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Comparison of Constitutive Models Using Different Yield Functions for Porous Shape Memory Alloy with Experimental Date

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Abstract: Several constitutive models with different yield functions for porous shape memory alloy (SMA) are compared with the experimental date. Different approaches such as upper bound theory and lower bound theory have been adopted and a new correction formula of the yield function is proposed in this work to study the behavior of porous SMAs. Numerical results are compared with the experimental date by Zhao et al (2005). It shows that the researches using upper bound and lower bound are nearly the same and the new correction formula is much closer to the experimental data than others.

Keywords: Porous shape memory alloy; Yield function; Constitutive model

1 Introduction

In the last two decades the number of innovative applications for advanced materials has been rapidly increasing. Porous shape Memory Alloys (SMAs) have been widely used in many engineering applications such as sensors, actuators, biomedical devices and surgical implant material as well as a surgical instrument, and micro-electro-mechanical systems due to their well-known good biocompatibility, unique shape memory properties, mechanical properties, superior damping capability, excellent corrosion resistance and wear resistance (Starosvetsky and Gotman, 2001; Teppei et al., 2005; Greiner et al., 2005).

In order to effectively use the porous SMA materials in various engineering areas, it is pertinent to know their mechanical and physical properties as a function of the pore volume fraction as well as the transformation. Moreover, the effective methods for analyzing porous SMA materials assist in designing new advanced materials with different properties. There are a great number of research papers available in the literature devoted to modeling of porous SMA materials. Several authors (Entchev and Lagoudas, 2002, 2004; Nemat-Nasser et al., 2005) have used

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micromechanical averaging technique to investigate the mechanical response of porous SMAs. Entchev and Lagoudas (2002) have obtained the prediction of the macroscopic response of the porous SMA material by treating the porous SMA as a composite in which the SMA is the matrix and pores are the inclusions. But the effect of hydrostatic stress was not considered. It is well known that the phase transformation characteristics of dense SMAs are independent of hydrostatic stress (Zhao et al., 2005), while the macroscopic behaviors of porous SMAs are significantly affected by hydrostatic stress. Because of stress concentration near pores, the measuring results showed that the initial onset of the phase transformation for the porous SMA appears earlier than that for the dense one and without evidently turning point in the phase transformation (Entchev and Lagoudas, 2002). Nemat-Nasser et al. (2005) have modeled the porous SMA as a three-phase composite with the parent phase (austenite) as the matrix and the product phase (martensite) and voids as the embedded inclusions and the prediction of the micromechanical modeling for superelastic response of the porous Ni-Ti SMA was obtained. However, the parameters have been used in the model were on the premise that the porosity is 12%. It is well know porous SMAs have been synthesized using many different methods such as combustion synthesis, hot isostatic processing, and so on. Material constants produced by different methods are significant different due to non-uniform pore distribution and irregular pore shape. That is to say experiment data are different even if material component and volume of porosity are the same. So, it is difficult to ensure the practicability of Nemat-Nasser's model when the pore volume fractions (PVFs) are changed and can not degenerate to dense material directly. Therefore, it is necessary to develop a proper model to describe the stress-strain curve of porous SMA.

Different approaches such as upper bound theory and lower bound theory have been adopted and a new correction formula of the yield function is proposed in this work to study the behavior of porous SMAs. In order to examine their reliability and accuracy, we compare them with experimental-based one (Zhao et al., 2005) under uniaxial compression condition. The advantage of the present work in describing the yield function of porous SMA is the effect of hydrostatic stress is considered. Results show that there is a good approaching between the result by upper bound theory and the upper bound one. The new proposing approach is much closer to the experimental result than others.

