 SL, vol.6, no.3, pp.155-185, 2011

Theory, Analysis and Design of Fluid-Shell Structures

Lakis A. A.[1,2], Toorani M. H.[1], Kerboua Y.[1]
Esmailzadeh M.[1] and Sabri F.[1]

Abstract: This paper review focuses mainly on development and application of a hybrid finite element approach used for linear and geometrically nonlinear vibration analysis of isotropic and anisotropic plates and shells, with and without fluid-structure interaction. Development of a hybrid element for different geometries of plates and shells is briefly discussed. In addition, studies dealing with particular dynamic problems such as dynamic stability and flutter of plates and shells coupled to flowing fluids are also discussed. This paper is structured as follows: after a short introduction on some of the fundamentals of the developed model applied to vibrations analysis of shells and plates in vacuo and in fluid, the dynamic analysis of anisotropic structural elements is discussed. Studies on dynamic response of plates in contact with dense fluid (submerged and/or subjected to liquid) follow. These studies present very interesting results that are suitable for various applications. Dynamic response of shell type structures subjected to random vibration due to a turbulent boundary layer of flowing fluid is reviewed. Aeroelasticity analysis of shells and plates (including the problem of stability; divergence and flutter) in contact with light fluids (gases) are also discussed.

Keywords: Plates, Shells, Hybrid Finite Element, Anisotropic, Vibration, Aeroelasticity, Turbulent Boundary Layer

1 Introduction

Shells and plates are widely used as structural elements in modern structural design i.e. aircraft construction, ship building, rocket construction, the nuclear, aerospace, and aeronautical industries, as well as the petroleum and petrochemical industries (pressure vessels, pipelines), etc. In addition, anisotropic, laminated composite shells are increasingly used in a variety of modern engineering fields (e.g.,

[1] Corresponding Author. Aouni.lakis@polymtl.ca.
[2] Mechanical Engineering Department, Ecole Polytechnqiue of Montreal, P.O. Box 6079, Station Centre-ville, Montreal (QC), H3C 3A7, Canada.

aerospace, aircraft construction) since they offer a unique advantage compared to isotropic materials. By optimizing the properties, one can reduce the overall weight of a structure. Also, it worthy of note that Fluid-Structure Interaction (FSI) occurs across many complex systems of engineering disciplines ranging from nuclear power plants and turbo machinery components, naval and aerospace structures, and dam reservoir systems to flow through blood vessels to name a few. The forces generated by violent fluid/structure contact can be very high; they are stochastic in nature (i.e. boundary layer of turbulent flow induces a random pressure field on the shell's wall) and thus difficult to describe. They do, however, often constitute the design loading for the structure. The problem is a tightly-coupled elasto-dynamic problem in which the structure and the fluid form a single system. Solution of these problems is obviously complex and technically challenging. One wide spread and complex FSI subclass is the category that studies non-stationary behavior of incompressible viscous flows and thin-walled structures exhibiting large deformations. Free surface motion of fluid often presents an essential additional challenge for this class of problems.

It is very important therefore, that the static and dynamic behavior of plate and shell structures when subjected to different loads be clearly understood in order that they are used safely in industry. The analysis of thin elastic shells under static and/or dynamic loads has been the focus of a great deal and research by Prof. Lakis and his research group for more than 40 years. These structural components (cylindrical, spherical, and conical shells as well as circular and rectangular plates) have been studied in light of such different factors as; large deformation (geometrical nonlinearity), thickness variation, residual stresses, rotary inertia, material anisotropy, initial curvature and the effect of the surrounding medium (air, liquid). We have developed a hybrid type of finite element, whereby the displacement functions in the finite element method are derived from Sanders' classical shell theory /or first order and higher order shear shell theory in the case of non-isotropic materials. This method has been applied with satisfactory results to the dynamic linear and non-linear analysis of plate and shell structures. The displacement functions are obtained by exact solution of the equilibrium equations of the structure instead of the usually used and more arbitrary interpolating polynomials. The structural shape function, mass and stiffness are derived by exact analytical integration. The velocity potential and Bernoulli's equation are adopted to express the fluid dynamic pressure acting on the structure. Integrating this dynamic pressure over the structural shape function results in the fluid-induced force components (inertia, Coriolic and centrifugal). In doing so, the accuracy of the formulation is less affected as the number of elements used is decreased, thus reducing computation time.

2 Background of the hybrid finite element method

Accurate prediction of the dynamic response (or failure characteristics) reached by the finite element displacement formulation depends on whether the assumed functions accurately model the deformation modes of structures. To satisfy this criterion, Lakis and Paidoussis (1971, 1972a) developed a hybrid type of finite element, whereby the displacement functions in the finite element method are derived from Sanders' classical shell theory. This allows us to use thin shell equations in full for determination of the displacement functions, mass and stiffness matrices, which are derived from precise analytical integration of equations of motion of shells. This theory is much more precise than the usual finite element method. The velocity potential and Bernoulli's equation have been adopted to describe an analytical expression for the fluid dynamic pressure whose analytical integration over the displacement functions of solid elements yields three forces (inertial, centrifugal and Coriolis) of the moving fluid. The shell is subdivided into several cylindrical elements (instead of the more commonly used triangular or rectangular elements) defined by two nodes and the boundaries of the nodal surface, see Figure 1.

The general displacement shape functions (in cylindrical co-ordinates in the axial, tangential and radial directions, taking into account their periodicity in the circumferential direction) are given by:

$$U = \int_n A e^{\lambda x/r} \cos(n\theta); \quad V = \int_n B e^{\lambda x/r} \sin(n\theta); \quad W = \int_n C e^{\lambda x/r} \cos(n\theta) \qquad (1)$$

Where n is the number of circumferential modes, x is the co-ordinate along the x-axis of the shell, θ is the co-ordinate in the circumferential direction, r is the average shell radius and A, B, C, and λ are the complex numbers. Substituting the above displacement functions into the equations of motion and solving for a non-trivial solution results in a characteristic eighth order equation [see Lakis and Paidoussis (1971)]:

$$h_8\lambda^8 + h_6\lambda^6 + h_4\lambda^4 + h_2\lambda^2 + h_0 = 0 \qquad (2)$$

Each root of (2) constitutes a solution of the equilibrium equations and the complete solution is a linear combination of these equations. After finding these solutions and carrying out a large number of the intermediate manipulations, which are not displayed here, the following equations can be derived that define the structural shape functions:

$$\left\{ \begin{array}{c} U \\ V \\ W \end{array} \right\} = [N] \left\{ \begin{array}{c} \delta_i \\ \delta_j \end{array} \right\} \qquad (3)$$

Where δ_i is the displacement vector of node i, see Fig. 1

$$\{\delta_i\} = \left\{ \begin{array}{c} u_{ni} \\ w_{ni} \\ (dw_n/dx)_i \\ v_{ni} \end{array} \right\} \tag{4}$$

The mass and stiffness matrices are then expressed as a function of (3)

$$[k_s] = \iint [B]^T [P] [B] \, dA \quad [m_s] = \rho t \iint [N]^T [N] \, dA \tag{5}$$

Where ρ and t are density and thickness of the shell. [N] and [B] are given by Lakis and Paidoussis (1971).

To model the fluid domain, a mathematical model has been developed based on the following hypotheses: the fluid is incompressible, the motion of the fluid is irrotational and inviscid, only small vibrations (linear theory) need to be considered, and the pressure of the fluid inside the shell is taken to be purely radial. The velocity function Φ, considering the aforementioned assumptions, in the cylindrical coordinate system is expressed as:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \Theta^2} + \frac{\partial^2 \Phi}{\partial x^2} = 0 \tag{6}$$

The components of the flow velocity are given by:

$$V_x = U_x + \frac{\partial}{\partial x}, \qquad V_\theta = \frac{1}{r}\frac{\partial \Phi}{?\theta}, \quad V_r = \frac{\partial \Phi}{\partial r} \tag{7}$$

Where U_x is the velocity of the fluid through the shell section and V_x, V_θ, and V_r are, respectively, the axial, tangential and radial components of the fluid velocity. Using Bernoulli's equation for steady flow:

$$\left(\frac{\partial \Phi}{\partial t} + \frac{V^2}{2} + \frac{P}{\rho_f} \right)_{r=\zeta} = 0 \tag{8}$$

Substituting for V^2 from (7), the dynamic pressure 'P' can be found as:

$$P_{i,e} = -\rho_{f_{i,e}} \left(\frac{\partial \Phi_{i,e}}{\partial t} + U_{x_{i,e}} \frac{\partial \Phi_{i,e}}{\partial x} + \frac{U_{x_{i,e}}^2}{2} + \frac{1}{2}\left[\left(\frac{\partial \Phi_{i,e}}{\partial x} \right)^2 + \frac{1}{r^2}\left(\frac{\partial \Phi_{i,e}}{\partial \Theta} \right)^2 + \left(\frac{\partial \Phi_{i,e}}{\partial \mathrm{r}} \right) \right] \right)_{r=R_{i,e}} \tag{9}$$

In which the subscript i, and e represent internal and external locations of the structure. A full definition of the flow requires that a condition be applied to the shell-fluid interface. The impermeability condition of the shell surface requires that the radial velocity of the fluid on the shell surface should match the instantaneous rate of change of the shell displacement in the radial direction. This condition implies a permanent contact between the shell surface and the peripheral fluid layer, which should be:

$$(V_r)_{r=R} = \left(\frac{?\Phi}{\partial r}\right)_{r=R} = \left(\frac{\partial W}{\partial t} + U_x \frac{\partial W}{\partial x}\right)_{r=R} \tag{10}$$

The radial displacement, from shell theory, is defined as:

$$W(x,\theta,t) = \int_{j=1}^{\infty} C_j \exp\left[\frac{\lambda_j x}{R} + i\omega t\right] cos\theta \tag{11}$$

The separation of variable method is used to obtain the velocity potential function, which is then substituted into (9) and results in the following Bessel's homogeneous differential equation:

$$\begin{gathered} r^2 \frac{d^2 R_j(r)}{dr^2} + r\frac{dR_j(r)}{dr} + R_j(r)\left[i^2 m_j^2 r^2 - n^2\right] = 0 \\ m_j^2 = \left(\frac{\lambda_j}{R}\right)^2 - \frac{1}{C_f^2}\left(\omega + U_x \frac{\lambda_j}{R}\right)^2 \end{gathered} \tag{12}$$

By solving the above differential equation, one can find the following explicit expression for dynamic pressure:

$$P = \int \left\{ \begin{array}{c} \left[\rho_e R_e Z_q^Y(m_q R) - \rho_i R_i Z_i^J(m_q R)\right] \frac{\partial^2 W_q}{\partial t^2} + \\ 2\left[\rho_e R_e U_{xe} Z_q^Y(m_q R) - \rho_i R_i U_{xi} Z_i^J(m_q R)\right] \frac{\partial^2 W_q}{\partial x \partial t} + \\ \left[\rho_e R_e U_{xe}^2 Z_q^Y(m_q R) - \rho_i R_i U_{xi}^2 Z_i^J(m_q R)\right] \frac{\partial^2 W_q}{\partial x^2} \end{array} \right\} \tag{13}$$

See Lakis and Paidoussis (1971, 1972a) for more details.

