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Phase-shifting Deflectometric Moiré Topography

Hongwei Guo[1]

Abstract: This paper presents a moiré-based technique for measuring the three-dimensional shapes of specular surfaces. In it, a Ronchi grating is closely located before the measured object and illuminated by diffusive light sources. When we observe the measured object through this grating, moiré fringes are generated by the superposition of this grating and its virtual image produced by the measured specular surface. With this scheme, the movements of the grating along the vertical direction introduce phase shifts in these moiré fringes so that their phases can be recovered by use of a self-calibrating phase-shifting algorithm. From the phases and phase shifts, the three-dimensional shape of the specular surface is further reconstructed. Compared with the existing techniques, this method gives a promising solution for measuring complex specular surfaces, because the measured objects can be placed much closer to the grating. The validity of this technique has been demonstrated by an experimental result.

Keywords: Three-dimensional measurement, specular surface measurement, moiré topography, phase-shifting technique.

1 Introduction

It is an urgent need in engineering to measure the three-dimensional (3-D) shape of an object with specular surfaces by using optical techniques [Chen, Brown, and Song (2000)], despite some attempts for performing this task. Deflectometry [Zhang and North (1998); Knauer, Kaminski, and Häusler (2004); Skydan, Lalor, and Burton (2007)], as a popularly used method, employs the diffusive screen of a planar monitor (or a retroreflective screen coupled with a projector) as structured light source, and therefore enables measuring the whole-field information of a specular surface. Its principle is that, periodic fringe patterns are generated in computer, displayed on the screen, reflected by the measured specular surface, and then captured with a camera. Analyzing these distorted patterns results in their phase maps,

[1] Lab of Applied Optics and Metrology, Department of Precision Mechanical Engineering, Shanghai University, Shanghai, China. Email Address: hw-guo@yeah.net

from which the slope distribution of the surface is reconstructed, and finally the 3-D shape is obtained by numerically integrating the slope distribution. Two problems arise with this technique. First, the phases of the fringes generally depend on the surface slopes coupled with the surface heights, and have ambiguities in their distributions (i.e. the same phase distribution may correspond to different surfaces), thus leading to prohibitive computational complexities in recovering the 3-D shapes. By using an approximation model or when measuring a simple planar surface, the impacts of the surface heights on the phase distributions can be neglected. Only in this case can the surface slopes be calculated with ease. Another method is to use two or more cameras in the measurement system, so the phase ambiguities are removed with the aid of photogrammetric algorithms. Second, the numerical integration for recovering the 3-D shape from slopes, from the view of signal processing, is equivalent to an inverse operation of high-pass filtering, in which the spectrum of a signal is divided by the high-pass filter. Because the denominator, i.e. the magnitude of this high-pass filter, is very much close to 0 within low frequency band, the problem is usually ill-conditioned, and in other words a small error of low frequency may induce a large distortion in the integration.

Another method is based on triangulation [Guo, Feng, and Tao (2010)]. Its measurement system is similar to that of deflectometry, but the diffusive light source (e.g. a planar monitor with fringes on its screen) can be moved vertically to two or more known positions. At each position the phase distribution of the deformed fringe pattern is measured. The orientation of incident ray for each pixel is directly tracked in the least squares sense via corresponding phases, and further the 3-D coordinates of the corresponding point on the specular surface are determined. With this technique, the restrictions in computational complexities, phase ambiguities are eliminated.

The shared drawback of both the aforementioned techniques is that, the planar light source cannot be placed close to the object due to the limited space for the camera and light source, and thus the light source usually has a very large size, especially when measuring a deep surface with relatively great slopes. To solve this problem, this paper presents a moiré-based technique for measuring the 3-D shapes of specular surfaces. In it, a Ronchi grating is closely located before the measured object and illuminated by diffusive light sources. When we observe the measured object through this grating, moiré fringes are generated by the superposition of this grating and its virtual image produced by the measured specular surface. With this scheme, the movements of the grating along the vertical direction introduce phase shifts in these moiré fringes so that their phases can be recovered by use of a self-calibrating phase-shifting algorithm. From the phases, the three-dimensional shape of the specular surface is further reconstructed. The validity of this technique has been demonstrated by experimental results.

