

## A Numerical Investigation on the Characteristics of the Radial Force in a Cycloid Gerotor Pump

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Received: 24 December 2019; Accepted: 04 August 2020

**Abstract:** In order to improve the performances of a cycloid gerotor pump, the variations of the radial force induced by different rotating speeds and outlet pressures are analyzed numerically. Using the numerical simulations as a basis, an improved oil inlet and outlet groove structure is proposed. The results show that the radial force decreases with the decrease of the outlet pressure and of the rotor speed. Compared with the original model, the large-end oil inlet line and pressure line of the new oil groove are claw-shaped. This configuration can effectively weaken the pressure changes inside the gerotor pump and reduce accordingly the radial force on the inner rotor.

**Keywords:** Cycloid gerotor pump; radial force; oil groove structure; pressure distribution

### 1 Introduction

Gerotor pump has many advantages like compact structure, small size and light-weight [1–3]. It is an important part of an automobile engine [4] and construction machinery lubrication system [5]. Its main function is to pump oil and provide a steady stream of motor oil for the lubrication system. Gerotor pumps can be divided into internal gear pump and external gear pump according to their structure. Cycloid gerotor pumps are widely used in automotive lubrication systems [6] because of their compact structure, high volumetric efficiency and low cost. The main disadvantage of the gerotor pump is the large internal leakage and radial force. The radial force on the internal rotor is composed of the radial force generated by the liquid pressure along the circumference of the inner rotor and the radial force generated by the meshing of the rotor. The periodic pressure shock in the rotor pump cavity can aggravate the radial force of the gerotor pump. The excessive radial force will cause the insufficient of rotor axis's rigidity, worse the meshing of the rotor, and affect the accuracy of the meshing. Eventually, it will reduce the life of the rotor pump [7,8].

At present, the research on cycloid gerotor pumps mainly focuses on analyzing the rotor's profile of cycloid gerotor pumps [9] and improving the volumetric efficiency [10,11]. The research on the radial force of gerotor pumps is mainly focused on the derivation of theoretical formulas. For example, Xing [12] deduced the calculation formula of the radial force of the cycloid pump by analyzing the radial force



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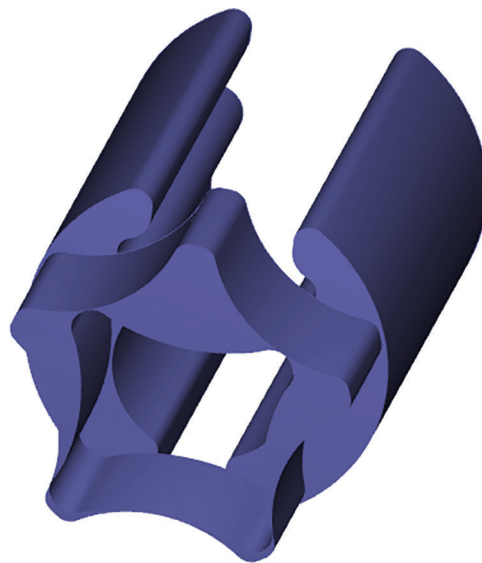
composition of the cycloid pump and taking the involute gear pump as a reference. Li et al. [13,14] deduced the calculation formulas of the maximum radial force on the gear shaft of the internal-meshing gear pump and the maximum bending moment of the gear teeth through computer drawing. Although there has been some research on the radial force of gerotor pumps, there is less research on improving the pulsation of radial force under different working conditions.

This article takes the cycloid gerotor pump as the research object, and uses the methods of numerical simulation to study the influence of rotating speed and outlet pressure on the radial force of the gerotor pump [15,16], analyzes the internal flow state of the gerotor pump, and delves into the causes and position of extreme values of radial force pulsations. Based on the above analysis, a new oil groove structure is proposed to reduce the radial force on the internal rotor in the gerotor pump.

## 2 Numerical Calculation Method of Cycloid Gerotor Pump Flow Field

### 2.1 Computing Models and Meshing

D30 cycloid gerotor pump was selected as research object, and its design parameters are that: outer diameter of external rotor  $D_2 = 69.7$  mm, inner diameter of external rotor  $d_2 = 42.7$  mm, outer diameter of internal rotor  $D_1 = 52.6$  mm, Inner diameter of internal rotor  $d_1 = 32.85$  mm, thickness  $h = 22$  mm, teeth number of internal rotor  $Z_1 = 4$ , teeth number of external rotor  $Z_2 = 5$ , rated speed  $n = 1500$  r/min, outlet pressure  $P_2 = 0.5$  MPa. The three-dimensional fluid domain model of the cycloidal rotor pump is established by using CATIA software, as shown in Fig. 1. Pumplinx is used for meshing and non-constant simulation.



