

## Determination of Yield Lines for Circular Tubular T-joints under Axial Compression

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**Abstract:** Yield line theory is commonly used for predicting the load carrying capacity of a tubular joint. Theoretical method and numerical investigation have both been carried out to determine the profile of yield lines on the chord surface for a circular tubular T-joint under axial compression. Two yield lines in a form of closed space curve are assumed to initiate when the axial compressive load reaches a critical value. The first yield line is located along the weld toe, and the second yield line is determined by minimum energy principle. Through finite element simulation and verification, the theoretical method is evaluated to be effective and reliable.

**Keywords:** Yield line, circular tubular T-joints, axial compression.

### 1 Introduction

Circular tubular structures are widely used in offshore platform, stadium, bridge and airport etc. In these structures, one or several smaller tubes namely brace are welded directly onto the outer surface of a bigger tube called chord. The brace/chord intersection forms a connection which is named as a tubular joint. The joint is a very critical position because high stress concentration is located at the weld toe due to discontinuous stiffness in this region (Chiew, Lie, Lee and Huang, 2004; Shao, 2007). In addition, the radial stiffness of the chord is generally much weaker than the axial stiffness of the brace when the tube wall thickness is very small. This fact causes failure commonly to occur on the chord surface near the brace/chord intersection for a tubular joint under static or fatigue loading (Shao, 2005; Shao, Lie and Chiew, 2010; Lie, Chiew, Lee, and Yang, 2006).

On the other hand, nonlinear structural mechanics and analysis in association with

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limit states approaches play a key role in the development of core technologies used in the design and safety assessment of various structures. It has been well recognized that limit states provide a much better basis for structural design and strength assessment than allowable working stresses since it is impossible to determine the true margin of structural safety if the limit states remain unknown (Paik and Seo, 2005).

To estimate the load carrying capacity of a tubular joint under axial load, yield line theory is used by many researchers in the literature (Cao, Packer and Yang, 1998; Mashiri and Zhao, 2004; Ye, Zhao, Do Van Binh and Riadh, 2007). In this theory, the ultimate state of a tubular joint is defined as the formation of a series of yield lines. In this case, plastic hinges initiate along the yield line to make a local surface of the chord to rotate freely, and thus to cause the failure of this joint. Although the above researchers have tried to apply this theory in calculating the static strength of tubular joints, their studying objects are focused on square tubular joints since the chord of the squared model has plain surfaces in which yield lines can be clearly observed. For circular tubular joint, quite few reports have been seen in the past although the research for this type of tubular joint is also very significant. Therefore, how to determine the profile and the location of the yield lines is significant in applying this theory to circular one.

Yield line theory is especially reliable for predicting the bending strength of plate or shell-like structure because yield lines easily initiate in these structures. For tubular joint structure, it just has the property of thin-walled thickness. Hence, using yield line theory to assess the safety and reliability of circular tubular joint is practical and original. Although the most accurate method to determine the yield line is by observing the failure mode of many specimens in experimental test, theoretical analysis and numerical simulation are still widely used because it is very expensive and time-consuming for conducting experiments. Finite element method is then acceptable for assessing the reliability or accuracy of theoretical model. This study aims to investigate the profile of yield lines in a circular tubular T-joint under axial compression to provide useful information to assessment of the longevity of tubular structures.

## **2 Yield lines in a circular tubular T-joint under axial compression**

Fig. 1 shows the typical failure mode of a tubular T-joint under axial compression at the brace end. Failure occurs on the chord surface near the brace/chord intersection as the radial stiffness of the chord is smaller than the axial stiffness of the brace due to the tubes with thin-wall.

For brevity, some geometric definitions are illustrated in Fig. 2. Upper case letters

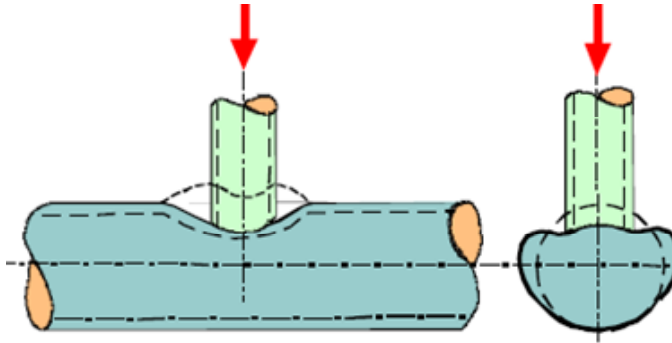


Figure 1: Common failure mode of a tubular T-joint under axial compression

are used to describe the geometric dimensions of the chord, and lower case letters are used for the brace. In addition, three normalized parameters, namely  $\beta$ ,  $\gamma$  and  $\tau$ , are also used in distinguishing the geometry of a circular tubular T-joint. The definitions of the three parameters are listed as:  $\beta=d/D$ ;  $\gamma=D/T$ ;  $\tau=t/T$

