Study on Regularities of the Dimensionless Pressure in Bending Pipes

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Abstract: This paper studies the distribution of dimensionless pressure in elbow pipes by dimensional analysis methods. The qualitative description method of fluid flow such as 90˚ elbow pipe is obtained. With Computational Fluid Dynamics (CFD), the effect of a number of dimensionless parameters such as the non-dimensional curvature, Reynolds number, dimensionless axial angle α and annular angle $\beta\hat{A}$ and other factors on the spatial distribution of pressure inside the elbow is analyzed and discussed in detail. This paper not only provides theoretical and numerical methods for understanding the dynamic behavior of the fluid in the elbow pipes, but also provides the reliable basis for designing thickness of elbow pipes with high temperature, high pressure and high velocity.

Keywords: Pressure in elbow pipes, Regularities of distribution, Dimensionless analysis, qualitative description method

1 Introduction

Elbows are critical components of piping system in petroleum industry, chemical industry, navigation industry, aviation industry and nucleus industry. However, in high temperature and high pressure conditions inhomogeneity of pressure distribution aggravates elbow stress concentration, the occurrence of damage to failure problems are becoming increasingly apparent [1]. Therefore, the development of relevant design methods, the qualitative analysis of flow characteristics and the formula of pressure distribution in curved pipe are of great scientific significance and application value [2,3].

In this paper the flow characteristics and regularities of distribution in the most common 90˚ elbow pipe is studied by numerical methods based on computational

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fluid dynamics primarily and the dimensional analysis theories for auxiliary. The qualitative formula of pressure distribution is obtained. The effect of multiple parameters on regularity of pressure distribution is discussed in details and some conclusions with guiding significance foe engineering is obtained in the paper.

2   Prediction model for pressure distribution in elbow pipes

Dimension is an important concept in theoretical analysis. Dimensional analysis method can be used to infer certain physical laws, which not only provides theoretical guidance in testing the process of scientific organizations and sorting results, but also provides a theoretical basis for fitting experience or semi-empirical formula [4].
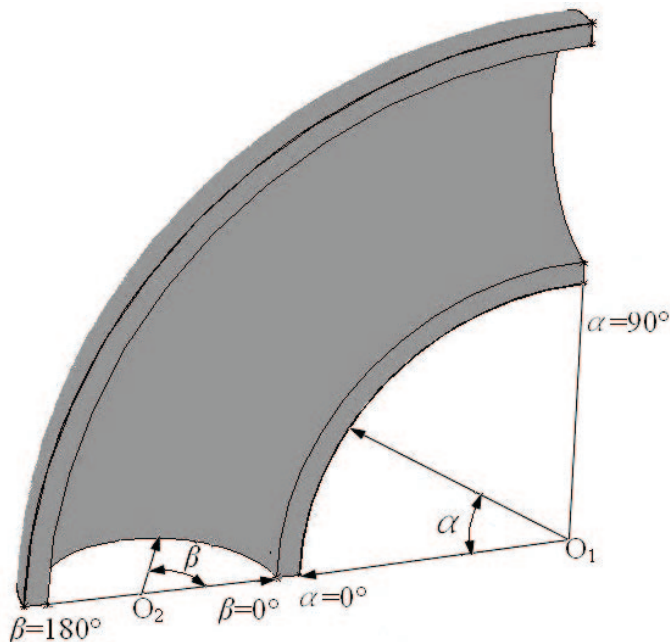

Figure 1: Coordinate system for elbow

Dimensional analysis is a method to establish mathematical model in physical fields. Based on experience and testing the physical relationship between variables can be determined by means of the harmony laws of dimensions of physics variables and dimensional analysis modeling for the Buckingham-$\Pi°$ theorem. A physical phenomenon have n influencing factors. There are m independent physical dimensions, the physical phenomenon can be expressed by $(n-m)$ dimensionless

parameters. The similarity theory and dimensional analysis can be applied to study the impact of many complex physical phenomena or engineering system in order to reduce the number of variables and to make expressions simple [5].

