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***Article***

# A Method of Disc Inclination Correction Based on the Inversion Model of Rotation Law

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**Abstract:** Under the traditional dynamic model, the conventional method for solving the rotation angle of a rigid body is to use the fixed-axis rotation law of the rigid body, but the known rotation shaft position must be used as a prerequisite. In practical work, for the rotation of a rigid body under multiple forces, solving the shaft is often a difficult problem. In this paper, we consider the rigid body of the disc is subjected to the force of uneven magnitude from multiple angles, the position of the rotating shaft is obtained by iterative inversion through the rigid body rotation law and the dichotomy method. After the position of the shaft is determined, we establish a differential equation model based on the law of rigid body rotation, the rotation angle of the rigid body thus being solved based on this model. Furthermore, an optimization algorithm such as genetic algorithm is used to search for a correction scheme to return the rigid body to equilibrium at any given deflection angle. The model and method are based on computer to explore the law of rotation, the practical application of them play an important role in studying the concentric drum movement and the balance of handling furniture.

**Keywords:** Dynamics; rigid body rotation law; iterative inversion; dichotomy; genetic algorithm.

## 1 Introduction

There are many studies on the rotation of rigid bodies [1–5], among which the law of rigid body is a very important knowledge point in the course of *University Physics*. Using this law and moment of inertia, the law of the rotation of a fixed rigid body's axis when subjected to many external forces can be solved [6–7]. There are some experiments about the rotation law and the inertia moment. In essence, the combined torque of the external force on the rotating shaft determines the rotation state of the rigid body. But if the rigid body is subjected to several external forces, its axis of rotation is not certain. At this time, the rotation of the rigid body is relatively complicated, but according to the rotation law, the rotation law can still be obtained. This paper takes the rigid body of a disc ignoring the thickness as an example, and uses the law of rotation to derive and calculate the specific position of the rotating shaft of the disc under the action of several known forces.

According to the law of rigid body rotation, the moment of the combined external force is divided into parallel to the direction of the rotation axis and perpendicular to the direction of the rotation axis. The torque component parallel to the direction of the rotation axis does not contribute to the rotation of the rigid body. For a fixed axis perpendicular to the direction of the rotation axis, the sum of all torque component perpendicular to the rotation axis is zero. Therefore, in the case of disc with uneven force at multiple angles, the position of the multiple rotating shafts can be continuously tried through the dichotomy until this condition

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is met, then the position of the rotating shaft can be reversed, and the rotation angle can be solved. In order to apply the model to reality, a variety of optimizing algorithms such as genetic algorithm, chaotic monkey algorithm, etc. can be further used to try various force-applying schemes, so as to reverse the correction scheme that returns the rigid body to its original position under the deflection angle. The practical application of this model method plays an important role in studying the coordination problems in the concentric drum movement and maintaining the balance of furniture when multiple people are carrying furniture.

## 2 Modeling

### *2.1 Moment of Rigid Body to Fixed Axis*

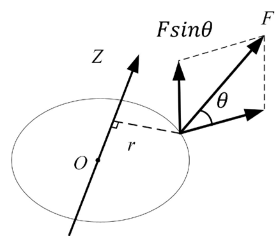


**Figure 1:** Disc moment analysis

As shown in Fig. 1, the force of the rigid body of the disc ignoring the thickness is F, and it is assumed that the disc can rotate around the axis z in its disc surface. According to the definition of torque, the torque generated by F on the z-axis can be obtained as follows.

