borehole data to establish geological model of normal strata, fold etc. could realize the full process automation, while for the complicated geological phenomena, such as fault and lens, it still need manual intervention.

3 Kriging interpolation methods

Let $Z(x)$ be a point load of regionalized variable, and it's second-order stationary (or intrinsic): $Z(x_i)(i = 1,2,.....,n)$, they are defined in the points on the $x_i(i = 1,2,...,n)$, now it's going to estimate the regionalized variable of point x_0, the esti-

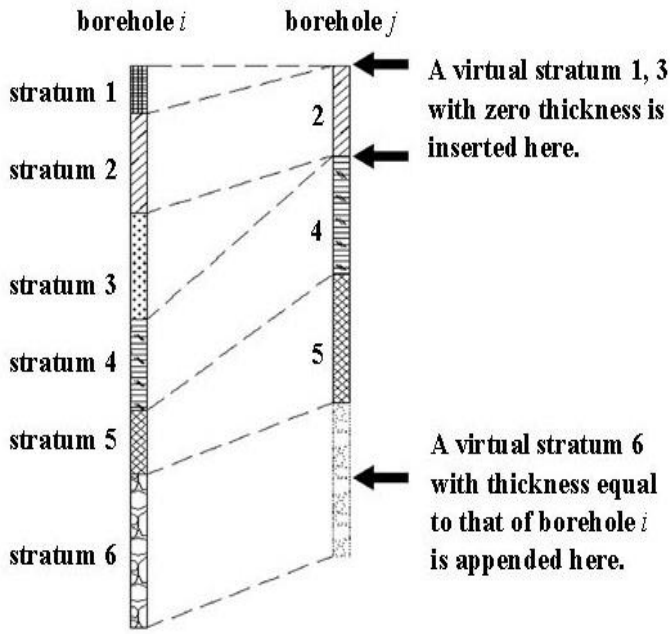

Figure 1: Borehole data pre-processing.

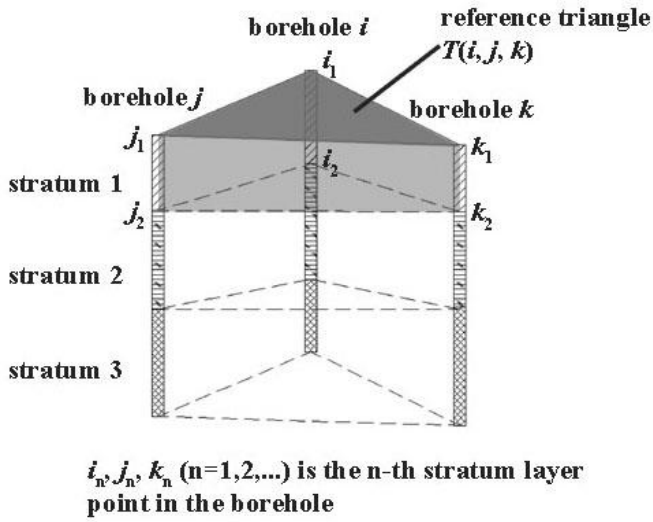

Figure 2: Tri-prisms generated from reference triangle.

mator is as follows:

$$\hat{Z}(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i) \tag{1}$$

To meet the constraint conditions:

$$\left.\begin{array}{l} E[\hat{Z}(x_0) - Z(x_0)] = 0 \Rightarrow \sum\limits_{i=1}^{n} \lambda_i = 1 \\ \sigma_E^2 = E[\hat{Z}(x_0) - Z(x_0)]^2 = \mathrm{M}in \end{array}\right\} \tag{2}$$

Finally, getting the Kriging interpolation equations of semi-variogram variables:

$$\left.\begin{array}{l} \sum\limits_{j=1}^{n} \lambda_j y_{i,j} + \mu = y_{i,0} \ , (i = 1, 2, ..., n) \\ \sum\limits_{j=1}^{n} \lambda_j = 1 \end{array}\right\} \tag{3}$$

The semi-variogram values $y_{i,j} = y(x_i - x_j)$, it is calculated from Kriging theoretical curve which is described by the measured data fitting.

