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Research on the Freezing Phenomenon of Quantum Correlation by Machine Learning

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Abstract: Quantum correlation shows a fascinating nature of quantum mechanics and plays an important role in some physics topics, especially in the field of quantum information. Quantum correlations of the composite system can be quantified by resorting to geometric or entropy methods, and all these quantification methods exhibit the peculiar freezing phenomenon. The challenge is to find the characteristics of the quantum states that generate the freezing phenomenon, rather than only study the conditions which generate this phenomenon under a certain quantum system. In essence, this is a classification problem. Machine learning has become an effective method for researchers to study classification and feature generation. In this work, we prove that the machine learning can solve the problem of X form quantum states, which is a problem of physical significance. Subsequently, we apply the density-based spatial clustering of applications with noise (DBSCAN) algorithm and the decision tree to divide quantum states into two different groups. Our goal is to classify the quantum correlations of quantum states into two classes: one is the quantum correlation with freezing phenomenon for both Rènyi discord ($\alpha = 2$) and the geometric discord (Bures distance), the other is the quantum correlation of non-freezing phenomenon. The results demonstrate that the machine learning method has reasonable performance in quantum correlation research.

Keywords: Machine learning, quantum correlation, freezing phenomenon, Rènyi discord, geometric discord.

1 Introduction

As the basic theories of quantum mechanics, the superposition principle and the tensorial structure of the Hilbert space have been widely applied to describe the composite quantum systems and the concept of entanglement [Einstein, Podolsky and Rosen (1935)],

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which are kinds of special superposition states involved that have triggered the great interest, especially in the field of quantum information [Nielsen and Chuang (2007)]. In the earlier studies, entanglement has always been considered equivalent to the quantum correlation. Until a few decades ago, the concept of “quantum discord”, which was introduced by Ollivier et al. [Ollivier and Zurek (2001); Henderson and Vedral (2001)] shows a universal consensus “entanglement entirely captures quantum correlation only for a global pure state” [Cianciaruso, Bromley, Roga et al. (2015)]. In other words, entanglement does not account for all nonclassical correlations, for example, the states with zero entanglements still contain quantum correlations [Modi, Brodutch, Cable et al. (2012)]. Therefore, in the past few years, many works have been presented on the subject matter [Franco, Bellomo, Maniscalco et al. (2013); Xu, Xu, Li et al. (2010); Breuer, Laine, Piilo et al. (2016); Aolita and De Melo (2015); Zhu, Ding, Wu et al. (2016)].

As a unique behavior, the freezing phenomenon of quantum correlation originates from the discovery of Maziero et al. [Maziero, Celeri, Serra et al. (2009)]. They found about the frozen behavior of the classical correlations for phase-flip, bit-flip, and bit-phase flip channels. Later, Mazzola et al. [Mazzola, Piilo and Maniscalco (2009)] demonstrated a similar behavior of quantum correlations under the non-dissipative independent Markovian reservoirs for special choices of the initial state. In the same year, Lang et al. [Lang and Caves (2010)] provided a complete geometric pictorial interpretation for Bell-diagonal states condition. After that, many researchers have devoted to studying the conditions of the frozen-discord for some Non-Markovian processes and initial states [Cianciaruso, Bromley, Roga et al. (2015); Zhu, Ding, Wu et al. (2016); Ding, Zhu, Wu et al. (2017); Li, Zhu, Zhu et al. (2019)], such as Bell-diagonal states, X states and SCI states. In general, the freezing discord shows a robust feature of a family with two-qubit models affected by non-dissipative decoherence.

Although the freezing phenomenon is shown from several quantification methods [Cianciaruso, Bromley, Roga et al. (2015); Zhu, Ding, Wu et al. (2016); Ding, Zhu, Wu et al. (2017); Li, Zhu, Zhu et al. (2019)], the different methods have led to different conditions. What conditions lead to freezing phenomenon is an open question, which is still the current research hotspot.

In this work, we calculate the value of two different discords for the X form initial states, then use the DBSCAN algorithm and the decision tree method to classify quantum states into two groups. One is the quantum correlation with freezing phenomenon for both Rènyi discord ($\alpha = 2$) and the geometric discord (Bures distance), the other is without.

2 The geometric and Rènyi discord

Cianciaruso et al. [Cianciaruso, Bromley, Roga et al. (2015)] have discussed the geometric measure of quantum correlation based on the Bures distance (D_{Br}), which is defined as Eq. (1),

$$D_{Br} \equiv \inf_{\chi'} d_{Bu}^2(\rho, \chi') = \inf_{\chi'} 2\left(1 - Tr\left(\left[\sqrt{\chi'}\rho\sqrt{\chi'}\right]^{1/2}\right)\right) \tag{1}$$

where the set of classical-quantum states is $\chi' = \sum_i p_i |i><i|^A \otimes \omega_i^B$, p_i is a probability distribution, $\{|i>^A\}$ denotes an orthogonal basis for subsystem A, ω_i^B is an arbitrary ensemble of states for subsystem B, and $d_{Bu}(\rho, \chi')$ is the Bures distance. Since it is difficult to obtain a mathematically analytic form for general models, some numerical calculation methods have been proposed in the paper [Girolami, Souza, Giovannetti et al. (2014)], which are also adopted in this work based on the relation between quantum Fisher information and the Bures distance. Therefore, the Bures distance can be rewritten as Eq. (2),

