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On Effective Permeability of Heterogeneous Porous Media

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Abstract: The conventional procedures for describing fluid flow in porous media largely rely on assumptions of spatial homogeneity and isotropy. In contrast, naturally occurring porous geologic media display heterogeneity and anisotropy that is scale-dependent. This paper summarizes results of recent research that uses experimental techniques, mathematical analyses of the experimental configurations and computational procedures for developing estimates for the effective permeability of a heterogeneous porous geomaterial.

Keywords: Permeability heterogeneity, effective permeability, surface permeability tests, computational developments

1 Introduction

Naturally occurring geologic media have heterogeneous material characteristics that are scale-dependent. The fluid transmissivity properties in such porous media are particularly sensitive to inhomogeneities that can exist at any scale. These can range from inhomogeneities at the micro-scale resulting from defects such as micro cracks and porous inclusions that can be present in the scale of a laboratory test specimen to inhomogeneities at the regional scale, which can result from fractures, stratifications, dissolution channels (Selvadurai et al., 2005). The methods that can be used to accurately characterize and define the permeability characteristics of such heterogeneous porous media become important to many areas of environmental geosciences and geomechanics with key applications that focus on deep geologic disposal of hazardous substances such as heat emitting nuclear waste, energy resources recovery, earthquake hazards along fault zones, geothermal energy extraction and geologic disposal of carbon dioxide as a means of mitigating climate change through reduction of greenhouse gases. In these endeavours, permeability of the geologic medium encountered in the geoenvironmental or geoscience activity is regarded as the parameter of critical interest and governs much of the

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dominant phenomena. There are various approaches that have been proposed in the literature for characterizing permeability in heterogeneous formations and references to these can be found in the texts and articles by Cushman (1990), Adler (1992), Gelhar (1993), Hornung (1997), Markov and Preziosi (2000), Selvadurai (2010), Suvorov and Selvadurai (2010) and Selvadurai and Selvadurai (2010). While the complete non-deterministic characterization of permeability heterogeneity is certainly possible, the use of such formulations in the computational solution of large scale porous media flow problems can be complex if both heterogeneity and anisotropy are simultaneously introduced to characterize such variability. Furthermore the parameter identification applicable to scale-dependent permeability that is both position-dependent and direction dependent is not a straightforward exercise. The more pragmatic approaches to characterization of permeability of heterogeneous porous geomaterials tend to retain the overall structure of a porous medium that is either both homogeneous and isotropic or homogeneous and anisotropic and then attempts to introduce the overall influences of heterogeneity by representing the permeability by effective estimates. The influences of heterogeneities are thus averaged out over a representative volume element. Even with such simplifications, the separation of the influences of heterogeneity and anisotropy is not straightforward. For example, a stratified porous medium that is isotropic and heterogeneous at one scale can be viewed as a porous medium that is homogeneous and anisotropic at a different scale. In heterogeneous porous media therefore, the choice of the Representative Volume Element (RVE) can also influence the model selected to simulate flow through such a porous medium.

The estimation of effective permeability of a porous medium continues to be a challenging research topic. The objective of this paper is to highlight recent developments (P.A. Selvadurai, 2010; Selvadurai and Selvadurai, 2010) that involved the combined application of experimental techniques, mathematical and computational methods for examining the experimental procedures with the objective of estimating the effective permeability of a cuboidal specimen of Indiana Limestone measuring 504 mm. The near surface permeability of the cuboidal region was examined using novel experimental techniques involving an annular permeameter. The experimental results were used to construct a spatial distribution of permeability heterogeneity within the cuboidal specimen. Upon verification of the absence of dominant pathways for fluid flow through the cuboidal region, estimates were obtained for the "Effective Permeability" of the cuboid using the mathematical relationships proposed by, among others, Wiener, Landau and Lifschitz, King, Matheron, Journel et al., Dagan. The results of these estimates are compared with the *geometric mean* based estimate for determining the effective permeability of a porous medium with heterogeneity proposed by Selvadurai and Selvadurai (2010), through their computational studies.