$$M_z = \vec{r} \times \vec{F} = rF\sin\theta \tag{1}$$

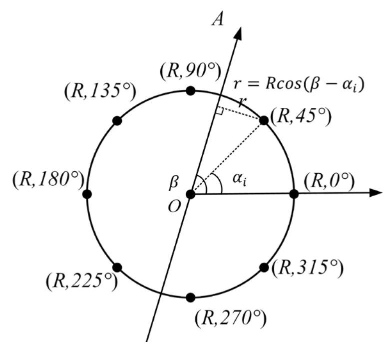


**Figure 2:** Schematic diagram of evenly distributed multiple forces

The function of the moment is to rotate the rigid body. If F is decomposed into a component perpendicular to the plane of rotation and a component located in the plane of rotation, then the F component located in the plane of rotation does not contribute to the rotation of the rigid body. So, it is only necessary for a certain point of the rigid body to calculate the component that is perpendicular to the plane of the disc, then the rotational moment of its force on a rotating shaft in the rigid body plane can be obtained. Thus, if the disc is subjected to several forces at the edge, the sum of the moments generated by all external forces can be calculated. As shown in Fig. 2, a polar coordinate system is established, and the coordinates of the force points are given respectively (In the figure, eight force points are evenly distributed on the edge of the disc as an example).

According to Eq. (1), the sum of all the external forces of the disc rigid body to the axis A with angle β can be calculated as:

$$\vec{M} = \sum_{i=1}^{n} \vec{M}_i = \sum_{i=1}^{n} F_i \sin\theta\, R \sin(\beta - \alpha_i) \tag{2}$$

where $\theta$ is the angle between the force and the plane of the disc whose value range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $\alpha_i$ is the azimuth angle in the polar coordinates of the force point, and the value range is $[0,2\pi]$.

### *2.2 Solution of the Rotation Axis of Rigid Body*

Normally, when a rigid body is subjected to a force, the torque required for a certain rotation axis is required. But the given model in this article is given the points and their forces, and is not sure about the rotation axis of the rigid body. As shown in Fig. 3, on the axis where the disc actually rotates, the moment $M$ is decomposed along an arbitrary angle $\delta$, and the rotation motion of the disc on the rotation axis can be regarded as the synthesis of the rotation motion on the two decomposition axes.

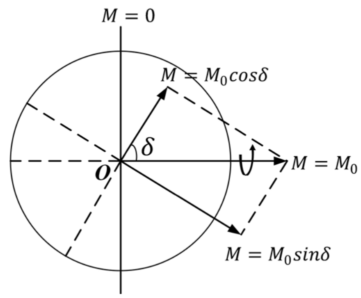


**Figure 3:** Torque decomposition

When $\delta = 90°$, $M = 0$ on this axis. According to this condition, if this axis is known, the position of the real axis of rotation can be determined by rotating it by 90°. Therefore, at the initial moment, we let the axis A locate at $\beta_0 = 0°$, and calculate the resultant torque $M_a$, and then the axis A is rotated by a slight angle $\Delta\beta$, and the resultant torque $M_b$ is calculated. If $M_a M_b > 0$, then we replace $M_a$ with $M_b$, and then the axis A go on rotating until $M_a M_b < 0$. Then there exists $\beta_c$ in the interval $[\beta_a, \beta_b]$ determined by a, b which makes $M_c = 0$. When the interval length meets the accuracy requirements, $\beta_c = \frac{\beta_a+\beta_b}{2}$ can be taken approximately. Then, the axis Z is taken as $\beta = \beta_c + 90°$, after that, the axis Z is the axis of rotation under all external forces.

### *2.3 Calculation of Deflection Angle γ*

It can be seen from both Fig. 1 and Fig. 2 that for any force $F_i$, it can be decomposed in two directions, horizontal and vertical. Since the component force in the horizontal direction only draws the rigid body to move without contributing to the deflection, only the moment $M_i$ of the component force $F_i sin\theta$ in the vertical direction is considered, which is equal to the magnitude of the force times the distance from the force point to the rotation axis. which is:

$$M_i = F_i R sin\theta sin(\beta - \alpha_i) \tag{3}$$

among them, $\beta - \alpha_i$ is the angle formed by the straight line from the force point to the center of the circle and the rotation axis, which is the difference between the position of the rotation axis and the position of the force point in the reference table of the model, so the value range is $[0, 2\pi]$. When its value exceeds $\pi$, $sin(\beta - \alpha_i)$ takes a negative value, therefore the role of this force is to drive the drum to rotate in the other direction, which is a negative contribution to the original direction. Sum up the positive and negative contributions of all forces to the rotating shaft, which is the moment $M$ of the external forces. Using the law of rotation, there are:

$$M = \sum_{i=1}^{n} M_i = \sum_{i=1}^{n} F_i R sin\theta sin(\beta - \alpha_i) = J\frac{d^2\gamma}{dt^2} = \frac{mR^2}{2}\frac{d^2\gamma}{dt^2} \tag{4}$$

among them, $J$ is the moment of inertia, its expression is very simple for regular shape, and can be calculated for a complex shape [8].

The calculation formula of deflection angle γ is:

$$\gamma = \iint \frac{d^2\gamma}{dt^2}(dt)^2 = \iint \frac{\sum_{i=1}^{n} F_i R sin\theta sin(\beta-\alpha_i)}{\frac{1}{2}mR^2}(dt)^2 = \frac{\sum_{i=1}^{n} F_i R sin\theta sin(\beta-\alpha_i)}{mR^2} t^2 \tag{5}$$

Substitute a very short time, such as t = 0.1 s, the deflection angle of the disc can be calculated.

### *2.4 Correction of Inversion Based on Optimization Algorithm*

For any given deflection angle γ, it still needs to rotate γ angle to correct it, which is to find a force application scheme that can deflect γ angle in fact. The existing model can only find the deflection angle according to the force situation. Therefore, it is still necessary to perform an inversion operation to search for a situation that can make the deflection angle γ among all possible force situations. This process can be inherited optimized algorithms such as genetic algorithms and monkey algorithms are implemented. There are more and more optimized algorithms were proposed recently [9–10].

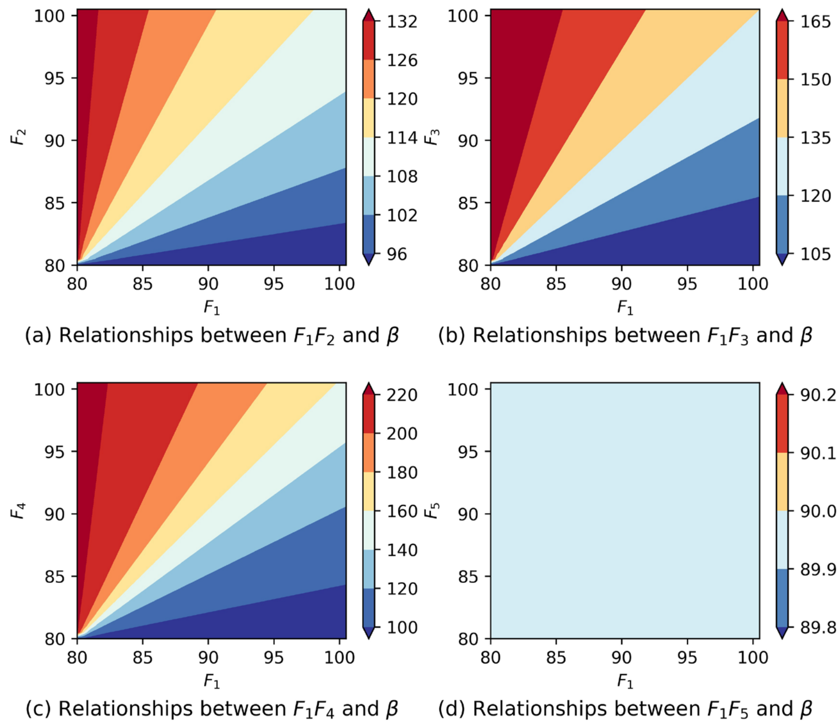


**Figure 4:** Relationship between force and shaft position

## 3 Model Parameters and Results Analysis

In order to sort out the relationship between the parameters of the model and the calculation results and evaluate the model, eight forces are evenly distributed in all directions of the disc and are numbered 1–8 in the polar coordinate system in sequence. The initial magnitude of all forces is 80N, the angle θ between the force and the level is a random fixed value, and the action time is 0.1 s. The control variable method is now used for research.

