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Flow Simulations by a Particle Method Using Logarithmic Weighting Function

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Abstract: The application of a particle method to incompressible viscous fluid flow problems is presented. The method is based on the MPS (Moving Particle Semi-implicit) scheme using logarithmic weighting function. Numerical results demonstrate the workability and the validity of the present approach through incompressible viscous fluid flow in a driven cavity and flow behavior in a liquid ring pump with rotating impeller blades.

Keywords: particle method, MPS, logarithmic weighting function, cavity flow, liquid ring pump, rotating impeller.

1 Introduction

From a simulation-based practical point of view, it is important to compute efficiently multi-physics problem and moving boundary/obstacle one in the wide fields of engineering and science. There are various meshless-based methods, such as SPH (Smoothed Particle Hydrodynamics) method [Lucy (1977);Gingold and Monaghan (1977)], MPS (Moving Particle Semi-implicit) one [Koshizuka and Oka (1996)], and MLPG (Meshless Local Petrov-Galerkin) one [Atluri and Zhu (1998);Lin and Atluri (2001)], to simulate effectively such problems.

The purpose of this paper is to present the application of a particle method using logarithmic weighting function to incompressible viscous fluid flow problems, namely flow in a driven cavity [Ghia, Ghia and Shin (1982);Kakuda and Tosaka (1992)] and flow in a liquid ring pump with rotating impeller [Kakuda, Ushiyama, Obara, Toyotani, Matsuda, Tanaka and Katagiri (2010)]. The cavity flow is the well-known typical problem in an incompressible viscous fluid flow. On the other hand, the phenomena in the liquid ring pump require the multi-physics problem including the moving interface boundary between gas and liquid, and the rotating impeller with blades. The pump has an impeller with blades attached to a center

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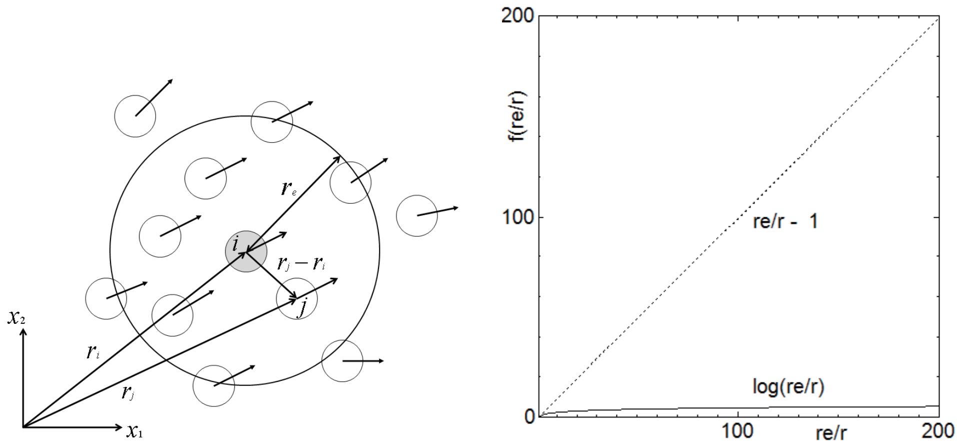
hub, located by the decentering in a cylindrical body. The workability and validity of the present approach are demonstrated through the driven cavity flow and flow in the liquid ring pump, and compared with experimental data and other numerical ones.



(a) Particle interaction models (b) Logarithmic weighting function

Figure 1: Particle interaction models and weighting functions

2 MPS formulation

Let us briefly describe the MPS as one of the particle methods [Koshizuka and Oka (1996)]. The particle interaction models as illustrated in Fig. 1(a) are prepared with respect to differential operators, namely, gradient, divergence and Laplacian. The incompressible viscous fluid flow is calculated by a semi-implicit algorithm, such as SMAC (Simplified MAC) scheme.

The particle number density n at particle i with the neighboring particles j is defined as

$$n_i = \sum_{j \neq i} w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{1}$$

in which the weighting function $w(r)$ is

$$w(r) = \begin{cases} \dfrac{r_e}{r} - 1 & (r < r_e) \\ 0 & (r \geq r_e) \end{cases} \tag{2}$$

where r_e is the radius of the interaction area as shown in Fig. 1(a).