2 Principle

2.1 Geometry of the measurement system

Fig. 1 shows the measurement system, which consists of a computer, a Ronchi grating, and a camera with its optical axis being perpendicular to the grating plane. The measured object is placed closely under the grating. This system is similar to that of the shadow moiré topography [Meadows, Johnson, and Allen (1970); Takasiki (1970)], but diffusive light sources instead of a point light source are used for illuminating the grating and the measured object. Through this grating, the camera captures moiré fringes, which are generated by the superposition of the grating and its virtual image produced by the measured specular surface. Like the situation in the shadow moiré topography, the horizontal shifting of grating within the grating plane does not induce a variation in the moiré pattern [Dirckx and Decraemer (1990)]. Therefore, a mechanism is used in our system to drive the grating shifting in vertical direction, thus introducing phase shifts in these moiré fringes.

Fig. 2 illustrates the geometry of the measurement system. C is the center of lens of camera. The optical axis of the imaging system, i.e. CO, is perpendicular to the grating plane, and the distance between C and the grating plane is H. We set a coordinate system ($Oxyz$) with Obeing the origin,x-axis coinciding with CO, and y-axis being perpendicular to the grating lines. The pitch of the grating is p.

Assume A is a general point on the measured surface, whose depth from the grating plane is h. D is a point on the grating. The incident ray from D(a point on the grating) is reflected by A, and finally goes to the imaging plane of the camera. The incident angle is α, and the reflect ray, AC, crosses the grating plane at the point B. The measured specular surface produces the virtual image of D at D'. What the camera captures is the moiré fringes generated by the superposition of the grating and its virtual image. For the point A, the phase of moiré fringe is

$$\phi = \frac{2\pi\overline{BD}}{p} = \frac{2\pi h[\tan\beta + \tan(2\alpha - \beta)]}{p}. \tag{1}$$

Noting that the slope of surface at the point A along x-direction is $\tan(\alpha\text{-}\beta)$, the phase in Eq.1 depends not only on the depth h but on the slope, and therefore it is not easy to directly determine h from the phase.

The horizontal shifting of grating, as the aforementioned, does not induce a variation in the moiré pattern, and grating movement in the vertical direction introduces