2 Theoretical results

The basis of the experimental procedure is that the porous medium will exhibit local isotropy and homogeneity such that the fluid flow through the medium can be defined in relation to Darcy's law

$$\mathbf{v}(\mathbf{x}) = -(K\,\gamma_w/\mu)\nabla\Phi \tag{1}$$

where $\mathbf{v}(\mathbf{x})$ is the velocity vector, K is the isotropic permeability, γ_w is the unit weight of water, μ is the dynamic viscosity, Φ is the reduced Bernoulli potential and ∇ is the gradient operator. The mass conservation equation during flow through the porous medium is given by$\nabla \cdot \mathbf{v}(\mathbf{x}) = 0$; this together with (1), gives rise to Laplace's equation, which takes the form

$$\nabla^2\,\Phi(\mathbf{x}) = 0 \tag{2}$$

where ∇^2 is Laplace's operator. The experimental configuration that is used to estimate the surface permeability of the cuboidal block of Indiana Limestone involves the application of steady flow to the central part of an annular sealed region (inner radius a and outer radius$b$) of a permeameter that can be placed at any location of a plane face of the cuboid. The dimensions of the permeameter in terms of the ratio b/a are such that the potential problem can be formulated as a three-part mixed boundary value problem for a halfspace region. When the annular permeameter is located centrally, the boundary conditions take the form

$$\Phi(r,0) = \Phi_0\ ,\ r \in (0,a); \quad (\partial\Phi/\partial z)_{z=0} = 0\ ,\ r \in (a,b); \quad \Phi(r,0) = 0\ ,\ r \in (a,\infty) \tag{3}$$

The three-part mixed boundary value problem defined by (3) can be solved using a Hankel transform development of (2) (Selvadurai, 2000a) and the reduction of the ensuing equations to a pair of Fredholm integral equations of the second-kind (Selvadurai and Singh, 1984a, b). The details of the procedure are also given by Selvadurai and Selvadurai (2010) and will not be repeated here. The result of primary importance to the research is the steady flow rate q from the central aperture that is subjected to the potentialΦ_0, which can be expressed in the form

$$q = 4a\Phi_0\left(\frac{K\gamma_w}{\mu}\right)\left\{\begin{array}{l} 1+\left(\frac{4}{\pi^2}\right)c+\left(\frac{16}{\pi^4}\right)c^2+\left(\frac{64}{\pi^6}+\frac{8}{9\pi^2}\right)c^3+\left(\frac{64}{9\pi^4}+\frac{256}{\pi^8}\right)c^4+ \\ \left(\frac{92}{225\pi^4}+\frac{384}{9\pi^6}+\frac{1024}{\pi^{10}}\right)c^5+\mathrm{O}(c^6) \end{array}\right\} \tag{4}$$

where$c = (a/b) << 1$ is the aperture ratio. Selvadurai and Selvadurai (2010) have shown that this analytical result, albeit approximate, is in excellent agreement with results obtained using alternative numerical solutions of the system of triple integral equations. Also it is noted that when $c \to 0$ the three-part boundary value problem (3) reduces to a classical two-part mixed boundary value problem in potential theory for a halfspace (Sneddon, 1972; Selvadurai, 2000b) and (4) reduces to $q = 4a\Phi_0(K\gamma_w/\mu)$,which is a compact result that can be used to estimate the local permeability of the tested region.