A specific coordinate system is established in Fig.1. Angel $\alpha(0–90°)$ along elbow axial direction is denoted the position of cross section along elbow axis. While angle $\beta(0–180°)$ is denoted the position of cross section along circumferential direction.

The major impact parameters of pressure distribution in the pipe are as[5,6], geometry of the elbow (bend radius R and the diameter D), the fluid physical parameters (density ρ and dynamic viscosity μ), position in the pipe (circumferential angle β and axial angle α), and fluid status (exit pressure p_0, the speed v). Therefore, the pressure distribution model $p = \phi(R, D, \alpha, \beta, \rho, \mu, p_0, v)$ can be written as

$$f(R, D, \alpha, \beta, \rho, \mu, (p - p_0), v) = 0 \tag{1}$$

Taking into account that α and β are angles, which can be dimensionless itself, therefore, $m=3$ basic physical quantities (the fluid density ρ, the diameter D, the fluid velocity v) is selected from the above $n=8$ physical quantities. These three physical quantities are related to three basic dimensions M, L, T. Because dimensional index determinant of the 3 physical quantities
$$\begin{cases} \rho = [M^1 L^{-3} T^0] \\ D = [M^0 L^1 T^0] \\ v = [M^0 L^1 T^{-1}] \end{cases}$$
is not zero $\begin{vmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} \neq 0$. So the above three basic physical dimensions are independent, can not form a dimensionless quantity. According to the Third Similarity Theorem, which can get $n - m=5$ dimensionless $\Pi°$-free items. So we can get,

$$F(\Pi_1, \Pi_2, \Pi_3, \alpha, \beta) = 0 \tag{2}$$

Where, $\Pi_1 = R\rho^{a_1} D^{b_1} v^{c_1}$, $\Pi_2 = \mu\rho^{a_2} D^{b_2} v^{c_2}$, $\Pi_3 = (p - p_0)\rho^{a_3} D^{b_3} v^{c_3}$.

According to dimensional harmonious principle the solution is $\Pi_1 = R/D$, $\Pi_2 = 1/Re$, $\Pi_3 = (p - p_0)/\rho v^2$

Then, we can get this the following relationship,

$$F\left(R/D, 1/Re, (p - p_0)/\rho v^2, \alpha, \beta\right) = 0 \tag{3}$$

As the dimensions of each parameter can only be obtained from the product and export business of basic dimensions, not from add and subtract. So according to the

Rayleigh law we can arrange the equation (3) and get the formula of dimensionless pressure and the difference of dimensionless pressure as follows,

$$(p - p_0)/(\frac{1}{2} \cdot \rho v^2) = \cdot f_1(R/D) \cdot f_2(Re) \cdot f_3(\alpha) \cdot f_3(\beta) \tag{4}$$

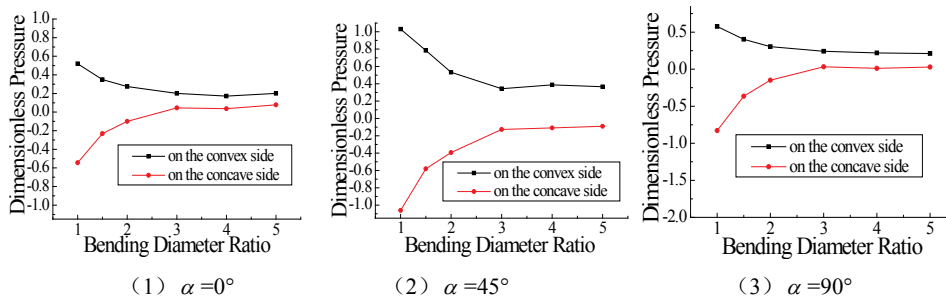$$p = p_0 + \frac{1}{2} \cdot \rho v^2 \cdot f_1(R/D) \cdot f_2(Re) \cdot f_3(\alpha) \cdot f_3(\beta) \tag{5}$$

Figure 2: Variation of the dimensionless pressure with the bending diameter ratio R/D

3 Regularities of the dimensionless pressure distribution

In order to study Regularities of the dimensionless pressure distribution, we analyze influencing factors based on the theoretical analysis and the numerical results of pressure distribution. Taking into account the 90 ˚ elbow angle geometric similarity, non-dimensional pressure distribution has geometric similarity with different R/D and Re. The following dimensionless pressure with the same α and β in the elbow pipe are taken as the object of analysis and research.

