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Size-Dependent Elastic Properties of Micro- and Nano-Open-Celled Foams

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Abstract: Simple analytical results for the size-dependent elastic properties of micro- and nano-sized regular open-celled foams are presented in this paper. The results indicate that at micrometer scale, strain gradient has a dominant effect and at nanometer scale, surface elasticity dominates the effect on the honeycomb elastic properties. In addition, very small micro- or nano- open-celled foams could have a negative Poisson ratio, the elastic properties of nano-sized open-cell foams can be controlled to vary over a range up to around 66% by the application of an electric potential.

Keywords: Size effect, Open cell foams, Elastic properties, Surface elasticity, Initial Stress.

1 General expressions of the independent elastic constants and the anisotropy factor

The knowledge about the elastic properties of macro-sized open-celled foams has well been established [Gibson and Ashby (1997); Zhu, Knott and Mills (1997); Zhu, Mills and Knott (1997); Zhu, Hobdell and Windle (2000)]. However, the relevant results for macro-sized foams may not apply to their micro- and nano-sized counterparts because of the strain gradient effect at micrometer scale [Mindlin and Tiersten (1962); Yang et al. (2002)] and surface elasticity [Miller and Shenoy (2000)] and initial stress [Zhu (2008)] at nanometer scale.

For perfect regular body-centered cubic cell foams (i.e. BCC foams) with uniform struts, there are only three independent elastic constants. The open-celled foams are assumed to be made of an isotropic solid material with Young's modulus E_S, Poisson's ratio v_S. For simplicity, v_S is fixed at 0.3 throughout the analysis and the struts are assumed to have a length L and a circular cross-section of diameter d.

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The relative density of the regular BCC open-celled foams is thus

$$\rho = \frac{3\pi}{8\sqrt{2}}(\frac{d}{L})^2 \tag{1}$$

Taking strut bending, torsion and axial compression/stretching as the deformation mechanisms, Zhu, Knott and Mills (1997) have obtained the closed results for the three independent elastic constants BCC open-celled foams, i.e. the Young's modulus E_{11}, shear modulus G_{12} and the Poisson ration v_{12}, and the Zener's anisotropy factor A^* [Reid (1973)], and given as

$$\overline{E_{11}} = \frac{E_{11}}{\alpha E_S \rho^2} = \frac{1}{\alpha E_S \rho^2} \cdot \frac{6\sqrt{2} D_b D_c}{L^2(D_c L^2 + 12 D_b)} \tag{2}$$

$$\frac{1}{\overline{\overline{G_{12}}}} = \frac{\alpha E_S \rho^2}{G_{12}} = \alpha E_S \rho^2 (\frac{2\sqrt{2}L^2}{D_c} + \frac{\sqrt{2}L^4}{6D_b} \cdot \frac{8D_b + D_t}{5D_b + D_t}) \tag{3}$$

$$v_{12} = 0.5 \cdot \frac{D_c L^2 - 12 D_b}{D_c L^2 + 12 D_b} \tag{4}$$

and

$$A^* = \frac{2(S_{11} - S_{12})}{S_{44}} = \frac{2(1 + v_{12}) G_{12}}{E_{11}} \tag{5}$$

where D_b, D_t, and D_c are the bending stiffness, torsional stiffness, and axial compression/stretching stiffness of the cell strut, respectively. In equations (2) and (3), the Young's modulus and shear modulus are normalised by $\alpha E_S \rho^2$, which is the conventional Young's modulus of an open-celled foam with a very low density (say, $\rho \leq 0.01$) when strut bending is the sole deformation mechanism [Gibson and Ashby (1997)]. For circular strut cross-section, $\alpha = 0.6$. Note that $\alpha = 1$ if the strut cross-section is a plateau border [Zhu, Knott and Mills (1997)].

As the transverse shear deformation of the struts is ignored, equations (2) and (3) slightly overestimate the Young's modulus and the shear modulus of regular open celled foams. However, if the foam relative density is small (say, $\rho < 5\%$), the effect of the shear deformation mechanism on the results would be small.

2 Size-dependent elastic properties of micro-open-celled foams

It is generally recognised that the elastic properties of micro structural elements are size-dependent [Mindlin and Tiersten (1962); Yang et al. (2002); Lam et al.

