

Improvement of Location Algorithm in Wireless Networks

Duolu Mao^{1,*}, Kaiyong Li¹ and Yaping Mao²

¹School of Physical and Electronic Information Engineering, Qinghai Nationalities University, Xining, 810007, China

²Qinghai Basic Surveying and Mapping Institute, Xining, 810001, China

*Corresponding Author: Duolu Mao. Email: 707882663@qq.com

Received: 30 August 2020; Accepted: 20 September 2020

Abstract: In order to improve the accuracy of wireless network positioning, the triangulation method of wireless network positioning technology is proposed, which is based on the linear least square fitting method. It makes the observed value and the fitting value very close, effectively solves the problem of significant contradiction between the fitting result and the observed value in the principle of least square method, and can realize the accurate measurement of geographic information by wireless network positioning technology.

Keywords: Geographic information measurement; least square method; fitting a straight line

1 Introduction

In geographic information measurement, accurate location technology is very important. At present, the most widely used Positioning technology at home and abroad is GPS (Global Positioning System). However, there are also some problems in the practical application of GPS technology. For example, when working in places with occlusion (indoor, tunnel, etc.) or strong interference sources (under strong magnetic environment, under high voltage line, etc.), GPS positioning technology will have certain blind spots and cannot provide high-precision positioning services. Thus wireless network location technology is applied.

The most common application of wireless network positioning technology is the positioning measurement of coal mines. Since coal mines are usually underground, it is difficult to obtain satellite signals. The movement of personnel and equipment in limited space also makes the acquisition of signals more difficult. The wireless network positioning system can solve this problem.

Wireless sensors receive satellite measurements, measure the user's position, and obtain positioning information. Based on the detected network data, the position of the mobile terminal is measured. Then the actual distance is determined by the Signal Strength detected by the mobile terminal box receiver. The specific location of the mobile terminal can be accurately determined by the measured data.

2 Principles of Wireless Network Positioning

The application of wireless network positioning technology follows the law of technology development, and usually triangulation is used to solve the positioning technology in wireless network according to the principle of least square. Wireless network positioning device is the main part of mobile node and the reference node, and the difference between the two nodes reference node does not participate in the process of positioning is calculated, and the mobile node can be performed within the node control freedom of movement, when the mobile node RSSI signal from the received within the scope of the reference node, according to the measured values accurately find the position. First of all, data-intensive calculation should be carried out, that is, after calculating the parameter data of the relevant computer node position, the information should be transmitted to the central data collection point to



calculate the node position, and finally the calculated data of the node position should be transmitted to the original node. Due to certain defects in the measurement method of this node, the traffic required in the process of calculation will increase continuously along with the number of nodes, which is not conducive to the expansion of its application scope and more suitable for small networks and situations with fewer nodes [1,2].

Engine positioning as an important part of the wireless network positioning technology, its technical principle is relatively simple, as a three-dimensional coordinate system needs to have 3–5 reference node, so as to determine the distance between the coordinate points, just according to the principle of minimum square method by means of triangulation, using computer to aerial triangulation. For example, a relatively simple positioning system is mainly composed of 1 positioning node and 3 reference nodes. This measurement method is also a relatively common positioning method, which finds the coordinate position by the distance between different points. However, the calculated value may not match the actual distance, and the fitting result is significantly inconsistent with the observed value. In this paper, the least square method is used to fit the observed data into a straight line, which effectively solves the contradiction between the two and improves the measurement accuracy.

3 Principle of Least Square Method

In two observations, there is always a quantity with higher accuracy, which can be used to represent the observation with higher accuracy, assuming that it has no error, and all the errors are summed up as the error of y . The function relation of x and y is expressed as follows:

$$y = f(x; c_1, c_2, \dots, c_m) \quad (1)$$

c_1, c_2, \dots, c_m is some parameter that needs to be determined experimentally. By observing each set of data (x_i, y_i) , $i = 1, 2, 3, \dots, N$, we can find a point on the plane (x, y) . If there is no measurement error, the data points will be arranged accurately on the theoretical curve. As long as m group of measured values is selected and substituted into Eq. (1), the following equations can be obtained [3]

$$y_i = f(x; c_1, c_2, \dots, c_m) \quad (2)$$

In the formula above, $i = 1, 2, 3, \dots, m$. The simultaneous solution of the equation gives the value of m parameters in case of $N < m$, Then the parameter cannot be determined.

