# Phase-Shift Estimation from Interferograms by Histogram of Phase Difference 

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#### Abstract

We propose a non-iterative approach to extract the unknown phase shift in phase shifting interferometry without the assumption of equal distribution of measured phase in $[0,2 \pi]$. According to the histogram of the phase difference between two adjacent frames, the phase shift can be accurately extracted by finding the bin of histogram with the highest frequency. The main factors that influence the accuracy of the proposed method are analyzed and discussed, such as the random noise, the quantization bit of CCD, the number of fringe patterns used and the bin width of histogram. Numerical simulations and optical experiments are also implemented to verify the effectiveness of this method.


Keywords: Interferometry, Fringe analysis, Phase measurement.

## 1 Introduction

Phase-shifting interferometry (PSI) has been widely used in surface testing of optical elements, especially the large-aperture mirrors in astronomical telescopes or satellite cameras. Owing to the large size of the measured surface, the length of optical path is set to be several meters or even longer to complete the testing. However, the mechanical vibration and the air turbulence will change the preset phase shifts and cause inevitable errors to the measurement [1]. To deal with the problem of the random phase shifts, iterative algorithms based on the least-square method have been proposed to determine the phase-shift amounts and the phase distribution simultaneously [2-5]. It needs three or more frames of interferogram and requires an iteration process that will lead to substantial computation loads. To further simplify the measurement and improve the computing efficiency, Cai et al [6], Xu et al[7-8], Meng et al[9] and Gao et al [10] introduced some algorithms to directly extract the unknown phase shifts on the assumption that the measured surface has an equal distribution in $[0,2 \pi]$ over the whole interferogram. If the assumption is

[^0]not fully met, iterative calculation is also needed to improve the extraction of phase shifts.

In this paper, we report a non-iterative approach to extract the unknown phase shift directly by the histogram of phase difference without the assumption of equal distribution of measured phase in $[0,2 \pi]$. Thus, the phase shifts and measured phase can be extracted accurately without consideration of optics quality.

## 2 Principle

Suppose N frames of random phase-shifting interferograms are collected and the intensity of an arbitrary pixel ( $\mathrm{x}, \mathrm{y}$ ) in the nth interferogram is expressed as
$I(x, y, n)=A(x, y)+B(x, y) \cos [\varphi(x, y)+\theta(n)]$
where $A(x, y)$ and $B(x, y)$ represent the background intensity and the modulation amplitude in pixel $(x, y)$, respectively. $\phi(x, y)$ is the measured phase distribution and $\theta(n)$ is the random phase shift for the $n$th frame. The intensity at $(x, y)$ can be ergodic over $N$ frames if $N$ is large enough [11]. Then the maximum and the minimum intensity, denoted as $I_{\text {max }}(x, y)$ and $I_{\text {min }}(x, y)$, can be easily determined from $N$ frames of interferograms. So $A(x, y)$ and $B(x, y)$ can be derived from
$A(x, y)=\left[I_{\max }(x, y)+I_{\text {min }}(x, y)\right] / 2$
$B(x, y)=\left[I_{\max }(x, y)-I_{\text {min }}(x, y)\right] / 2$
Then the phase of the $n$th frame, denoted as $\Phi(x, y, n)=\varphi(x, y)+\theta(n)$, can be solved from the normalized fringe pattern with arc cosine function
$\Phi_{0, \pi}(n)=\arccos \left[\frac{I(n)-A}{B}\right]$
Here $\Phi_{0, \pi}(n)$ means the unrecovered phase whose range is $(0, \pi)$. Firstly, we define the phase difference between the $n$th and the $(n-1)$ th frames as
$\Delta \Phi_{0, \pi}(n)=\Phi_{0, \pi}(n)-\Phi_{0, \pi}(n-1)$
Figure 1(a) shows the phases of the $n$th and the ( $n-1$ )th frames and the phase difference between these two frames. Secondly, we assume that $\Delta \boldsymbol{\theta}(n)$ is less than $\pi / 2$ and the peak-to-valley ( PV ) value of the measured phase $\phi$ is larger than $\pi$, which is usually met in practical experiment. Thus there is always more than half of the area over the interferogram where $\left|\Delta \Phi_{0, \pi}(n)\right|=\Delta \theta(n)$. In the other parts of the interferogram, $\left|\Delta \Phi_{0, \pi}(n)\right|$ varies nearly uniform from zero to $\Delta \theta(n)$. Thus, the
frequency of $\left|\Delta \Phi_{0, \pi}(n)\right|=\Delta \theta(n)$ is much larger than that of $\left|\Delta \Phi_{0, \pi}(n)\right|$ equaling any other values. We divide $\left|\Delta \Phi_{0, \pi}(n)\right|$ into many groups, named as bins in the context of histograms. By setting the bin width $\varepsilon$, we can calculate the histogram of $\left|\Delta \Phi_{0, \pi}(n)\right|$. Figure 1(b) is the histogram of Fig. 1 (a) and it shows the frequency distribution of the absolute value of the phase difference. From the histogram, we can easily find the bin with the highest frequency. So the phase shift $\Delta \theta(n)$ can be estimated as the average of $\left|\Delta \Phi_{0, \pi}(n)\right|$ in the bin, and then the principal phase can be easily derived from two adjacent frames [7]
$\Phi_{-\pi, \pi}(n)=\tan ^{-1}\left[\cot (\Delta \theta(n))-\frac{I(n)-A}{\sin (\Delta \theta(n))(I(n-1)-A)}\right]$