(2003)]. For uniform strut with a circular cross-section, the bending stiffness is given by [Zhu (2010)]

$$D_b = E_S \frac{\pi d^4}{64}[1 + 8(1 + \nu_S)(\frac{l_m}{d})^2] \tag{6}$$

The torsional stiffness is given by [Yang et al. (2002)]

$$D_t = G_S \frac{\pi d^4}{32}[1 + 24(\frac{l_m}{d})^2] \tag{7}$$

where, l_m is the material length scale parameter for strain gradient effect. For most engineering materials, l_m is in the range from submicron to microns. When a straight uniform strut undergoes axial compression or stretching, strain gradient effect is absent. The compression/stretching stiffness is thus the same as the conventional result, i.e. $D_C = E_S \pi d^2/4$. Substituting D_b, D_t, D_c into equations (2), (3), (4) and (5) the size-dependent elastic properties are plotted against the foam relative density in Figure 1 (a, b, c and d).

Figures 1a and 1b show clearly that the dimensionless Young's modulus and shear modulus of micro- open-celled foams depend significantly on the diameter d of struts or on the cell size $\sqrt{2}L$ if the foam relative density ρ is fixed. The thinner the struts or the smaller the cell size, the larger will be the dimensionless Young's modulus and shear modulus. While the larger the foam relative density, the smaller the dimensionless Young's modulus and shear modulus [Zhu et al. (2000)]. These are very similar to the findings for micro-honeycombs [Zhu (2010)]. Figure 1c indicates that the Poisson ratio reduces with the increase of the foam relative density, and that the thinner the struts or the smaller the cell size, the smaller the Poisson ratio. If the strut diameter or the cell size is small enough, the Poisson ratio could become negative. This may be of very interesting applications. Figure 1d suggest that micro- lower density open-celled foams are nearly isotropic and gradually become anisotropic with the increase of the foam relative density or the reduction of the strut diameter or the cell size. When strut diameter d is much larger than the material length scale parameter l_m, i.e. $l_m/d = 0$, all the results become those of the conventional (i.e. macro-sized) open-celled foams, as obtained by Zhu, Knott and Mills (1997).

3 Size-dependent elastic properties of nano-open-celled foams

At nanometer scale, both surface elasticity [Miller and Shenoy (2000)] and initial stress [Zhu (2008)] greatly affect the elastic properties of nano-sized structural elements. For simplicity, the surface is assumed to be isotropic, with surface elastic

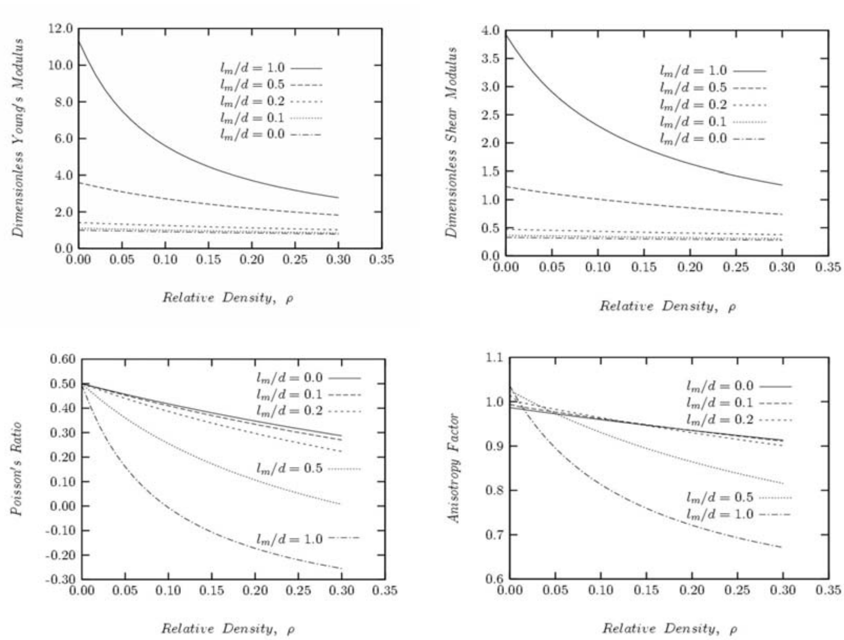

Figure 1: Elastic properties vs. the relative density of micro- open-celled foams: a) dimensionless Young's modulus $\overline{E}_{11}$; b) dimensionless shear modulus $\overline{G}_{12}$; c) the Poisson ratio v_{12}; and d) the Zener's anisotropy factor A^*