4 Multi-fractal theories

Multi-fractal theory is proposed by Mandelbrot who studied the turbulence, and then its mathematical model is gradually established and has been applied in various fields. Fractal is the local singularity distribution of physical quantity connected with the geometric support.

Multi-fractal characteristics could be depicted by the statistics moment, which uses the partition function to calculate the multi-fractal dimension; the formula is as follows [Mandelbrot (1983)]:

$$\alpha = \lim_{\varepsilon \to 0} \frac{\lg \langle \chi(\varepsilon) \rangle}{\lg \varepsilon} = \lim_{\varepsilon \to 0} \frac{\lg \left\langle \sum\limits_{\mathrm{i}=1}^{N(\varepsilon)} \mu_i(\varepsilon) \right\rangle}{\lg \varepsilon} \tag{4}$$

In above equation: $\langle\rangle$ is the statistical moment of $\mu_i(\varepsilon)$ in the partitioned data set S, ε is the unit size and $N(\varepsilon)$ is the number of whole data set.

Current methods of spatial interpolation and wave filtering are the sliding weighted average values of field points, so, the equation (1) could be change into:

$$\hat{Z}(x_0) = \sum_{\Omega(\mathrm{x}_0, \varepsilon)} \omega(||x_0 - x||) Z(x) \tag{5}$$

In above equation, $\Omega(x_0,\varepsilon)$ is the small sliding area of radius ε around the center pointx_0, $\omega(||x_0 - x||)$ is the distance $(|x_0 - x||)$ weighted function between point x_0 and x which comes from$\Omega(\mathrm{x}_0,\varepsilon)$, it could be determined by Kriging method, but does not involve the local singularity measurement. So, multi-fractal method proposed by Cheng could change equation (5) into:

$$\hat{Z}(x_0)=\varepsilon^{\alpha-2} \sum_{\Omega(x_0,\varepsilon)} \omega(||x_0 - x||)Z(x) \tag{6}$$

In above equation, α is the local multi-fractal dimension of point x_0.

The above expression contains not only the spatial correlation of the components, but also has the factor of local singularity. Thus, the multi-fractal is suitable for local anomaly description, the weighted average interpolation method in usually is the special case of multi-fractal method.

5 engineering application

5.1 Borehole data sources

There are number of 98 regional engineering geological boreholes, and they are representing 12 strata as shown in Fig.3-4, the statistical data [Zhu, Wang and Li (2007a, b)] is shown in tab.1.

Table 1: main stratum parameters.

Stratum name	Elevation/ m		
	maximum	minimum	mean
Ground surface	7.00	3.00	4.26
(1) Artificial fill soil	4.61	-2.65	1.91
(2)1 Silty clay	1.70	-0.36	0.68
(3) Muddy silty clay	-1.71	-6.49	-4.0
(4) Muddy clay	-9.03	-14.76	-12.11
(5)1 Silty clay	-11.7	-45.29	-18.66
(6)2 Sandy silt	-13.82	-63.99	-33.65
(7) 3 Silty clay	-19.92	-57.56	-39.52
(8) 4 Silty clay	-27.72	-51.70	-41.95
(9) 2 Silt	-53.68	-81.19	-62.17
(10)1 Fine sand	-65.22	-83.25	-78.53
(11) 2 Pebbly fine sand	-90.49	-105.26	-96.77
(12) Silty clay	-95.69	-103.59	-99.31

As shown in Fig.3, boreholes are sparsely distributed in the space; the irregular 3D stratum model of the original borehole data set would be unable to meet the needs of research.

To facilitate subsequent research on digital strata, virtual boreholes in the rectangular region need spatial interpolation as shown in Fig.3, the geometric parameters of the rectangular region as listed in tab.2.