Substituting the nodal interpolation functions of the empty shell (3) into the dynamic pressure expression (13) and carrying out the necessary matrix operations using the proposed method, the mass, damping, and stiffness matrices for the fluid are obtained by integrating the following integral with respect to x and θ:

$$F_f = \iint [N]^T \{P\} r dr d\theta = [m_f]\left\{\ddot{\delta}\right\} + [C_f]\left\{\dot{\delta}\right\} + [k_f]\{\delta\} \tag{14}$$

After superimposing the mass, damping and stiffness matrices for each individual element, and applying the given boundary conditions the following dynamic equation is obtained for the coupled fluid-structure system. The dynamic response of the system can be investigated by solving this equation:

$$([M_s]-[M_f])\left\{\ddot{\delta}\right\}-[C_f]\left\{\dot{\delta}\right\}+([K_s]-[K_f])\{\delta\}=0 \tag{15}$$

Extensive results are given by Lakis and Paidoussis (1971, 1972a) to illustrate the dynamic behavior of uniform and non-uniform cylindrical shells partially or completely filled with liquid, as well as subjected to internal and external flowing fluid.

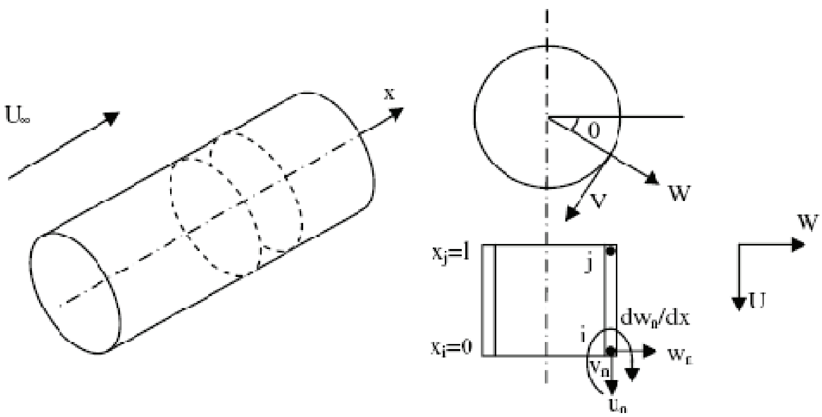

Figure 1: Geometry of cylindrical frustum element

3 Dynamic behavior of anisotropic plates and shells coupled with fluid

Use of advanced composite materials is expanding into a variety of industries due to their high strength and stiffness-to-weight ratios; this has led to a rapid increase in the use of these materials in structural applications during the past decades. Structural elements made of advanced fiber-reinforced composite materials offer unique advantages over those made of isotropic materials. They are now extensively used in high and low technology areas, e.g. the aerospace industry, where complex shell configurations are common structural elements. The filament-winding techniques for manufacturing composite shells of revolution have recently been expanded in aircraft, shipbuilding, petroleum and other industries. In general, these materials are fiber-reinforced laminate, symmetric or anti-symmetric cross- and angle-ply, which consist of numerous layers each with various fiber orientations. Although the total laminate may exhibit orthotropic-like properties, each layer of the laminate is usually anisotropic; thus the individual properties of each layer must be taken into account when attempting to gain insight into the actual stress and stress fields. By optimizing the properties we can reduce the overall weight of a structure since stiffness and strength can be designed only where they are required. A lower weight structure translates into higher performance. Since optimized structural systems are often more sensitive to instabilities, it is necessary to exercise caution. The designer would be much better able to avoid any instabilities if, when predicting a maximum load capacity, he either knew the equilibrium paths of the structural elements or had accurate modeling of the load-displacement behavior of the structure. Anisotropic laminated plates and shells have a further complication which must be considered during the design process: potentially large directional variations of stiffness properties in these structures due to tailoring mean that three-dimensional effects can become very important. The classic two-dimensional assumptions may lead to gross inaccuracies, although they may be valid for an identical shell structure made up of isotropic materials. Although they have properties that are superior to isotropic materials, advanced composite structures present some technical problems in both manufacturing and design. For computational reasons, the study of composite materials involves either their behaviors on the macroscopic level such as linear and non-linear loading responses, natural frequencies, buckling loads, etc., or their micro-mechanical properties, including cracking, delaminating, fiber-matrix debonding etc.

The general equations of motion of anisotropic plates and shells are derived by Toorani and Lakis (2000). The equations, which include the effect of shear deformation and rotary inertia as well as initial curvature (included in the stress resultants and transverse shear stresses), are deduced by application of the virtual work principle, with displacements and transverse shear as independent variables. These equations are applied to different shell type structures, such as revolution, cylindrical, spherical, and conical shells as well as rectangular and circular plates.

In the following sections, a new hybrid element method combining the first-order shear shell theory, classical finite element approach, and potential flow theory has been developed for linear and non-linear vibration analysis of multi-layer composite open and closed cylindrical shells coupled with dense fluid (liquid). A multidirectional laminate with co-ordinate notation of individual plies is shown in Figure 2. For mathematical modeling of the structure, the equations of motion of the shell are derived based on first order shear shell theory, and then the shape function, stiffness and mass matrices are developed by exact analytical integration. The shear deformations, rotary inertia, and initial curvature have been taken into account. The velocity potential, Bernoulli's equation and impermeability condition imposed at the fluid-structure interface have been used to develop the fluid model and derive the dynamic pressure and fluid force components including the inertia, centrifugal and Corilois forces. Once these fluid forces are derived, they are combined with those of the structure in order to develop the dynamic equations of motion for a coupled fluid-structure system. The non-linear differential equations of motion are solved by a fourth-order Runge-Kutta numerical method.

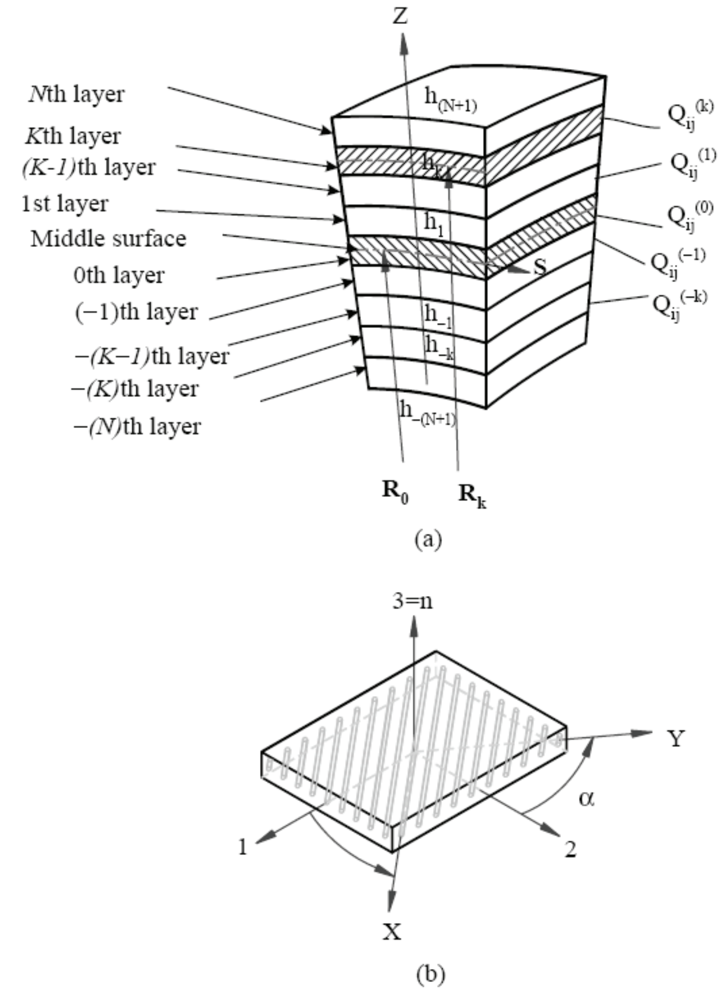

Figure 2: (a) Multi-directional laminate with co-ordinate notation of individual plies, (b) a fiber reinforced lamina with global and material co-ordinate system

The shell is subdivided into finite segment panels, Figure 3, with two nodal lines having five degrees of freedom at each node. The general strain-displacement relations are expressed in arbitrary orthogonal curvilinear coordinates to define the strain-displacement relations. The same approach as described in Section 2 is followed to develop the equations of motion. Note that in the case of isotropic materials, the five differential equations of motion can be reduced to three equations since two rotations can be expressed in terms of other displacement components. For structural components made of composite materials, in which the shear deformation effect plays an important role, the rotations of tangents to the reference surface are considered as independent variables therefore there are five degrees of freedom at each nodal line compared to three DOFs for classical materials as explained in Section 2. The proposed model is capable of solving the equations of motion of fluid-filled shells for any combination of boundary conditions without necessitating changes to the displacement functions. See Toorani and Lakis (2000, 2001b) for more details including a presentation of extensive results considering various physical and geometrical parameters as well as the liquid depth ratios. These are presented to show the reliability and effectiveness of the developed formulations. A satisfactory agreement is seen between the numerical results predicted by this theory and those of other available theories.