modulus S and Poisson ratio v_S=0.3. For a uniform strut with a circular cross-section, its initial surface stresses in both the circular (or the radial) and the axial directions are assumed to be the same as τ_0. The strut diameter is assumed to be d when the initial surface stress $\tau_0 = 0$ (or absent). When the initial stresses τ_0 is present (or not 0), the initial strain in the axial direction is $\varepsilon_0^x = -\frac{4\tau_0}{E_S d}(1-v_S)$ and the initial strain in the radial direction is $\varepsilon_0^r = -\frac{2\tau_0}{E_S d}(1-3v_S) = \frac{1-3v_S}{2(1-v_S)}\varepsilon_0^x$. The bending, torsional and comprssion/stretching rigidities of a nano-sized strut with a circular cross-section are thus respectively

$$D_b = E_S \frac{\pi d^4}{64}(1+8\frac{l_n}{d}+\frac{v_S \varepsilon_0^x}{1-v_S})[1+\frac{1-3v_S}{2(1-v_S)}\varepsilon_0^x]^4 \tag{8}$$

$$D_t = G_S \frac{\pi d^4}{32}[1+16(1+v_S)\frac{l_n}{d}][1+\frac{1-3v_S}{2(1-v_S)}\varepsilon_0^x]^4 \tag{9}$$

$$D_C = E_S \frac{\pi d^2}{4}(1+4\frac{l_n}{d})[1+\frac{1-3v_S}{2(1-v_S)}\varepsilon_0^x]^2 \tag{10}$$

where $l_n = S/E_S$ is the nanoscale intrinsic length of the bulk material of the nano-struts. For most solid material, l_n is in the order of 10^{-10}m. When the initial stresses τ_0 is zero (i.e. $\varepsilon_0^x = 0$), the results of equations (8), (9) and (10) reduce to those of the bending, torsional and xompression/stretching rigidities of a nanowire with a circular cross-section [Miller and Shenoy (2000); Zhu (2008)].

Substituting equations (8), (9) and (10) into (2), (3), (4) and (5), the effects of the surface elasticity and initial surface stresses on the relationships between the size-dependent elastic properties of nano-sized open-celled foams and the foam relative density can be obtained. When the initial surface stress is absent (i.e. $\tau_0 = 0$ or $\varepsilon_0^x = 0$), the effects of the surface elasticity on the elastic properties are shown in Figure 2 (a, b, c and d).

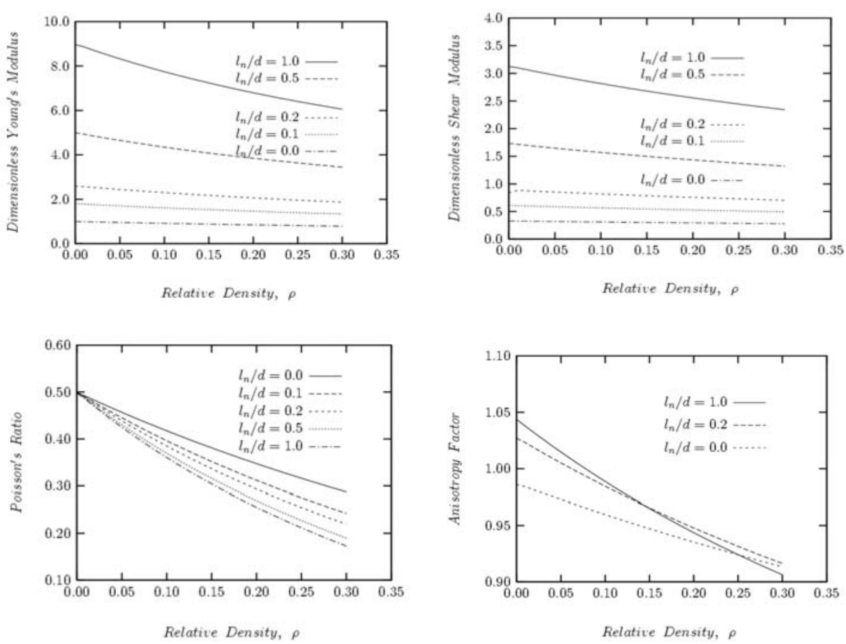

Figure 2: Effects of surface elasticity on the relationships between elastic properties and the relative density of nano-sized open-celled foams: a) dimensionless Young's modulus $\overline{E}_{11}$; b) dimensionless shear modulus $\overline{G}_{12}$; c) the Poisson ratio v_{12}; and d) the Zener's anisotropy factor A^*.

As can be clearly seen from figures 2a and 2b, the thinner the cell wall struts or the smaller the cell size (if the foam relative density is fixed), the larger will the dimensionless Young's modulus and shear modulus of nano-sized open-celled foams.