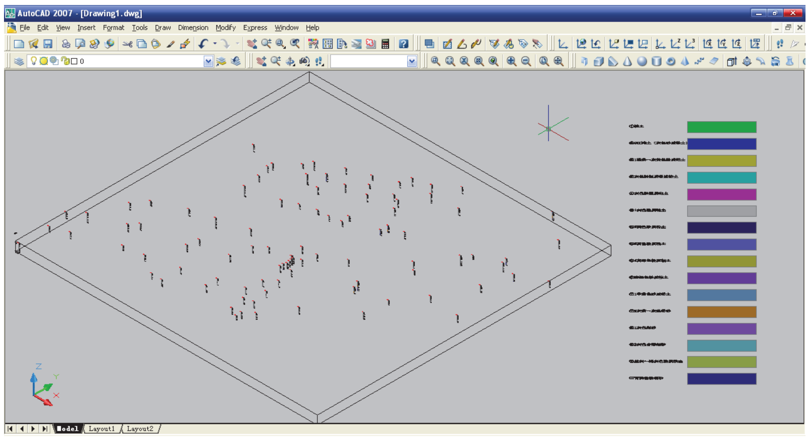

Figure 3: Global coordinates of boreholes(m).

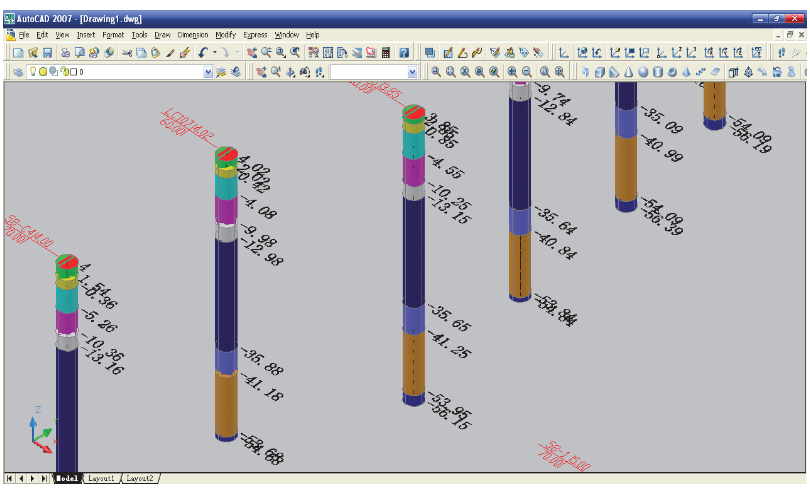

Figure 4: Local coordinates of boreholes(m)

Table 2: Spatial interpolation area.

Sequence	Point name	X/m	Y/m
1	P1	-93.0	-6 721.0
2	P2	3 757.0	-6 721.0
3	P3	3 757.0	-2 789.0
4	P4	-93.0	-2 789.0

The length of rectangular region is 3850m, and the width is 3932m, surface reference triangulation consists of number $20 \times 20 \times 2$ triangles.

We takes ground surface elevation interpolation as an example to discuss the algorithm efficiency, however, the other strata properties would be interpolated in same principle. We compare the validity between self-compiled kriging method and the results are provided by commercial software Winsurf; moreover, we compare the results whether considering the multi-fractal theory; lastly, we compare the results are provided by engineering experience.

5.2 comparisons with commercial software

Fig.5 is the semi-variogram curve of surface elevation, three parameters: range is $a = 2\ 137.76$ m, nugget is $C_0 = 0$, and arch height is $C = 0.389$ [Li, Wang and Zhu (2009)].

In Fig.5-8, comparing self-compiled kriging method and the results are provided by commercial software in the most areas, the calculated differences of the two software are very small, the elevation difference average value is $M = 0.018$ m, standard deviation is $S = 0.08$, the reliability of the self-compiling algorithm is as same as the commercial software Winsurf, this example guarantees the Kriging method could be used in engineering practice.