### *3.1 Effect of Force on the Position of the Shaft*

When controlling the other six forces to keep 80N unchanged, study the influence of the two forces at different positions on the position of the rotating shaft, which are represented by color charts. As shown in Fig. 4, the abscissa and ordinate indicate the magnitude of the two forces under study, and the colors in the figure represent the position of the rigid body's axis of rotation. It can be generalized from the figures a, b, and c that when the magnitudes of the two forces $F_m$ and $F_n$ in the adjacent position or the intermediate position change, if the positions corresponding to $F_m$ and $F_n$ are respectively recorded as $\alpha_m$, $\alpha_n$, the position of the rotating shaft is distributed in within the interval of $[\alpha_m + 90^{\circ}, \alpha_n + 90^{\circ}]$, and the greater the side of the two forces, the closer the axis of rotation is to the vertical position of this force. When the two forces are in the opposite position, as shown in Figure d, no matter how the two forces change, the axis of rotation is located in the middle of the two, which is $90^{\circ}$. These conclusions are consistent with common sense, so it can explain the accuracy of the inversion model of the shaft.

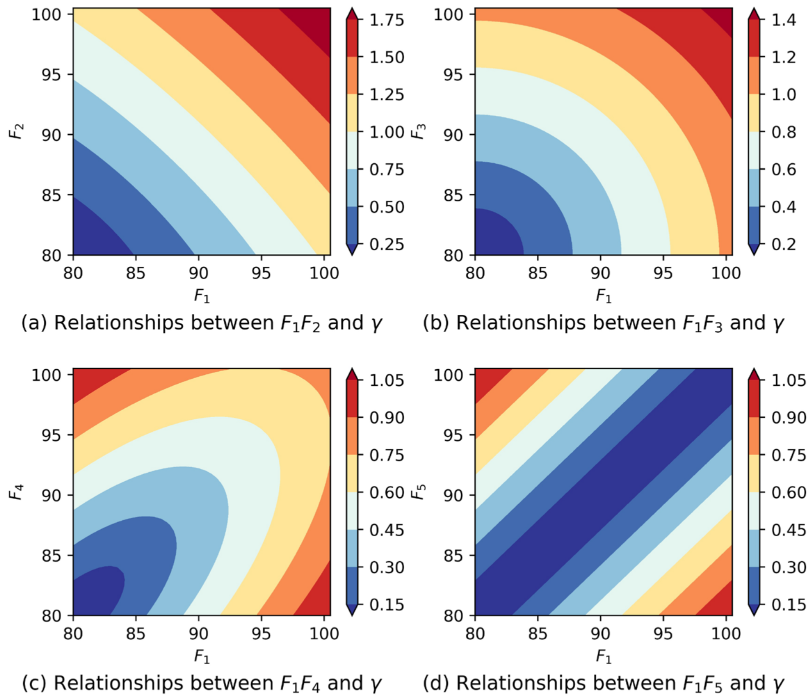


**Figure 5:** The relationship between the angle of forces and the deflection angle

### *3.2 Effect of Force on Deflection Angle*

As shown in Fig. 5, the influence of the two forces at different positions on the deflection angles is studied, and it is represented by a colored map. It can be seen from Figures a, b, and c that increasing the two forces in the adjacent position has the greatest effect on the deflection angle of the rigid body of the disc, followed by the intermediate position. It can be seen from the diagonal of Fig. 5d that because the two forces are in the opposite position, When the two forces increase in the same proportion, they cancel each other out. These conclusions are consistent with the experience of daily life. To be rigorous, our model verifies its rationality from a more scientific perspective.