**Figure 1:** Schematic diagram of calculation model of cycloid gerotor pump

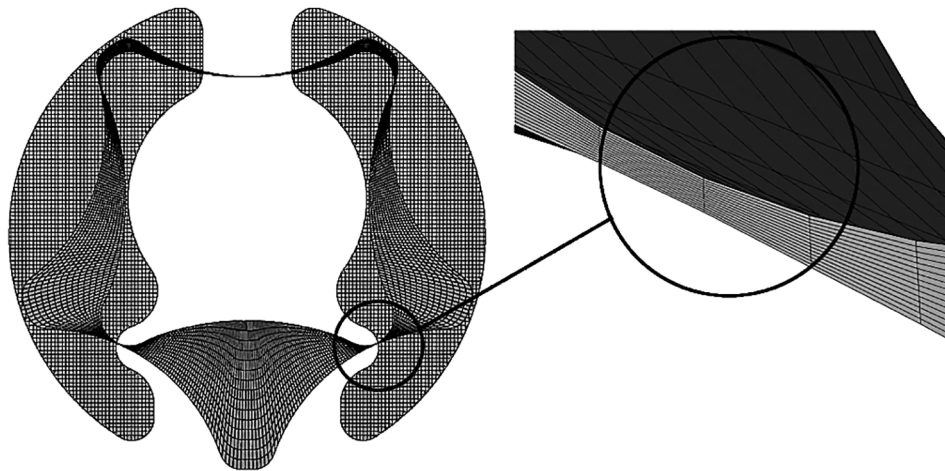
Because there is a very small gap, 0.1 mm, between the internal and external rotors, the grid of this gap should be refined to accord with the calculation accuracy. However, such a small grid will lead to a sharp increase of the number of grid points, and the solving time will be too long. In the view of the above problems, structured meshing of the rotor and oil groove is taken into consideration to save the number of meshes. PumpLinX includes an automated Cartesian grid generator. It can quickly generate high-quality meshes that match the CFD solver and can be solved efficiently. The grid generator uses a geometry Conformal Adaptive Binary-tree (CAB) algorithm. Near the geometric boundary, the mesh

generator automatically splits the large mesh, integrates the small mesh to fit the geometric surface shape and the geometric boundary line, so as to capture the details of the geometric boundary. This method can efficiently use the smallest grid to resolve detail features.

In order to take into account the reliability of numerical simulation and the time it takes to calculate, it is necessary to select an appropriate number of grid points. When the correlation between the outlet flow of the cycloidal rotor pump and the number of grid points is not large, it can be regarded as grid-independent. Five sets of grids with different scales were used to perform numerical simulation under the working condition of  $n = 3000$  r/min and  $P_2 = 0.5$  MPa. The simulated outlet flows corresponding to the number of different grids are shown in Tab. 1. It can be seen from the table that when the grid number is greater than 221250, the volumetric efficiency of the cycloidal rotor pump is basically stable, and the error is within 1%. Finally, the total number of meshes is 242369. The meshes of the internal and external rotors are divided into 12 layers. The meshing is shown in Fig. 2. Volumetric efficiency in a pump refers to the percentage of actual fluid flow out of the pump compared to the flow out of the pump without leakage.

**Table 1:** Mesh sensitivity analysis

Mesh number	Volumetric efficiency (%)
221250	87.44
242369	87.60
271023	87.82



**Figure 2:** Meshing of cycloid gerotor pump

## 2.2 Mathematical Model

### 1. Governing equations

The oil of the cycloid gerotor pump is an isothermal fluid, so it does not need to be described by the energy conservation equation. The law of conservation of mass of the cycloid gerotor pump is expressed as: The mass of fluid flowing into the control body per unit time is equal to the sum of the mass of fluid flowing out of the control body and the increase in the mass of fluid in the control body. The mass balance equation [17] is:

$$\frac{\partial(u_x)}{\partial x} + \frac{\partial(u_y)}{\partial y} + \frac{\partial(u_z)}{\partial z} = 0 \quad (1)$$

where  $u_x$  is fluid velocity in the  $x$  direction,  $u_y$  is fluid velocity in the  $y$  direction,  $u_z$  is fluid velocity in the  $z$  direction.

The fluid inside the cycloid gerotor pump is an incompressible viscous fluid and the momentum conservation equation [17] is:

$$\rho \frac{\partial(u_i)}{\partial t} + \rho \frac{\partial(u_i u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] - \frac{\partial p}{\partial x_i} + f_i \quad (2)$$

where  $u_i$  is fluid three-dimensional velocity component,  $x_i$  is each coordinate component,  $f_i$  is external source,  $\mu$  is hydrodynamic viscosity,  $\rho$  is fluid density,  $t$  is time.

## 2. Turbulence model

In this paper, the RNG  $k$ - $\varepsilon$  turbulence model is selected to simulate the three-dimensional unsteady flow of a cycloidal rotor pump. The RNG  $k$ - $\varepsilon$  turbulence model not only fully considers the rotation and swirling conditions in the fluid flow of the cycloidal rotor pump, but also can accurately simulate the flow near the wall of the cycloidal rotor pump.