$$\mathcal{P}^{\mathcal{A}}(\rho_{AB}|\Gamma) = \frac{1}{4} min_{H_A^\Gamma} F(\rho_{AB}; H_A^\Gamma) \tag{2}$$

where F denotes the quantum Fisher information,

$$F(\rho_{AB}; H_A^\Gamma) = 4 \sum_{i<k: q_i+q_k \neq 0} \frac{(q_i+q_k)^2}{q_i+q_k} \left| \langle \psi_i | (H_A^\Gamma \otimes I_B) | \psi_k \rangle \right|^2 \tag{3}$$

which q_i and $|\psi^i\rangle\}$ denote the eigenvalues and eigenvectors of ρ_{AB}, respectively. The minimum is taken over the set of all local Hamiltonians H_A^Γ.

The Rènyi quantum discord of ρ_{AB} is an extension of quantum discord, which is defined for $\alpha \in (0,1) \cup (1,2]$ as Eq. (4) [Mario, Kaushik and Mark (2015); Seshadreesan, Berta, and Wilde (2015)],

$$D_\alpha(\rho_{AB}) = inf_{\Pi_k^A} I_\alpha(E; B \mid X)_{\tau_{XEB}} \tag{4}$$

where the Rènyi conditional mutual information $I_\alpha(E; B \mid X)_{\tau_{XEB}}$ satisfies Eq. (5):

$$I_\alpha(E; B \mid X)_{\tau_{XBE}} = \frac{\alpha}{\alpha-1} log\, Tr \left(\rho_X^{\frac{\alpha-1}{2}} Tr_E (\rho_{EX}^{\frac{1-\alpha}{2}} \rho_{EBX}^{\alpha} \rho_{EX}^{\frac{1-\alpha}{2}}) \rho_X^{\frac{\alpha-1}{2}} \right)^{\frac{1}{\alpha}} \tag{5}$$

where the classical output X denotes the measurement acting on system A, E is an environment for the measurement map. In this paper, we choose the von Neumann measurement $\{\Pi_{i'} = |i'\rangle\langle i'| (i = 0,1)\}$with two angular parameters θ and ϕ: $|0'\rangle = cos\,(\theta/2)|0\rangle + e^{i\phi} sin\,(\theta/2)|1\rangle$ and $|1'\rangle = sin\,(\theta/2)|0\rangle - e^{i\phi} cos\,(\theta/2)|1\rangle (0 \leq \theta \leq \pi/2; 0 \leq \phi \leq \pi)$. The properties of the Rènyi quantum discord are shown in Tab. 2 of paper [Seshadreesan, Berta and Wilde (2015)].

3 Classification of the quantum states with freezing phenomenon

From the computer science perspective, the above freezing phenomenon, which is caused by features that allow these states to generate freezing phenomenon using different quantification methods, can be transformed into an equivalent classification problem, i.e., "the classification of quantum states". As part of both artificial intelligence and statistics, machine learning originated from the field of computer science in which the goal is to learn potential patterns from data sets previously given, and make the decision or prediction for unknown future situations using learned patterns. Nejad et al. [Nejad and Shiri (2019)] proposed a new learning approach based on the salp swarm algorithm that was implemented and evaluated on learning algorithm Decision Tree, K-Nearest Neighbors and Naive Bayes. Hossain et al. [Hossain, Morooka, Okuno et al. (2019)] proposed two prediction methods along with their sub-classes and evaluated by a leave-one-out cross-validation procedure. Recently, this tool has been used to study some quantum problems such as, quantum state tomography [Torlai, Mazzola, Carrasquilla et al. (2018)], quantum many-body problem [Carleo and Troyer (2017)] and quantum correlation problem

[Li, Zhu, Zhu et al. (2019); Su, Sheng, Xie et al. (2019)]. Similarly, we apply two machine learning methods including DBSCAN and the decision tree in the classification of quantum states of freezing phenomenon on the two open quantum systems.

3.1 System 1

Here, we consider the anisotropic coupling two-qubits system, which is coupled by two correlated Fermi-spin environments. The Hamiltonian of the total system has the following form [Li, Zhu, Meng et al. (2019)] as Eq. (6),

$$H = H_s + \sum_{i=1,2}\left(H_{E_i} + H_{sE_i}\right) + qS_1^z S_2^z \tag{6}$$

$$H_s = J_1\left(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y\right) + J_2\sigma_1^z\sigma_2^z + \sum_{i=1,2}\omega_i\sigma_i^z\,; H_{E_t} = \alpha_i S_i^z; H_{sE_t} = \gamma_i\sigma_i^z S_i^z \tag{7}$$

where J_1 and J_2 are the anisotropic coupling parameters between two spin particles; ω_i and α_i are the frequencies of spin particle and environmental spin particle, respectively; q describes an Ising-type correlation between the environments. $S_i^z = \sum_{k=1}^{N_i}\frac{\sigma_z^{k,i}}{2}$ is the collective spin operator; $\sigma_z^{k,i}$ is the Pauli matrix; the environment E_i is consisted of N_i particle with spin $1/2$.