5.3 Multi-fractal interpolation results

Based on Kriging method, the multi-fractal theory is used to measure the local singularity of elevation distribution, first of all, it's necessary to calculate the local singularity exponent of virtual borehole; therefore, we could define multi-fractal measurement $\mu(\varepsilon)$ which is volume of surrounded elevation in the range of $\varepsilon \times \varepsilon$. Using squares to calculate $\mu(\varepsilon)$ with different length of 0.99 m, d, $2d$, $3d$, $4d$(d is the small grid size, in this case $d = 192.5$ m), such as the double logarithmic shown in Fig.9, the slope of the straight line regression is the singularity index or the fractal dimension $a(x_0)$ of the special point, When the linear correlation

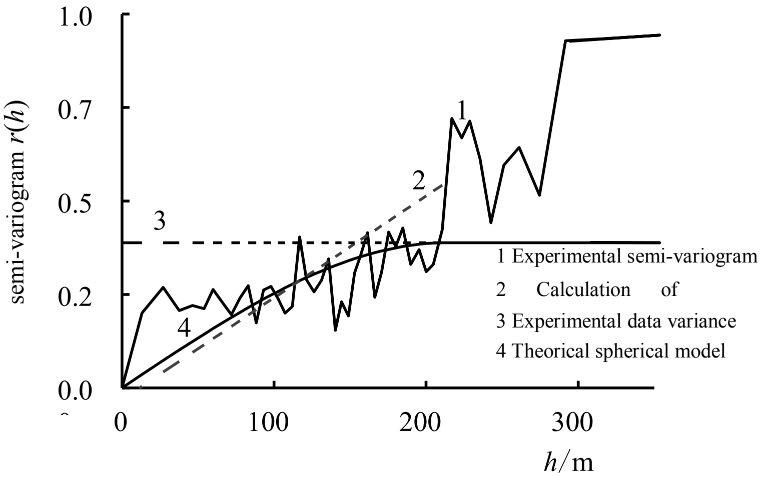

Figure 5: Semi-variation of surface elevation.

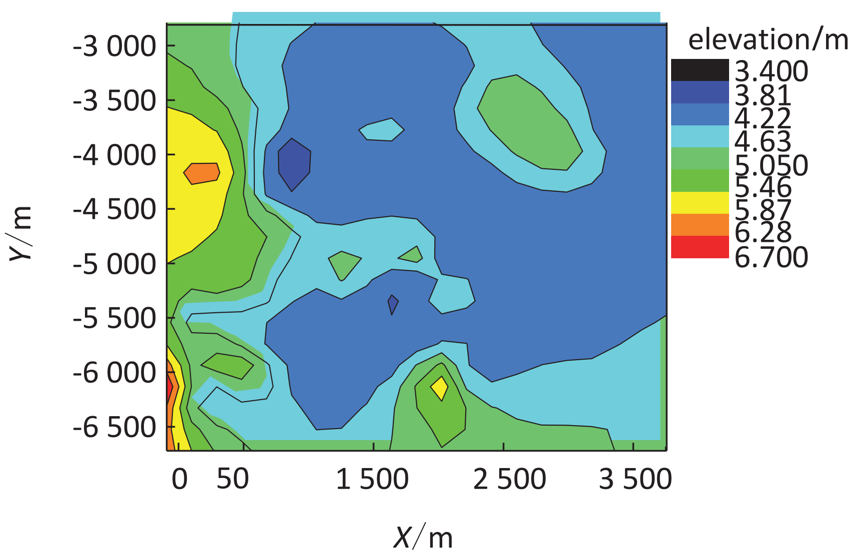

Figure 6: Semi-variation of surface elevation.

coefficient of $\lg\varepsilon - \lg\mu(\varepsilon)$ is greater than the specified threshold (R=0.9), $\mu(\varepsilon)$ and ε are in exponent relation, otherwise, $a(x_0) = 2$.