The turbulent kinetic energy equation used in the RNG  $k$ - $\varepsilon$  turbulence model [18] is:

$$\rho \frac{\partial(k)}{\partial t} + \rho \frac{\partial(k u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \alpha_k \mu_e \frac{\partial k}{\partial x_j} \right) + \rho (P_k - \varepsilon) \quad (3)$$

$$\rho \frac{\partial(\varepsilon)}{\partial t} + \rho \frac{\partial(\varepsilon u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \alpha_\varepsilon \mu_e \frac{\partial \varepsilon}{\partial x_j} \right) + \rho \frac{\varepsilon}{k} (C_{1\varepsilon}^* P_k - C_{2\varepsilon} \varepsilon) \quad (4)$$

In this equation,  $C_{1\varepsilon}^* = C_{1\varepsilon} - \frac{\eta(1 - \eta/\eta_0)}{1 + \beta\eta^3}$ ,  $\eta = (2E_{ij} \cdot E_{ij})^{1/2} \frac{k}{\varepsilon}$ ,  $E_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ .  $\mu_e$  is the effective viscosity coefficient.  $E_{ij}$  is the time-average strain rate. Model constant:  $\alpha_k = \alpha_\varepsilon = 1.39$ ,  $C_{1\varepsilon} = 1.42$ ,  $C_{2\varepsilon} = 1.68$ ,  $\eta_0 = 4.377$ ,  $\beta = 0.012$ .

## 2.3 Numerical Method

Pressure-based solution is used to solve basic equations. The pressure-velocity coupling equation is solved by PRESTO algorithm [19]. The pressure term is discretized in SIMPLE format [20], and the remaining terms are discretized in first-order upwind style.

## 2.4 Setting of Condition

In order to examine the influence of the speed and outlet pressure on the radial force of the rotor pump, five speeds of  $n = 1000, 1500, 2000, 2500, 3000$  r/min and five different outlet pressures  $P_2 = 0.1, 0.2, 0.3, 0.4, 0.5$  MPa were selected for numerical calculation.

Boundary conditions include: The end faces of the inlet and outlet tanks of the pump are the boundary conditions of pressure inlet and pressure outlet, the inlet pressure is 1 standard atmospheric pressure, and the other surfaces are set to the boundary conditions of the wall surface. The convergence accuracy during the calculation is  $1 \times 10^{-3}$ . The temperature change of the oil is not considered in the calculation, and the energy equation is not added to the model. The oil density is  $800 \text{ kg/m}^3$  and the saturation pressure is 400 Pa.

## 2.5 Validation of Numerical Method

Select 8 operating conditions,  $n = 2000, 3000$  r/min,  $P_2 = 0.2, 0.3, 0.4, 0.5$  MPa, to perform numerical simulation on the cycloidal rotor pump and obtain the external characteristic parameters of the cycloidal rotor pump, that is, the outlet flow  $Q_2$ . In Fig. 3, it can be found that the numerical simulation performance curve is consistent with the measured performance curve by comparing experimental and simulated values. The outlet flow increases with the increase of the speed, and is basically proportional to the speed. This is because the increase in speed directly causes more oil in the interdental volume to be pumped out of the oil channel per unit time. Outlet flow decreases with increasing outlet pressure. This is because the increase in outlet pressure leads to an increase in the pressure difference with the inlet pressure, that is, the pressure difference at the rotor gap also increases accordingly, resulting in an increase in leakage at the gap, which reduces the outlet flow. In addition, the numerical simulation of the outlet flow rate is higher than the test value under various operating conditions, because the simulation does not consider the actual rotor end face leakage and axial leakage caused by the deviation. Comparing the test value with the simulation value, it can be seen that the agreement is generally good, and the deviation of calculation is about 3%, which indicates that the accuracy of the numerical simulation is acceptable.

