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A Novel Constitutive Equation for Inverse Analysis Method and its Application in Sheet Metal Forming

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Abstract: The Traditional Inverse Analysis Method (TIAM) of sheet metal stamping has the shortcomings of neglecting the effects of deformation history on stress prediction. An Updated Inverse Analysis Method (UIAM) is proposed based on the final workpiece in Euler coordinate system. The UIAM uses the constitutive equation based on flow theory of plasticity to consider the loading history. In order to avoid numerous iterations to ensure the numerical stability in Newton-Raphson scheme to obtain plastic multiplier $\Delta\lambda$, the equation in unknown stress vectors is transformed into a scalar equation using the notion of the equivalent stress. Thus a scalar equation of two orders and only one unknown factor $\Delta\lambda$ is obtained. A simple transformation matrix is introduced to reverse this matrix, so that the multiplier $\Delta\lambda$ can be solved explicitly. Results obtained with the TIAM based on deformation theory of plasticity and the updated one based on flow theory of plasticity are compared with those of the incremental forward finite element solver LS-DYNA. The comparisons of blank configurations and the effective strain distribution show that the proposed plasticity integration algrithm is effective and reliable.

Keywords: Sheet metal stamping; Inverse analysis method; Constitutive equations; Deformation theory of plasticity; Flow theory of Plasticity

1 Introduction

The sheet metal forming is a method widely used in the automotive industry for body panels and structures (truck lids, rails ...). In general, the process of developing a stamping part is long and expensive. Indeed, it is based on a series of trial-and-errors that heavily depend on the experience of the designer to eliminate defects, such as indentation, cracks and folds. As a result, manufacturers are very interested to reduce the number of trials and replaced by numerical simulations. In

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recent years, thanks to the development of computer technology, umerical analysis methods have been widely used in the field of design and manufacturing.

There are now several numerical simulation software (FASTSTAMP, FastForm, Autoform One-step, etc..) using simplified methods based on the geometry of final stamped workpiece to determine the unkowns, such as the contours of intial blank, strains and stresses in the final workpiece only in one step. The simplified assumptions on the law of material behavior (theory of total plastic strain) and the action of the tools are used. The method known as "The ideal forming theory" proposed in the early 90's by Chung and Richmond (1992a,b,c) calculated the displacement field by means of minimize the plastic work. The method called "Inverse Approach" [Guo et al (1990, 2000)] developed by Batoz and Guo calculated the contours of initial blank based on the desired final shape with the hypothesis of a radial loading. The method called "multi-step inverse analysis" proposed and developed by Lee and Huh (1998) is based on the same concept as the simplified Inverse Approach except that the finite element analysis of deformation paths is done in several steps instead of one. Tang et al (2007a,b) improved the simplified method with a double section curve expanding method to get initial solutions of intermediate configurations which can take into account of plastic deformation characteristics.

Because of the constitutive equation based on deformation theory of plasticity, the TIAM can not well reflect the loading history, such as bending/unbending condition. To overcome this difficulty, UIAM has been developed to inherit the advantages of TIAM and conventional incremental approach: speed and taking into account the deformation history.

In order to take account of the deformation history, UIAM uses the flow theory of plasticity instead of the deformation theory of plasticity. The use of Simo's Radial Return Mapping Algorithm [Simo and Hughes (1998)] requires a number of iterations and sometimes leads to a considerable CPU time. In the paper, a special matrix and a scalar equation using the notion of the equivalent stress are used and thus leads to a direct resolution of the plastic multiplier. This algorithm greatly reduces the computation time.

2 Radial return algorithm

In the UIAM, the theory of plastic flow must be used to reflect the loading history. The assumptions of plane stress and isotropic hardening are adopted.