3.1 *The variation of the dimensionless pressure (difference) with the bending diameter ratio R/D*

3.1.1 *The variation of the dimensionless pressure with R/D*

In the CFD simulation, the pipe inlet velocity is defined 60 m/s, the of elbow outlet is treated as pressure outlet boundary condition. The diameter $D = 0.1$m. The medium is water, whose temperature is 25°, density is 997kg/m3, dynamic viscosity μ is $\mu = 0.0008899$kg / (m·s), the Reynolds number is $Re = \frac{vD\rho}{\mu} = \frac{60 \times 0.1 \times 997}{0.008899} = 6.72 \times 10^6$. Under the premise of keeping Reynolds number unchanged, we analyze

the dimensionless pressure changes with the change rule of the bending diameter ratio *R/D*, as is shown in Fig.2. It can be seen from Fig.2, (1) The dimensionless pressure on the elbow wall of convex side decreases with the bending diameter ratio R/D increases, which tends to a constant value when $R/D \geq 3$; (2) The dimensionless pressure on the concave side of elbows increases with R/D increases, which tends to a constant value when $R/D \geq 3$; (3) When $R/D \geq 3$, the dimensionless pressure on the concave side of elbows corresponding to the axial angle $\alpha = 0°$ and $\alpha = 90°$ is slightly greater than zero and that of the axial angle $\alpha = 45°$ is slightly less than zero.
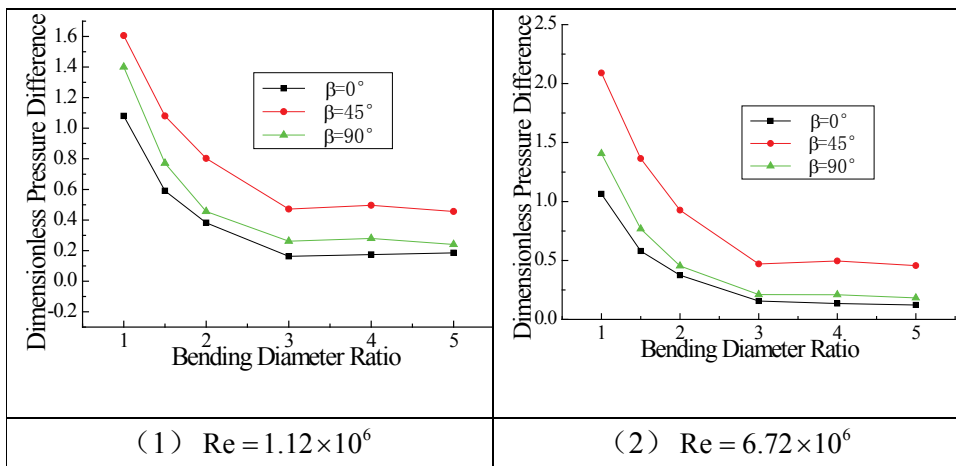

Figure 3: Variation of the dimensionless pressure difference with R/D

3.1.2   *The variation of the dimensionless pressure difference with R/D*

Fig.3 shows that the dimensionless pressure difference on the concave and convex side as the bending diameter ratio at different axial position of diameter. It can be seen from that: (1) The dimensionless pressure difference is maximum when axial angle $\alpha = 45°$; (2) The dimensionless pressure on the concave side is lower than that on the convex side, and the pressure difference becomes smaller as R/D increases; (3) when $R/D \geq 3$ the pressure difference tends to a constant value; (4) The larger the Reynolds number, the greater the dimensionless pressure difference inside and outside at the position of axial angle $\alpha = 45°$.