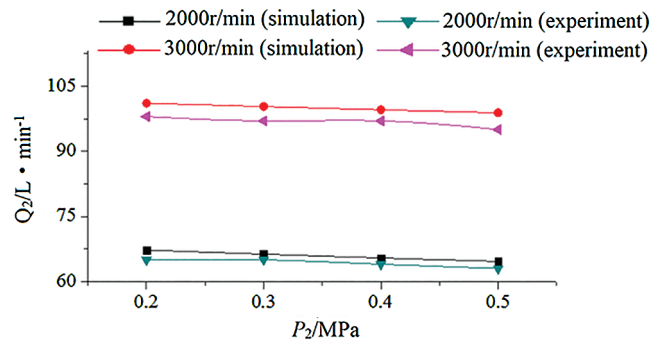
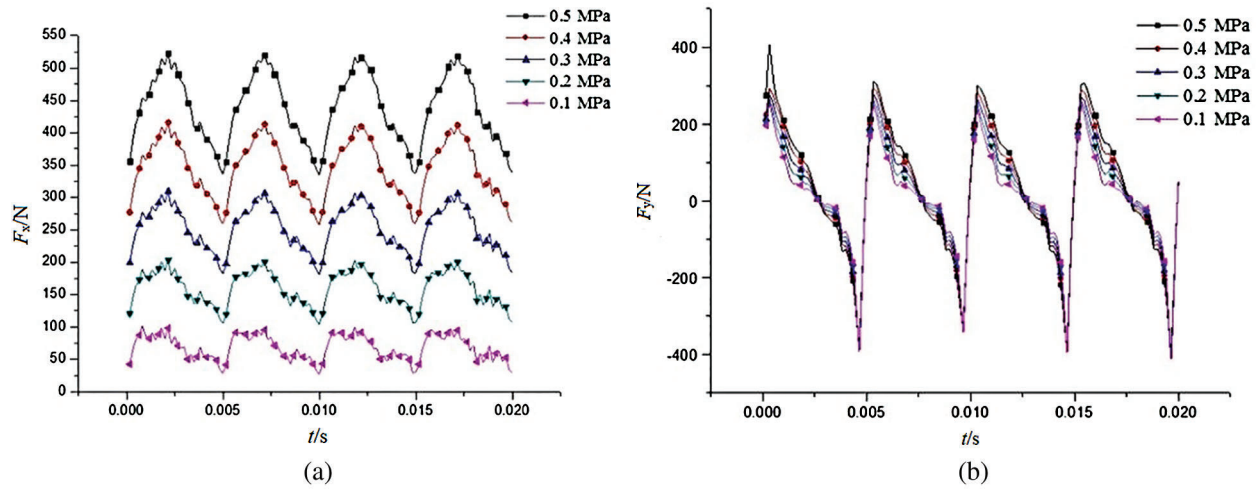


Figure 3: External characteristic curve of Gerotor pump

## 3 Analysis of Characteristics of Radial Force

### 3.1 The Effect of Outlet Pressure on Radial Force

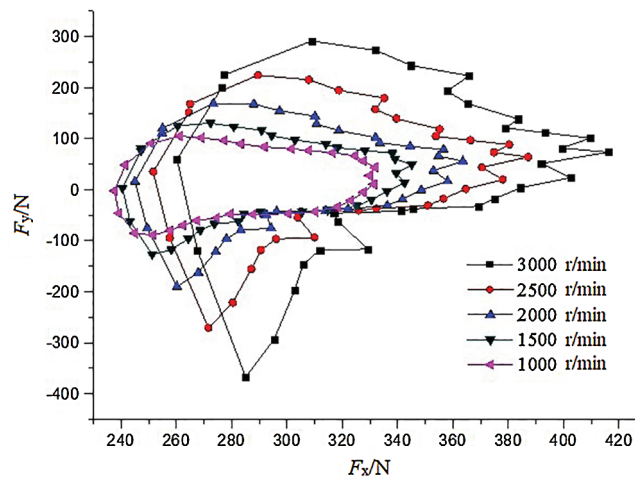
The higher the speed of the gerotor pump, the more the radial force on the internal rotor. In order to intuitively show the change of radial force of the internal rotor under different outlet pressures, take  $n = 3000$  r/min to analyze the change of radial force under 5 types of outlet pressure. As shown in Fig. 4, Fig. 4a is a change curve of a radial force  $F_x$  received in a lateral direction and Fig. 4b is a change curve of a radial force  $F_y$  received in a longitudinal direction. It can be seen from Fig. 4 that  $F_x$  and  $F_y$  both decrease as the outlet pressure decreases, but their sensitivity to the outlet pressure is different.  $F_x$  varies sharply with the size of the outlet pressure and changes linearly. Each time the outlet pressure decreases by 0.1 MPa,  $F_x$  decreases by 20%, and the pulsation amplitude of  $F_x$  also decreases accordingly. However, the change in outlet pressure has little effect on  $F_y$ . For each decrease of 0.1 MPa in outlet pressure,  $F_y$  decreases by about 6%. It can be seen that the change in outlet pressure has a greater impact on  $F_x$ . The reason is that the size of  $F_x$  is determined by the pressure difference between the oil inlet and the oil outlet. So, the effect on  $F_x$  is greater.  $F_y$  is mainly determined by the pressure difference in the y-direction, that is, the trapped oil pressure of the meshing volume of the rotor, so its correlation with the outlet pressure is small.



**Figure 4:** The radial force of internal rotor under different outlet pressure. (a)  $F_x$  and (b)  $F_y$ .

### 3.2 Effect of Speed on Radial Force

It can be known from Fig. 4 that at a certain speed, the greater the  $P_2$ , the greater the radial force of the internal rotor. Therefore, in the process of analyzing the influence of the speed, analyze the change of radial force under five kinds of speed under the condition of  $P_2 = 0.5$  MPa. It is shown in Fig. 5 that  $F_x$ ,  $F_y$  both become smaller with reduction of  $n$ , and the amplitude of pulsation also becomes smaller. When  $n$  is small, the change of  $n$  has a small effect on  $F_x$  and  $F_y$ , but as  $n$  increases, the change becomes drastic. When  $n$  decreases sequentially from 3000, 2500, 2000, 1500, 1000 r/min, reduction rate of  $F_x$  were 5.51%, 4.74%, 3.89%, 2.97%; and reduction rate of  $F_y$  were 20.35%, 18.91%, 17.98%, 16.11%. It can be seen that the change of  $n$  has a greater impact on  $F_y$ .