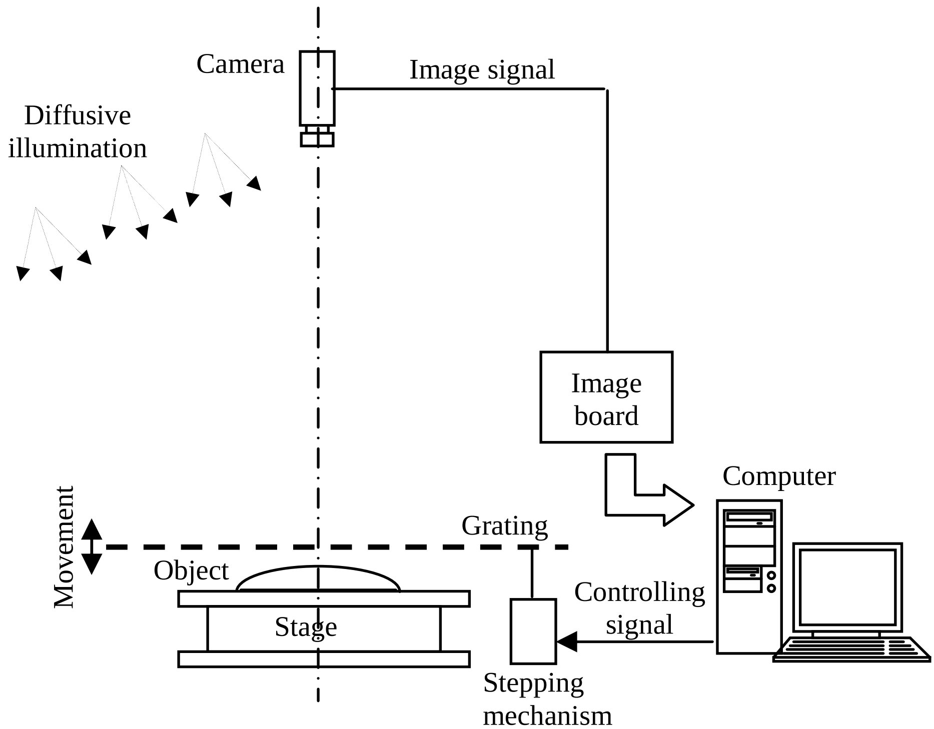

Figure 1: Measurement system

a phase shift in the moiré fringes. If the grating movement distance in the –z direction is Δ, the angles α and β keep invariant for the point A, and the phase becomes

$$\phi+\delta=\frac{2\pi(h+\Delta)[\tan\beta+\tan(2\alpha-\beta)]}{p}, \tag{2}$$

where δ denotes the phase shift induced by the grating movement. It is independent of h, and can be denoted with

$$\delta=\frac{2\pi\Delta[\tan\beta+\tan(2\alpha-\beta)]}{p}. \tag{3}$$

If we shift the grating to K different positions with the equal step of Δ, the resulting phase shifts are $(k\text{-}1)\delta$ with k=1, 2, ..., K. In order to remove the influence of the surface slope, we deduce from Eqs. 1 and 3 that

$$\frac{\phi}{\delta}=\frac{h}{\Delta}. \tag{4}$$

Eq. 4 implies a possibility to recover the 3-D shape from the moiré fringe patterns. The next subsection introduces a phase shifting algorithm for calculating the phase and phase shift.

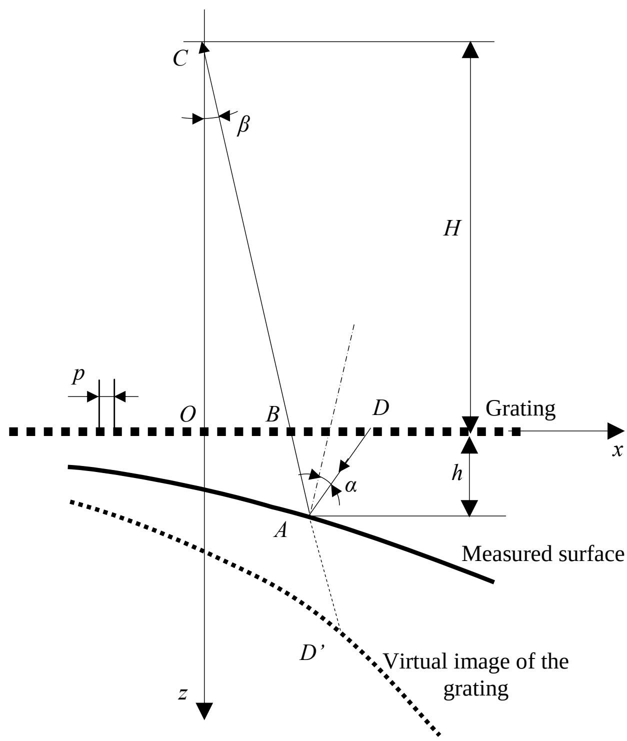

Figure 2: Geometry of the measurement system

2.2 Phase-shifting algorithm

After a low-pass filtering, the kth moiré pattern with phase shift (k-1)δ can be represented with

$$I_k = I_0\{1 + M\cos[\phi + (k-1)\delta]\} \quad k = 1, 2, \cdots, K \tag{5}$$

where I_0 is the background intensity and M is the modulation. The problem is that, the phase shift, δ, depends on the slope and hence is unknown. We have to employ a self-calibrating algorithm for determining the phase and phase shift simultaneously.