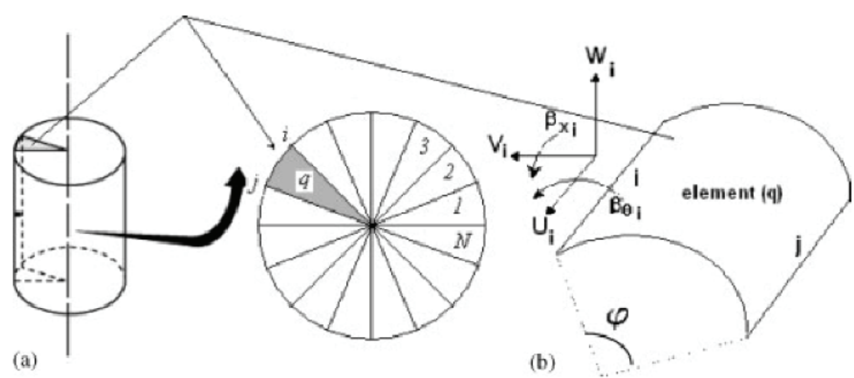

Figure 3: (a) Finite element discretization (N is the number of elements), (b) Nodal displacement at node 'i' of a typical element

To develop the hybrid finite element method, the following displacement functions are assumed:

$$\begin{cases} U(x,\theta) = A\left[cos\left(\frac{m\pi}{L}\right)x\right]\left[e^{\eta\theta}\right], & \beta_x(x,\theta) = D\left[cos\left(\frac{m\pi}{L}\right)x\right]\left[e^{\eta\theta}\right] \\ V(x,\theta) = B\left[cos\left(\frac{m\pi}{L}\right)x\right]\left[e^{\eta\theta}\right], & \beta_\theta(x,\theta) = E\left[cos\left(\frac{m\pi}{L}\right)x\right]\left[e^{\eta\theta}\right] \\ W(x,\theta) = C\left[cos\left(\frac{m\pi}{L}\right)x\right]\left[e^{\eta\theta}\right] & \end{cases} \quad (16)$$

The derived equations for the stress resultants and stress couple resultants for anisotropic shell type structures are given as:

$$\{N_{11}N_{12}Q_{11}N_{22}N_{21}Q_{22}M_{11}M_{12}M_{22}M_{21}\}^T = [P]_{(10\times10)}\{\varepsilon_1^0\gamma_1^0\mu_1^0\varepsilon_2^0\gamma_2^0\mu_2^0\kappa_1\tau_1\kappa_2\tau_2\}^T \quad (17)$$

The P_{ij}'s elements are given in the Appendix and the interested reader is referred to Toorani and Lakis (2000) for full details.

For the case of a coupled fluid-structure system, elastic structures subjected to fluid flow can undergo excessive vibrations and consequently a considerable change in their dynamic behavior. They may also lose their stability. Therefore, the influence of fluid velocity on structural stability has been also investigated. Both static 'buckling' and dynamic 'flutter' instabilities are verified. Figure 4 shows the non-dimensional frequency of a cylindric shell made of four symmetric cross-ply laminates as a function of the circumferential and axial wave numbers. Mechanical properties used for this example are E_1=25E_2; G_{23}=0.2E_2; G_{13}=G_{12}=0.5E_2, ν_{12}=0.25; ρ=1.

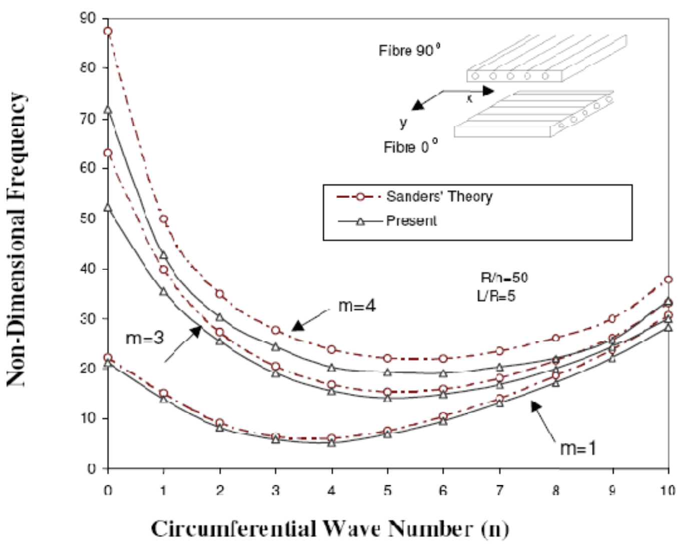

Figure 4: Variation of non-dimensional natural frequencies in conjunction with variation of m

The dynamic behavior of axisymmetric, beam-like and shell modes of anisotropic cylindrical shells, Figure 5, have been investigated by Toorani and Lakis (2002a) under different physical and geometrical parameters while they are subjected to mechanical and flowing fluid loads. Toorani and Lakis (2006a) studied the free vibrations of non-uniform composite cylindrical shells as well.

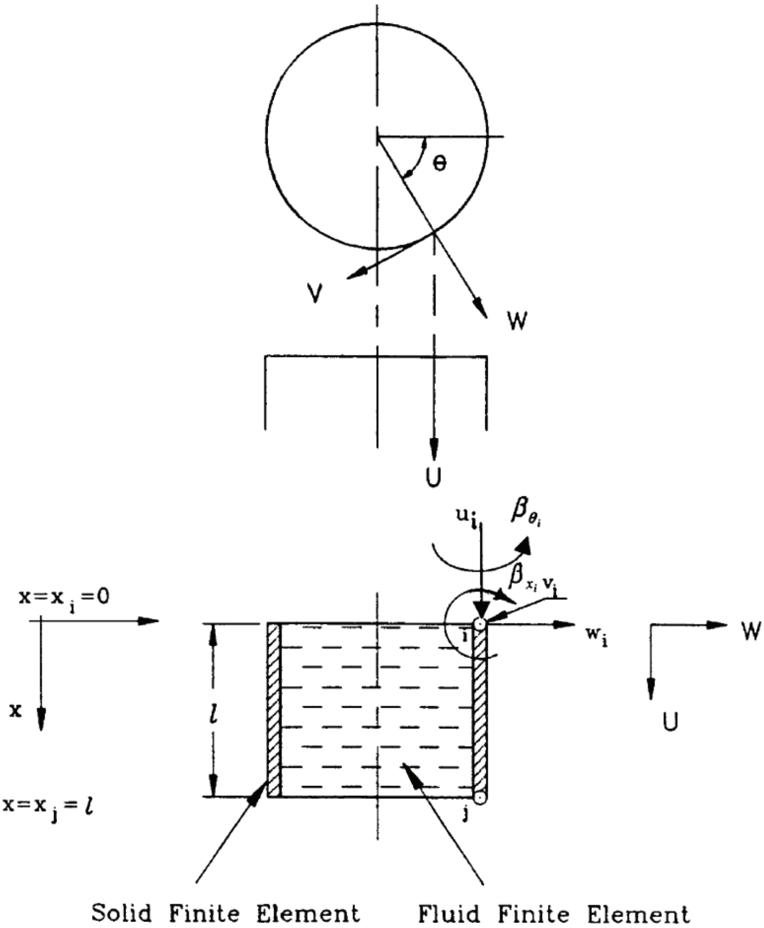

Figure 5: Displacement and degrees of freedom at a circular node

Nuclear power plant reliability depends directly on its component performance. The higher energy transfer performance of nuclear plant components often requires higher flow velocities through the shell and tube heat exchanger and steam generator. Excessive flow-induced vibration, which is a major cause of machinery downtime, fatigue failure and high noise, limits the performance of these structures. Therefore, calculating the safety of a nuclear power plant's components requires analysis of several possibilities of accident events. Considering a tube structure carrying high-velocity flow under high pressure, examples of these events could be: pressure oscillations in a nuclear reactor cavity, velocity oscillations of fluid in a pipe due to external excitations and fluid-elastic instabilities etc. These tubes could be subjected to a diodic leak condition (internal pressurization to the point of tube yielding/swelling) that results in contact with their supports and an associated risk of structural degradation. Tube lock-up as a result of tube swelling due to diodic leakage could potentially result in tubes being locked at the supports and subject to wear. Locked supports will result in a loss of damping since the support damping is no longer active. The swelled tube will therefore be subjected to fluid-elastic instability. The mathematical model developed by Toorani and Lakis (2006b) for isotropic /and anisotropic cylindrical shells is also capable of modeling structures that may be non-uniform in the circumferential direction. This allows the model to address the effect of tube swelling caused by external and internal flowing fluid on its dynamic response. The dynamic behavior of an open cylindrical shell, empty or filled with liquid as a function of the number of circumferential modes is shown in Figure 6a. For a given axial wave number 'm' the frequencies decrease to a minimum before they increase as the number of circumferential waves 'n' is increased. This behavior was first observed for a shell in vacuo by considering the strain energy associated with bending and stretching of the reference surface. At low 'n' the bending strain energy is low and the stretching strain energy is high, while at higher 'n' the relative contributions from the two types of energy are reversed. A stability analysis of a distorted cylindrical shell simply supported at both ends and subjected to internal flow is shown in Figure 6b. The natural frequencies of the system are examined as a function of flow velocity. As the velocity increases from zero, the frequencies associated with all eccentricity cases decrease. They remain real (the system being conservative) until, at sufficiently high velocities, they vanish, indicating the existence of buckling type (static divergence) instability. At higher flow velocity the frequencies become purely imaginary. The results show that the first loss of stability occurs for e=1mm. It is concluded that distortion in the cylindrical shells decreases the critical flow velocity and renders the system less stable.