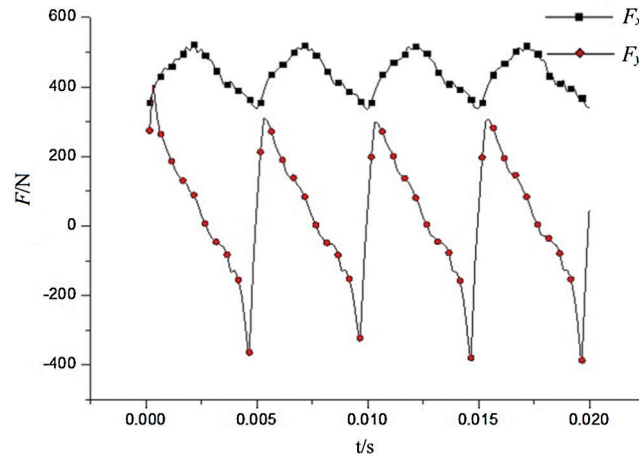
**Figure 5:** Radial force of internal rotor at different speed

### 3.3 Analysis of Extreme Position of $F_x$ and $F_y$

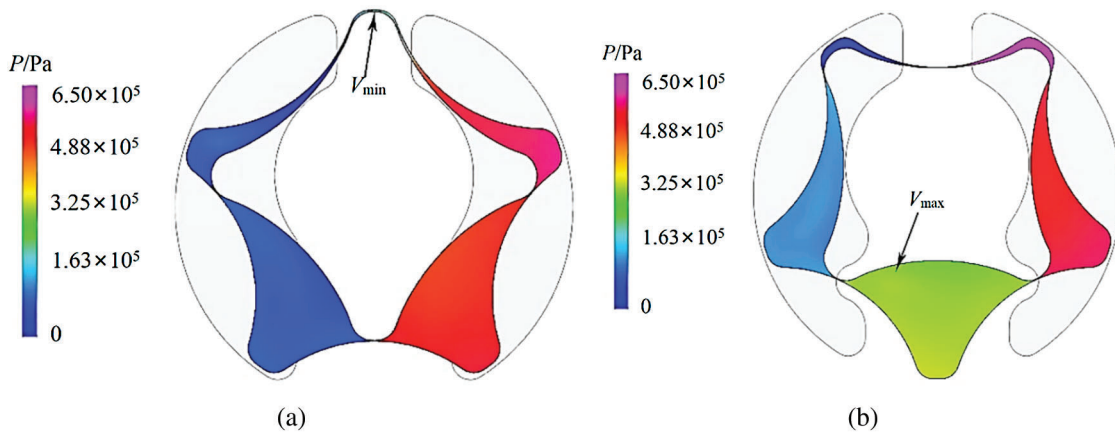
Fig. 6 shows the radial force pulsation of the internal rotor at the condition of  $n = 3000$  r/min and  $P_2 = 0.5$  MPa. It can be seen from Fig. 6 that the amplitude of the pulsation of  $F_x$  is much smaller than that of  $F_y$  and  $F_x$  is a positive value. In addition to the large amplitude, the value of  $F_y$  has sudden changes from negative to



positive. Radial force shocks and sudden changes can seriously affect the stiffness of the pump shaft. When  $F_x$  reaches the maximum value,  $F_y$  is about zero. At this time, the rotor is symmetrically distributed along the y-axis, so  $F_y$  is close to zero. when  $F_x$  reaches the minimum value,  $F_y$  changes from negative to positive and it is in this time that the maximum volume between the teeth is changed from oil absorption to oil extraction. The pressure distribution cloud diagrams of the rotor position where  $F_x$  and  $F_y$  take extreme values in Fig. 6 are shown in Figs. 7 and 8.