First, the relative phase shift is calculated with [Guo and Chen (2005)]

$$\delta = \arccos\left(\frac{Q-\sqrt{Q^2+4P^2}}{4P}-\frac{1}{2}\right) \tag{6}$$

where

$$P = \sum_{k=4}^{K}[(I_{k-2}-I_{k-1})(I_k-I_{k-3})] \tag{7}$$

and

$$Q = \sum_{k=4}^{K}[(I_{k-2}-I_{k-1})^2-(I_k-I_{k-3})^2]. \tag{8}$$

Second, the phase is calculated by using the least squares phase-shifting algorithm [Greivenkamp (1984)]. Solving the following linear system of equations for the unknowns, c_0, c_1, and c_2,

$$\begin{bmatrix} K & \sum\cos[(k-1)\delta & \sum\sin[(k-1)\delta \\ \sum\cos[(k-1)\delta & \sum\cos^2[(k-1)\delta & \sum\sin[(k-1)\delta\cos[(k-1)\delta \\ \sum\sin[(k-1)\delta & \sum\sin[(k-1)\delta\cos[(k-1)\delta & \sum\sin^2[(k-1)\delta \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}$$

$$= \begin{bmatrix} \sum I_k \\ \sum I_k\cos[(k-1)\delta] \\ \sum I_k\sin[(k-1)\delta] \end{bmatrix}, \tag{9}$$

where Σ denotes $\Sigma_{k=1}^{K}$ for short, then we have

$$\phi = \arctan\frac{-c_2}{c_1}. \tag{10}$$

After implementing phase unwrapping, the actual phase distribution can be obtained.

By the two steps, the phase and phase shift for each point are obtained, from which the corresponding depth can be recovered. The overall measurement procedure is described in the next subsection.

2.3 Measurement procedure

From the above subsections we have known that it is possible to recover the depths of the measured surface from the moiré fringe patterns, and that the phases and phase shifts of the moiré fringes can be calculated by use of a self-calibrating phase-shifting algorithm. The overall measurement procedure is summarized as follows:

First, adjust and calibrate the measurement system, and measure the system parameters, e.g. H and p, as well as the camera parameters. Second, mount the measured specular surface, and vertically move the grating to K different positions with an equal interval Δ which is controlled and therefore known; at each position, capture the moiré pattern. Third, calculate the phase shift and phase of each point from the moiré patterns by using the method in Section 2.2. Finally, calculate the depth of each point with the formula

$$h = \frac{\phi}{\delta}\Delta. \tag{11}$$

In addition, a camera calibration is performed. Using its result, we calculate the lateral coordinates of the object points from their pixel coordinates in images.

3 Experiment

An experiment was carried out to verify the validity of this technique, and the experimental system has been sketched in Fig. 1. The distance between the optical center of the camera and the grating plane is H=1004mm, and the pitch of grating is p=0.65mm. We select a concave mirror with a diameter of 115mm as the measured specular surface. The grating is vertically shifted to 64 positions with an equal interval Δ=0.15mm. Fig. 3 shows the first eight frames of the 64 moiré patterns captured by the camera, which are generated by the superposition of this grating and its virtual images. From this figure, the phase shifts, induced by the movements of grating, between frames are observed. Fig. 4 shows the measurement result, where the reconstructed 3-D shape of the concave mirror has been transformed to the horizontal position.

An issue worth discussion is regarding the effect of error-inducing factors. First, the moiré fringes, as popularly known, are approximately triangular signals rather than standard sinusoid ones, so we have to implement a low-pass filtering to the moiré patterns in advance in order to eliminate the high-order harmonics. In this work, we simply used a 2-D Gaussian smooth filter, but using a more advanced filtering technique is helpful for improving the measurement accuracy, and some methods based on wavelet have been reported and demonstrated to be more effective in processing moiré fringe patterns. Another method for solving this problem is to increase the number of phase shifts. Second, we must select the grating pitch properly. On the one hand, using a finer pitch can enhance the measurement resolution, on the other hand phase-unwrapping becomes more difficult, and the moiré fringes may become denser and even hard to distinguish from each other when measuring a surface with large slopes. Third, illumination variation during the grating shifting results in ununiformities in background intensities and modulations, thus causing errors in the