As shown in Fig.9-12, multi-fractal theory combines with spatial interpolation Kriging methods would makes up the defect of semi-variogram local smoothing,

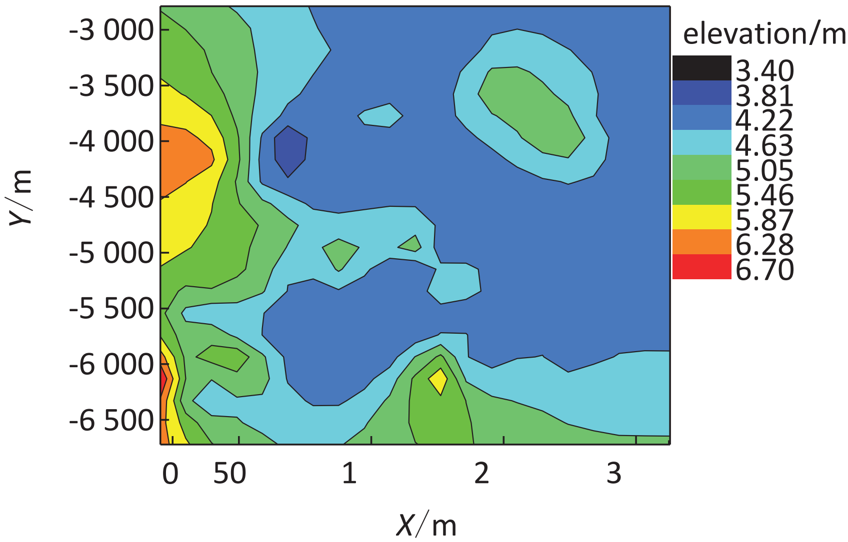

Figure 7: Contours of surface elevation by Winsurf.

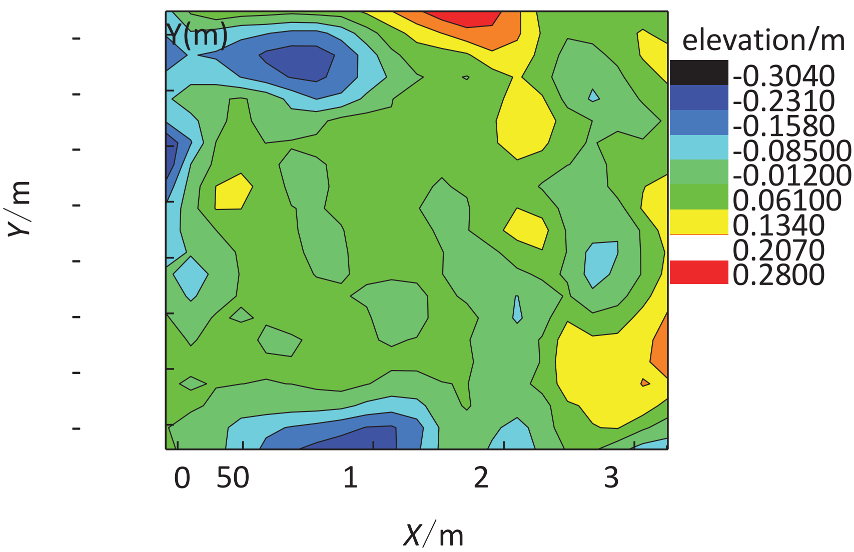

Figure 8: Difference of Interpolation.