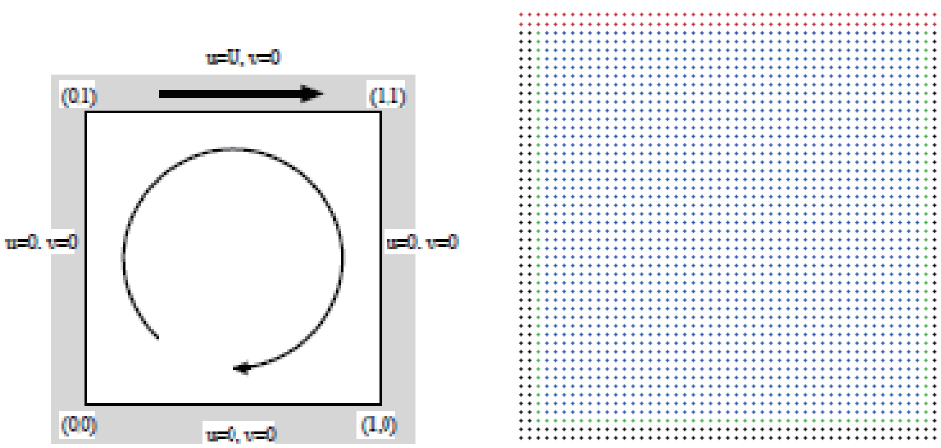
The model of the gradient vectors at particle i between particles i and j are weighted with the kernel function and averaged as follows :

$$< \nabla\phi >_i = \frac{d}{n^0}\sum_{j\neq i}\left[\frac{\phi_j - \phi_i}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2}(\boldsymbol{r}_j - \boldsymbol{r}_i)w(|\boldsymbol{r}_j - \boldsymbol{r}_i|)\right] \tag{3}$$

where d is the number of spatial dimensions, ϕ_i and ϕ_j denote the scalar quantities at coordinates $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$, respectively, and n^0 is the constant value of the particle number density. The Laplacian model at particle i is also given by

$$< \nabla^2\phi >_i = \frac{2d}{n^0\lambda}\sum_{j\neq i}(\phi_j - \phi_i)w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{4}$$

where λ is an ad hoc coefficient.

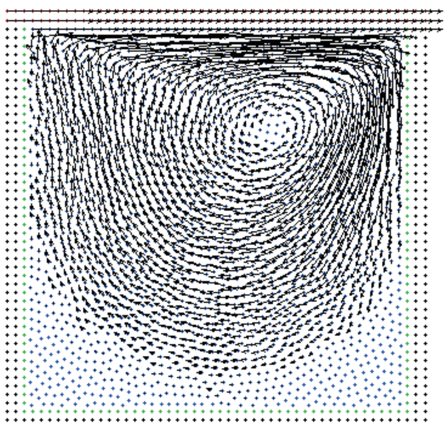


(a) Geometrical configuration (b) Initial state of particles

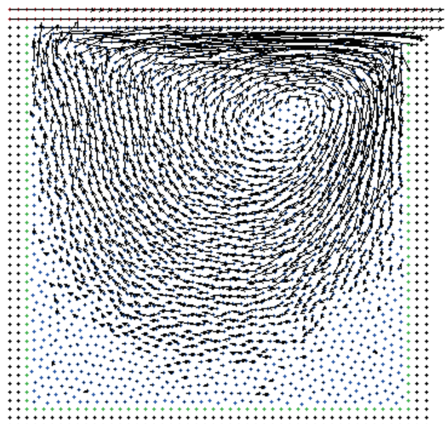
Figure 2: Flow in a driven cavity

3 Logarithmic weighting function

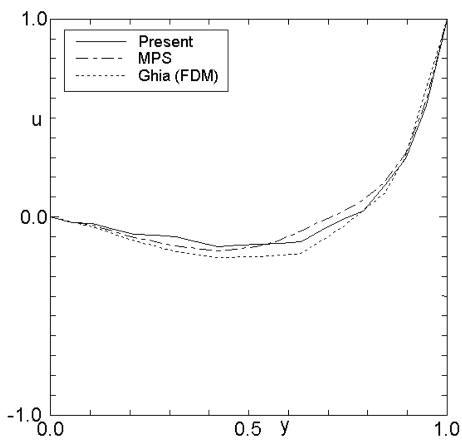
For the MPS formulation mentioned above, the weighting function of Eq. 2 is a key factor in the particle framework. If the distance r between the coordinates $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$ is very close, then there is a possibility that the computation fails suddenly.



(a) MPS with Eq. 2



(b) Present



(c) Comparisons with FDM

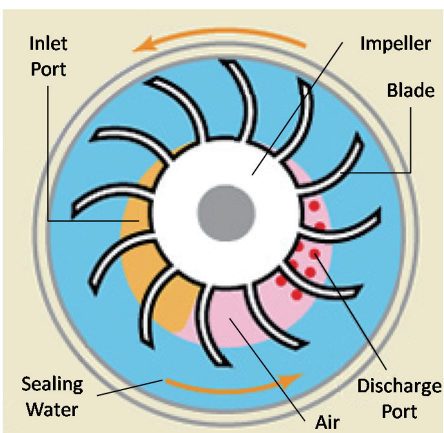
Figure 3: Velocity vector fields and comparisons with the other numerical data

Therefore, we propose the following logarithmic-type weighting function instead of Eq. 2 as shown in Fig. 1(b).