### *3.3 The Influence of the Included Angle of Force on the Deflection Angle*

When controlling the other seven forces to keep 80N constant, as shown in Fig. 6, the relationship between the angle between the force and the disc plane and the deflection angle of the rigid body is given under the action of different magnitudes of force. When the angle between the force and the level is less than $90^{\circ}$, it is outward along the disc, and it is inward when it is greater than $90^{\circ}$, so it is not difficult to understand that it is a symmetrical process when the angle is from $0^{\circ}$ to $180^{\circ}$. It can be seen from the figure that the relationship between the angle of the force and the deflection angle is a sinusoidal relationship with a period of $2\pi$. The magnitude of the force can affect its amplitude, and the greater the force, the greater the amplitude. In addition, as can be seen from the iso-$\theta$ line, the greater the magnitude of the force at the same angle, the greater the deflection angle. It can be seen from the iso-γ line that the closer the angle is to 90°, the smaller the force is required.

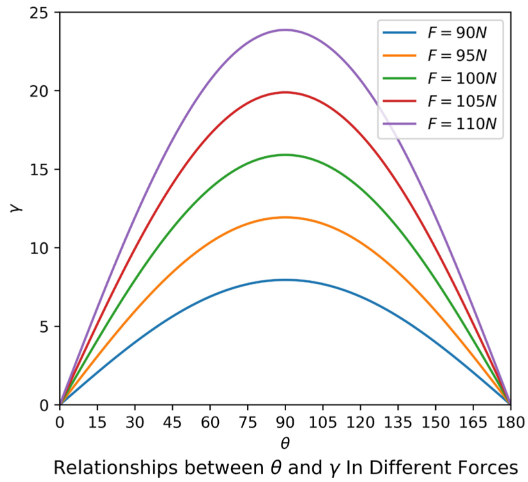


**Figure 6:** Relationship between included angle and deflection angle

## 4 Examples of Concentric Drum Applications

The concentric drum sport is a fun-based sport. Multiple players pull the rope to pull the drum to hit the volleyball. The goal of the team is to make the ball turn as many times as possible. For this reason, the drum should be as balanced as possible. Here is an example of the application of this model to the concentric drum problem. The problem stems from the Problem B of China Undergraduate Mathematical Contest in Modeling in 2019 ,and has been modified for more in-depth consideration.

### *4.1 The Solution of Concentric Drum Deflection Angle*

Examples of problems: Suppose the number of players is 8, the length of the rope is 1.7 m, and the drumhead is horizontally still at the initial moment. The initial position is 11 cm lower than when the rope is horizontal. Tab. 1 shows the different timing and strength of several groups of players. Find the inclination angle of the drumhead at 0.1 s.

Solution to the problem: The angle of the available force according to the title is $\arctan\frac{0.11}{1.7}$ rad, and then the force of the 8 players is substituted into the model, the action time is 0.1 s, and the drum deflection angles can be calculated as 0.64°, 1.21°, 0.39°

**Table 1:** Drum deflection angle

| Forces | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Drum deflection angle |
|---|---|---|---|---|---|---|---|---|---|
| Group1 | 95 | 81 | 87 | 83 | 82 | 85 | 84 | 86 | 0.64 |
| Group2 | 95 | 95 | 87 | 83 | 82 | 85 | 84 | 86 | 1.21 |
| Group3 | 95 | 81 | 87 | 95 | 82 | 85 | 84 | 86 | 0.39 |