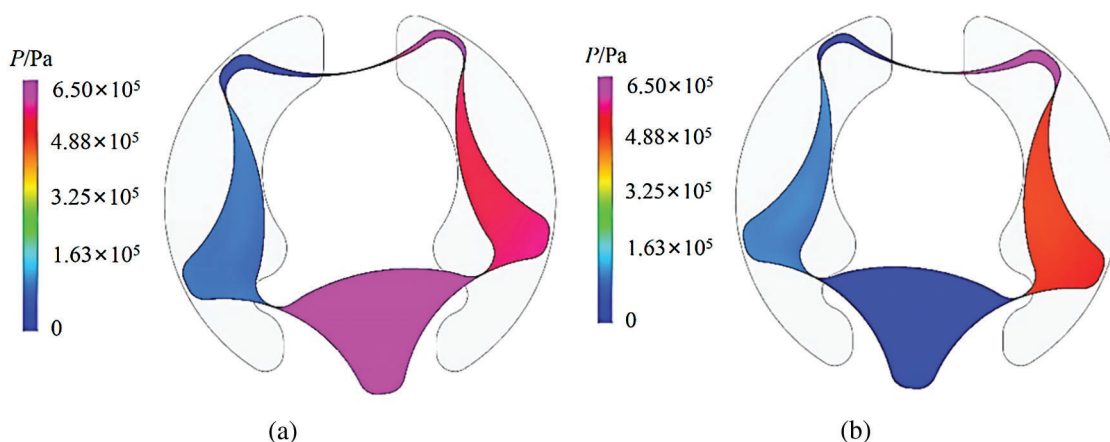
**Figure 6:** Radial force of internal rotor when  $n = 3000$  r/min,  $P_2 = 0.5$  MPa



**Figure 7:** Pressure distribution of  $F_x$  extreme position. (a) When  $F_x$  is minimum and (b) When  $F_x$  is maximum

In Fig. 7, the position of the extreme value of  $F_x$  is shown. Fig. 7a shows the pressure distribution when  $F_x$  is the smallest, and Fig. 7b shows the pressure distribution when  $F_x$  is the largest. As can be seen from Fig. 7, the position where  $F_x$  reaches the minimum value is the position where the rotor turns to the maximum inter-tooth area  $V_{max}$ . At this time, the oil absorption process has been completed, but oil pressure has not been performed. The negative pressure in the oil absorption area has been eliminated. The pressure in the oil drainage area has not increased. The pressure difference between the oil discharge area and the oil absorption area is not too big. So  $F_x$  reaches the minimum value. The position where  $F_x$  reaches the maximum value is the position where the rotor turns to the minimum inter-tooth area  $V_{min}$ . At

this time, the oil pressure process is completed. The pressure of the oil discharge zone reaches the Maximum, the pressure in the oil absorption zone is relatively low, the pressure difference between the oil discharge zone and the oil absorption zone are symmetrical along with the y-axis distribution, so  $F_y = 0$ . It can be clearly seen that the high-pressure area near the oil discharge cavity in Fig. 7b is significantly more than the high-pressure area in Fig. 7a, and the pressure is greater; the low-pressure area near the oil inlet cavity in Fig. 7b is also significantly more than the low-pressure area in Fig. 7a, and the negative pressure is greater.



**Figure 8:** Pressure distribution of  $F_y$  extreme position. (a) Negative maximum of  $F_y$  and (b) Positive maximum of  $F_y$

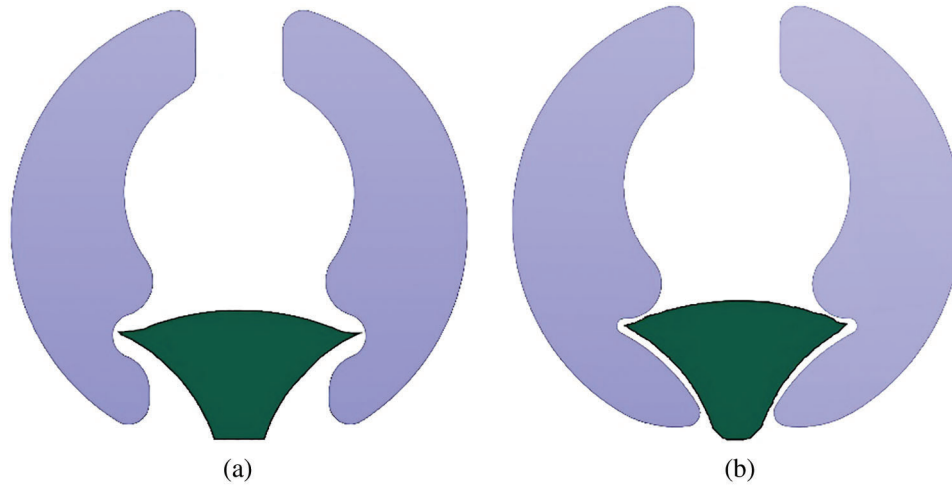
In Fig. 8, the position where  $F_y$  extremes occur is shown. The Fig. 8a is the pressure distribution when  $F_y$  is at the maximum in the negative direction, and Fig. 8b is the pressure distribution when  $F_y$  is at the maximum in the positive direction. As can be seen from Fig. 8, the position where  $F_y$  reaches the negative maximum value is the position where the rotor is about to turn to the maximum inter-tooth area  $V_{\max}$ . At this time, the inter-tooth area continues to increase, resulting in a decrease in pressure, and the oil in the oil groove is too late to fill the inter-tooth area. When the inter-tooth area is close to  $V_{\max}$ , the pressure drops to the lowest value, so that the negative pressure difference along the y-axis reaches the maximum, so  $F_y$  reaches the negative maximum value; the position where  $F_y$  reaches the positive maximum value is that the rotor has just passed the maximum inter-tooth area  $V_{\max}$ . The area between the teeth is constantly decreasing, resulting in an increase in pressure, and this part of the high-pressure oil is too late to be discharged through the oil groove, and the oil pressure rises to the highest value so that the pressure difference in the positive direction along the y-axis reaches the maximum. Therefore,  $F_y$  reaches a positive maximum. From the pressure distribution in the figure, it can be seen that during the process of the rotor changing from the position shown in Fig. 8a to the position shown in Fig. 8b, the maximum inter-tooth volume suddenly changed from extremely low negative pressure to extremely high positive. This is because when the maximum inter-tooth volume connects with the drain groove, the high-pressure fluid in the groove flows back into the inter-tooth volume, causing a sudden rise in pressure. At the same time, this part of the return fluid does not form an effective flow rate, which causes fluctuations in the outlet flow rate.