The influence of non-linearities associated with the wall (geometry non-linearity) of the shell and with the fluid flow on the elastic, thin, orthotropic and non-uniform cylindrical shells submerged and/or subjected simultaneously to an internal and external fluid has been also studied by Toorani and Lakis (2002b, 2006c, 2009b). For the case of anisotropic or laminated composite materials, first order and higher-order shear theory have been applied in deriving the equations of motion of all shell type structures. The exact Green strain relations are used in order to describe the

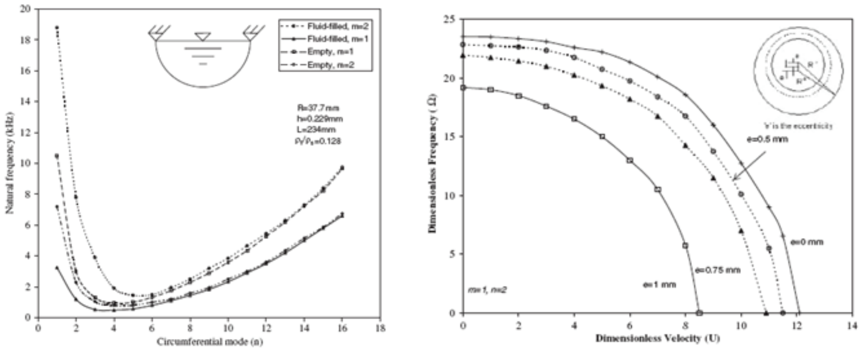

Figure 6: (a) Natural frequencies of an empty and fluid-filled open cylindrical shell as a function of circumferential mode number (b) Stability of a distorted cylindrical shell as a function of flow velocity

non-linear terms, including large displacement and rotation, for anisotropic cylindrical shells. The coefficients of modal equations are obtained using the Lagrange method. Thus, the non-linear stiffness matrices of the second- and third-order are superimposed on the linear part of the equations to establish the non-linear modal equations.

To develop the non-linear stiffness matrices of the second and third order, the following shell displacements are used as generalized products of coordinate sums and spatial functions:

$$u=\int_i q_i(t)\,U_i(x,\theta)\,;; \qquad \beta_x=\int_i q_i(t)\,\beta_{x_i}(x,\theta)\,;$$
$$v=\int_i q_i(t)\,V_i(x,\theta)\,;; \qquad \beta_\theta=\int_i q_i(t)\,\beta_{\theta_i}(x,\theta)\,; \tag{18}$$
$$w=\int_i q_i(t)\,W_i(x,\theta)\,.$$

And, the deformation vector is written as a function of the generalized coordinate by separating the linear part from the non-linear part:

$$\{\varepsilon\}=\{\varepsilon_L\}+\{\varepsilon_{NL}\}=\left\{\varepsilon_x^o,\gamma_x^o,\mu_x^o,\varepsilon_\theta^o,\gamma_\theta^o,\mu_{\theta,}^o\kappa_x,\tau_x,\kappa_\theta,\tau_\theta\right\}^T \tag{19}$$

Using (**??**) and Hamilton's principle leads to Lagrange's equations of motion in the generalized coordinate system q_i (t):

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial V}{\partial q_i}=Q_i \tag{20}$$

Where T is the total kinetic energy, V the total elastic energy of deformation and the Q_i's are the generalized forces. After developing the total kinetic and strain energy and then substituting into the Lagrange equation and carrying out a large number of the intermediate manipulations (not displayed here), the following non-linear modal equations are obtained.

$$\int_j m_{ij}\ddot{\delta}_j + \int_j k_{ij}^{L}\delta_j + \int_j\int_k k_{ijk}^{NL2}\delta_j\delta_k + \int_j\int_k\int_s k_{ijks}^{NL3}\delta_j\delta_k\delta_s = Q_i, \qquad i = 1,2,\ldots \quad (21)$$

Where m_{ij}, ${k_{ij}}^{L}$, are the terms of mass and linear stiffness matrices and the terms ${k_{ijk}}^{NL2}$, and ${k_{ijks}}^{NL3}$ represent the second-order and third order non-linear stiffness matrices. These terms, in the case of anisotropic laminated cylindrical shells, are given by Toorani and Lakis (2002b). The same approach explained in Section 2 is applied to develop the fluid equations and then derive the coupled fluid-structure's dynamic equations.

Sloshing is a free surface flow problem in a structure which is subjected to forced oscillation. Clarification of the sloshing phenomenon is very important in the design of vessels destined to contain liquid. Violent sloshing creates localized high impact loads on the structure which may cause damage. An analytical approach has also been presented by Lakis et al. (1997, 2009a) to investigate the effect of free surface motion of fluid (sloshing) on the dynamic behavior of thin walled, both horizontal and vertical, cylindrical shells. The free surface has been modeled for different fluid heights; Figures 7 and 8. The structure is modeled as explained in Section 2 but the displacement functions change in the case of horizontal shells to become:

$$U(x,\theta) = Ae^{\eta\theta}\cos\left(\frac{m\pi}{L}\right)x;\; V(x,\theta) = Be^{\eta\theta}\sin\left(\frac{m\pi}{L}\right)x;\; W(x,\theta) = C\sin\left(\frac{m\pi}{L}\right)x \quad (22)$$

For sloshing analysis of a vertical shell, the following model is considered and the same displacement functions as reported in Section 1 are used to develop the mathematical model.

4 Dynamic analysis of plates in interaction with fluid

Structural components (like nuclear power plant components, piping systems and tube heat exchangers) that are in contact with fluid can fail due to excessive flow-induced vibrations which continue to affect their performance and reliability. Fluid-elastic vibrations have been recognized as a major cause of failure in shell and tube type heat exchangers and steam generators. Fluid elastic vibrations result from coupling between fluid-induced dynamic forces and motion of the structure. Depending on the boundary conditions, static (buckling) and dynamic (flutter) instabilities

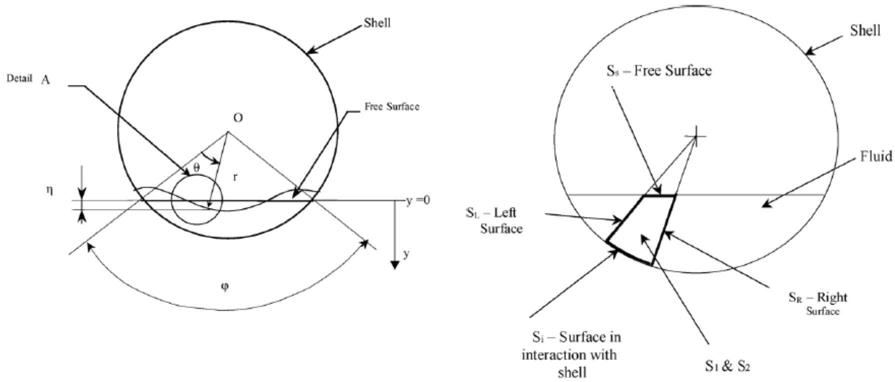

Figure 7: Sloshing model of a horizontal shell (a) Modeling of free surface, (b) Free surfaces of a fluid finite element

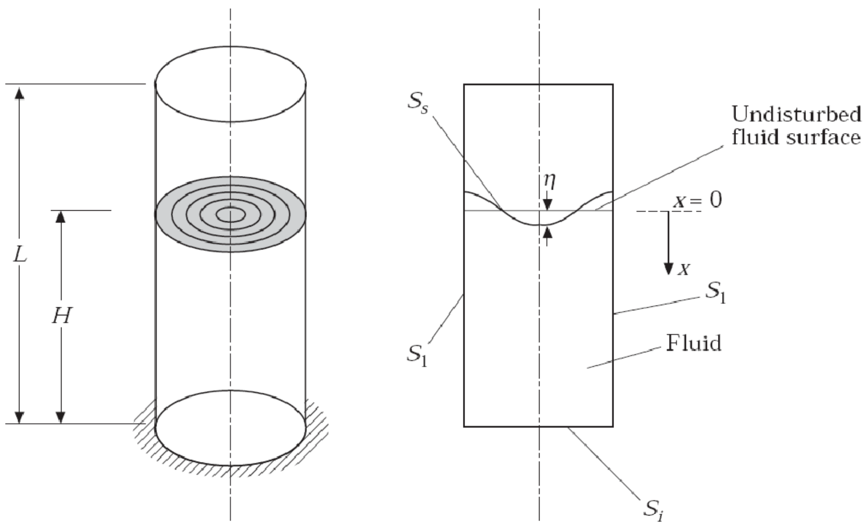

Figure 8: Sloshing model of a vertical shell

are possible in these structures at sufficiently high flow velocities. The nature of fluid-elastic instability can be illustrated as a feedback mechanism between structural motion and the resulting fluid forces. A small structural displacement due to fluid forces or an alteration of the flow pattern induces a change in the fluid forces; this in turn leads to further displacement, and so on. When the flow velocity becomes larger an impact phenomenon occurs that can lead to unacceptable tube damage due to fatigue and /or fretting-wear at tube support plate locations in critical process equipment. Therefore, evaluation of complex vibrational behavior of these structural components is highly desirable to avoid such problems.