Figure 1: (a) The relation between two phases and their phase difference in one dimension and (b) the histogram of the absolute value of the phase difference

## 3 Numerical simulation

$N$ frames are generated according to Eq. (1) by setting the parameters as follows. $A=130 \exp \left[-0.02\left(x^{2}+y^{2}\right)\right], B=120 \exp \left[-0.02\left(x^{2}+y^{2}\right)\right]$, and $\varphi=2 \pi \cos \left[\left(x^{2}+\right.\right.$ $\left.\left.y^{2}\right)+3 x\right]$ where $-1 \leq x \leq 1$ and $-1 \leq y \leq 1 . \theta_{1}(n)=n \pi / 4$ and $\theta_{2}(n)$ is random rational number ranging from -0.3 to 0.3 . Here the phase-shift extraction error is defined as the difference between the extracted phase shift and the given phase shift. The main factors(random noise, quantization error, number of frame used and bin width) that influence the accuracy of the proposed method are analyzed and the results are shown in Figure 2.


Figure 2: The relation between the average phase-shift extraction error and (a) the signal-to-noise ratios of interferogram, (b)the Bit of CCD ,(c) the number of frame used and (d) the bin width.

## 4 Experiment

An optical flat with aperture of 100 mm is measured in a standard phase-shifting Fizeau interferometer with the active vibration isolation workstation turned off. A total of 100 frames of interferograms, whose preset phase shift is $n \pi / 4$, are captured at a frame frequency of 20 fps . Two typical adjacent interferograms are shown in Figs. 3 (a) and (b). The real phase shift between them is calculated by our proposed method and equals to 1.120 rad . According to Eq. (6), the principal phase is obtained. Then by use of phase unwrapping and tilt removal, the recovered phase, with PV and RMS values of 1.1316 and 0.1446 rad, is shown in Fig.3(c). Meanwhile, the optical flat is also measured by ZYGO interferometer with vibrationisolating platform and calibrated PZT. The recovered phase, with PV and RMS values of 1.1398 and 0.1685 rad , is shown in Fig. 7 (d). The difference between Figs.3(c) and (d), with PV and RMS values of 0.2715 and 0.0453 rad , is shown in Fig.3(e).From the comparison between Figs.3(c) and (d), it shows that the result
from our proposed method coincides well with that from standard ZYGO interferometer. However, with the proposed method, we can accurately extract phase shift and thus relax the requirements on the accuracy of PZT and the performance of active vibration isolation workstation, which has potential application for test and measurement of large-aperture optical elements.


Figure 3: Optical experiment results :(a)-(b) two typical adjacent interferograms,(c)-(d) the phase recovered by the proposed method and ZYGO's standard method respectively and (e) the difference between (c) and (d).

## 5 Conclusion

In conclusion, we have presented a non-iterative method to extract the unknown phase shift directly without the assumption of equal distribution of measured phase in $[0,2 \pi]$. The phase shift between two adjacent frames can be accurately extracted by calculating the histogram of the phase difference. Simulated and experimental results demonstrate the effectiveness of the proposed algorithm. In order to improve the accuracy of the proposed method, we should capture more fringe patterns with high signal-to-noise ratio, use the CCD with high quantization bit, and choose the bin width of histogram properly according to the practical experiment condition. The average phase-shift extraction error is less than 0.01 rad when $N=50$, $S N R=60 \mathrm{~dB}, 0.001<\varepsilon<0.03 \mathrm{rad}$ and the bit of CCD is 8 . This method is well implemented in existing interferometers simply by incorporating a high-speed camera and choosing a proper preset phase shift. It has potential application for test and measurement of large-aperture optical elements.

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