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# Calculation on the Ultimate Vertical Strength of Steel Tube in CFST Stub Column

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**Abstract:** This paper concentrates on the compressive strength of steel tube in Concrete-filled steel tube (CFST) column. Through an elasto-plastic limit analysis based on Twin Shear Unified Strength Theory (TSUST), the vertical load capacity of the steel tube alone was analyzed by adopting a parameter *b*, which varies from 0 to 1. And the strength reduction factor has been derived by total theory of plasticity. A new analytical formula, capable of predicting the axial bearing capacity of the CFST stub columns subjected to axial compression, has been developed based on the linear approximation of Mises yield criterion among TSUST. And There are good agreement between the experimental and analytical results.

**Keywords:** Steel tube, Axial compressive strength, Twin Shear Unified Strength Theory, Theory of plasticity .

## 1 Introduction

In recent years, Concrete-Filled Steel Tubular (CFST) has been applied widely in civil engineering for its predominant mechanical behavior. This increase in use is due to the significant advantages that concrete filled steel columns offer in comparison to more traditional construction methods (*Kenj,* 2004).

There are two kinds of methods to calculate the ultimate compressive strength of CFST. One is the bearing capacity superposition superimposition of its components (*Cai,* 2003), the other is regression analysis based on the tests considering steel ratio and confinement factor (*Zhong,* 2000). This paper adopts the first superposition method to analyze the ultimate strength of CFST column. For the concrete filled

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in CFST, it is under three-dimensional state, and there are well-developed theories (*Richart*, 1928*; Garder,* 1967) to calculate its ultimate compressive strength. However for steel tube, no uniform achievement can be used to determinate the vertical strength contributing to the whole CFST column. In this paper, the ultimate compressive strength of steel tube was derived by Twin-Shear Unified Strength Theory (TSUST) and theory of plasticity, Then, with the principle of limiting equilibrium, this paper presented one new formula for axial bearing capacity of the stub CFST column. Compared with the experimental results, the applicability of the formulas was testified

## 2 Twin-Shear Unified Strength Theory

The TSUST (*Yu*, 2004) considers the two larger principal shear stresses and the corresponding normal stresses and their different effects on the failure of materials. When the relationship function between them reaches one ultimate value, the material can be defined as failure at this state which is formulated as follow

$$F = \tau_{13} + b\tau_{12} + \beta(\sigma_{13} + b\sigma_{12}) = C \text{ when } \tau_{12} + \beta\sigma_{12} \geq \tau_{23} + \beta\sigma_{23} \tag{1a}$$

$$F' = \tau_{13} + b\tau_{23} + \beta(\sigma_{13} + b\sigma_{23}) = C \text{ when } \tau_{12} + \beta\sigma_{12} \leq \tau_{23} + \beta\sigma_{23} \tag{1b}$$

where $\tau_{12}$, $\tau_{23}$ and $\tau_{13}$ are the principal shear stresses, $\tau_{13} = (\sigma_1 - \sigma_3)/2$, $\tau_{12} = (\sigma_1 - \sigma_2)/2$ and $\tau_{23} = (\sigma_2 - \sigma_3)/2$; $\sigma_{12}$, $\sigma_{23}$ and $\sigma_{13}$ are the corresponding normal stresses on the principal shear stress element; $\sigma_1$, $\sigma_2$ and $\sigma_3$ are the principal stresses and $\sigma_1 \geq \sigma_2 \geq \sigma_3$. $b$ is a weighting coefficient reflecting the relative effect of the intermediate principal shear stress $\tau_{12}$ or $\tau_{23}$ on the strength of materials; $C$ equals to the material strength. Denoting the tension-compression strength ratio as $\alpha = \sigma_s/\sigma_c$, we rewrite Eqs.1a and 1b in terms of principal stresses as follow

$$F = \sigma_1 - \frac{\alpha}{1+b}(b\sigma_2 + \sigma_3) = \sigma_s \quad \sigma_2 \leq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha} \tag{2}$$

$$F' = \frac{\sigma_1 + b\sigma_2}{1+b} - \alpha\sigma_3 = \sigma_s \quad \sigma_2 \geq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha} \tag{3}$$

## 3 Limit Analysis of the Steel Tube

Under the state of ultimate balance of the whole column, steel tubes are in three dimensional stress states including of axial compression, radial compression and circumferential tension. In most CFST structure, diameter-thickness ratio of steel tubes is generally not smaller than 20, so the tubes can be regarded as thin-walled cylinder. The tube can be considered as under the plane stress state, i.e., radial compressive stress $\sigma_r$=0.