In the Cartesian coordinate system as shown in Fig. 2, the brace/chord intersection curve can be expressed by using the following equation (Cao, Yang, Packer and

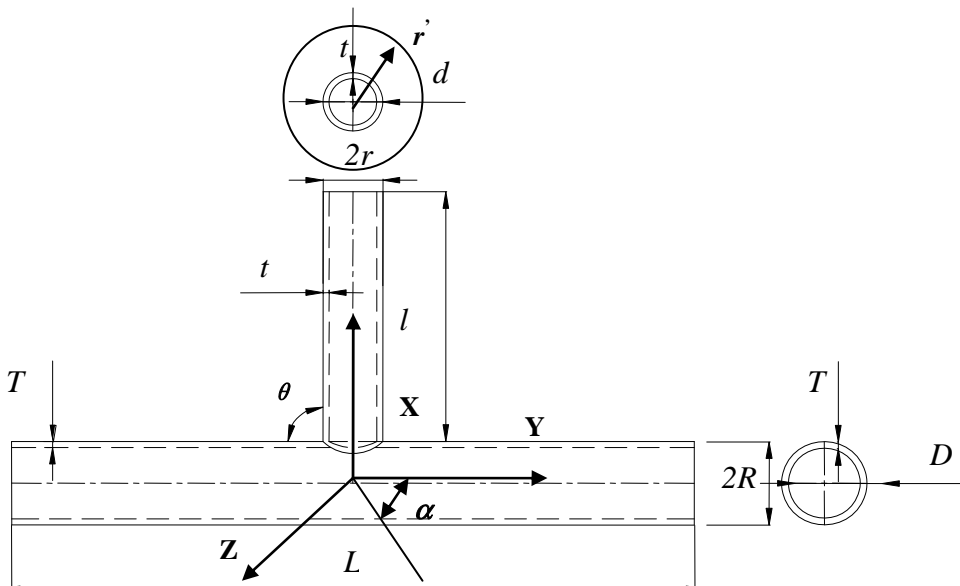


Figure 2: Configuration of a circular tubular T-joint

Burdekin, 1998)

$$\begin{cases} X = R \cos \left[ \arcsin \left( \frac{r}{R} \sin \alpha \right) \right] \\ Y = r \sin \alpha \\ Z = r \cos \alpha \end{cases} \quad (1)$$

According to the deformation of the T-joint in failure state as shown in Figure 1, two yield lines in a form of closed space curve are assumed. The first yield line is located around the brace/chord intersection (the weld size here is ignored), and it is expression in Equation (1). The second yield line is determined by the intersection between an assuming a virtual brace with a bigger radius  $r'$  and the chord, as shown in Figure 2.

Similarly, the second yield line can be expressed by the following equation

$$\begin{cases} X' = R \cos \left[ \arcsin \left( \frac{r'}{R} \sin \alpha \right) \right] \\ Y' = r' \sin \alpha \\ Z' = r' \cos \alpha \end{cases} \quad (2)$$

In determination of the coordinates of the second yield line, virtual work principle is used. In critical state, it is assumed that a virtual displacement in axial direction at the brace end,  $\Delta$ , is applied. Then the external axial load,  $P$ , will produce a virtual work  $W = P\Delta$ . According to the virtual work principle,  $W$  is equal to the virtual potential energy in the joint. As the formation of the yield lines, it is assumed that all the virtual potential energy is generated by the plastic moment along the yield lines and the rotation angle of the local chord surface between the two yield lines. According to this assumption, the following equation can be used for describing the virtual work principle

$$\begin{aligned} & \int_0^{2\pi} \sqrt{\frac{r^4 \sin^2 \alpha \cdot \cos^2 \alpha}{R^2 - r^2 \sin^2 \alpha} + r^2} d\alpha \times \frac{T^2 f_y}{4} \times \frac{\Delta}{\sqrt{(r' - r)^2 + (X' - X)^2}} \\ & + \int_0^{2\pi} \sqrt{\frac{r'^4 \sin^2 \alpha \cdot \cos^2 \alpha}{R^2 - r'^2 \sin^2 \alpha} + r'^2} d\alpha \times \frac{T^2 f_y}{4} \times \frac{\Delta}{\sqrt{(r' - r)^2 + (X' - X)^2}} \\ & = P\Delta \end{aligned} \quad (3)$$

In Equation (3),  $f_y$  is the yield stress of the steel material.