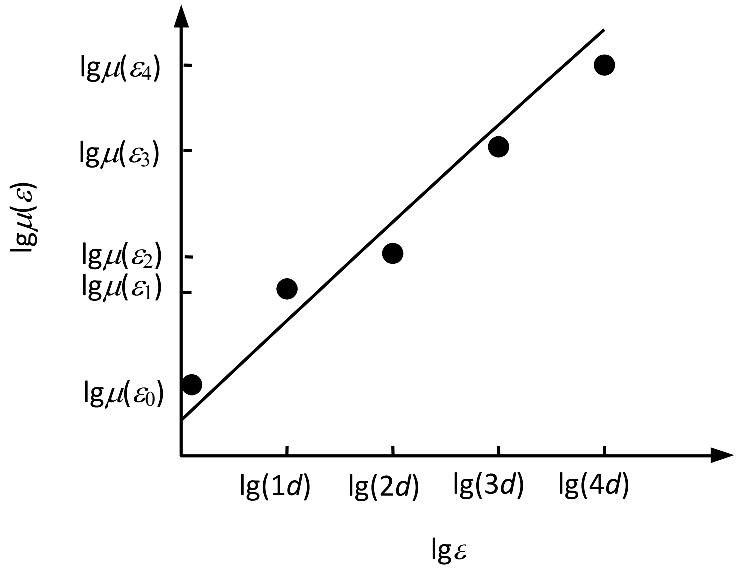

Figure 9: Regression of singularity index.

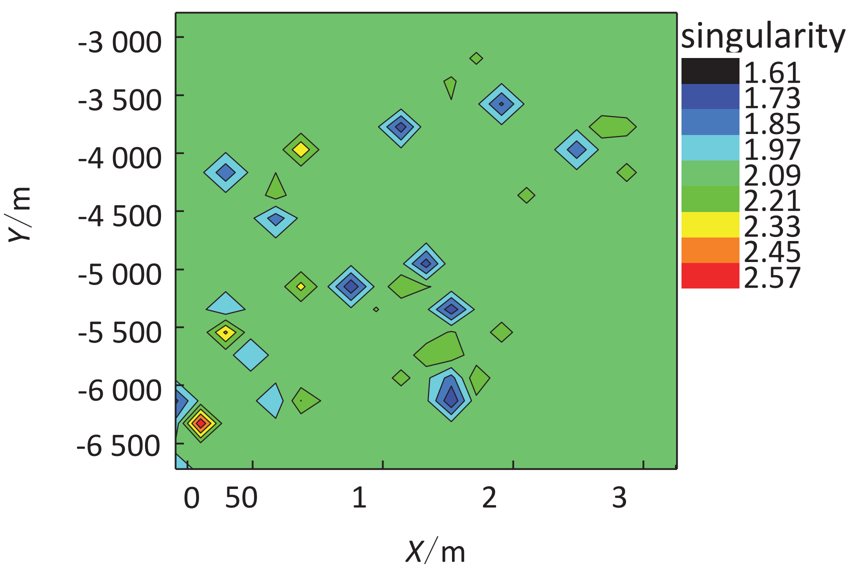

Figure 10: Spatial distribution of singularity.

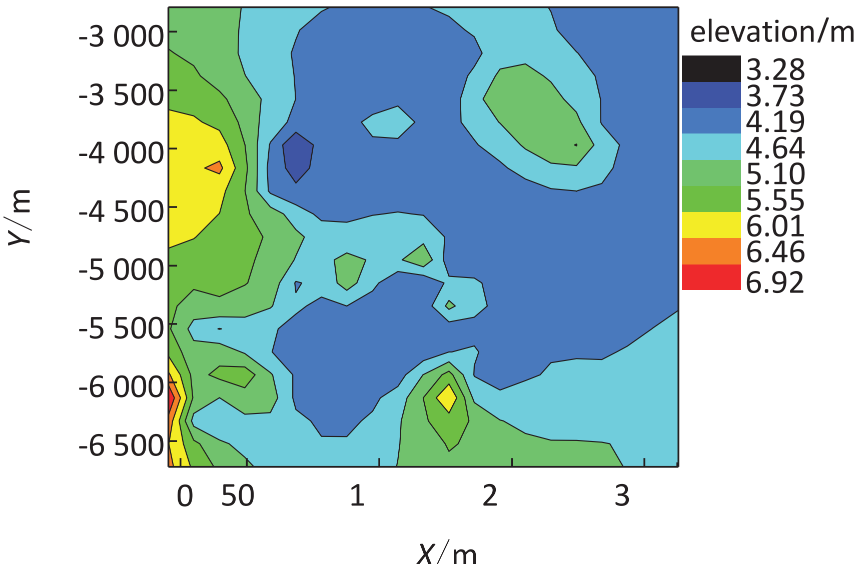

Figure 11: Contours of surface elevation based on local singularity.