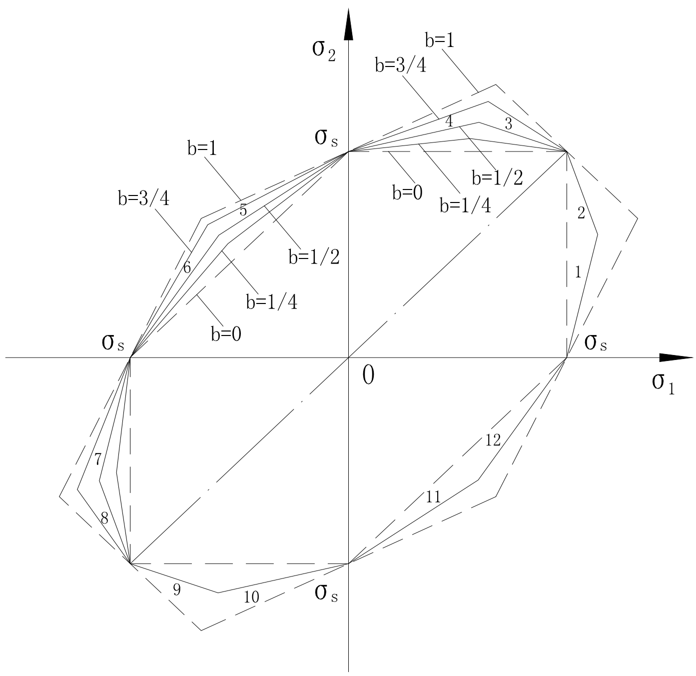


Figure 1: Yield criterion in plane state

According to the TSUST, the stresses of steel tube in plate state are shown in Fig. 1. The TSUST include or approximate an existing strength criterion or slip-line field theory by adopting a parameter *b*, which varies from 0 to 1. Therefore, a family of convex yield criteria suitable for any kind of materials, are deduced. In particular, the UST becomes the Tresca criterion when $\alpha$=1.0 and *b*=0.The von Mises criterion can be linear approximated by the TSUST with $\alpha$=1.0 and *b*=0.366, and the Mohr–Coulomb criterion is obtained with *b*=0 (*Yu*,2004). No matter what b takes for different strength theory, if circumferential tensile stress $\sigma_\theta$ arrives the yield strength $f_y$, $\sigma_z$axial compressive stress nearly becomes zero, and vice versa. Actually, the steel tube can not reach this ideal state under ultimate state of the composite column. The steel tube can still bear some vertical load when the CFST column is under the ultimate state, and assume $\beta_f$ as the strength reduction factor of the steel tube, i.e.

$$\sigma_3 = \sigma_z = \beta_f f_y \tag{4}$$

The main stresses of circular steel tube can be explicated by $\sigma_3 = \sigma_z = -\beta_f \sigma_s$, $\sigma_2 = \sigma_r = 0$, $\sigma_1 = \sigma_\theta$. According to the test results (*Pei*, 2005), since |O$\varepsilon_z$|O>$\varepsilon_\theta$, there is $|\sigma_3| > \sigma_1$. Hence, $\sigma_2 \geq \frac{\sigma_1+\sigma_3}{1+\alpha}$. Substitution of this relation into Eq.2*b* gives

$$\frac{\sigma_\theta}{1+b} + \sigma_z = f_y \tag{5}$$

Hencky stress-strain relation among the total theory of plasticity can be written as

$$\varepsilon_z = \frac{1+\phi}{2G}[\sigma_z - \frac{\phi + 3(m+1)^{-1}}{\phi+1}\sigma_m] \tag{6a}$$

$$\varepsilon_\theta = \frac{1+\phi}{2G}[\sigma_\theta - \frac{\phi + 3(m+1)^{-1}}{\phi+1}\sigma_m] \tag{6b}$$

where $m = \frac{1}{v}$, $G = \frac{E}{2(1+v)}$, and $\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$. $\phi$ is Non-negative scalar factor. By taking poisson ratio $v$Í =0.5, m=2. When $\sigma_2 = 0$, these equations can be simplified as follows