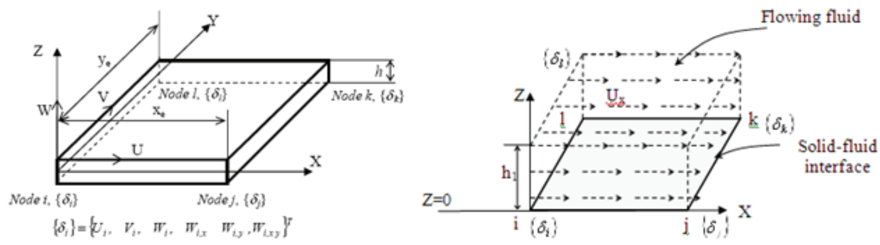

Figure 9: (a) Geometry and displacement field of a typical element, (b) Fluid-solid element

To address the aforementioned design issues, a semi-analytical approach has been developed by Kerboua et al (2007, 2008a to 2008c) for dynamic analysis of rectangular plates. The mathematical model is developed based on a combination of Sanders' shell theory and the classic finite element method. A typical finite element in its local coordinate is shown in Figure 9. Each element is represented by four nodes and six degrees of freedom at each node consisting of three displacements and three rotations. The in-plane membrane displacement components are modeled by bilinear polynomials. The out-of-plane, normal to mid-surface displacement component is modeled by an exponential function that represents a general form of the exact solution of equations of motion. The displacement field used in this model is defined as:

$$
\begin{aligned}
U(x,y,t) &= C_1 + C_2\frac{x}{A} + C_3\frac{y}{B} + C_4\frac{xy}{AB} \\
V(x,y,t) &= C_5 + C_6\frac{x}{A} + C_7\frac{y}{B} + C_8\frac{xy}{AB} \qquad (23) \\
W(x,y,t) &= \int_{j=9}^{24} C_j e^{i\pi\left(\frac{x}{A}+\frac{y}{B}\right)} e^{i\omega t}
\end{aligned}
$$

The shape functions, mass and stiffness matrices are determined by exact analytical integration to establish the plate's dynamic equations. The velocity potential and Bernoulli's equation are adopted to express the fluid dynamic pressure acting on the structure for various boundary conditions of the fluid and structure. The product of the dynamic pressure expression and the developed structural shape function is integrated over the structure-fluid interface to assess the inertial, Coriolis and centrifugal fluid forces. The dynamic pressure has been derived for different fluid-structure interfaces e.g. (i) fluid-solid element subject to flowing fluid with infinite level of fluid; (ii) fluid-solid finite element subject to flowing fluid bounded by rigid wall and (iii) fluid-solid model subject to flowing fluid bounded by elastic plate. The developed theory is also capable of modeling a set of parallel /or radial plate assemblies, Figure10. These types of systems are used in many industrial applications such as turbine blades. Parallel plates consist of many thin plates stacked in parallel between which there are channels to let fluid flow through. When the channel height is relatively low the kinetic energy of the solid travels through the fluid from one plate to another. Vibrations of the plates modify the distributions of pressure and velocity along the channel. Therefore, the fluid in the channels enters in interaction simultaneously with the higher and lower plates. The effect of various geometrical parameters and boundary conditions, fluid height and velocity (which strongly influence the dynamic response of the plates used as hydraulic turbine /or turbo reactor blades) on the dynamic responses of the rectangular plates has been explored.

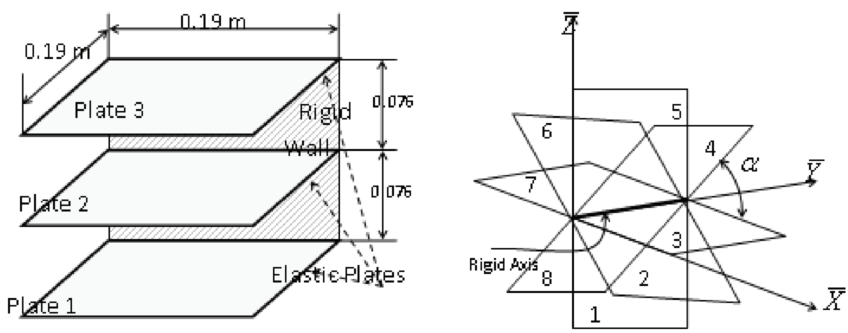

Figure 10: (a) A set of parallel plates fixed at one side, (b) A set of radial plates

The dimensionless frequency variation of a clamped plate subjected to axial flowing fluid is plotted as a function of the dimensionless velocity of flow for the first three modes, Figure 11. Note that the plate becomes increasingly vulnerable to static instability as the rate of flow increases. Beyond the critical velocity, we expect a large deflection of the plate to occur.

Circular plates are widely used in engineering. Some examples are; by the aerospace and aeronautical industry in aircraft fuselage, rocket and turbo-jets, by the nuclear industry in reactor vessels, by the marine industry for ship and submarine parts, by the petroleum industry in holding tanks, and by civil engineering in domes and thin

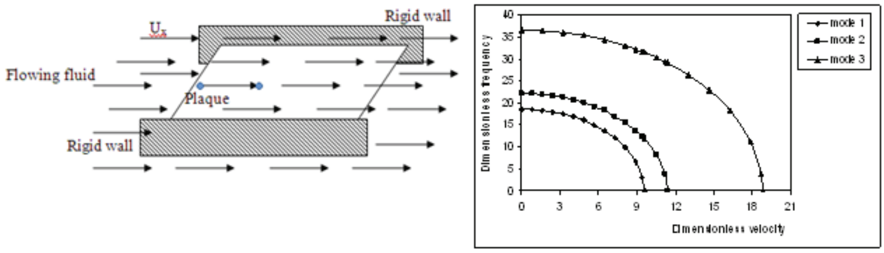

Figure 11: (a) Plate clamped on two opposite edges subjected to flowing fluid, (b) Variation of frequency versus fluid velocity

shells. To respond to these needs, the static and dynamic analysis of thin, elastic, isotropic non-uniform circular and annular plates has been conducted by Lakis and Selmane (1997). The displacement functions for circular element, Figure 12, are defined as:

$$
\begin{aligned}
U(r,\theta) &= \int_{n=0}^{\infty} C y^{(\lambda-1)/2} cos(n\theta) \\
V(r,\theta) &= \int_{n=0}^{\infty} B y^{(\lambda-1)/2} sin(n\theta) \\
W(r,\theta) &= \int_{n=0}^{\infty} \left(C_3 y^n + C_4 y^{n+2}\right) cos(n\theta)
\end{aligned}
\tag{24}
$$

The displacement functions for an annular element, Figure 12, are defined as:

$$
\begin{aligned}
U(r,\theta) &= \int_{n=0}^{\infty} C y^{(\lambda-1)/2} cos(n\theta) \\
V(r,\theta) &= \int_{n=0}^{\infty} B y^{(\lambda-1)/2} sin(n\theta) \\
W(r,\theta) &= \int_{n=0}^{\infty} \left(C_5 y^n + C_6 y^{-n} + C_7 y^{n+2} + C_8 y^{-n+2}\right) cos(n\theta)
\end{aligned}
\tag{25}
$$

The dynamic behaviour of 3D thin shell structures partially or completely filled with or submerged in inviscid incompressible quiescent fluid was studied numerically by Esmailzadeh et al (2008). A finite element was developed using a combination of classic thin shell theory and finite element analysis, in which the finite elements are rectangular four-noded flat shells with five degrees of freedom per node; three displacements and two rotations about the in-plane axes. The displacement functions were derived from Sanders' thin shell equations. The structural

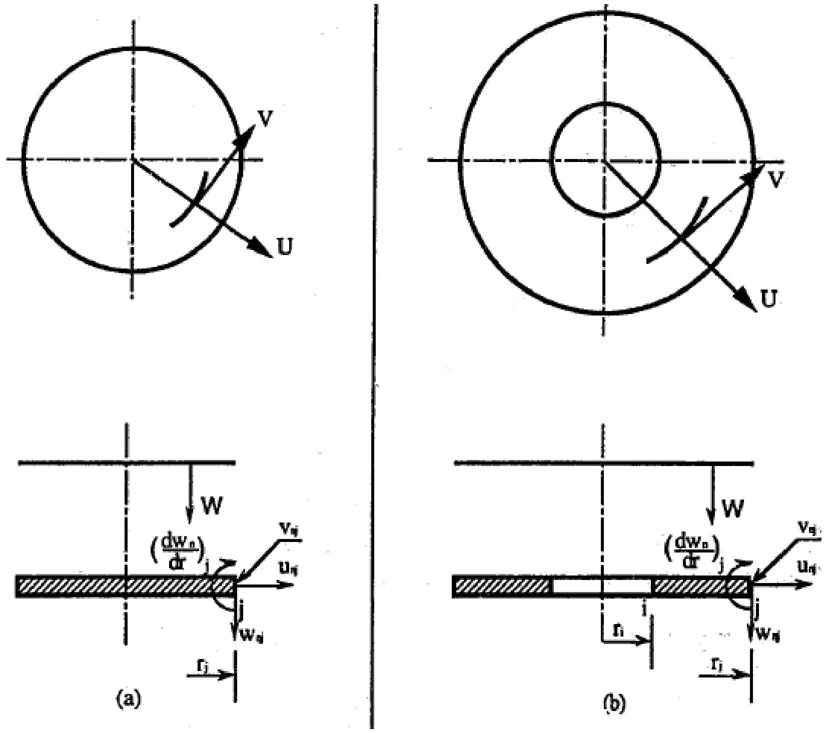

Figure 12: Displacement and degrees of freedom (a) Finite element of the circular plate type (b) Finite element of the annular plate type

mass and stiffness matrices were determined by exact analytical integration. Since the transverse displacement function is derived from thin shell theory, this method may easily be adapted to take hydrodynamic effects into account. The fluid pressure applied on the structure was determined by combining potential flow theory and an impermeability condition, and expressed as a function of the acceleration of the normal displacement of the structure. Analytical integration of the fluid pressure over the element produced the virtual added-mass matrix of the stationary fluid. An in-house program was developed to calculate eigenvalues and eigenvectors of 3D thin shell structures in a vacuum, containing and/or submerged in fluid. A rectangular reservoir partially and completely filled with fluid as well as a submerged blade was studied. The developed method can be utilized to investigate non-uniform structures under various boundary conditions.