In case of $N > m$, Eq. (2) becomes the contradictory system, The method of solving equations cannot be used to get the parameter value, The curve fitting analysis method is as follows: Suppose there is no systematic error in the measurement, or it's been corrected. The observed values that distributed around the expected value of $f(x; c_1, c_2, \dots, c_m)$ are satisfies the normal distribution of the function.

Then the probability density of y_i is [4]:

$$p(y_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left\{-\frac{[y_i - \langle f(x_i; c_1, c_2, \dots, c_m) \rangle]^2}{2\sigma_i^2}\right\} \quad (3)$$

σ_i is the standard error of the distribution. So if we use C for (c_1, c_2, \dots, c_m) , Because each measurement is independent of each other, So the likelihood function the observed value of (y_1, y_2, \dots, y_N) is:

$$L = \frac{1}{(\sqrt{2\pi})^N \sigma_1 \sigma_2 \dots \sigma_N} \exp\left\{-\frac{1}{2} \sum_{i=1}^N \frac{[y_i - f(x; C)]^2}{\sigma_i^2}\right\} \quad (4)$$

Take the maximum value of likelihood function L to estimate parameter C :

$$\sum_{i=1}^N \frac{1}{\sigma_i^2} [y_i - f(x_i; C)]^2 = \min \quad (5)$$

When the function minimizes, the distribution of y is not limited to a normal distribution, Formula (5) is called the least square rule. If it has a normal distribution, the maximum likelihood method is consistent with the least square method. Let the weight factor be $\omega_i = 1/\sigma_i^2$, The least square method is used to estimate the parameters, The weighted sum of squares of the deviation of each measured value y is the minimum. we can get [5]:

$$\frac{\partial}{\partial c_k} \sum_{i=1}^N \frac{1}{\sigma_i^2} [y_i - f(x_i; C)]^2 \Big|_{c=\hat{c}} = 0 \quad (k=1,2,\dots,m) \quad (6)$$

So we get the system of equations:

$$\sum_{i=1}^N \frac{1}{\sigma_i^2} [y_i - f(x_i; C)] \frac{\partial f(x; C)}{\partial C_k} \Big|_{c=\hat{c}} = 0 \quad (k=1,2,\dots,m) \quad (7)$$

Solve the equations (7), This is the estimate of m parameters $\hat{c}_1, \hat{c}_2, \dots, \hat{c}_m$, Thus, the fitted curve equation is obtained: $f(x; \hat{c}_1, \hat{c}_2, \dots, \hat{c}_m)$. If y_i meets the normal distribution, the fitting x^2 quantity can be introduced:

$$x^2 = \sum_{i=1}^N \frac{1}{\sigma_i^2} [y_i - f(x_i; C)]^2 \quad (8)$$

Substitute the estimated value $\hat{c} = (\hat{c}_1, \hat{c}_2, \dots, \hat{c}_m)$ of m parameters into the above equation and compare equation (3). The minimum value of x^2 is obtained.

$$x_{\min}^2 = \sum_{i=1}^N \frac{1}{\sigma_i^2} [y_i - f(x_i; \hat{c})]^2 \quad (9)$$

It can be concluded from the above analysis, x_{\min}^2 obeys the $V = N - m$ distribution of dOF x^2 , so x^2 test can be performed on the fitting results.

As you can see from the x^2 distribution, the expected value of the random variable x_{\min}^2 is $N - m$. According to Eq. (9), x_{\min}^2 is close to $N - m$ ($x_{\min}^2 \leq N - m$ is a example). The fitting result is in accordance with the requirements, if $\sqrt{x_{\min}^2} - \sqrt{N - m} > 2$, There is a significant contradiction between the fitting results and the observed values.