3.2 *The variation of the dimensionless pressure (or difference) with the Reynolds number* Re

3.2.1 *The variation of the dimensionless pressure with Re*

Fig.4 shows that the dimensionless pressure difference on the concave and convex side varies with Re. It can be seen from fig.4 that, (1) the dimensionless pressure has little change with the Reynolds number. (2)When $R/D = 3$ and $\alpha = 0°$, $\alpha = 45°$ and $\alpha = 90°$ respectively pressure on the concave side is lower than that on the convex side, and the pressure difference becomes smaller as the bending diameter ratio increases, then tends to a constant value. (3) When $\alpha = 45°$, pressure on the convex side is maximum. (4) The pressure difference tends to a constant value when $R/D \geq 3$. The results coincide with ones in reference [7].
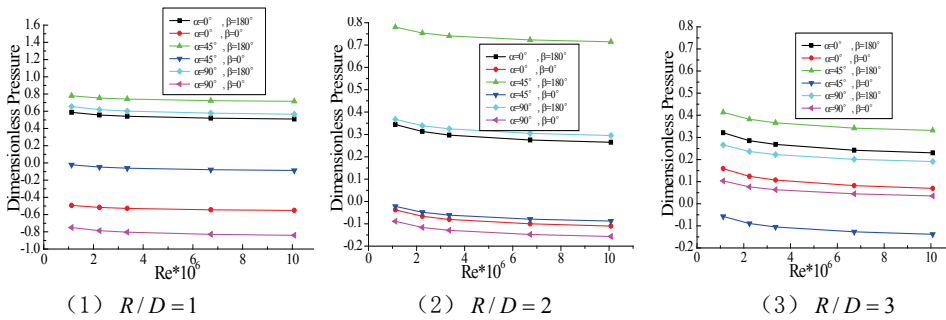

Figure 4: Variation of the dimensionless pressure with *Re*

3.2.2 *The variation of the dimensionless pressure difference with Re*

Fig.5 shows that the variation of the dimensionless pressure difference on the concave and convex side varies with Reynolds number when $R/D = 1$, 2 and 3. It can be seen from that, (1) In case of large Reynolds number, the dimensionless pressure difference on the concave and convex side is basically a constant value and has nothing to do with the Reynolds number. (2) When $R/D = 2$ the dimensionless pressure difference with $\alpha = 90°$ is slightly larger than that with $\alpha = 0°$, and the dimensionless pressure difference with $\alpha = 45°$ is maximum; (3) The dimensionless pressure difference in different Reynolds number corresponding to the different axial angle $\alpha = 0°$ and $\alpha = 90°$ are almost the same when $R/D = 3$. This indicates that the lager the bending diameter ratio, the smaller the pressure difference on inlet and outlet. (4) The smaller the bending diameter ratio, the greater the pressure difference on the concave and convex side, so resistance coefficient in the bend is greater.
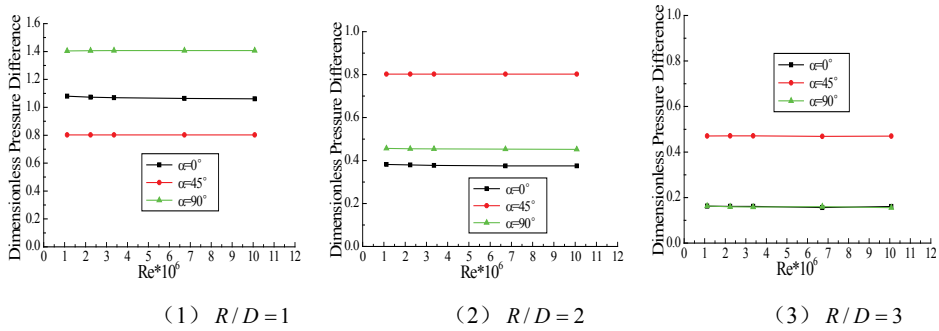
（1）$R/D = 1$ （2）$R/D = 2$ （3）$R/D = 3$