The rate of deformation is decomposed into an elastic part and a plastic part:

$$\{\dot{\varepsilon}\} = \{\dot{\varepsilon}^e\} + \{\dot{\varepsilon}^p\} \tag{1}$$

The associated flow rule of plasticity gives the normality of plastic strain rate:

$$\{\dot{\varepsilon}^p\} = \dot{\lambda}\frac{\partial f}{\partial\{\sigma\}} = \dot{\lambda}\,[P]\{\sigma\}/\sigma_{eq} \tag{2}$$

with σ_{eq} the equivalent stress defined by:

$$\sigma_{eq} = (\langle\sigma\rangle\,[P]\{\sigma\})^{1/2} \tag{3}$$

And the rate of equivalent plastic strain defined by:

$$\dot{\bar{\varepsilon}}^p = \left(\langle\dot{\varepsilon}^p\rangle\,[P]^{-1}\{\dot{\varepsilon}^p\}\right)^{1/2} = \left(\frac{\dot{\lambda}\,\langle\sigma\rangle\,\langle P\rangle}{\sigma_{eq}}[P]^{-1}\frac{\dot{\lambda}\,[P]\{\sigma\}}{\sigma_{eq}}\right)^{1/2} = \dot{\lambda} \tag{4}$$

$[P]$ is constant matrix defined by the coefficients of anisotropy.
The elastic constitutive equation is given by Hook's law:

$$\{\dot{\sigma}\} = [H_e]\{\dot{\varepsilon}^e\} \tag{5}$$

Combining Eq.s (2), (4) and (5) gives the strain rate with the rate of plastic multiplier λ:

$$\{\dot{\sigma}\} = [H_e](\{\dot{\varepsilon}\} - \{\dot{\varepsilon}^p\}) = [H_e]\left(\{\dot{\varepsilon}\} - \frac{\dot{\lambda}\,[P]\{\sigma\}}{\sigma_{eq}}\right) \tag{6}$$

The stress vector at step $n+1$ can be expressed as an incremental form:

$$\{\sigma_{n+1}\} = \{\sigma_n\} + [H_e]\{\Delta\varepsilon\} - \frac{\Delta\lambda\,[H_e]\,[P]\{\sigma_{n+1}\}}{\sigma_{eq,n+1}} \tag{7}$$

Eq. (7) is written as:

$$\{\sigma_{n+1}\} = \left([I] + \frac{\Delta\lambda}{\sigma_{eq,n+1}}[H_e]\,[P]\right)^{-1}\left\{\sigma_{n+1}^{\text{Tria}}\right\} \tag{8}$$

with

$$\left\{\sigma_{n+1}^{\text{Tria}}\right\} = \{\sigma_n\} + [H_e]\{\Delta\varepsilon\} \tag{9}$$

The stress vector at step $n+1$ $\{\sigma_{n+1}\}$ is determined using an elastic prediction and plastic correction method. We assume that the strain increment is purely elastic to estimate the stress.

Let $\gamma\{\Delta\varepsilon\}$, $(1-\gamma)\{\Delta\varepsilon\}$ are partitions of elastic and plastic deformations, respectively. In order to predict elastic strain increment, it is considered fully elastic. $\left\{\sigma_{n+1}^{\gamma}(\gamma)\right\}$ is then the function of γ and $\{\Delta\varepsilon\}$. Then the Hill's yielding criterion is written as:

$$f\left(\left\{\sigma_{n+1}^{\gamma}(\gamma)\right\}\right)=0 \tag{10}$$

with

$$\left\{\sigma_{n+1}^{\gamma}(\gamma)\right\}=\{\sigma_n\}+\gamma[H_e]\{\Delta\varepsilon\} \tag{11}$$

This nonlinear equation will be solved by the Newton-Raphson:

$$\gamma_{k+1}=\gamma_k-\left(\frac{df}{d\gamma}\right)_{\gamma=\gamma_k}^{-1} f(\gamma_k) \tag{12}$$

After γ is obtained, the equivalent plastic deformation of step $n+1$ can be calculated by:

$$\bar{\varepsilon}_{n+1}^{p}=\bar{\varepsilon}_{n}^{p}+(1-\gamma)\left(\langle\Delta\varepsilon_{n+1}\rangle[P]^{-1}\{\Delta\varepsilon_{n+1}\}\right)^{1/2}=\bar{\varepsilon}_{n}^{p}+(1-\gamma)\Delta\bar{\varepsilon}_{n+1} \tag{13}$$