4 Linear Least Squares Fitting

In order to solve the above contradiction, the method of linear least squares fitting can be adopted. Let the equation of the line between x and y be:

$$y = a_0 + a_1 x \quad (10)$$

a_0 and a_1 are intercept and slope. Data $(x_i, y_i), i=1,2,\dots,N$ is N sets of equal precision measurements, Let's say x_i is the exact value, The resulting error is related to the observed value y_i . In this way, the least square method can be used to fit a line.

4.1 Estimation of Line Parameters

When the least square method is used to estimate parameters, the weighted sum of the y_i deviation of the observed value is required to be the minimum. For the line fitting of the observation values of the same accuracy, formula (5) can be obtained [6]:

$$\sum_{i=1}^N [y_i - (a_0 + a_1 x_i)]^2 \Big|_{a=\hat{a}} \quad (11)$$

If a (representing a_0, a_1) takes the minimum as the best estimate of the parameter, then the sum of squares of the deviation of y_i is also the minimum. According to Eq. (11), the following equation can be obtained:

$$\begin{aligned} \frac{\partial}{\partial a_0} \sum_{i=1}^N [y_i - (a_0 + a_1 x_i)]^2 \Big|_{a=\hat{a}} &= -2 \sum_{i=1}^N (y_i - \hat{a}_0 - \hat{a}_1 x_i) = 0 \\ \frac{\partial}{\partial a_1} \sum_{i=1}^N [y_i - (a_0 + a_1 x_i)]^2 \Big|_{a=\hat{a}} &= -2 \sum_{i=1}^N (y_i - \hat{a}_0 - \hat{a}_1 x_i) x_i = 0. \end{aligned} \quad (12)$$

The system of equations is obtained after sorting out:

$$\begin{cases} \hat{a}_0 N + \hat{a}_1 \sum x_i = \sum y_i, \\ \hat{a}_0 \sum x_i + \hat{a}_1 \sum x_i^2 = \sum x_i y_i. \end{cases} \quad (13)$$

By solving the system of equations, the best estimators of the parameters \hat{a}_0 and \hat{a}_1 of the line a_0, a_1 , it is:

$$\begin{aligned} \hat{a}_0 &= \frac{(\sum x_i^2)(\sum y_i) - (\sum x_i)(\sum x_i y_i)}{N(\sum x_i^2) - (\sum x_i)^2} \\ \hat{a}_1 &= \frac{N(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{N(\sum x_i^2) - (\sum x_i)^2} \end{aligned} \quad (14)$$

4.2 Deviation of Fitting Results

As the estimated values of line parameters \hat{a}_0 and \hat{a}_1 are calculated based on the observation data points with errors, there is inevitably certain deviation. Since the observed data points do not fall on the fitting line accurately, there is also a certain deviation between the observed value y_i and the fitted line \hat{y}_i .

So let's talk about standard deviation s of y_i . Since all σ_i values of equation (9) are the same for equal precision measurement value y_i , the standard deviation s of y_i is used for estimation. Therefore, in line fitting of equal precision measurement value, it can be expressed as [7]:

$$x_{\min}^2 = \frac{1}{S^2} \sum_{i=1}^N [y_i - (\hat{a}_0 + \hat{a}_1 x_i)]^2 \quad (15)$$

If the measured value satisfies the normal distribution, then x_{\min}^2 obeys the x^2 distribution of $V = N - 2$, and its expected value is:

$$\langle x_{\min}^2 \rangle = \left\langle \frac{1}{S^2} \sum_{i=1}^N [y_i - (\hat{a}_0 + \hat{a}_1 x_i)]^2 \right\rangle = N - 2. \quad (16)$$

The standard deviation of y_i can be obtained through the above equation:

$$S = \sqrt{\frac{1}{N-2} \sum_{i=1}^N [y_i - (\hat{a}_0 + \hat{a}_1 x_i)]^2} \quad (17)$$

S is the standard deviation of the fitting line. Make two lines parallel to the fitted line in the x, y planes: $y' = \hat{a}_0 + \hat{a}_1 x - S, y'' = \hat{a}_0 + \hat{a}_1 x + S$

Of all the distributed observation data points (x_i, y_i) , about 68.3% fall within the range between these two lines (see Fig. 1).