#### 4 The Improvement of Inlet and Outlet Oil Groove

Based on the above analysis of the locations and causes of the extreme values of  $F_x$  and  $F_y$ , an improved structure of the inlet and outlet groove is proposed. As shown Fig. 9, the improved large-end oil inlet line and oil pressure line are claw-shaped, so that when the cycloid gerotor pump sucks oil, the oil in the oil inlet

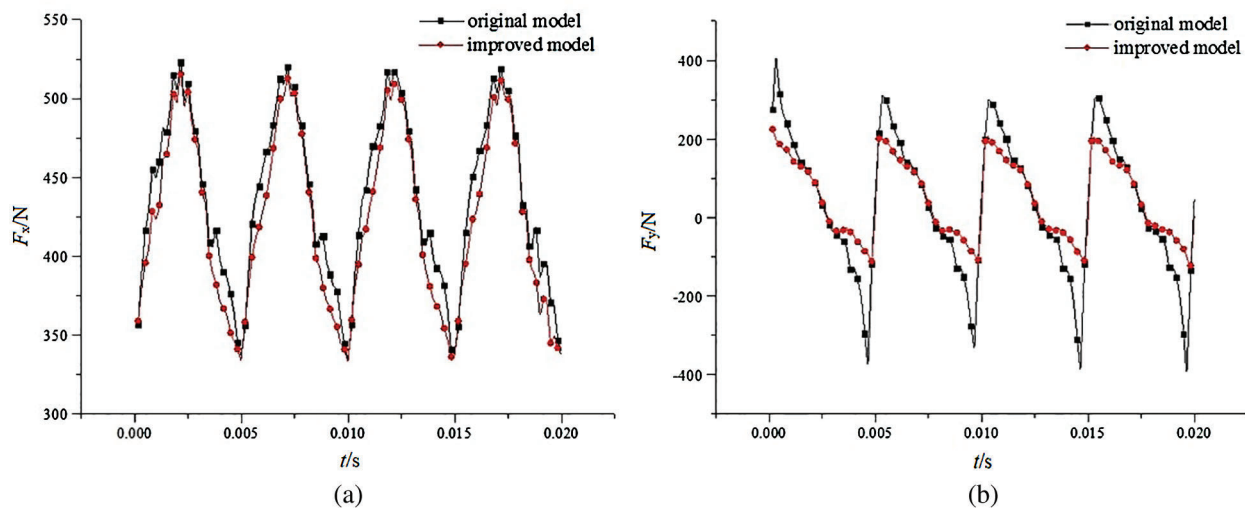


groove can fill the inter-tooth volume earlier and when the pump pressurizes oil, the high-pressure oil can discharge the inter-tooth volume earlier. To conclude, the new structure can alleviate the sudden pressure charge in the flow field of the gerotor pump and reduce the radial force.



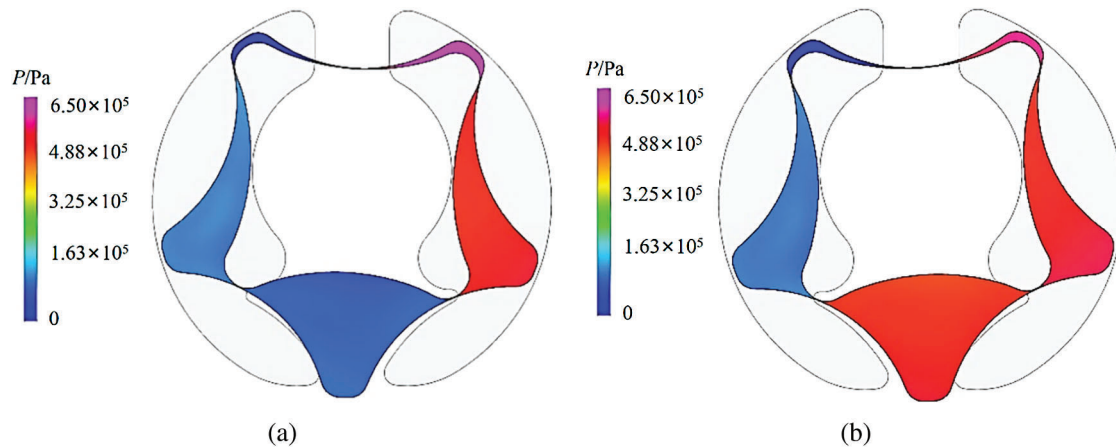
**Figure 9:** Improved oil groove structure. (a) Before improvement and (b) After improvement