Similarly, the nonlinear equation of $\Delta\lambda$ can be written as:

$$f(\Delta\lambda)=\sigma_{eq,n+1}-\bar{\sigma}_{n+1}=\left(\langle\sigma_{n+1}\rangle[P]\{\sigma_{n+1}\}\right)^{1/2}-\bar{\sigma}_{n+1} \tag{14}$$

A Newton-Raphson iterative algorithm is required to sovle this nonlinear equation:

$$\Delta\lambda_{k+1}=\Delta\lambda_k-\left(\frac{df}{d\lambda}\right)_{\lambda=\lambda_k}^{-1} f(\Delta\lambda_k) \tag{15}$$

Typically, the iterative method converges with the initial value $\Delta\lambda_0=0$. However it must use small increments and many iterations to ensure digital stability. In the following section, a robust and efficient algorithm is proposed to obtain $\Delta\lambda$ without iterations.

3 New algorithms for the plasticity integration

The idea of this algorithm is to introduce a transformation equation into a scalar equation whose only unknown is $\Delta\lambda$. Then a direct solver is used to find $\Delta\lambda$.

A transformation matrix is introduced:

$$[Q] = \frac{\sqrt{2}}{2}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix} \tag{16}$$

Then Eq.(8) becomes:

$$[Q]^{-1}\{\sigma_{n+1}\} = \left([I] + \frac{\Delta\lambda}{\sigma_{eq,n+1}}[Q]^{-1}[H_e][P][Q]\right)^{-1}[Q]^{-1}(\{\sigma_n\} + [H_e]\{\Delta\varepsilon\}) \tag{17}$$

Then Eq. (17) gives:

$$[I] + \frac{\Delta\lambda}{\sigma_{eq,n+1}}[Q]^{-1}[H_e][P][Q] = \left(1 + \frac{b\Delta\lambda}{\sigma_{eq,n+1}}\right)[I] + \frac{\Delta\lambda}{\sigma_{eq,n+1}}(a-b)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{18}$$

with $a = \frac{E(1+\nu)}{(1-\nu^2)(1+r)}$, $b = \frac{E(1-\nu)(1+2r)}{(1-\nu^2)(1+r)}$.
Some denotions:

$$\{\sigma^*_{n+1}\} = [Q]^{-1}\{\sigma_{n+1}\} = \frac{\sqrt{2}}{2}\langle\ \sigma_x + \sigma_y \quad \sigma_y - \sigma_x \quad \sqrt{2}\tau_{xy}\ \rangle \tag{19}$$

$$[P^*] = [Q]^{-1}[P][Q] \tag{20}$$

with σ_x, σ_y and τ_{xy} the stress of step $n+1$
Then Eq. (17) is expressed as:

$$\left([I] + \frac{\Delta\lambda}{\sigma_{eq,n+1}}[Q]^{-1}[H_e][P][Q]\right)\{\sigma^*_{n+1}\} = [Q]^{-1}(\{\sigma_n\} + [H_e]\{\Delta\varepsilon\}) \tag{21}$$

As the stress of step n and the strain increment of step $n+1$ are known, the value of the right of Eq. (21) is determined. The right of Eq.(21) is defined as matrix $[A]$, then

$$A = \langle A\rangle[P^*]\{A\} = (\langle\sigma_n\rangle + \langle\Delta\varepsilon\rangle[H_e])[P](\{\sigma_n\} + [H_e]\{\Delta\varepsilon\}) \tag{22}$$

The above equation can prove the value A is the equivalent stress of elastic trial stress $\{\sigma^e_{n+1}\}$. According to Eq. (8), in order to obtain $\{\sigma_{n+1}\}$, $\Delta\lambda$ should be firstly defined. An efficient and robust method is proposed to obtain $\Delta\lambda$ without iterations.