2 Constitutive equation for porous SMA

The stress-strain relation of the system is given by (Lagoudas and Qidwai 1996)

$$\dot{\varepsilon}_{ij}^{p} = \bar{M}\dot{\sigma}_{ij}^{p} + \dot{\varepsilon}_{ij}^{pt} \tag{1}$$

Where $\bar{M}$ is overall compliance tensor of porous SMA, based on Eshelby inclusion method and Mori-Tanaka scheme, the compliance tensor can be expressed by

$$\bar{M} = L_0^{-1} + f[(1-f)L_0(I-\bar{S})]^{-1} \tag{2}$$

where I is fourth-order identity tensor, f is the PVF, $\bar{S}$ is the average Eshelby's tensor, ξ is the martensitic volume fraction. L_0 is the stiffness tensor of the matrix SMA. For simple computation, the effective modulus of SMA is assumed to vary with martensitic volume fraction ξ as follows

$$L_0 = L_A + \xi(L_M - L_A) \tag{3}$$

Setting the thermodynamic driving force equal to the resistance force, a balance equation that governs the martensitie transformation is obtained

$$\frac{1}{2}(1-2\xi)L_0(S-I)\varepsilon^t\varepsilon^t + \sigma_{ij}^{p}\varepsilon^t + (T-T_0)\Delta S_{A\to M} = 2\gamma_s/t + h(b_0 e^{b_0\xi} - b_1 e^{-b_1\xi}) \tag{4}$$

where h, b_0 and b_1 are material constants defining the nature of energy dissipation. $\Delta S_{A\to M}$ is the change of entropy from austensite to martensite. T_0 is the equilibrium reference temperature and T is the current temperature. γ_s is the surface energy density, t is the average thickness of martensite. This equation gives the equilibrium volume fraction of martensite ξ for given external stress, temperature. Following the same method, the volume fraction for the reverse transformation from full martensite to anstenite will be obtained.

The increment of overall transformation strain during the forward or reverse transformation is expressed by the following equation

$$\dot{\varepsilon}_{ij}^{pt} = H\frac{\partial \Phi}{\partial \sigma_{ij}^{p}} \tag{5}$$

where H is the Lagrange multiplier given by the consistency conditions.

$$\dot{\Phi} = 0\frac{\partial\Phi}{\partial\sigma_{ij}^{p}}\dot{\sigma}_{ij}^{p} + \frac{\partial\Phi}{\partial T}\dot{T} + \frac{\partial\Phi}{\partial\varsigma}\dot{\varsigma} = 0 \tag{6}$$

where Φ is the yield function for porous SMAs.

According to the research by (Yin et al., 2001), the equivalent form of the lower bound yield function is

$$\Phi = 3J_2 + 2(1 - \frac{1}{4}\ln f)f\sigma_0^2\cosh(\frac{1}{2}\frac{\sigma_{kk}}{\sigma_0}) - 1 - f(1+\ln f) = 0 \tag{7}$$

where J_2 is the second invariant of the deviatoric stress. σ_0 is the reference stress. σ_{kk} is the hydrostatic stress. In the same way, the equivalent form of the upper bound yield function is

$$\Phi = 3J_2 + 2f\sigma_0^2\cosh(\frac{1}{2}\frac{\sigma_{kk}}{\sigma_0}) - 1 - f^2 = 0 \tag{8}$$

However, both the above theories are usually used to simulate the rigid plastic porous materials. For porous SMAs, since the phase transformation stage for SMA material is similar to the "plastic" stage of viscoplastic material, we analysis the SMA matrix by dilatational plasticity theory with a matching strain hardening constant (Wang et al 1995). Here we propose a new modified formula

$$\Phi = 3J_2 + 2f\sigma_0^2\cosh((\frac{1}{2}\frac{\sigma_{kk}}{\sigma_0})^{\frac{1}{qn+1}}) - 1 - f^2 = 0 \tag{9}$$

where $q = 0.8$ is an adjusted coefficient, $n = 0.8$ is the strain hardening constant.

