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Modeling and Simulation of the Nonlinear Computed Torque Control in Simulink/MATLAB for an Industrial Robot

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Abstract: The paper presents a conceptually simple nonlinear controller, commonly called computer torque controller, which can fully compensate the nonlinear forces: Coriolis and centripetal forces (natural and continuous nonlinearities) and at the same time the program in Simulink can fully compensate the natural and discontinuous nonlinearities (hard nonlinearities): friction and backlash utilizing the intentional nonlinearities artificially introduced in system and lead to high accuracy control for a very large range of robot speeds and a large workspace.

Keywords: robot dynamics, nonlinearities, linear and nonlinear control of robots.

1 Introduction

In order to achieve a pre-specified accuracy in robot tasks such as pick-and-place, arc welding and laser cutting the speed of robot motion and thus productivity, has to be kept low. In this case it is considered the simplest type of position control strategy, namely: *independent joint control* or classical linear control. In this type of control each axis of the manipulator is controlled as a Single-Input/Single Output (SISO) system and any coupling effects due to the motion of the other links is treated as a disturbance. The classical manipulator control schemes are based on independent joint design using P, PD or PID controller. In reality the dynamic equations: Euler-Lagrange of robot form a nonlinear and multivariable system and in this case the robot control is named *multivariable control*: Multi-Input/Multi-Output(MIMO). A conceptually simple nonlinear controller, called computed-torque controller, can fully compensate the nonlinear Coriolis and centripetal forces (continuous nonlinearities) in the robot motion and lead to high accuracy control for a very large of robot speeds and a large workspace. However, in control systems there are many nonlinearities whose discontinuous nature does not allow linear approximation. These so-called “hard nonlinearities” or discontinuous nonlinearities

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include friction, saturation, dead-zones, backlash and hysteresis and are often found in control engineering. Usually, the continuous and discontinuous nonlinearities, named *natural nonlinearities*, have undesirable effects and control system have to properly compensate for them and the *intentional nonlinearities* are artificially introduced by the designer.

2 Classical linear and nonlinear control in Simulink/MATLAB

The position control system is a system that converts a position input command to a position output response. A schematic layout of the servomotor (permanent magnet d.c. motor) and gear reduction is shown in Fig.1.

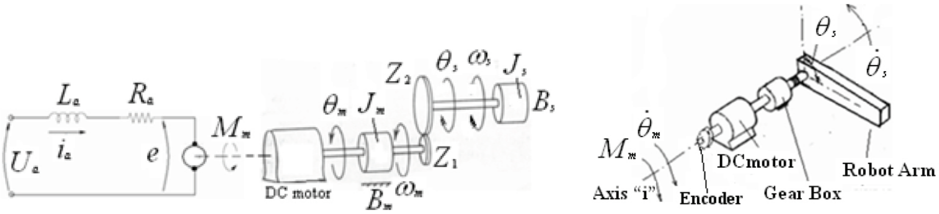


Figure 1: Servo plant schematic (D.C. motor and gear reduction).

The electrical parameters are as follows:

$R_a[\Omega]$-armature resistance; L_a[H]-armature inductance; K_m[Nms/rad]-motor torque constant; K_e[V/rad/s]-motor voltage constant.

The mechanical parameters are as follows:

$J_m[\text{Nms}^2]$-armature inertia; $J_s[\text{Nms}^2]$-load inertia; B_m[Nms/rad]-armature frictional coefficient;B_s[Nms/rad]-load frictional coefficient; $i = Z_2 / Z_1$–low gear ratio; M_r-resistant moment refered to the rotor shaft.

From the Fig.1, we can write the following equations based on Newton's law combinated with Kirchhof's law:

$$u_a = R_a\, i_a + L_a \frac{d\, i_a}{dt} + e \tag{1}$$

$$M_m - M_r = J \frac{d\, \omega_m}{dt} + B\, \omega_m \tag{2}$$

where: $u_a$armature voltage , $e = K_m\, \omega_m$ the motor e.m.f., ω_m armature speed, $M_m = K_m\, i_a$ motor torque, i_a armature current and equivalent inertia and friction refered to the rotor shaft(dynamic model)are:

$$J{=}\, J_m + J_s \frac{1}{i^2}; B{=}\, B_m + B_s \frac{1}{i^2} \tag{3}$$

Using Laplace Transforms the above equations can be expressed in terms of ”s”:

$$U_a(s) = (s\,L_a + R_a)\,I_a(s) + K_e\,\Omega_m(s)$$

$$K_m\,I_a(s) - M_r(s) = (s\,J + B)\,\Omega_m(s) \tag{4}$$

and it results the scheme of nonlinear model in Fig.2, where:

-*Transf Fcn2* is the transfer function of d.c. motor: *1/(sL*$_a + R_a$);

-*Transf Fcn 4* is the transfer function of mechanical transmission: *1/(sJ+B)*

-*Saturation* (voltage and current limiters);

-*LuGre F. M.* is a dynamic model of friction with low speed and the friction moment M_f is given by the expression:

$$M_f = \lambda_0\,z + \lambda_1\,\frac{dz}{dt} + \alpha_2\,\omega$$

$$\frac{dz}{dt} = \omega - \frac{\lambda_0\,|\omega|}{g\,(\omega)}$$

$$g\,(\omega) = \alpha_0 + \alpha_1\,e^{-\,\omega/\,\omega_{sk}} \tag{5}$$

where: α_0-Coulomb friction; α_1-Stribeck friction; α_2-viscous friction; λ_0-bristles stiffness; λ_1-bristles damping; ω_{sk}-Stribeck velocity; ω-angular velocity.

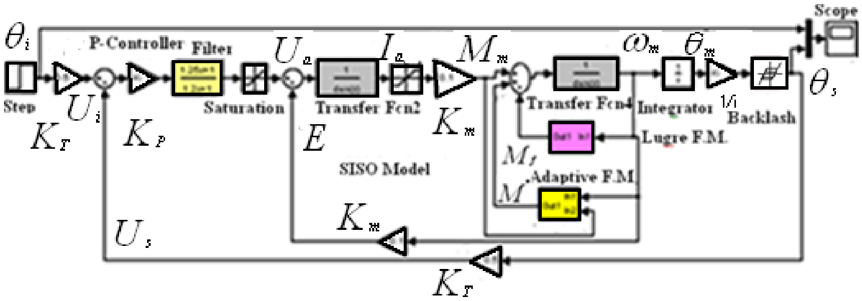


Figure 2: Software Simulink/MATLAB for the nonlinear independent control position in an industrial robot (model of position controlled robot link drived by permanent magnet DC motor).

-*Adaptive F. M.* (friction compensation) is adaptive friction model based on a Coulomb model:

$$M^* = a^*\,sign\,(\omega)$$

$$\dot{z} = K\, u_c\, sign\,(\omega)$$

$$a^* = z - K\, J\, |\omega| \tag{6}$$

where: K is an adaptation gain; J is inertia, z is a internal variable; $M^*$is the estimate friction moment, $u_c\,(M_m)$ is the conventional controller output, ω is the relative velocity and a^* is the estimated Coulomb model parameter.

-*Backlash*, also known as play, is the difference between tooth width and, for example, a lead compensator, named *Filter*, has the expression:

$$F\,(s) = \frac{T_2\,.s+1}{T_1\,.s+1} \tag{7}$$

where: T_1, T_2 are constants.

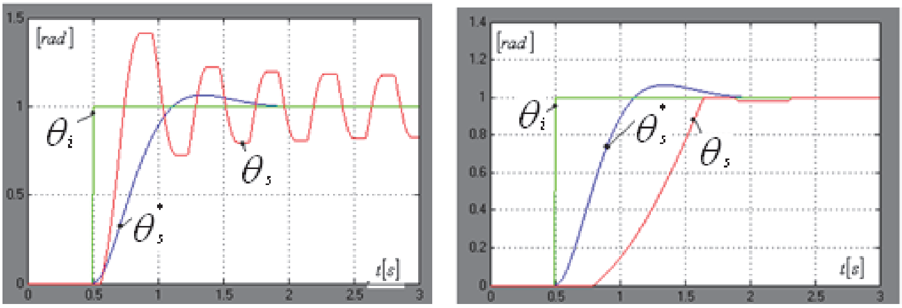
-The unit *Step,* as:

$$\theta_i = \begin{cases} 0 & t = 0.5s\ (for\ example) \\ 1 & t > 0 \end{cases} \tag{8}$$

is widely used in studying input/output systems and *step response* of linear or nonlinear system is defined as the output $\theta_s(t)$ starting from initial condition

-The constant for the *conversion: angle-voltage* is K_T in the case input θ_i or output θ_s (encoder). The scheme of servomechanism has a P-*Controller* with proportional gain K_P.