Eq. (20) is transformed into:

$$\left\langle \sigma_{n+1}^{*} \right\rangle \left([I] + \frac{\Delta\lambda}{\sigma_{eq,n+1}} [Q]^{-1} [P] [H_e] [Q] \right) [P^{*}] \left([I] + \frac{\Delta\lambda}{\sigma_{eq,n+1}} [Q]^{-1} [H_e] [P] [Q] \right) \left\{ \sigma_{n+1}^{*} \right\}$$
$$= \left(1 + \frac{b\Delta\lambda}{\sigma_{eq,n+1}} \right)^{2} \sigma_{eq,n+1}^{2} + \frac{\Delta\lambda (a-b)(1+b\Delta\lambda)(\sigma_x+\sigma_y)^2}{1+r} + \frac{\Delta\lambda^2 (a-b)^2 (\sigma_x+\sigma_y)^2}{2(1+r)} \tag{23}$$

Then the unknown stress of step $n+1$ is expressed by its equivalent $\sigma_{eq,n+1}$. Based on Hill's yield criterion and uniaxial stress-strain curves $\sigma_{eq,n+1} = \bar{\sigma}_{n+1} = \bar{\sigma}\left(\bar{\varepsilon}_{n+1}^{p}\right)$, Eq.(21) is changed from vector expression to a quadratic equation in one variable $\Delta\lambda$:

$$\Delta\lambda^2 \left(b^2 + \frac{1}{2} \frac{(a^2-b^2)(\sigma_x+\sigma_y)^2}{\sigma_{eq,n+1}^{2}(1+r)} \right) + \Delta\lambda \left(2b + \frac{(a-b)(\sigma_x+\sigma_y)^2}{\sigma_{eq,n+1}^{2}(1+r)} \right) + 1 = \frac{A}{\sigma_{eq,n+1}^{2}} \tag{24}$$

The Eq. (24) cannot be solved since stress σ_x¡¢σ_y of step $n+1$ are unknown. According to Eq. (19), the relation $[H_e] = E[P]^{-1}$ can be obtained. Thus,

$$a = b = E \tag{25}$$

Eq. (24) is combined with Eq. (25) to give:

$$\left(1 + \frac{\Delta\lambda E}{\sigma_{eq,n+1}} \right)^{2} \sigma_{eq,n+1}^{2} = A \tag{26}$$

Then $\Delta\lambda$ can be obtained directly:

$$\Delta\lambda = \left(\sqrt{\langle A \rangle [P] \{A\}} - \bar{\sigma}_{\mathrm{n+1}} \right) \Big/ \mathrm{E} \tag{27}$$

4 Case study

The drawing of a square box is simulated by our in-house code INVERSTAMP based on flow theory of plasticity and LS-DYNA for validation of the novel constitutive equation (Fig.1). The geometry and material data can be found in Tab.1. The uniaxial stress-strain relationship is defined:

$\bar{\sigma} = 545(0.004 + \bar{\varepsilon})^{0.263}$MPa

The geometry of square cup discretized by 6163 shell elements and 3156 nodes is shown in Fig.1. The Pentium(R) 4 CPU 2.26GHz, 512MB memory PC takes 62

Table 1: Material parameters

Elastic Modulus E / GPa	Pssion ratioμ	Average anisotropic coef-ficient r	Friction coef-ficient μ	Sheet thickness δ_0/mm	Binder force F_b/kN
210	0.3	1.87	0.15	0.80	40.0

iterations and 5 minutes to calculate the blank shape by TIAM and 60 iterations and 6 minutes by the UIAM which is shown in Fig.2. The computing time is compared to the incremental finite element analysis. The elasto-plastic incremental analysis takes 30 minutes to calculate the flange contour. The above comparisons show that the UIAM is more appropriate to calculation of accurate blank shapes than TIAM and incremental analysis.