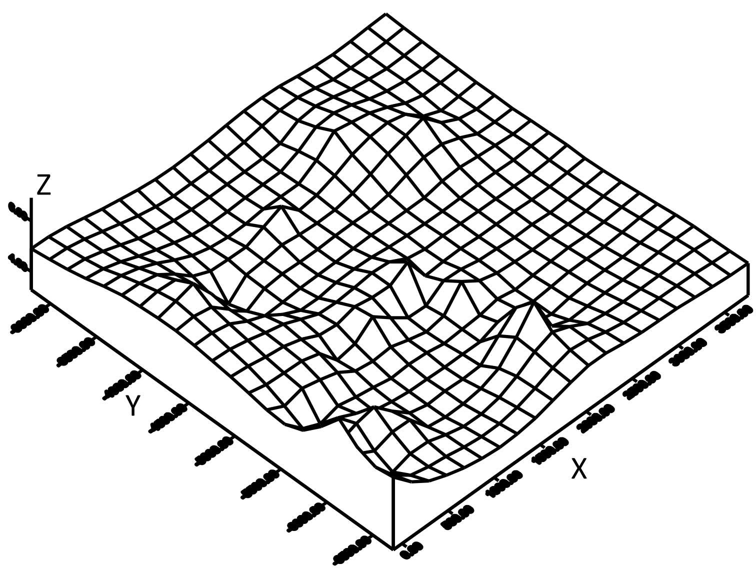

Figure 12: 3D surface elevation (m).

and depict the local singular of surface elevation. Computing the singular coefficient is based entirely on the original borehole distribution; therefore, the drilling position needs consider engineering geological sampling convenience and should be arranged in the point of elevation change.

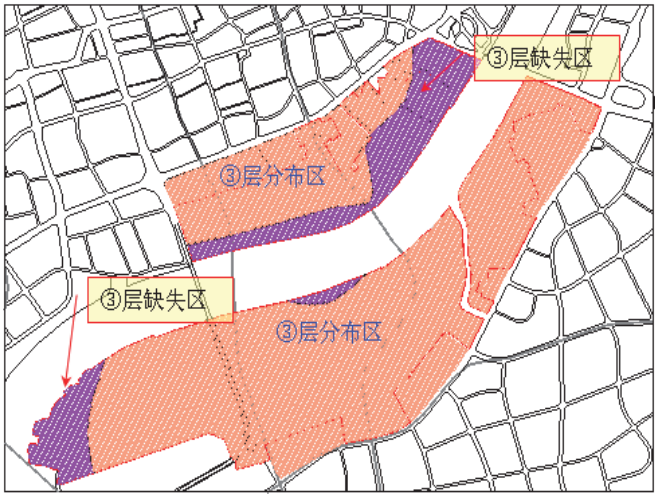

Figure 13: empirical estimation of No. (3) stratum.