$$\varepsilon_z = \frac{1+\phi}{6G}[2\sigma_z - \sigma_\theta] \tag{7a}$$

$$\varepsilon_\theta = \frac{1+\phi}{6G}[2\sigma_\theta - \sigma_z] \tag{7b}$$

Hence, there is

$$\frac{\varepsilon_\theta}{\varepsilon_z} = \frac{2\sigma_\theta - \sigma_z}{2\sigma_z - \sigma_\theta} \tag{8}$$

where $\varepsilon_z$, $\varepsilon_\theta$ are the vertical and circumferential strain of the steel tubes. The same results can also be obtained by stress-strain relation in term of total theory of plasticity. The strain value of steel tube has been obtained at the ultimate state through the specimens test. Letting $u = \frac{\varepsilon_\theta}{\varepsilon_z}$ and transposing in Eqs.4 and 7

$$\sigma_\theta = \frac{(2u+1)(1+b)}{3(1+u)+b(u+2)} f_y, \quad \sigma_z = \frac{(u+2)(1+b)}{3(1+u)+b(u+2)} f_y \tag{9}$$

Therefore, $\beta_f$ can be obtained as

$$\beta_f = \frac{(u+2)(1+b)}{3(1+u)+b(u+2)} \tag{10}$$

The strain obtained from different gauging points in the experiment (*Pei*, 2005) is shown in Tab. 1 when b takes different value. For steel material, we can take the approximation of von Mises criterion as the final answer. In this paper, $\beta_f$ was taken as 0.65 in the calculation of bearing capacity of the composite column. Substitute $\sigma_3 = \sigma_z = -0.65\sigma_s$ into Eq.4 ,the following expression can be derived as

$$\sigma_1 = \sigma_\theta = 0.4774 f_y \tag{11}$$

Then the confining pressure on the concrete under ultimate state of the CFST column is

$$p = \frac{2t}{D}\sigma_\theta = 0.4774\frac{2t}{D}f_y \tag{12}$$

where $D$, $t$ are the diameter and thickness of the steel tube.

## 4 Calculation of Ultimate Bearing Capacity

In the state of limit equilibrium, the ultimate axial bearing capacity can be calculated by$N = N_s + N_c$,where $N, N_s$ and $N_c$ are vertical bearing capacity of the whole column, steel tube and concrete respectively.

The concrete was filled in steel tube, its main stresses can be explicated by $0 > \sigma_1 = \sigma_2 > \sigma_3$, $\sigma_1 = \sigma_2 = p$. For $\sigma_2 \geq \frac{\sigma_1+\alpha\sigma_3}{1+\alpha}$, substitute them into the stress expression of TSUST, the following expression can be obtained.

$$\sigma_3 = f_c + k_c p \tag{13}$$

where $f_c$ is the axial strength of concrete when $p = 0$;$k_c$ Is a the strength improvement coefficient. In TSUST, $k_c$can be calculated by cohesion and friction angle at material failure state, while $k_c$ as a the improvement coefficient about concrete under fixed lateral compressive force has been studied much. According to the test of Richart (1928),$k_c$ has been taken as the constant 4.1 simply here.

Therefore, synthesize the results both steel tube and concrete, the ultimate axial bearing capacity of the stub column can be calculated as