3 Experimental modelling

An extensive set of experiments were conducted by P.A. Selvadurai (2010) to determine the surface permeability of the cuboidal region of Indiana Limestone. The experimental developments, the theoretical interpretations and the estimates obtained for the surface permeability distributions are also documented by Selvadurai and Selvadurai (2010). The surface permeability was determined at 9 locations on each of the six surfaces of the cuboidal block of Indiana Limestone, to generate surface estimates of permeability. The surface point estimates were in turn used to generate, via a kriging procedure, estimates for the distribution of permeabilities at the interior of the cuboidal block of Indiana Limestone (Figure 1). The fundamental assumption in the estimation of the local permeability of the porous medium is that the permeability inhomogeneity in the cuboidal region as a whole has little or no influence on the locally estimated value of the permeability. To assess this assumption a computational model of a sub cube region was developed, with the annular permeameter located on one surface of the region (Figure 2).

Two extreme cases of computational solutions were developed where the surfaces were subjected to the following separate sets of boundary conditions in relation to the sub-cuboidal region shown in Figure 2:i.e.

The boundary conditions simulate the extreme conditions that are likely to influence the hydraulic response of the permeameter. It is found (Selvadurai and Selvadurai, 2010) that the difference between the two sets of extreme boundary conditions is less than 5.5% for the two extreme cases. Consequently, the theoretical developments that are used for the interpretation of local permeabilities using the

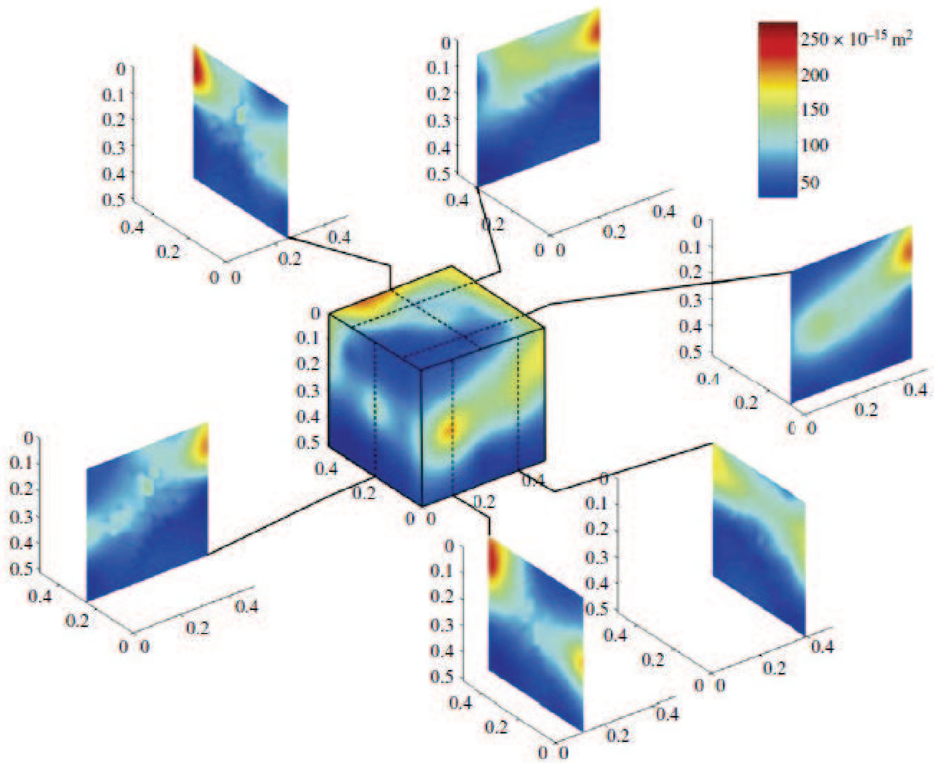

Figure 1: The permeability inhomogeneity in the cuboidal block of Indiana Limestone [Selvadurai and Selvadurai, 2010]

concept of local homogeneity are regarded as a suitable approximation, even for cases involving extreme inhomogeneity.