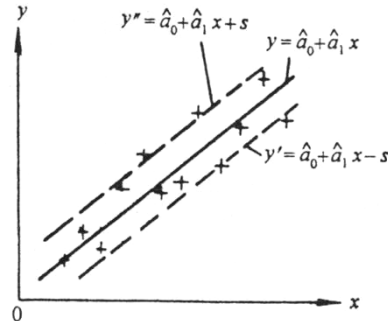


Figure 1: distribution of data points on both sides of the fitted line

In Eq. (11), the two parameter estimates fitted by the line \hat{a}_0 and \hat{a}_1 are functions of y_i . Moreover, it is assumed that the value of x_i is accurate and the error of all measured values is related to y_i , so the standard deviation of \hat{a}_0 and \hat{a}_1 can be obtained by using the uncertainty transfer formula, it is:

$$S_{a_0} = \sqrt{\sum_{i=1}^N \left(\frac{\partial \hat{a}_0}{\partial y_i} S \right)^2}; \quad S_{a_1} = \sqrt{\sum_{i=1}^N \left(\frac{\partial \hat{a}_1}{\partial y_i} S \right)^2} \tag{18}$$

Further information can be obtained:

$$S_{a_0} = S \sqrt{\frac{\sum x_i^2}{N(\sum x_i^2) - (\sum x_i)^2}}; \tag{19}$$

$$S_{a_1} = S \sqrt{\frac{N}{N(\sum x_i^2) - (\sum x_i)^2}}.$$

4.3 Correlation Coefficient and Its Significance Test

When fitting the observed data point (x_i, y_i) as a straight line, the correlation coefficient $\rho(x, y)$ is used to judge if it is not clear how close the linear relationship between x and y is. Instead, the correlation coefficient can be expressed as:

$$r = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\left[\sum_i (x_i - \bar{x})^2 \cdot \sum_i (y_i - \bar{y})^2 \right]^{1/2}} \tag{20}$$

where \bar{x} and \bar{y} are the arithmetic mean values of x and y respectively, and $r \in [-1, 1]$.

- (1) When $r > 0$, the slope of the line is positive, indicating a positive correlation;
- (2) When $r < 0$, the slope of the line is negative, indicating a negative correlation;
- (3) When $|r| = 1$ time points (x_i, y_i) all fall on the fitting line;

(4) If $r = 0$, then there is no correlation between x and y . The closer r is to ± 1 , the closer the linear relationship between them is.

5 Discussion and Conclusion

Aiming at the application of wireless network positioning technology in actual measurement, this

paper analyzes the limitations of positioning technology, wireless sensor network and RSSI positioning method in actual application process, and theoretically analyzes that the calculated value may not match the actual distance. There is a significant contradiction between the fitting results and the observed values. An improvement is proposed to the original algorithm. Through data analysis, about 68.3% of all distributed observation data points can fall within the range between the two expected lines, so as to accurately determine the specific location of the mobile terminal.

To sum up, the development of positioning technology is bound to be more advanced and more accurate, so that wireless positioning technology can be well combined with wireless sensor network, and the accuracy and coverage will be greatly improved.

Funding Statement: This paper has no funding.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References

- [1] T. Wan, G. Zhou and X. Ma, "Combination and application of engineering survey and geographic information," *Urban Construction Theory Research (Electronic Version)*, vol. 19, no. 2, pp. 106, 2018.
- [2] D. Yu and Z. Shao, "Information mapping in the new geographic information age," *Residential and Real Estate*, vol. 16, no. 1, pp. 233, 2018.
- [3] Y. Zhang, "Research on line Fitting based on least square method," *Information Communication*, vol. 11, no. 2, pp. 44-45, 2014.
- [4] J. Zhu and Y. Li, *Numerical Calculation Method*. Higher Education Press, 2012.
- [5] Y. Li, "Application of Doppler radar data in weather forecasting," University of Electronic Science and Technology of China, 2008.
- [6] G. Cai and S. Cui, "Research on measuring accuracy of arc welding robot's straight line trajectory," *Computer Engineering and Application*, vol. 46, no. 1, 2010.
- [7] J. Luo and J. Hung, "Two-dimensional small-size precision measurement system based on machine vision," *Computer Measurement and Control*, vol. 15, no. 1, 2007.