Fig.3. presents the input system θ_i-unit step and the step response θ_s^* for the ideal linear control position and θ_s for the nonlinear control position with a) backlash or b) friction plus backlash and compensations.



(a) with backlash, without compensation; (b)with friction + backlash and compensations

Figure 3: Step response of the linear or nonlinear control position model in Simulink/MATLAB.

The backlash induces oscillations or steady state errors and the effect of friction is the existence of a considerable steady state error. These effects disappear in presence of "lead compensator"-filter and model based friction compensation (Fig.3.b)

3 Nonlinear computed torque control in Simulink/MATLAB

In reality, the dynamic equations of a robot manipulator form a complex, nonlinear and multivariable system. A basic problem in controlling robots is to make the manipulator follow a preplanned desired trajectory. Suppose that the end of the arm should trace out the *circular workspace path* shown in Fig.4., which described (point **E**) by:

$$x(\varphi) = x_0 + R\cos\varphi$$

$$y(\varphi) = y_0 + R\sin\varphi \tag{9}$$

In the same time the characteristic point **E** has the Cartesian coordinates:

$$x = l_{12}\cos\theta_1 + l_2\cos\theta_2$$

$$y = l_{12}\sin\theta_1 + l_2\sin\theta_2 \tag{10}$$

where: l_{12}, l_2, θ_1, θ_2 are the parameters of the elbow manipulator.

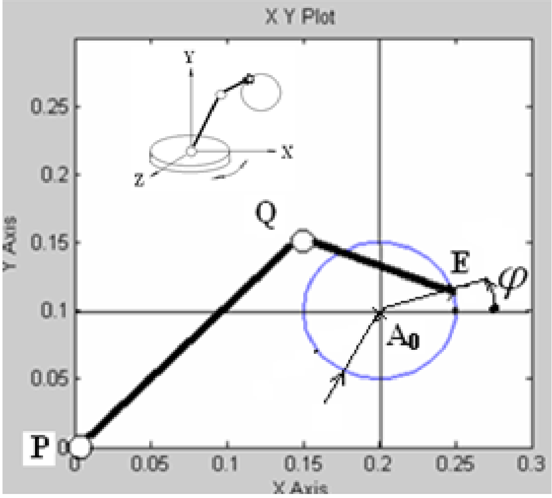


Figure 4: Desired Cartesian trajectory.

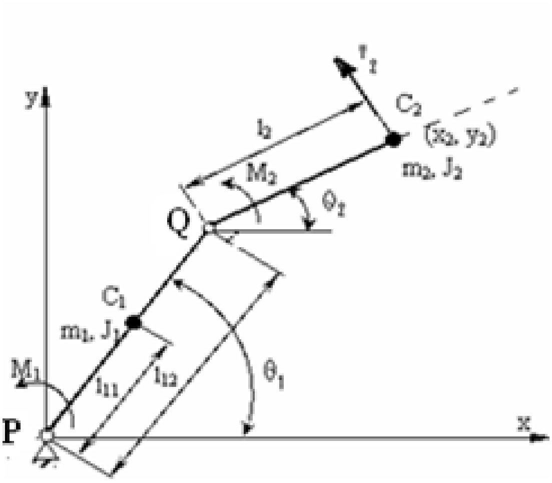


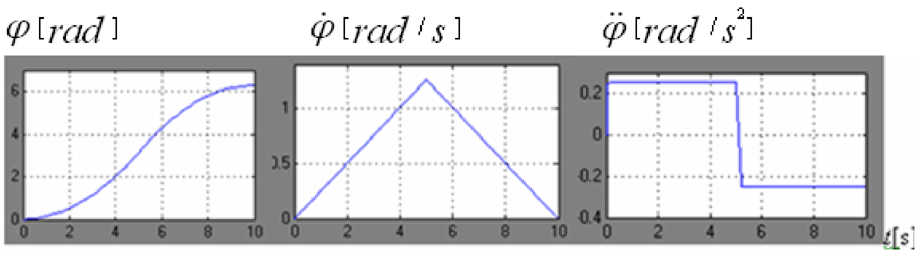
Figure 5: Two-link plannar elbow arm.

Therefore, it is important to be able to find the desired joint space trajectory $q_d(t)$ given the desired Cartesian trajectory(inverse kinematics):

$$\theta_{1d} = arctg\frac{y(\varphi) - l_2 \sin\theta_2}{x(\varphi) - l_2 \cos\theta_2}$$

$$\theta_{2d} = arctg\frac{y(\varphi) - l_{12} \sin\theta_1}{x(\varphi) - l_{12} \cos\theta_1} \tag{11}$$

where: $q_d(\theta_{1d}, \theta_{2d})$ are the joint variables in terms of the x and y coordinates of **E**.