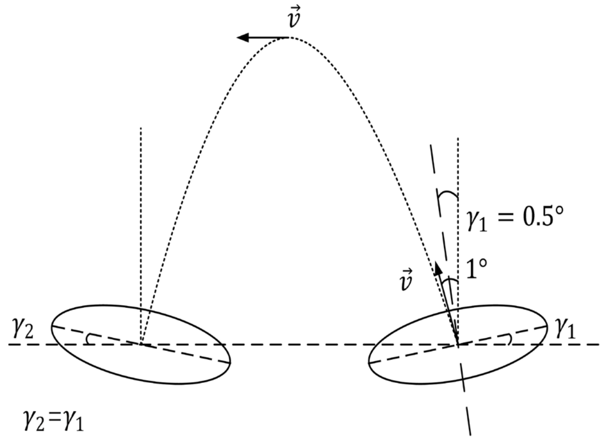


**Figure 7:** Volleyball process

### *4.2 Inversion of Concentric Drum Correction Strategy*

Example of the problem: Suppose the number of people is 10, the length of the rope is 2 m, the rebound height of the ball is 60 cm, a tilt angle of 1 degree relative to the vertical direction is generated, and the projection of the tilt direction on the horizontal plane points between a certain two players. The angle ratio of the players is 1:2. In order to adjust the ball to bounce vertically, find the timing and strength of all players under precise control conditions.

Solution to the problem: As shown in Fig. 7 and Fig. 8.

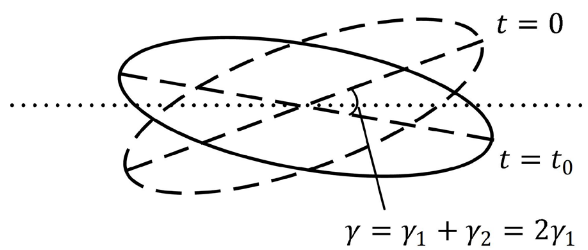


**Figure 8:** Schematic diagram of drum rotation

The ball deviates by 1 degree from the vertical. According to the analysis of the oblique throwing movement, it shows that the drum surface is inclined by 0.5 degree. We need to turn the drum 1 degree in the reverse direction to –0.5 degree to catch the ball to correct it. Considering the actual situation, some players are allowed not to participate in the adjustment, and only the size of the specified number of forces is searched for the optimal solution by genetic algorithm. In this example, strategies for 1, 4, and 8 people to participate in the adjustment are given, as shown in Tab. 2.

**Table 2:** Correction strategy

| Forces | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Scheme 1 | 90 | 80 | 80 | 80 | 80 | 80 | 80 | 80 |
| Scheme 2 | 109 | 65 | 76 | 90 | 80 | 80 | 80 | 80 |
| Scheme 3 | 82 | 88 | 77 | 76 | 78 | 83 | 76 | 79 |

## 5 Summary and Outlook

### *5.1 Summary*

In this paper, based on the fixed-axis rotation law of the rigid body, an inversion model that can solve the position of the rotating shaft of the rigid body rotation under various stress conditions is used, and based on this model, a genetic algorithm is used to invert the model of the force application scheme under various specified deflection angles. Multiple iterations of dichotomy and genetic algorithm are implemented in Python language. This computer solution process simulates the rotation of the disc under various complex forces, which can numerically demonstrate this physical process.

### *5.2 Advantages and Disadvantages of Models and Algorithms*

In this paper, the rotation law is used to switch the way of thinking when solving the position of the rotating shaft, which is a novel idea. The fixed torque of the fixed shaft at different positions is continuous and monotonous, so the dichotomy is well-used because of this feature, which greatly resolves the complexity of the problem.

Since the objects studied in this paper are very complicated under real conditions, the model will have errors. In actual situations, the action time is longer, the position, magnitude and angle of the applied force change from moment to moment, and the irregular shape of the rigid body is the main source of model errors.

### *5.3 Model Improvement and Promotion*

When considering more complex actual situations, the model can be improved by splitting, the duration is decomposed into several shorter time steps, and then the angular accelerations at each moment are vector superimposed to obtain the deflection angle. With this improved approach, the error can be reduced.

This model and method to study the process of the disc rigid body rotating due to the force has a certain reference value for the teaching research of university physics, and also provides more examples for the computer method to solve physics problems. In addition, it can also be extended to other rotating physical models involving dynamic forces. The promotion in practical applications includes concentric drum sports, carrying furniture, table tennis sports, etc.

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