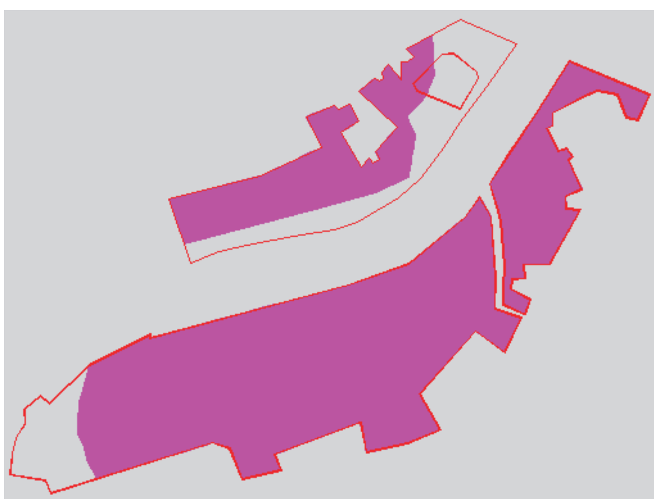

Figure 14: Spatial Interpolation of No. (3) stratum

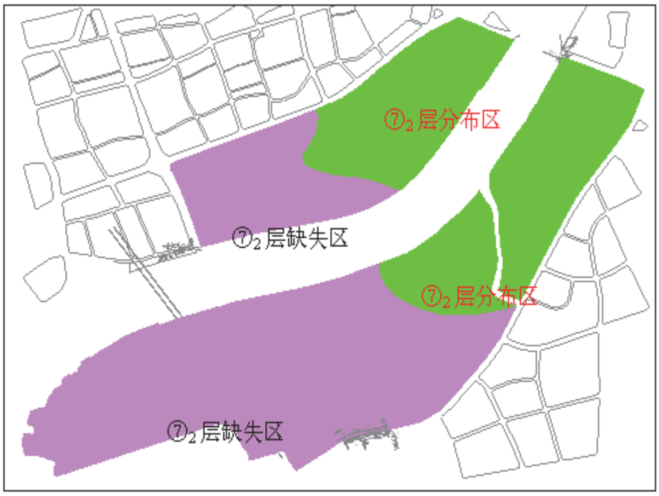

Figure 15: Empirical estimation of No. (7) stratum.

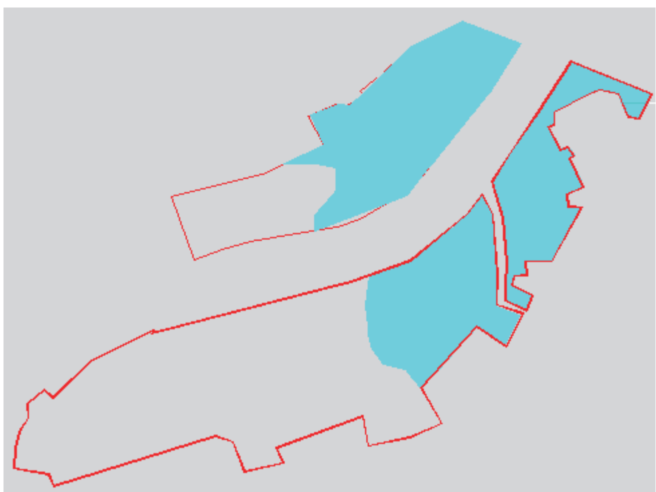

Figure 16: Spatial Interpolation of No.(7) stratum

5.4 comparing the results are provided by engineering experience.

We use Kriging method and multi-fractal spatial interpolation theory in a region, and comparing with engineering geological map of practical experience, as shown in Fig.13-16.

The difference between spatial interpolations of Kriging method combined with multi-fractal theory with engineering practice is less than 5%, and it satisfies the geotechnical engineering requirements.

6 conclusions

In recent years, computer technology combines geotechnical engineering more and more closely, borehole information interpolation and extrapolation based on Kriging method and multi-fractal theory have two features:

(1) Through the semi- variogram function depiction, Kriging method calculates the weight coefficients of observation points for virtual point prediction; it reflects the global characteristics of data space.

(2) Through spatial self-similarity filtering, multi-fractal theory overcomes the shortcomings of semi-variogram local smoothing of Kriging method, it could measure local singularity of spatial data distribution, through digital strata re-construction example of engineering geological borehole data, our composite method could obtain good results in the spatial interpolation.

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