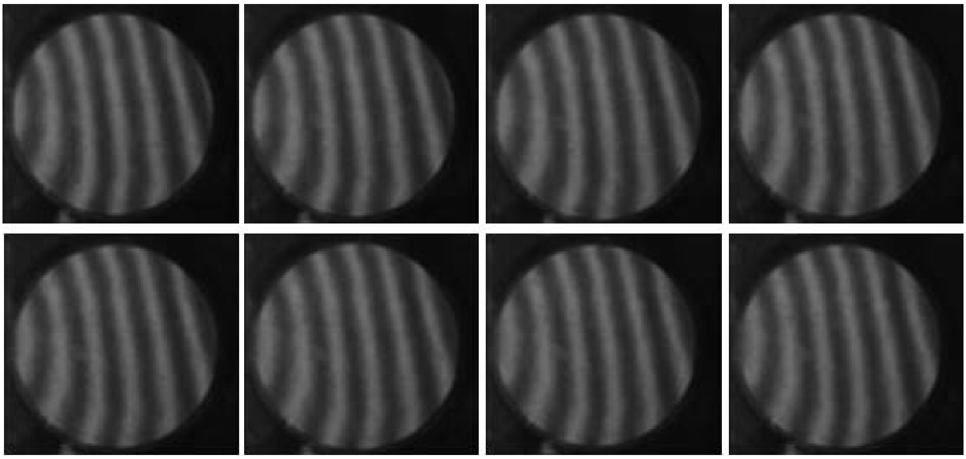

Figure 3: Moiré fringe patterns with phase shifts

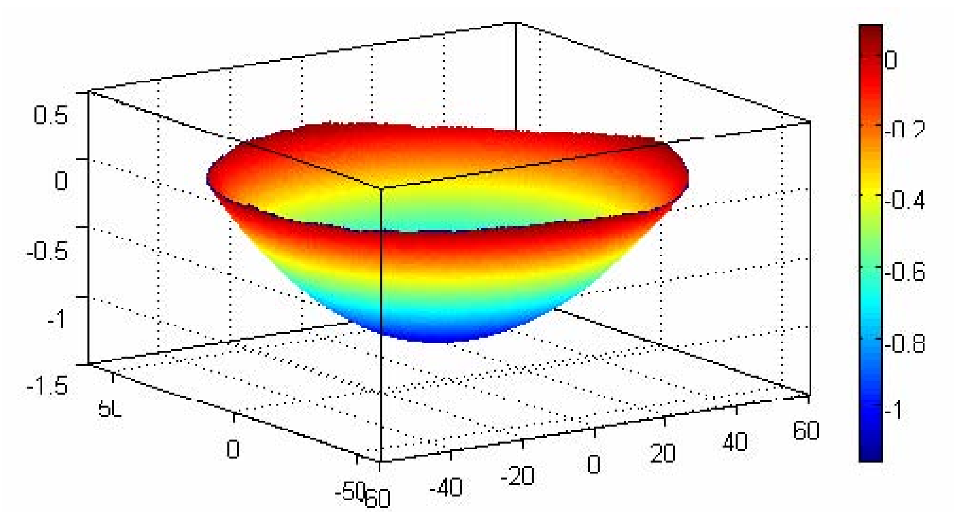

Figure 4: Measurement result (mm^3)

recovered phases and phase shifts. The self-calibrating phase-shifting algorithm in Section 2.2 models the illumination as a constant during the grating shifting, but it is necessary to use a more complex model to deal with this problem in order to increase the accuracies in estimating the phases and phase shifts.

4 Conclusion

In conclusion, we have proposed phase-shifting deflectometric moiré topography for measuring the 3-D shapes of specular surfaces. With this technique, a Ronchi grating is closely located before the measured object and illuminated by diffusive light sources. Through this grating, a camera captures moiré patterns. Vertically shifting the grating introduces phase shifts in the moiré patterns, and a self-calibrating phase-shifting algorithm is employed to calculate the phases and phase shifts, from which the 3-D shape of the specular surface is further reconstructed. Compared with deflectometry or triangulation-based techniques mentioned in the introduction section, the proposed technique gives a promising solution for measuring specular objects with more complex surfaces, because the measured objects can be placed much closer to the grating.

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