In Equation (3), the left part is the virtual potential energy, and the right part denotes the virtual work done by the external load. As the integral in the virtual potential

part is very difficult to obtained from theoretical analysis, numerical summation can be used to calculate the virtual potential energy as following equation

$$\begin{aligned} & \sum_{n=1}^N \sqrt{\frac{r^4 \sin^2(n\alpha) \cdot \cos^2(n\alpha)}{R^2 - r^2 \sin^2(n\alpha)} + r^2} \cdot \alpha \cdot \frac{T^2 f_y}{4} \cdot \frac{\Delta}{\sqrt{(r' - r)^2 + (X' - X)^2}} \\ & + \sum_{n=1}^N \sqrt{\frac{r'^4 \sin^2(n\alpha) \cdot \cos^2(n\alpha)}{R^2 - r'^2 \sin^2(n\alpha)} + r'^2} \cdot \alpha \cdot \frac{T^2 f_y}{4} \cdot \frac{\Delta}{\sqrt{(r' - r)^2 + (X' - X)^2}} \\ & = P\Delta \end{aligned} \tag{4}$$

In Equation (4),  $N$  is the division number of segment for the yield lines.

In Equation (4), the radius  $r'$  is determined from the minimum energy principle. That means the derivative of the left part in Equation (4) should be equal to zero. Then the following equation is obtained:

$$\begin{aligned} & - \sum_{n=1}^N \sqrt{\frac{r^4 \sin^2(n\alpha) \cdot \cos^2(n\alpha)}{R^2 - r^2 \sin^2(n\alpha)} + r^2} \cdot \alpha \cdot \frac{T^2 f_y}{4} \cdot [(r' - r)^2 + (X' - x)^2]^{\frac{3}{2}} \\ & \times \left[ (r' - r) - (X' - X) \cdot \frac{r' \sin^2(n\alpha)}{\sqrt{R^2 - r'^2 \sin^2(n\alpha)}} \right] \\ & + \left\{ \begin{aligned} & \left[ \frac{r'^4 \sin^2(n\alpha) \cdot \cos^2(n\alpha)}{R^2 - r'^2 \sin^2(n\alpha)} + r'^2 \right]^{-\frac{1}{2}} \times \\ & \left\{ \frac{2r'^3 \sin^2(n\alpha) \cdot \cos^2(n\alpha) \cdot [R^2 - r'^2 \sin^2(n\alpha)] + r'^4 \sin^4(n\alpha) \cdot \cos^2(n\alpha)}{[R^2 - r'^2 \sin^2(n\alpha)]^2} \right. \\ & \left. + r'^2 \right\} - \sqrt{\frac{r'^4 \sin^2(n\alpha) \cdot \cos^2(n\alpha)}{R^2 - r'^2 \sin^2(n\alpha)} + r'^2} \cdot \alpha \cdot \frac{T^2 f_y}{4} \cdot [(r' - r)^2 + (X' - X)^2]^{\frac{3}{2}} \\ & \times \left[ (r' - r) - (X' - X) \cdot \frac{r' \sin^2(n\alpha)}{\sqrt{R^2 - r'^2 \sin^2(n\alpha)}} \right] \end{aligned} \right\} \tag{5} \\ & = 0 \end{aligned}$$

It is not easy to obtain the exact solution of  $r'$  from Equation (5), and thus iterating process can be used for finding a very close solution.

### 3 Finite element investigation

#### 3.1 FE model

In the finite element (FE) modelling, 3-D hexahedral elements are used to produce the mesh of the entire tubular T-joint. The commercial software ABAQUS is used



positions on the predicted yield line (calculated from Equation (5)) are marked by A and B respectively in these figures.