5 Vibration analysis of plates and shells subjected to a turbulent boundary-layer-induced random pressure field

Thin shells are major components in industrial structures such as skins of aircraft fuselage, hulls of ships and blades of turbines. These structures are commonly subjected to excitation forces such as turbulence, which are intrinsically random. Random pressure fluctuations induced by a turbulent boundary layer are a frequent source of excitation and can cause small amplitude vibration and eventual fatigue failure, therefore determination of the response of shell structures to these pressures is of importance. An investigation was carried out by Lakis and Paidoussis (1972b) to determine the total root mean square displacement response of cylindrical shells to turbulent flow. Esmailzadeh et al. (2009) studied the dynamic response of shell type structures subjected to random vibration due to a turbulent boundary layer of flowing fluid. They introduced a method that is capable of predicting the total root mean square (rms) displacement response of a thin plate to an arbitrary random pressure field. The method was then specialized for application to the case where the pressure field originates from a turbulent boundary layer of subsonic flow. This method uses a combination of classical thin shell theory and finite element analysis in which the finite elements are flat rectangular elements with six degrees of freedom per node, representing the in-plane and out-of-plane displacements and their spatial derivatives. This method is also capable of calculating both high and low frequencies with high accuracy. Wetted natural frequencies and mode shapes in a vacuum obtained using the method previously developed by the authors are incorporated into the calculation of random response. A continuous random pressure field is transformed into a discrete force field acting at each node of the finite element. Structural response to turbulence-induced excitation forces is calculated using random vibration theory. Description of the turbulent pressure field is based on the Corcos formulation for the cross spectral density of pressure fluctuations. Root mean square displacement is found in terms of the cross spectral density of the pressure. A theoretical-numerical approach is proposed to obtain the magnitude of the random response of shell structures. Exact integration over surface and frequency leads to an expression for the response in terms of the structure and flow characteristics. The total root mean square displacement response is obtained by summation over all significant modes of vibration. The total root mean square displacements of a thin plate under different boundary conditions subjected to a turbulent boundary layer are then calculated. Accuracy of the proposed method is also verified for a cylindrical shell. To validate the method, a thin cylindrical shell subjected to internally fully developed turbulent flow is also studied and compared favorably with the results obtained by Lakis and Paidoussis (1972b) using cylindrical elements and a hybrid finite element. It is observed that the maximum total

RMS displacement is directly proportional to free stream velocity and inversely proportional to the damping ratio. It is noted that the maximum total RMS displacements are small for the set of calculations. Such small amplitudes are mainly of concern for fatigue considerations and must be below acceptable levels. Furthermore, the proposed method is capable of predicting the power spectral density (PSD) of the displacement. The power spectral densities of the membrane and radial displacements of an SFSF plate subjected to fully developed turbulent flow is studied. The spectrum shows the dominant peaks representing the coupled natural frequencies of the system. It is observed that the lower natural frequencies contribute significantly to the PSD of response. An in-house program based on the presented method is developed to predict the RMS displacement response of thin shell structure to a random pressure field arising from a turbulent boundary layer.

The dynamic behavior of a structure subjected to arbitrary loads is governed by the following equation:

$$[[M_s]-[M_f]]\left\{\ddot{\delta}\right\}+[[C_s]-[C_f]]\left\{\dot{\delta}\right\}+[[K_s]-[K_f]]\left\{\delta g134\right\}=\left\{F(x,\ y,\ t)\right\} \quad (26)$$

where F(x, y, t) is a vector of external forces as a function of space and time. The continuous random pressure field of the deformable body is approximated using a finite set of discrete forces and moments acting at the nodal points. The plate is subdivided into finite elements, each of which is a rectangular flat element, Figure 13. A pressure field P is considered to be acting on an area S_c surrounding the node c of the coordinates l_c and d_c as shown in Figure 13. This area S_c is delimited by the positions l'_c *and* l''_c with respect to the origin in the x-direction and d'_c and $d''{}_c$ with respect to the origin in the y-direction. It is therefore possible to determine the pressure distribution acting over the area S_c in terms of a lateral force. The lateral force acting at an arbitrary point, A, on the area S_c is given by (see Fig. 13):

$$F_A(t)=\int_{d'_c}^{d''_c}\int_{l'_c}^{l''_c}P(x,y,t)\,dxdy \quad (27)$$

Where P(x, y, t) is the instantaneous pressure on the surface. The force F_A(t) acting at point A is transformed into one force and two moments acting at node c as illustrated in Figure 13.

The external load vector acting at a typical node c, F_c associated with the nodal

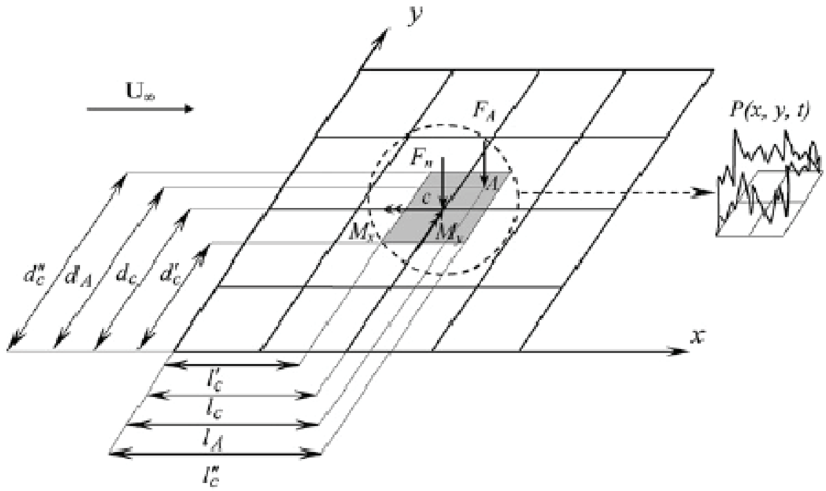

Figure 13: Transformation of a continuous pressure field into a discrete force field and the equivalent discrete force field acting at node c. Pressure fluctuations are also illustrated laterally on the area surrounding node c.

displacements can be written in the following form:

$$F_c(t) = \begin{Bmatrix} 0 \\ 0 \\ F_n \\ M_y \\ M_x \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -\int_{d'_i}^{d''_i}\int_{l'_i}^{l''_i} P(x_i, y_i, t)\, dx_i dy_i \\ -\int_{d'_p}^{d''_p}\int_{l'_p}^{l''_p} (x_p - l_p) P(x_p, y_p, t)\, dx_p dy_p \\ -\int_{d'_j}^{d''_j}\int_{l'_j}^{l''_j} (x_j - l_j) P(x_j, y_j, t)\, dx_j dy_j \\ 0 \end{Bmatrix} \tag{28}$$

Where F_n is the lateral force in the z-direction, and M_x and M_y are the moments in the x- and y-directions acting at node c, respectively.

The computational process used in determining the dynamic response of structure to turbulent boundary-layer pressure fields is presented in Figure 14.

The power spectral density of the radial displacement of an SFSF plate subjected to fully developed turbulent flow from one side where flow is along its long sides is plotted against excitation frequency in Figure 15. The PSD of the radial displacement is calculated for a free stream velocity of 30 ms^{-1} and a damping ratio of 0.001 at the node at which the maximum total rms radial displacement is obtained.

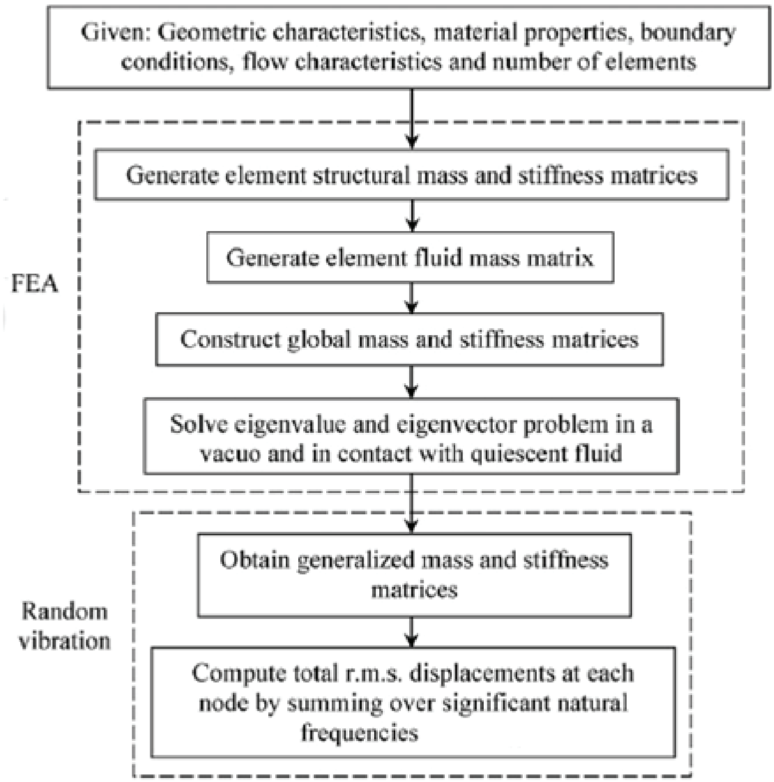

Figure 14: Flow chart of the computational process for calculation of root-mean square displacement response

6 Aeroelasticity analysis of plates and shells

It is notable that shells and plates are among the key structures in aerospace vehicles. For example, large numbers of these elements are used in the fuselage and engine nacelles of airplanes and in the skin of the space shuttle. As they are exposed to external air flow and particularly supersonic flow, dynamic instability (flutter) may occur, and is therefore one of the practical considerations in the design and analysis of skin panels. Cylindrical shells can also show this kind of aeroelastic instability, and prevention of this behavior is one of the primary design criteria and technical challenges faced by aeronautical engineers. An investigation of supersonic flutter of an empty or partially fluid-filled truncated conical shell, cylindrical shells (under combined internal pressure and axial compression) and panels is also required. The motivation for conducting research in this field stems from the need