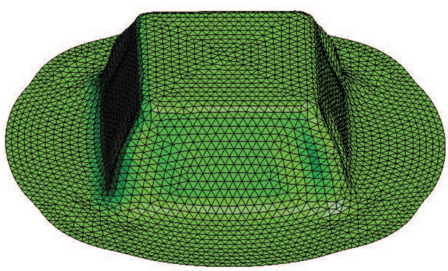

Figure 1: CAD modeling of drawn part

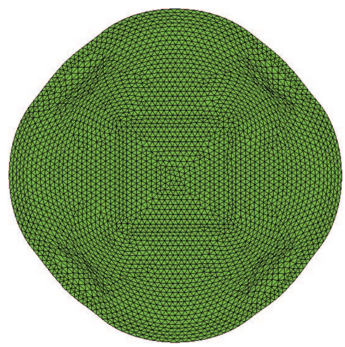

Figure 2: Blank configuration with inverse analysis method

The calculated initial blank shape obtained from the TIAM based on deformation theory of plasticity is compared with that obtained from the updated one based on flow theory of plasticity in Fig.3. The blank size of TIAM is smaller than that of UIAM in the direction of 45°and almost the same in the direction of 0°and 90°of square cup drawing. The elasto-plastic incremental finite element simulation using LS-DYNA with the blank shapes obtained from the two inverse analysis, the flange contours are compared in Fig.4. The flange contour using the blank shape from UIAM is in better agreement with the desired contour than that of TIAM. It also shows that the UIAM calculates the optimum blank shape better than the TIAM.

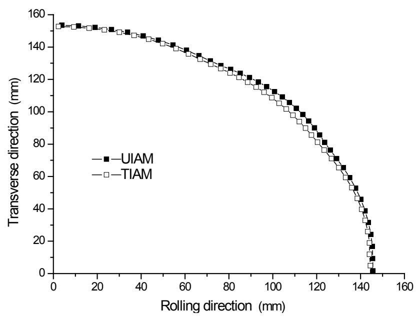

Figure 3: Blank contours with TIAM and UIAM

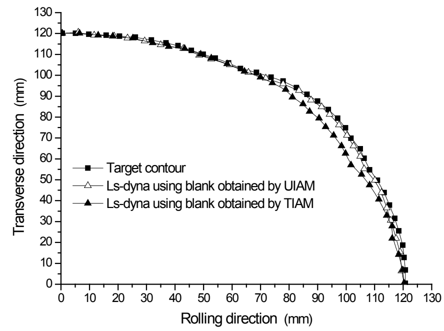

Figure 4: Workpiece contours with different blanks

Strain distribution is one of the most important factors in the final shape of a part. Homogenous distribution displays the high quality of the part. Fig.5 and Fig.6 show the thickness strain distributions calculated by UIAM, TIAM and LS-DYNA along diagonal and transverse direction respectively. Though both figures confirm similar pattern in the strain distributions with different analysis method, the results with UIAM are significantly close to those of LS-DYNA.

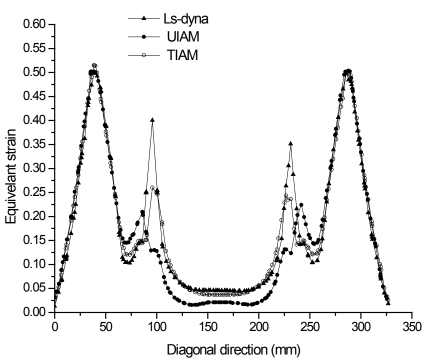

Figure 5: Equivalent strain distribution along diagonal direction

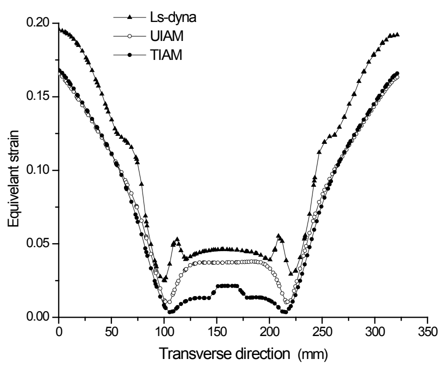

Figure 6: Equivalent strain distribution along transverse direction

5 Conclusion

A novel constitutive equation based inverse analysis method has been developed recently in order to consider the loading history and to improve the stress estimation in keeping the simplicity and rapidity of the TIAM. A new direct algorithm based on a scalar method is proposed for the plastic integration. In square cup example, the results of the present method have been compared with those of the TIAM and conventional forward incremental analysis. The predicted blank obtained by UIAM is somehow larger than that obtained by TIAM. Both inverse methods have similar strain patterns but the updated one shows more severe thickness variations as that of forward incremental FEM. This could be due to the bending effect that is considered in UIAM.

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