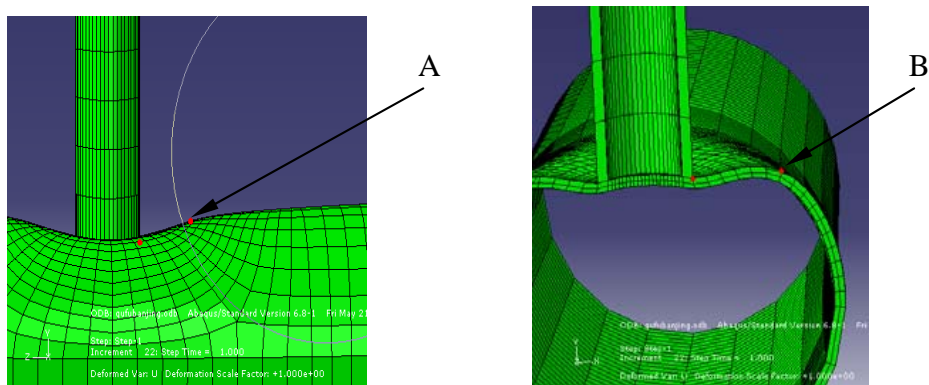


Figure 4: Failure mode of Model 1

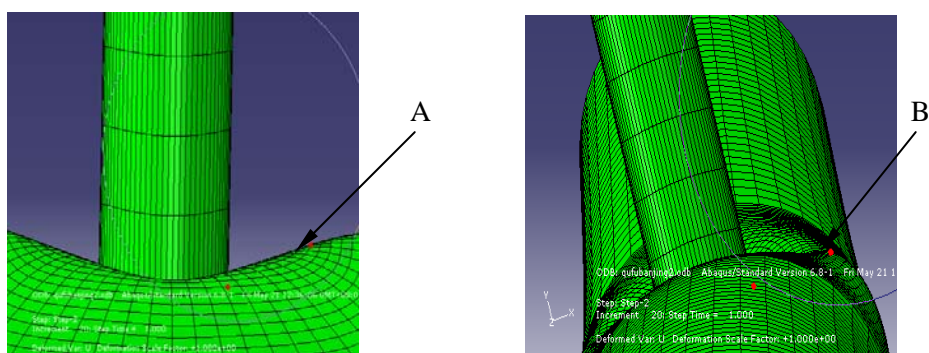


Figure 5: Failure mode of Model 2

From Figs. 4 and 5, it is found that the predicted points on the yield line seem to be quite reasonable compared with FE results. This means Equation (5) can be reliably used for estimating the profile of the yield lines of the two tubular T-joints under axial compression. However, more T-joint models should be analyzed to evaluate the accuracy of the theoretical model before the application of the presented theory. This will be done in the future work.

The reliability of using yield line theory to analyze a tubular T-joint can also be evaluated from the stress distribution obtained from FE analysis. Figs. 6 and 7 show the stress distribution of the two models when yielding occurs. It is easy to find that high stresses only distribute in a local region around the brace/chord intersection. This region seems to be reasonable to be estimated by using the presented yield line method which is introduced in previous section. In the region far away from the intersection, the stresses are much smaller, and thus failure can not occur here.

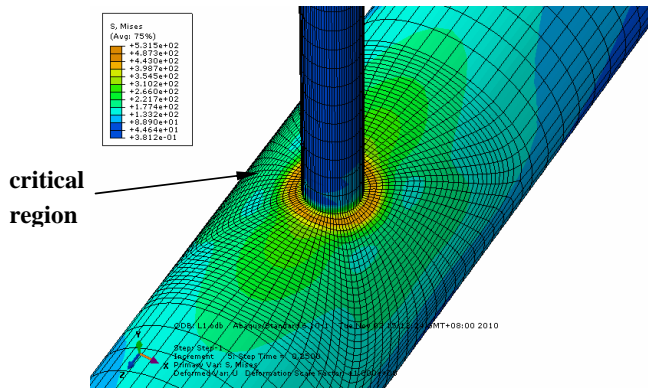


Figure 6: Critical region of model 1

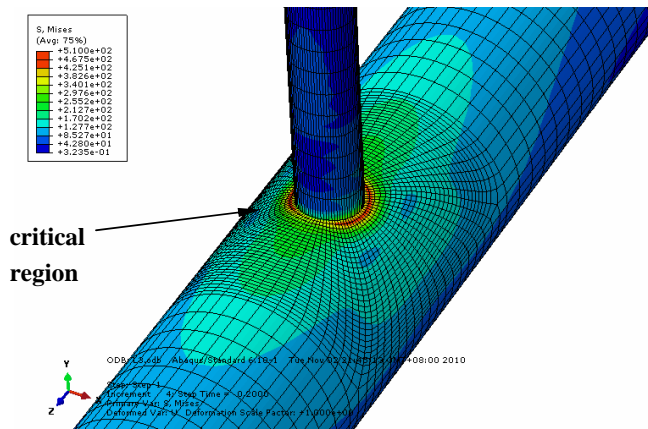


Figure 7: Critical region of model 2



## 4 Conclusions

This paper presents theoretical analysis and finite element verification for the location of yield lines in a tubular T-joint under axial compression. The results show that the proposed method is reliable for predicting the position of the yield lines for two presented T-joint models. Therefore, it can be applied in the design and safety assessment of tubular structures. The reliability and accuracy of this method should be evaluated through more T-joint models with different geometric parameters in the future work.

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