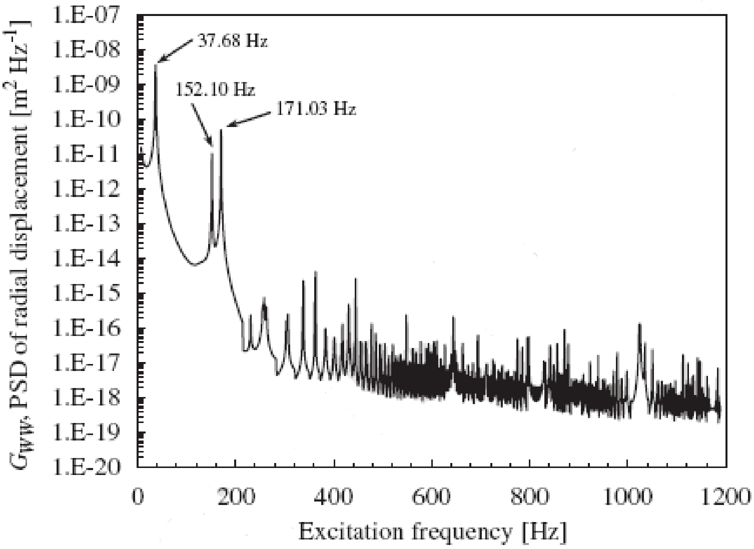

Figure 15: PSD of radial displacement of an SFSF plate subjected to fully developed turbulent flow

for precise and fast convergence of finite element computer codes for aeroelastic analysis of shell components used during the design of aerospace structures. The developed mathematical model can be used very effectively for aeroelastic analysis of shells of revolution, cylindrical and truncated conical shells and permits the designer; i) to predict the buckling condition of shells of revolution due to external pressure and axial compression, ii) to model the fluid-structure interaction effect in the presence of fluid inside the container, iii) to describe the effect of shell and flow parameters on the flutter boundaries and iii) to model the aerodynamic loading without the complexity of CFD methods. In mathematical modeling, the Piston theory with and without a correction factor for curvature is applied to derive the aerodynamic damping and stiffness matrices while also taking into consideration the influence of stress stiffness due to internal pressure and axial loading.

Sabri and Lakis (2010a) have conducted aeroelastic analysis of a truncated conical shell, Figure 16, subjected to external supersonic airflow. The structural model is based on a combination of linear Sanders' shell theory and the classical finite element approach as explained in Section 2. Linearized first-order potential (piston) theory with the curvature correction term is coupled with the structural model to account for pressure loading. The influence of stress stiffening due to internal and/or external pressure and axial compression is also taken into account. The fluid-filled effect is considered as a velocity potential variable at each node of the shell elements at the fluid-structure interface in terms of nodal elastic displacements. Aeroelastic equations using the hybrid finite element formulation are derived and solved numerically. The analysis is accomplished for conical shells of different boundary conditions and cone angles. In all cases the conical shell loses its stability through coupled-mode flutter. This developed hybrid finite element method can be used efficiently for design and analysis of conical shells employed in high speed aircraft structures. The displacement functions used in this model are given by Sabri and Lakis (2010a):

$$U(r,x,\theta) = \int_{n=0}^{\infty} A\left(\frac{x}{l}\right)^{(\lambda-1)/2} cos(n\theta)$$
$$V(r,x,\theta) = \int_{n=0}^{\infty} B\left(\frac{x}{l}\right)^{(\lambda-1)/2} cos(n\theta) \quad (29)$$
$$W(r,x,\theta) = \int_{n=0}^{\infty} C\left(\frac{x}{l}\right)^{(\lambda-1)/2} sin(n\theta)$$

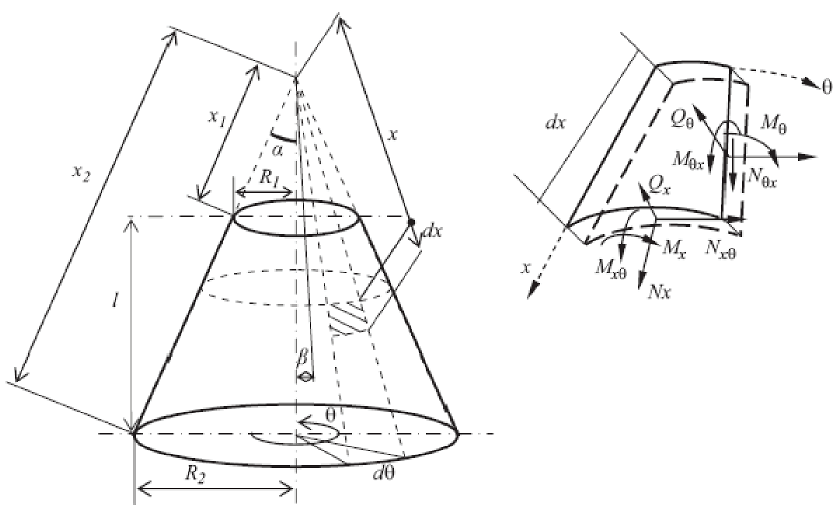

Figure 16: Geometry of a truncated conical shell

Figure 17 shows the frequency and damping (in terms of pressure) of a shell that is free at both ends. As seen, the real part of the complex frequency for the first mode decreases as the freestream static pressure increases, while the imaginary part remains positive. The existence of a zero real part and a negative imaginary part of the complex frequency indicates that the shell diverges statically. Further increasing the freestream static pressure, the second mode remains stable but the real parts of third and fourth modes merge into a single mode and their imaginary parts bifurcate into two branches and one of them becomes negative. At this point, the shell loses stability due to coupled-mode flutter because a negative imaginary part makes the vibration amplitudes grow.

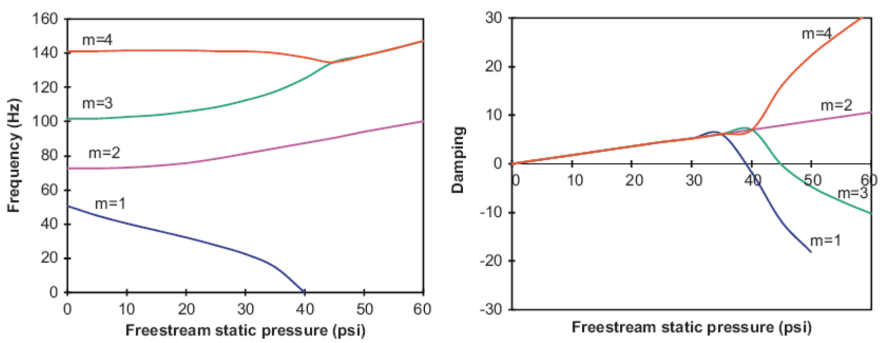

Figure 17: (a) Real part and (b) imaginary part of the complex frequencies versus freestream static pressure

Another model has been developed by Kerboua et al. (2010) to predict the dynamic behaviour of anisotropic truncated conical shells conveying fluid. It is a combination of the finite element method and classical shell theory.

Sabri and Lakis (2010b) have applied the hybrid finite element model to supersonic flutter analysis of circular cylindrical shells as shown in Figure 1. The displacement functions used for this model are defined as:

$$
\begin{aligned}
U(x,r,\theta) &= \int_n u_n \cos(n\theta) \\
V(x,r,\theta) &= \int_n v_n \cos(n\theta) \qquad (30) \\
W(x,r,\theta) &= \int_n w_n \cos(n\theta)
\end{aligned}
$$

Aeroelastic equations in the hybrid finite element formulation are derived and solved numerically. Different boundary conditions of the shell geometry and flow parameters are investigated. In all study cases, the shell loses its stability due to coupled-mode flutter and a travelling wave is observed during this dynamic instability. The results are compared with existing experimental data and other analytical and finite element solutions. This comparison indicates the reliability and effectiveness of the proposed model in aeroelastic design and analysis of shells of revolution in aerospace vehicles. Figure 18 shows some typical complex frequencies versus free stream static pressure, P_∞, for n=25. Only the first and the second axial modes are shown (m=1, 2). In Figure 18, the real part of the complex frequency for the first mode increases, whereas for the second mode it decreases as P_∞ increases. For higher values of P_∞ these real parts eventually merge into a single mode. If P_∞ is increased still further, the shell loses stability at P_∞=3592 Pa. This instability is due to coupled-mode flutter because the imaginary part of the complex frequency (which represents the damping of the system) crosses the zero value, Figure 18b, and makes the vibration amplitude grow.

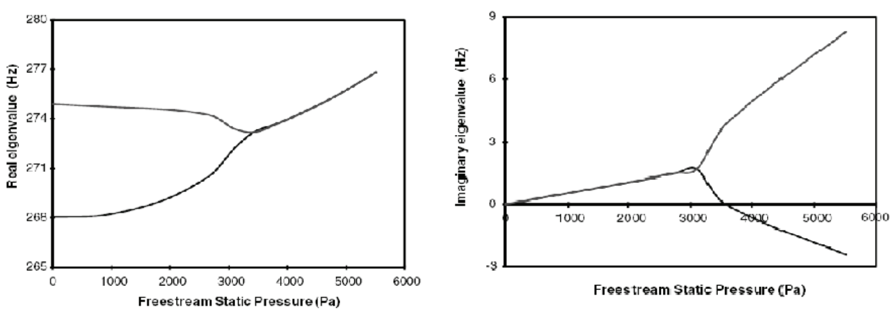

Figure 18: Eigenvalues of system vs freestream static pressure

Sabri and Lakis (2010c) adopted the hybrid finite element approach to investigate the dynamic stability of a partially fluid-filled circular cylindrical shell under constant lateral pressure and a compressive load. The shell model is shown in Figure 1 and displacement functions used in this formulation are defined by (**??**). Nodal displacement functions are derived from exact solution of Sanders' shell theory. Initial stress stiffness in the presence of shell lateral pressure and axial compression are taken into account. The parameter study is carried out to verify the effect of shell geometries, filling ratios of fluid, boundary conditions, and different combinations of lateral pressure and axial compressions on the stability of structure. The effect of shell internal pressure on the mode shape is reported in Figure 19 for different liquid filling ratios.