However, they both reduce with the increase of the foam relative density. Figure 2c shows that the Poisson ratio of a nano-open-celled foam reduces with the increase of the foam relative density and with the reduction in strut diameter or the cell size. Figure 2d indicates that when the foam relative density is small, the Zener's anisotropy factor is around 1 (i.e. the foam is nearly isotropic) and it reduces with the increase of the foam relative density. When the strut diameter d is much larger than the nanoscale intrinsic length of the bulk material, all the results become those of the conventional regular open-celled foams [Zhu, Knott and Mills (1997)].

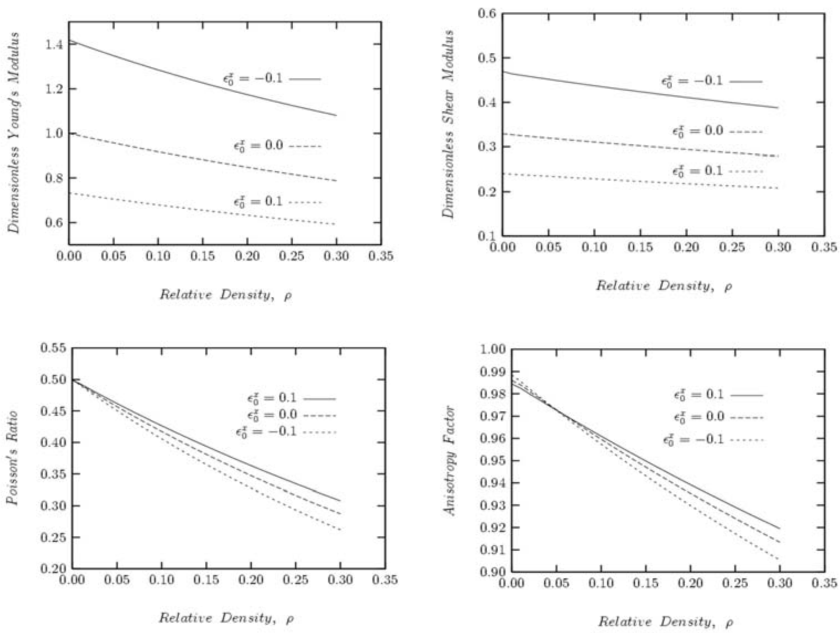

Figure 3: The elastic properties vs. the relative density of nano-sized open-celled foams: a) dimensionless Young's modulus $\overline{E}_{11}$; b) dimensionless shear modulus $\overline{G}_{12}$; c) the Poisson ratio v_{12}; and d) the Zener's anisotropy factor A^*.

When the effect of the surface elasticity is absent (i.e. $S = 0$or $l_n = 0$), the effects of the initial stress or strain on elastic properties of nano-sized open-celled foams are shown in Figure 3 (a, b, c and d). Thin nano-wires or nano-struts are usually single crystal material whose yield strength (or von Mises stress) can well reach $0.1E_S$ or larger [Diao et al. (2006); Zhu (2010)]. It is generally recognized that the amplitude of the surface stress of a material is correlated to the applied electric potential [Cammarata and Sieradzki (1994)]. In this paper, the initial strain ε_0^x

of the bulk material in axial direction is assumed to be controlled to vary over a range from -0.1 to 0.1 while the bulk material remains elastic (i.e. no plastic deformation). As can be seen from figure 3a and 3b, applying an initial negative strain $\varepsilon_0^x = -0.1$ increases the dimensionless Young's modulus and shear modulus around 40%, while a positive strain $\varepsilon_0^x = 0.1$ reduce them around 26%. When the foam relative density is small, the initial surface stress or strain has very little effects on the Poisson ratio and the anisotropy factor. While when the foam relative density is large, the larger the applied initial strain ε_0^x, the larger will be the Poisson ratio and the Zener's anisotropy factor.

4 Conclusion

The size-dependent elastic properties of micro- and nano-sized regular open-celled foams are obtained. At micrometer scale, strain gradient dominates the effects on the elastic properties, while at nanometer scale, both surface modulus and the initial stress/strain can greatly affect the elastic properties of nano-foams. Generally, the thinner the cell struts or the smaller the cell size, the larger will be the dimensionless Young's modulus and shear modulus and the smaller will be the Poisson ratio. All the elastic properties studied here reduce with the increase of the foam relative density. What more interesting is that for nano-sized open-celled foams, the dimensionless Young's modulus and shear modulus can be controlled to vary over a range around 66% by the application of an electric potential; and that very small micro- or nano- open-celled foams could have a negative Poisson ratio. The elastic properties explored in this paper are very interesting and of important practical applications.

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