(a) (b) (c)

Figure 6: Minimum-time trajectory: (a) position of the point **E** in the plane OXY; (b)velocity; (c)acceleration.

A minimum-time trajectory is shown in Fig.6. This is the sometimes called a Bang-Bang trajectory. In this condition, Fig 8. presents the "Trajectory planner" which converts the circular trajectory of the end of the arm (with mid point $\mathbf{X}_o$**,** $\mathbf{Y}_o$ and radius **R**) in joint space trajectory$q_d\,(\theta_{1d},\theta_{2d})$.

Using Lagrange's equation to compute the dynamical equations it results a special form (without viscous friction and disturbances):

$$M(q)\,\ddot{q}+C(q,\dot{q})+G(q)=T \tag{12}$$

where:

$q,\dot{q}$–joint-variable and joint velocity vectors;

$T=[\ \tau_1\quad \tau_2]^T$ –generalized force vector;

M–inertia matrix is symmetric and positive definite;

$C(q,\dot{q})$–Coriolis/centripetal vector;

$G(q)$– gravity vector.

The nonlinear term is:

$$N(q,\dot{q})=C(q,\dot{q})+G(q) \tag{13}$$

and the arm dynamics becomes:

$$M(q)\,\ddot{q}+N(q,\dot{q})=T \tag{14}$$

An independent joint PD-control scheme can be written in vector form as:

$$u(t)=-K_D\,\dot{e}-K_P\,e \tag{15}$$

where: $e(t)=q_d(t)-q(t)$ is the error between the desired joint displacements $q_d(t)$ and the actual joint displacements $q(t)$ and K_P, K_D are diagonal matrices (positive) of proportional and derivative gains, respectively.

Then the overall robot arm input becomes:

$$\mathrm{T}=M(q)\,(\ddot{q}_d+K_D\,\dot{e}+K_P\,e)+N(q,\dot{q}) \tag{16}$$

We call this the *computer-torque control law.* It is important to realize that computed-torque depends on the inversion of the robot dynamics, and indeed is called *inverse dynamics control* and it results the real joint acceleration vector:

$$\ddot{q}=M^{-1}\{T_-N\} \tag{17}$$

Successively integrating it results the $q,\dot{q}$ joint-variable and joint velocity vector.

The outer-loop signal $u(t)$ can be chosen using many approaches, including robust and adaptive control techniques.

The resulting control scheme appears in Fig.7. It consists of an inner nonlinear loop plus an outer control signal $u(t)$ (with PD-control).

Therefore, the PD gains are usually selected for critical damping $\xi = 1$. In this case: $k_{D_i} = 2\sqrt{k_{P_i}}$ and $k_{P_i} = k_{D_i}^2/4$ (for joint i).

In the computed-torque control scheme it is introduced the backlash.

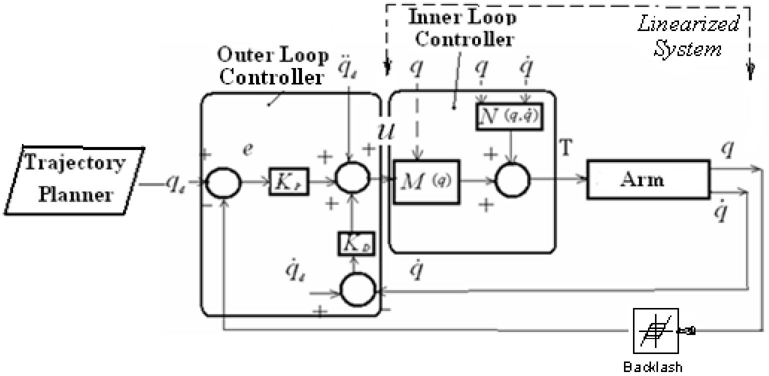


Figure 7: Inner loop/outer control architecture.

Computed-torque control scheme. PD-control scheme.