Fig. 10 is a comparison diagram of the radial force before and after the oil groove is improved. It can be seen from Fig. 10 that  $F_x$  does not change much before and after the model is improved. From the above analysis, it is known that  $F_x$  is mainly affected by the outlet pressure. The outlet pressure determines the pressure difference between the oil inlet area and the oil outlet area. The extreme value of  $F_y$  in the improved model has decreased significantly, and the fluctuation is more stable. The drop position is mainly the peak point of  $F_y$ , a process of the sudden change from negative to positive so that the impact on the rotor shaft is reduced. According to the data analysis, the positive maximum value of  $F_y$  decreases by 35.8%, the negative maximum value of  $F_y$  decreases by 69.1%, and the overall pulsation amplitude decreases by 53.8%, indicating that the improved oil groove structure has a significant relief effect on  $F_y$ .



**Figure 10:** The comparison of radial force before and after advance. (a)  $F_x$  and (b)  $F_y$

Because the improvement effect of  $F_x$  is not large, the flow field analysis is not performed at its corresponding time, and the analysis is mainly performed on the flow field at the extreme time of  $F_y$ . Fig. 11 shows the pressure distribution of the improved model. As can be seen from Fig. 11, when  $F_y$  reaches the negative maximum value, that is, the rotor is about to turn to the maximum inter-tooth area  $V_{\max}$ , the inter-tooth volume is constantly increasing, and the pressure is continuously decreasing. The improved claw-shaped oil groove can ensure the entire process of oil can be in contact with the volume between the teeth so that the oil can fully fill the interdental volume and avoid the formation of excessive negative pressure. Compared with Fig. 8a, it can be clearly seen that the flow field pressure in the  $V_{\max}$  region has increased, and decreased the negative maximum value of  $F_y$ . When  $F_y$  reaches the positive maximum value, that is, the rotor has just passed the maximum inter-tooth area  $V_{\max}$ , the inter-tooth volume is continuously reduced, and the pressure is continuously increased. The improved claw-shaped oil groove can be connected to the inter-tooth volume earlier so that the compressed high-pressure oil can be discharged from the oil groove in time to avoid the formation of excessively high pressure. Compared with Fig. 8b, it can be clearly seen that the flow field pressure in the  $V_{\max}$  region has reduced, thereby reducing the positive maximum value of  $F_y$ .



**Figure 11:** Pressure distribution of the improved model. (a) The negative maximum of  $F_y$  and (b) The positive maximum of  $F_y$ .

## 5 Conclusion

The radial forces of  $F_x$  and  $F_y$  on the rotor shaft both decrease as the outlet pressure decreases, and  $F_x$  changes sharply with the magnitude of the outlet pressure, which changes linearly, and the change in outlet pressure has little effect on  $F_y$ .  $F_x$  and  $F_y$  both increase with the increase of  $n$ , and change dramatically as the increase of  $n$ . The effect of  $n$  on  $F_x$  is small, but the effect on  $F_y$  is large.

The position where  $F_x$  reaches the minimum value is the position where the rotor is turned to the maximum inter-tooth area  $V_{\max}$ , and the position where  $F_x$  reaches the maximum value is the position where the rotor is turned to the minimum inter-tooth area  $V_{\min}$ . The position where  $F_y$  reaches the negative maximum value is the position where the rotor is about to turn to the maximum inter-tooth area  $V_{\max}$ , and the position where  $F_y$  reaches the positive maximum value is the position where the rotor has just passed the maximum inter-tooth area  $V_{\max}$ .

Based on the analysis of the radial force of the cycloid gerotor pump, we come up with an improved oil inlet and outlet groove with a claw-shaped large-end oil inlet line and oil pressure line. After the model improving,  $F_x$  changes a little. The extreme value of  $F_y$  after the improvement is significantly reduced,

and the fluctuation is more stable. The positive maximum value of  $F_y$  decreases by 35.8%, the negative maximum value of  $F_y$  decreases by 69.1%, and the overall pulsation amplitude decreases by 53.8%. It shows that the improved oil groove structure has an obvious effect on decreasing  $F_y$ .

**Funding Statement:** This work was supported by the Zhejiang Provincial Natural Science Foundation of China (No. LY19E050003); and the National Natural Science Foundation of China (No. 51779226).

**Conflicts of Interest:** The authors declare that they have no conflicts of interest to report regarding the present study.

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