Table 1: Calculation of vertical stress of steel tube under the ultimate state

| specimen | $t$ | $D$ | $\sigma_s$ | $\varepsilon_z$ | $\varepsilon_\theta$ | $\mu$ | $b$=0 | | $b$=0.366 | | $b$=1 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | $\sigma_z$ | $\beta_f$ | $\sigma_z$ | $\beta_f$ | $\sigma_z$ | $\beta_f$ |
| G1-2 | 1.4 | 58.5 | 352.5 | 2917 | 1232 | 0.43 | 200.1 | 0.57 | 226.2 | 0.64 | 255.3 | 0.72 |
| G1-2 | 1.4 | 58.5 | 352.5 | 3375 | 1606 | 0.48 | 197.1 | 0.56 | 223.4 | 0.63 | 252.8 | 0.72 |
| G1-2 | 1.4 | 58.5 | 352.5 | 3214 | 1477 | 0.46 | 198.0 | 0.56 | 224.2 | 0.64 | 253.6 | 0.72 |
| G1-2 | 1.4 | 58.5 | 352.5 | 3767 | 1983 | 0.53 | 194.5 | 0.55 | 220.9 | 0.63 | 250.7 | 0.71 |
| G1-3 | 0.9 | 74 | 680 | 2818 | 1218 | 0.43 | 384.9 | 0.57 | 435.3 | 0.64 | 491.6 | 0.72 |
| G1-3 | 0.9 | 74 | 680 | 3382 | 1475 | 0.44 | 384.5 | 0.57 | 435.0 | 0.64 | 491.2 | 0.72 |
| G1-3 | 0.9 | 74 | 680 | 3165 | 1172 | 0.37 | 392.1 | 0.58 | 442.0 | 0.65 | 497.4 | 0.73 |
| G1-3 | 0.9 | 74 | 680 | 3655 | 1397 | 0.38 | 390.7 | 0.57 | 440.7 | 0.65 | 496.2 | 0.73 |
| G1-4 | 0.9 | 83 | 597 | 3127 | 823 | 0.26 | 356.5 | 0.60 | 399.5 | 0.67 | 446.4 | 0.75 |
| G1-4 | 0.9 | 83 | 597 | 3495 | 1020 | 0.29 | 353.0 | 0.59 | 396.3 | 0.67 | 443.7 | 0.74 |
| G1-4 | 0.9 | 83 | 597 | 3910 | 1206 | 0.31 | 351.1 | 0.59 | 394.4 | 0.66 | 442.2 | 0.74 |
| G1-4 | 0.9 | 83 | 597 | 3799 | 934 | 0.25 | 358.7 | 0.60 | 401.5 | 0.67 | 448.2 | 0.75 |

$$N = 0.65 f_s A_s + (f_c + 3.91 \frac{t}{D} f_s) A_c \quad (14)$$

where $A_s$ and $A_c$ are cross-sectional area of steel tube and concrete respectively.

The predicted results ($N_c$) derived from Eq.13 are compared with the experimental values ($N_t$) in Table 2. As can be seen from the table, the calculated values are very close to the experimental ones with reference to the literatures.

## 5 Conclusion

Through an elasto-plastic limit analysis based on TSUST, the ultimate strengths of the steel tube under axial compression are analyzed. It can be concluded that the vertical strength of steel tube contributing to the whole column is neither zero nor its yield strength. Actually, the steel tube can bear some vertical load that has been deduced from TWUST and theory of plasticity. In this paper, the strength reduction factor $\beta_f$ was taken as 0.65 when the TSUST with $\alpha$=1.0 and b=0.366 was linear approximated by von Mises criterion. Then, A new analytical formula, capable of predicting the axial bearing capacity of the CFST columns subjected to axial compression, has been developed. And there was good agreement between theoretical and experimental results.

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Table 2: Comparison of experimental results and computed results

| Specimens | Steel tube $Ds \times t \times l$mm$^3$ | $f_c$ /MPa | $f_{so}$ /MPa | Test results $N_t$ /kN | Calculated results $N_c$ /kN | $\frac{N-N_t}{N_t} \times 100\%$ | Literatures |
|---|---|---|---|---|---|---|---|
| Z-69-84 | 100×2. 5×300 | 39. 2 | 442. 0 | 845 | 891.6 | 5.5 | (*Tang*,1982) |
| Z-70-102 | 100×2. 5×300 | 43. 4 | 249. 0 | 684 | 690.9 | 1.0 | |
| Z-70 -106 | 100×2. 0 ×300 | 43. 4 | 241. 0 | 548 | 512.1 | 6.6 | |
| Z-70-107 | 100 ×1. 5 ×300 | 43. 4 | 237. 0 | 515 | 499.2 | 3.1 | |
| Sccs1-1 | 131×2. 3 ×395 | 53. 4 | 323. 3 | 1250 | 1276.4 | 2.1 | (*Han*, 2000) |
| Sccs2-1 | 111×2. 0 ×339 | 53. 4 | 353. 6 | 894 | 901.8 | 0.9 | |
| Sccs3 -1 | 114×3. 2 ×337 | 53. 4 | 353. 6 | 1140 | 1121.6 | 1.6 | |

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