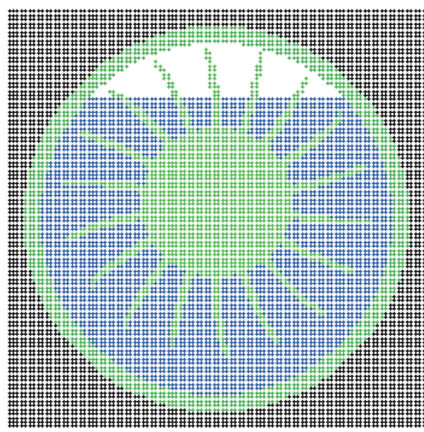
$$w(r) = \begin{cases} log(\frac{r_e}{r}) & (r < r_e) \\ 0 & (r \geq r_e) \end{cases} \tag{5}$$

4 Numerical examples

In this section we present numerical results obtained from applications of the above-mentioned numerical methods to incompressible viscous flow problems, namely flow in a driven cavity and flow in a liquid ring pump with rotating impeller blades from a practical point of view. The initial velocities are assumed to be zero everywhere in the interior domain. In both cases, we set the CFL condition $u_{max}\Delta t/l_{min} \leq C$, where C is the Courant number $(= 0.1)$. The kernel size for the particle number density and the gradient/Laplacian models is also $r_e = 4.0l_0$ in which l_0 is the distance between two neighboring particles in the initial state. In this case, we set $l_0 = 0.002333$.



(a) Geometrical configuration

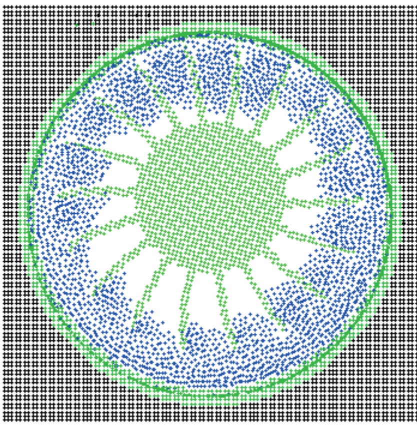


(b) Initial state of particles

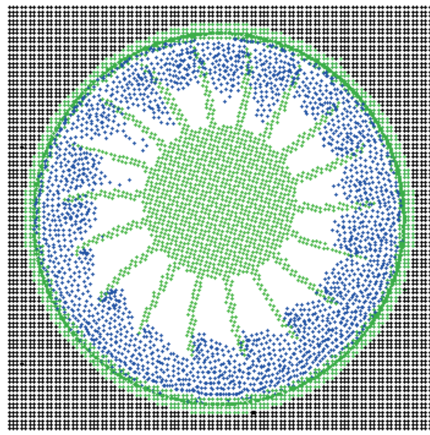
Figure 4: Flow in a liquid ring pump

4.1 Flow in a driven cavity

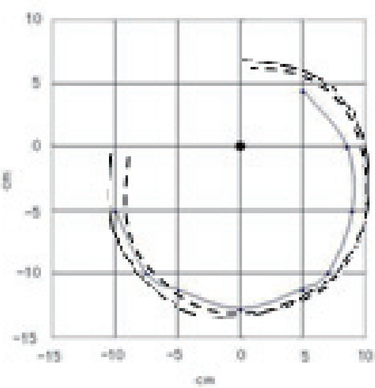
Let us first consider the flow in a square cavity driven by a lid sliding at a uniform velocity. The geometry, the boundary conditions and the initial state of particles



(a) MPS with Eq. 2



(b) Present



(c) Comparisons with exp.

Figure 5: Particle behaviors and comparisons with the experimental data

are shown in Fig. 2. In the initial configuration, we set $2,500$ particles and the Reynolds number of 100. Fig. 3 shows the velocity vector fields using MPS with Eq. 2 and present approach, and the velocity profiles on the centerline of the cavity. Our numerical results are generally comparable to the finite difference solutions [Ghia, Ghia and Shin (1982)].

4.2 Flow in a liquid ring pump

As the second example, Fig. 4 shows the geometry and the initial state of particles for flow in a liquid ring pump with rotating impeller. In Fig. 4(a) the blades near the top of the pump are very closer to the outside wall than at the side and bottom of the pump. The impeller with blades is attached to a center hub and located in off-set from the center of the cylindrical body. In this two-dimensional simulation, we set $9,527$ particles in the initial configuration and $2,400$rpm as the speed of the rotating impeller. Fig. 5 shows the instantaneous particle behaviors using MPS with Eq. 2 and present approach, and we compare our results with the air-water interface line obtained from the experiment (see Fig. 5(c)). Our results are qualitatively similar to the experimental data.

5 Conclusions

We have presented the MPS approach using logarithmic weighting function for solving numerically incompressible viscous fluid flow problems. The MPS scheme has been widely utilized as a particle strategy for free surface flow, the problem of moving boundary, and multi-physics/multi-scale ones. As the numerical examples, the driven cavity flow and the flow in a liquid ring pump with rotating impeller are carried out and compared with experimental data and other numerical ones.

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