It is assumed the effective transformation strain is proportional to the maximum transformation strain of porous SMA during a uniaxial tension test is

$$\dot{\varepsilon}_e^{pt} = \frac{\gamma\dot{\varsigma}}{1-f} \tag{10}$$

where γ is the maximum transformation strain of dense SMA, ε_e^{pt} is called the effective transformation strain for porous SMA, and provides a scalar measure of the total transformation strain. This quantity is defined as

$$\dot{\varepsilon}_e^{pt} = \sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}^{pt}\dot{\varepsilon}_{ij}^{pt}} \tag{11}$$

Substituting Eq. (4) in Eq. (11) and using the Eq. (10), the increment of the transformation strain for porous SMA is

$$\dot{\varepsilon}_{ij}^{pt} = \frac{\frac{\gamma\dot{\varsigma}}{1-f}}{\sqrt{\frac{2}{3}\frac{\partial\Phi}{\partial\sigma_{kl}^{p}}\frac{\partial\Phi}{\partial\sigma_{kl}^{p}}}}\frac{\partial\Phi}{\partial\sigma_{ij}^{p}} \tag{12}$$

The overall strain behavior of porous SMA can be obtained by using (12), (4), (5) and the yield functions.

3 Numerical results

The above developed theory will be applied in modeling the constitutive response of porous SMA. As a simple application of constitutive equations, we get the response of porous SMA under uniaxial load with different porosities at isothermal condition. We use the stable material parameters of dense Ni-Ti SMA by Zhao et al (2005) to compare the obtained predictions with experimental data.

Table 1: Parameters used in calculation for porous Ni-Ti SMA by Zhao et al (2005)

υ	L_A	L_M	σ_0	γ	σ_{As}	σ_{Af}
0.33	75Gpa	31Gpa	400Mpa	0.025	600Mpa	300Mpa

Figure.1 shows comparisons between simulation results of the stress-strain response of 13% porosity and experimental data (Zhao et al., 2005) for uniaxial compression. Triangle right curve represents experimental result published by Zhao et al. (2005). Sphere curve is the model result of the J_2 theory. Upward-pointing triangle and pentagram curves are the result of lower bound and upper bound, respectively. Square curve corresponds to the present new model. As shown in Figure.1, first, because the effect of hydrostatic stress is considered, the results by upper bound and lower bound are closer to the experimental result than J_2 theory and agree very well with each other. Besides, the new proposing approach is much closer to the experimental result than others. What's more, in the present model, the needed material constants are only for dense SMA, which are obtained easily, and reliable.

Figure. 2 illustrates the effect of porosity for the isothermal pseudoelastic response of porous NiTi. The predicted hysteresis curves reflect the fact that raising the

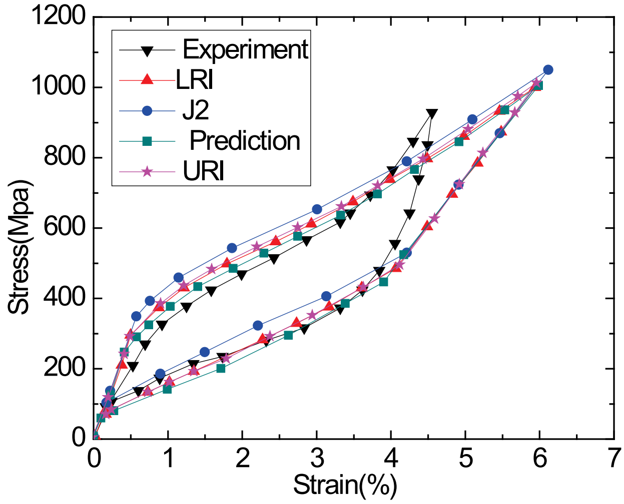

Figure 1: Comparison of present models and Zhao's experimental date for a PVF of 0.13