For a *two-link planar elbow arm*, the inertia matrix and the mentioned vectors are:

$$M(q) = \begin{bmatrix} J_1' & \dot{J}\cos(\theta_1 - \theta_2) \\ \dot{J}\cos(\theta_1 - \theta_2) & J_2' \end{bmatrix}$$

or

$$M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

with:

$$J_1' = J_1 + m_1 l_{11}^2 + m_2 l_{12}^2; \quad J_2' = J_2 + m_2 l_2^2; \quad \dot{J} = m_2 l_{12} l_2$$

$$N(q,\dot{q}) = \begin{bmatrix} -\dot{J}\dot{\theta}_2\left(\dot{\theta}_1 - \dot{\theta}_2\right)\sin(\theta_1 - \theta_2) + \dot{J}\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) + M_1'\cos\theta_1 \\ -\dot{J}\dot{\theta}_1\left(\dot{\theta}_1 - \dot{\theta}_2\right)\sin(\theta_1 - \theta_2) - \dot{J}\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) + M_2'\cos\theta_2 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} M_1'\cos\theta_1 \\ M_2'\cos\theta_2 \end{bmatrix}$$

with: $M_1^{'} = g\,(l_{11}\,m_1 + l_{12}\,m_2)$; $M_2^{'} = g\,m_2\,l_2$
m_1, J_1, m_2, J_2 the masses and masses moment of inertia (1) and (2)

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \left(\begin{bmatrix} \ddot{\theta}_{1d} \\ \ddot{\theta}_{2d} \end{bmatrix} + \begin{bmatrix} k_{D1} & 0 \\ 0 & k_{D2} \end{bmatrix} \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} k_{P1} & 0 \\ 0 & k_{P2} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \right) + \begin{bmatrix} N_{11} \\ N_{21} \end{bmatrix}$$

$$M^{-1} = \frac{1}{\det M} \begin{bmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{22} \end{bmatrix}$$

and $\det M = M_{11}\,M_{22} - M_{12}\,M_{21}$

$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \frac{1}{\det M} \begin{bmatrix} M_{22} & -M_{21} \\ -M_{21} & M_{11} \end{bmatrix} \begin{bmatrix} \tau_1 - N_{11} \\ \tau_2 - N_{21} \end{bmatrix} \tag{18}$$

In *Simulink/MATLAB* it is realized the simulation of PD Computed-Torque Control for a two-link elbow arm in plane circular motion (Fig.8).

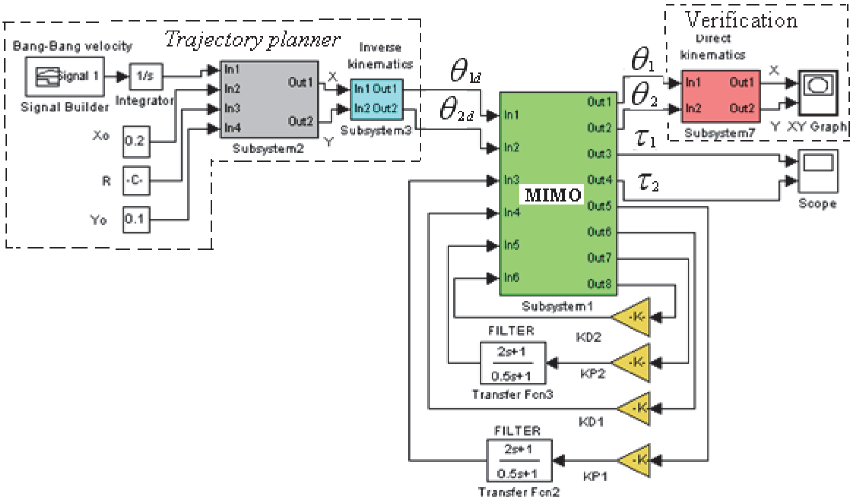
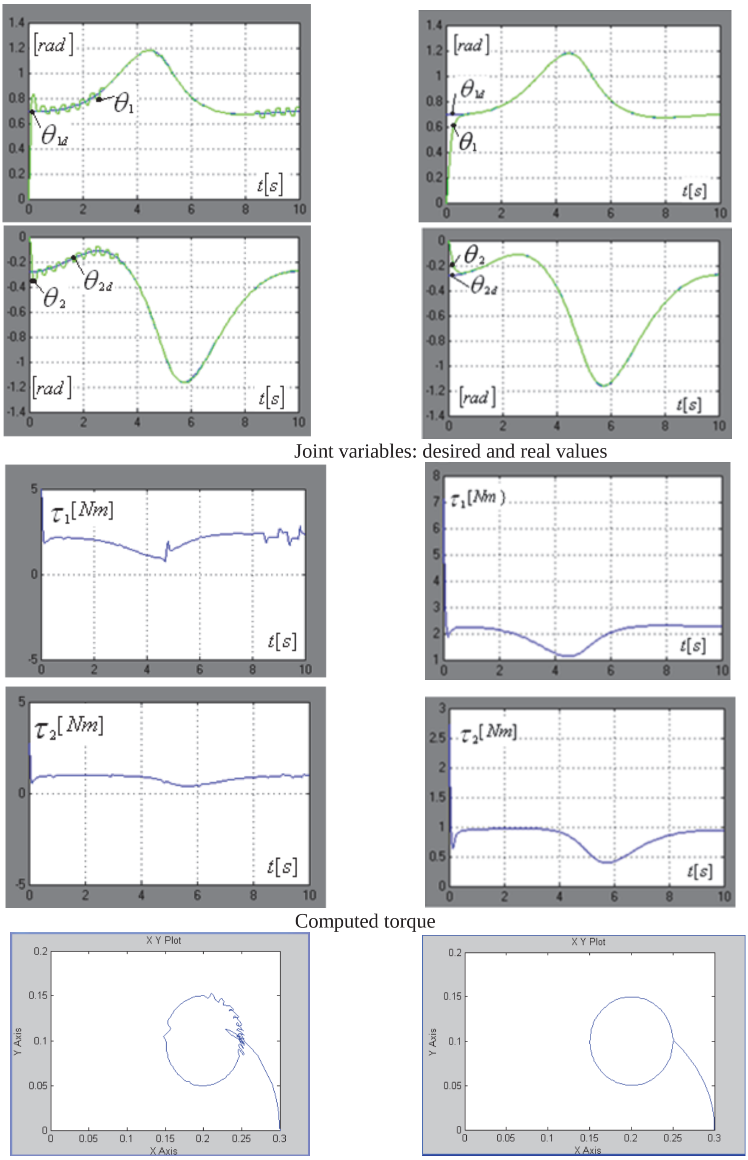