4 Effective permeability

The spatial distribution of permeability determined through experiments (Figure 1) can be used to estimate the effective permeability of the porous medium. Several researchers have proposed theoretical relationships that can be used to estimate the "effective permeability" of a heterogeneous porous medium and a complete discussion of these estimates is given by Selvadurai and Selvadurai (2010). For a region V_0 with a permeability distribution $K(\mathbf{x})$ the effective permeability is *bounded* by the Wiener (1912) estimates for the effective values defined by the harmonic mean (K_h) and the arithmetic mean (K_a).i.e.

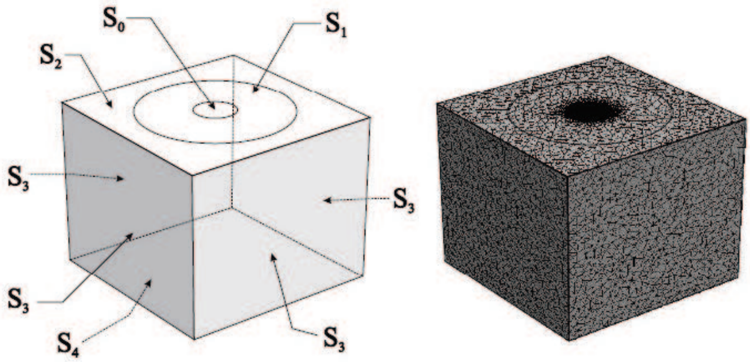

Figure 2: The sub-cube model of the porous medium with the annular permeameter. [Selvadurai and Selvadurai, 2010]

$$\left(\iiint_{V_0} dV \Big/ \iiint_{V_0} [K(\mathbf{x})]^{-1}\, dV\right) = K_h \leq K_{\text{eff}} \leq \; K_a = \left(\iiint_{V_0} K(\mathbf{x})\, dV \Big/ \iiint_{V_0} dV\right) \tag{5}$$

Similar relationships for estimating the effective permeability of inhomogeneous permeable media have been proposed by a number of researchers including of Landau and Lifshitz, Matheron, Journel et al., King and Dagan. The associated expressions for K_{eff} are presented by Selvadurai and Selvadurai (2010) and will not be repeated here. Selvadurai and Selvadurai (2011) also propose the following conjecture for estimating the effective permeability: Consider a region V_0, with a permeability distribution $K(\mathbf{x})$, which is *lognormal*. We consider one-dimensional permeabilities, measured in sub regions $V_s \in V_0$ along any set of n arbitrary directions that do not display marked variations. The one-dimensional permeabilities along the n directions are denoted by K_1, K_2, ..., K_n. The effective permeability can be estimated from the geometric mean. i.e.

$$K_{\text{eff}}^{\text{SS}} = \sqrt[n]{K_1 K_2 ....K_n} \tag{6}$$

The data for the spatial distribution of permeability within the cuboidal block of Indiana Limestone and the expressions in (8) were used to determine the effective permeability of the cuboidal specimen of Indiana Limestone. The results obtained were as follows (the actual estimate for the permeability can be obtained by multiplying the values by 10^{-15} m^2):

$$(K_{\mathrm{eff}}^{\mathrm{W}})_{\mathrm{LB}}(=\ 62.16) \leq K_{\mathrm{eff}}^{\mathrm{SS}} = 73.75 \leq (K_{\mathrm{eff}}^{\mathrm{W}})_{\mathrm{UB}}(=\ 80.10) \tag{7}$$

5 Concluding remarks

The estimation of the effective permeability of a heterogeneous porous medium continues to be a problem of major interest and importance to environmental geosciences and geoenvironmental engineering. The possibility of developing a single isotropic measure of the permeability of a heterogeneous porous medium is particularly attractive if the model of the effective porous medium is to be used in other types of problems including poroelasticity and in the study of advective transport of contaminants in heterogeneous porous media. While several techniques for estimating the effective permeability of heterogeneous porous media have been proposed in the literature, a canonical result relates to the geometric mean obtained through one-dimensional flow properties within a representative volume element. Within this representative volume element, the spatial distribution of permeability should conform to a lognormal distribution.

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