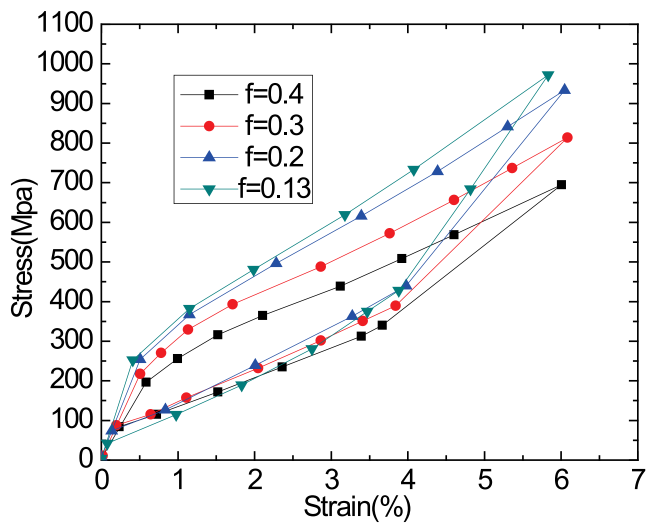

Figure 2: Curves of stress-strain with different PVFs

porosity decreases the start and the finish values of martensite transformation and reverse martensite transformation.

4 Conclusion

Several yield functions have been adopted in this work to study the behavior of porous SMAs. Numerical results have been compared with the experimental result under uniaxial compression. Comparison has shown that the new proposing approach is much closer to the experimental result than others. And the results by upper bound and lower bound are in good agreement with each other and the stage of start transformation is much closer to the experiment than that studied by J_2 theory.

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References

Entchev, P.B.; Lagoudas, D.C. (2002): Modeling porous shape memory alloys using micromechanical averaging techniques. *Mech. Mater*. vol. 34, pp. 1-24.

Entchev, P.B.; Lagoudas, D.C. (2004): Modeling of transformation induced plasticity and its effect on the behavior of porous shape memory alloys. Part II: porous SMA response. *Mech. Mater*. vol. 36, pp. 893-913.

Greiner, C.; Oppenheimer, S.M.; Dunand, D.C. (2005): High strength, low stiffness, porous Ni-Ti with superelastic properties. *Acta Biomater*. vol. 1, pp. 705-716.

Gurson, A.L. (1977): Continuum theory of ductile rupture by void nucleation and growth Part I. Yield cirteria and flow rules for porous media. *J. Eng. Mater. Technol*. vol. 99, pp. 2-15.

Lagoudas, D.C.; Bo, Z.; Qidwai, M.A. (1996): A unified thermodynamic constitutive model for SMA and finite element analysis of active metal matrix composites. *Mech. Compos. Mater. Struct.* vol. 3, pp. 153-179.

Nemat-Nasser, S.; Su, Y.; Guo, W.G.; Isaacs, J. (2005): Experimental characterization and micromechanical modeling of superelastic response of a porous Ni-Ti shape memory alloy. *J. Mech. Phys. Solids.* vol. 53, pp. 2320-2346.

Starosvetsky, D.; Gotman, I. (2001): Corrosion behavior of titanium nitride coated Ni-Ti shape memory surgical alloy. *Biomaterials*. vol. 22, pp. 1853-1859.

Teppei, W.; Yoshimi, W.; Hiroshi, O. (2005): Development of Fe-Mn-Si-Cr shape memory alloy fiber reinforced plaster-based smart composites. *Mater. Sci. Forum.* vol. 475, pp. 2063-2066.

Wang, Z.P.; Lam, K.Y.; Cotterell, B. (1995): An approximate yield criterion for voided nonlinear materials. *Mech. Mater*. vol. 22, pp. 291-300.

Yin,Y.J.; Xue, M.D.; Yu, S.W. (2001). Lower bound and upper bound rigid plastic constitutive models for porous materials: comparison and examination. *JSME*. vol. 44, pp. 45-56.

Zhao, Y.; Taya, M.; Kang, Y.S.; Kawasaki, A. (2005). Compression behavior of porous Ni-Ti shape memory alloy. *Acta. Mater*. vol. 53, pp. 337-343.