Effects of sloshing on flutter prediction of partially liquid-filled circular cylindrical shells and aerothermoelastic stability of functionally graded circular cylindrical shells have also been studied by Sabri and Lakis (2010d, e).

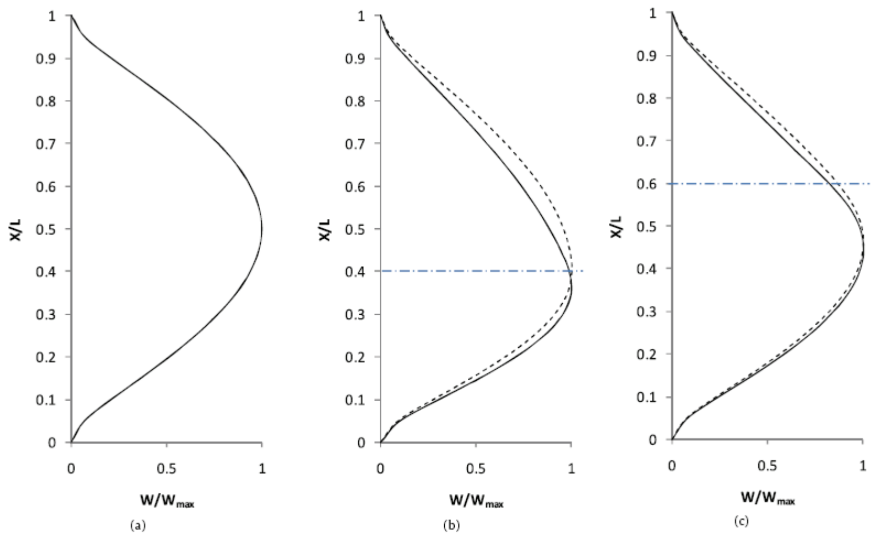

Figure 19: Variation of radial model shape (n=5, m=1) with filling ratio of a clamped-clamped shell under internal pressure P_m=1000 Pa; a) H/L=0; b) H/L=0.4; c) H/L=0.6; solid line: pressurized shell; dashed line: unpressurized shell

7 Conclusion

The dynamic analysis of the shell type structures subjected to flowing fluid is highly desirable in different sectors of industry, e.g. nuclear, aerospace. The study presented in this paper shows an analytical approach that has been developed to study the linear and non-linear flow-induced vibrations of these structures. This method is also capable to predict the total root-mean-square displacement response of a thin structure to an arbitrary random pressure field originated from a turbulent boundary layer of a subsonic flow. In addition, the semi-analytical model developed in this paper is applied to analyze the aeroelastic stability of different shell geometries subjected to a supersonic flow.

An efficient hybrid finite element method, Sanders and shearable shell theories and linear potential flow have been used to develop the dynamic equations of the coupled fluid-structure system. This theory has been developed for both isotropic and anisotropic shell type structures in which the rotary inertia and shear deformation effects are taken into consideration that tend to reduce the frequency parameters specially for laminated anisotropic shell. The shell can be uniform or non-uniform in the axial and/or circumferential direction.

The predicted results by this theory, which are in good agreement with those of other theories and experiments, show the reliability and effectiveness of the developed model.

References

Esmailzadeh, M.; Lakis, A. A.; Thomas, M.; Marcouiller, L. (2008): Three-Dimensional Modeling of Curved Structures Containing and/or Submerged in Fluid. *Finite Elements in Analysis and Design*, vol. 44 (6-7), pp. 334-345.

Esmailzadeh, M.; Lakis, A. A.; Thomas, M.; Marcouiller, L. (2009): Prediction of the Response of a Thin Structure Subjected to a Turbulent Boundary-Layer-Induced Random Pressure Field. *Journal of Sound and Vibration*, vol. 328 (1-2), pp. 109-128.

Kerboua, Y.; Lakis, A. A.; Thomas, M.; Marcouiller, L. (2007): 'Hybrid Method for Vibration Analysis of Rectangular Plates' *Nuclear engineering and design*. Vol. 237 (8), pp. 791-801.

Kerboua, Y.; Lakis, A. A.; Thomas, M.; Marcouiller, L. (2008a): Vibration Analysis of Rectangular Plates Coupled with Fluid. *Applied Mathematical Modelling*, vol. 32 (12), pp 2570-2586.

Kerboua, Y.; Lakis, A. A.; Thomas, M.; Marcouiller, L. (2008b): Computational Modeling of Coupled Fluid-Structure System with Applications. *Structural Engineering and Mechanics*, vol. 29 (**??**), pp 91-111.

Kerboua, Y.; Lakis, A.A.; Thomas, M.; Marcouiller, L. (2008c): Modelling of Plates Subjected to a Flowing Fluid under Various Boundary Conditions. *Engineering Applications of Computational Fluid Mechanics*, vol. 2 (4), pp 525-539.

Kerboua, Y.; Lakis, A. A.; Hmila, M. (2010): Dynamic Analysis of Truncated Conical Shells Subjected to Internal Flowing Fluid. *Applied Mathematical Modelling*, vol. 34 (3), pp. 791-809 2010.

Lakis, A. A.; Paidoussis M. P. (1971): Free Vibration of Cylindrical Shells Partially Field with Liquid. *Journal of Sound and Vibration,* vol. 19, pp. 1-15.

Lakis, A. A.; Paidoussis M. P. (1972a): Dynamic Analysis of axially Non-Uniform Thin Cylindrical Shells. *Journal of Mechanical Engineering Science*, vol. 14(**??**), pp. 49-71.

Lakis, A. A.; Paidoussis M. P. (1972b): Prediction of the Response of a Cylindrical Shell to Arbitrary or Boundary-Layer-Induced Random Pressure Fields. *Journal of Sound and Vibration*, vol. 25(**??**), pp. 1-27.

Lakis, A. A.; Neagu, S. (1997): Free Surface Effects on the Dynamics of Cylindrical Shells Partially Filled with Liquid. *Journal of Sound and Vibration*, vol. 207(2), pp. 175-205.

Lakis, A. A.; Selmane, A. (1997): Classical Solution Shape Functions in the Finite Element Analysis of Circular and Annular Plates. *International Journal for Numerical Methods in Engineering*, vol. 40, pp. 969-990.

Lakis, A. A.; Bursuc, G.; Toorani, M. H. (2009a): Sloshing Effect on the Dynamics Behaviour of Horizontal Cylindrical Shells. *International Journal of Nuclear Engineering and Design, vol. 239, pp. 1193-1206.*

Lakis, A. A.; Toorani, M. H (2009b): Non-Linear Axial Flow-Induced Vibrations of Composite Cylindrical Shells. *7th EUROMECH Solid Mechanics Conference*, Lisbon, Portugal.

Sabri, F.; Lakis, A. A (2010a) : Hybrid Finite Element Method Applied to Supersonic Flutter of an Empty or Partially Liquid-Filled Truncated Conical Shell. *Journal of Sound and Vibration*, vol. 329 (3), pp 302-316.

Sabri, F.; Lakis, A.A. (2010b): Finite Element Method Applied to Supersonic Flutter of Circular Cylindrical Shells. *AIAA Journal*, vol.48 (**??**), pp 73-81.

Sabri, F.; Lakis, A.A. (2010c): Hydroelastic Vibration of Partially Liquid-Filled Circular Cylindrical Shells under Combined Internal Pressure and Axial Compression. *Aerospace Science and Technology,* (In press).

Sabri, F.; Lakis, A.A. (2010d): Effects of Sloshing on Flutter Prediction of Partially Liquid-Filled Circular Cylindrical Shells. *Journal of Aircraft, (*Submitted April 2010).

Sabri, F.; Lakis, A.A., (2010e): Aerothermoelastic Stability of Functionally Graded Circular Cylindrical Shells. *Journal of Vibration and Acoustics*, ASME, (Submitted Dec. 2010).

Toorani, M. H.; Lakis A.A. (2000): General Equations of Anisotropic Plates and Shells Including Transverse Shear Deformation and Rotary Inertia Effects. *Journal of Sound and Vibration,* vol. 237 (4), pp. 561-615.

Toorani, M. H.; Lakis A.A. (2001a): Dynamic Analysis of Anisotropic Cylindrical Shells Containing Flowing Fluid. *Journal of Pressure Vessel Technology,* vol. (**??**), pp.1-7.

Toorani, M. H.; Lakis A.A. (2001b): Shear deformation theory in dynamic analysis of anisotropic laminated open cylindrical shells filled with or subjected to a flowing fluid. *Journal of Computer Methods in Applied Mechanics and Engineering,* vol. 190(37-38), pp. 4929-4966.

Toorani, M. H.; Lakis, A. A. (2002a): Dynamics Behaviour of Axisymmetric and Beam-Like Anisotropic Cylindrical Shells Conveying Fluid. *Journal of Sound and Vibration,* vol. 259(2), pp. 265-298.

Toorani, M. H.; Lakis, A. A. (2002b): Geometrically non-linear dynamics of anisotropic open cylindrical shells with a refined shell theory. *Technical Report, EPM-RT-01-07, Polytechnique de Montreal, Quebec, Canada*

Toorani, M. H.; Lakis, A. A. (2006a): Free Vibrations of Non-Uniform Composite Cylindrical Shells. *Nuclear Engineering and Design,* vol. 236(17), pp. 1748-1758.

Toorani, M. H.; Lakis, A. A. (2006b): Swelling Effect on the Dynamic Behaviour of Composite Cylindrical Shells Conveying Fluid. *International Journal for Numerical Methods in Fluids,* vol. 50(4), pp. 397-420.

Toorani, M. H.; Lakis, A. A. (2006c): Large Amplitude Vibrations of Anisotropic Cylindrical Shells. *Journal of Computers and Structures,* vol. 82 (23-26), pp. 2015-2025.