Figure 8: Software Simulink/MATLAB for the nonlinear multivariable control in an industrial robot (two-link planar elbow arm).

The software contains the trajectory planner with inverse kinematics, the MIMO system which includes PD computed-torque control, the backlash compensators (filters) and a block for verification which comprises the direct kinematics.

The diagrams of the joint variables: desired and real values: $\theta_{1d}, \theta_{2d}, \theta_1, \theta_2$, computed torque τ_1, τ_2 and circular workspace path (x, y), after simulation, are presented in Fig.9.



Joint variables: desired and real values

Computed torque

Circular workspace path

(a) with backlash/without compensation

(b) with backlash and compensation

Figure 9: The results of the simulation: joint variables, computed torque and circular path

The Fig.9.a) presents these diagrams when it is backlash and without compensation For our example the PD gains: $K_{P1}, K_{P2}, K_{D1}, K_{D2}$ and T_1, T_2 constants of the "filters" are selected to obtain minimum errors and a trajectory without oscillations Fig.9. b).

Another natural discontinuous nonlinearities produce the following effects:

-The effects of saturation is the decrease and delay of output. These effects are not compensated.

-The effect of friction is the existence of a considerable steady state error. When the friction is compensated, the steady state error approximately disappears but the settling time increases after simulations.

4 Conclusions

In the analysis, a nonlinear closed-loop system is assumed to have been designed and it is necessary to determine the characteristic of the system's behavior.

In the design it is given a nonlinear plant to be controlled and some specifications of closed-loop system meets the desired characteristics.

When a linear controller is used to control robot motion, it neglects the inherent nonlinear forces associated with the motion of the robot links. The controller's accuracy thus quickly degrades as the speed of motion increases, because many of the dynamic forces, such as Coriolis, centripetal forces, vary as square of the speed.

However, in control systems there are many nonlinearities whose discontinuous nature does not allow linear approximation (friction, saturation, dead-zone, hysteresis and backlash). These so-called"hard nonlinearities" produce: oscillations (instability) or steady state errors. Their effects cannot be derived from linear methods and nonlinear analysis techniques must be developed to predict a system's performance in the presence of these nonlinearities. The intentional nonlinearities are artificially introduced by the designer and this activity is named the compensation of the natural nonlinearities.

The Simulink/MATLAB software is in accordance with the design and the analysis of a nonlinear closed-loop system and with help of the Win Con software, a data acquisition board, it is possibly to control in real-time a servomechanism or an industrial robot.

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