Figure 5: Variation of the dimensionless pressure difference with *Re*

3.3   The variation of the dimensionless pressure with α
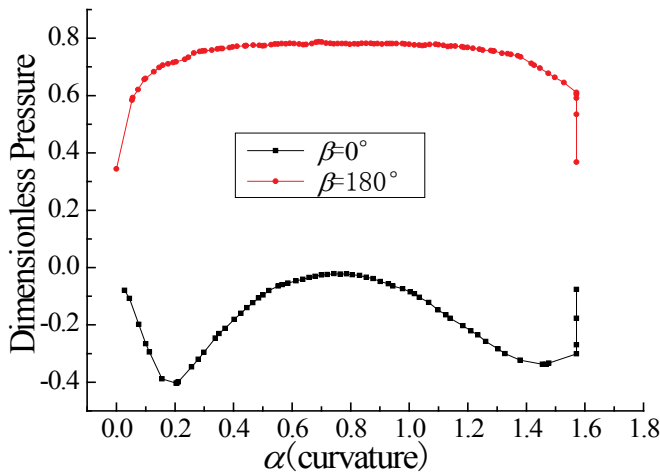

Figure 6: Variation of the dimensionless pressure with α ($R/D = 2$, $Re = 1.12 \times 10^6$)

Fig.6 shows the variation of the dimensionless pressure on the concave and convex side with α. It can be seen from fig.6 that, (1) The pressure on the concave side begins to decrease and then increases, decreases and then increases with α. (2) The pressure on the convex side begins to increase quickly and then tends constant, and decreases quickly with α.

3.4   *The variation of the dimensionless pressure with* β

Fig.7 shows that the variation of the dimensionless pressure with β in different bending diameter ratio and Reynolds number. It can be seen from that: (1) The dimensionless pressure changes inverse cosinely with β. (2) The increment of the profile shows that the pressure on the convex side is greater than that on the concave side. (3) The average pressure of cross-section as $\alpha = 45°$ is smaller than that as $\alpha = 0°$, $90°$ when $R/D = 1$, 2. This indicates that cross-section as $\alpha = 45°$ has small velocity and high pressure. (4) The average pressure of cross-section as $\alpha = 0°$ is higher than that as $\alpha = 45°$, $90°$ when $R/D = 3$. (5) The smaller the bending diameter ratio, the greater the slope of each profile, the greater the pressure difference between convex and concave side. Otherwise, we have inverse regularity.
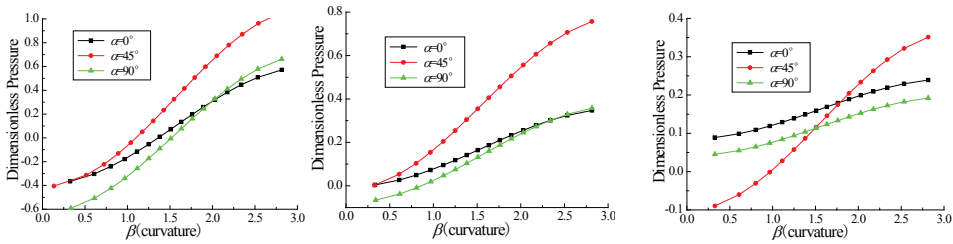

Figure 7: Variation of the dimensionless pressure with β

4   Conclusions

(1) The dimensionless pressure on the convex side decreases with R/D, that on the concave side increase with R/D, and these tend constant when $R/D \geq 3$.

(2) The dimensionless pressure has nothing to do with the Reynolds number and the dimensionless pressure difference as $\alpha = 45°$ is maximum.

(3) The pressure difference becomes smaller as R/D increases, and it tends to a constant value when $R/D \geq 3$ gradually.

(4) The pressure on the concave side begins to decrease and then increases, decreases and then increases with α. The pressure on the convex side begins to increase quickly, then tends constant, and decreases quickly with α.

(5) The dimensionless pressure changes inverse cosinely with β.

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