For the initial state $\rho(0) = \rho_s(0)\otimes\rho_E(0)$, the reduced density matrix $\rho_s(t)$ of the system can be calculated as Eq. (8).

$$\rho_d(t) = \frac{1}{Z}\sum_{j_1=0}^{N_1/2}\sum_{m_1=-j_1}^{j_1}\sum_{j_2=0}^{N_2/2}\sum_{m_2=-j_2}^{j_2}\frac{\nu(N_1,j_1)\nu(N_2,j_2)}{e^{\beta q m_1 m_2}e^{\beta\alpha_1 m_1}e^{\beta\alpha_2 m_2}}V^\dagger\, U^\dagger(t)\rho_s(0)U(t)V \tag{8}$$

3.2 System 2

Considering the system with two independent spin particles' (1 and 3) under each with spin $1/2$, which forms a two-qubits system that each spin particle interacts with one Fermi-environment 1 and 3 under spin 1/2 for each particle. Simultaneously, these two Fermi-environments interact with the third Fermi-environment 2 with spin 1/2 for each particle, where time-independent interaction parameters have been denoted by q_{12} and q_{23}, respectively. The Hamiltonian of this open systems is given as Eq. (9) under ($\hbar = 1$) [Ding, Zhu, Wu et al. (2017)],

$$H = \sum_{a=1,3}(\omega_a s_a^z + b_a s_a\cdot S_a) + \sum_{d=1,2,3}\alpha_d S_d^z + q_{12}S_1^z S_2^z + q_{23}S_2^z S_3^z \tag{9}$$

where $\omega_a(a = 1,3)$ and $\alpha_d(d = 1,2,3)$ are the frequencies of the qubit (characterized by spin operators s_i) and the environments, respectively; q_{12} and q_{23} are the time-independent coupling parameters between the environment 1(2) and the environment 2(3); Each environment is consisted of the N_d particles; The interaction intensity between the spin particle 1(3) and the environment 1(3) is $b_1(b_3)$; $S_d^z = \sum_{k=1}^{N_d}\frac{\sigma_{k,d}^z}{2}, S_d^x = \sum_{k=1}^{N_d}\frac{\sigma_{k,d}^x}{2}$ and $S_d^y = \sum_{k=1}^{N_d}\frac{\sigma_{k,d}^y}{2}$ are the collective spin operators of the environment; $\sigma_{k,d}^z$, $\sigma_{k,d}^y$, and $\sigma_{k,d}^x$ are the Pauli matrices.

When we integrate the degree of freedom in environment 2 under the initial state $\rho_0 = \rho_s(0)\otimes\rho_E(0)$, the reduced density operator $\rho_s(t)$ of the system can be obtained as Eq. (10),

$$\rho_s(t) = \prod_{d=1}^{3} \frac{1}{Z_d} e^{-\beta_d \alpha_d m_d} Tr_{E'} \Big(e^{-i\int_0^t H' dt'} (A^\dagger) \rho_s(0)(A)$$

$$\times |j_1, m_1 >< m_1, j_1||j_3, m_3 >< m_3, j_3| e^{i\int_0^t H' dt'} \Big) \tag{10}$$

where $|j_d, m_d\rangle$ denotes the orthogonal basis states of environments; $A^\dagger = \Big|\phi > \Big[\Big|\frac{1}{2}\Big\rangle_1 \Big|\frac{1}{2}\Big\rangle_3, \Big|\frac{1}{2}\Big\rangle_1 \Big|\frac{-1}{2}\Big\rangle_3, \Big|\frac{-1}{2}\Big\rangle_1 \Big|\frac{1}{2}\Big\rangle_3, \Big|\frac{-1}{2}\Big\rangle_1 \Big|\frac{-1}{2}\Big\rangle_3\Big], \Big|\pm\frac{1}{2}\Big\rangle_a$ $(a = 1,3)$ denotes the spin of particle a; $\beta_d = \frac{1}{k_B T_d}$; Z_d is the partition function. Finally, according to the algebraic dynamical method [Wang and Cen (1998); Zhu, Kuang and Tan (2005)], we obtain the analytical solution Eq. (3) in the paper [Ding, Zhu, Wu et al. (2017)].

Notice, when the initial density matrix $\rho_s(0)$ has X form [Franco, Bellomo, Maniscalco et al. (2013); Xu, Xu, Li et al. (2010)], the Eq. (11) can be satisfied by $a, b, c, d \geq 0, a + b + c + d = 1, \big||\delta|\big|^2 \leq ad$ and $\big||\beta|\big|^2 \leq bc$. It is easy to check that the matrix form of $\rho_s(t)$ is the X form for the above systems.

$$\rho_s(0) = \begin{bmatrix} a & 0 & 0 & \delta \\ 0 & b & \beta & 0 \\ 0 & \beta^* & c & 0 \\ \delta^* & 0 & 0 & d \end{bmatrix} \tag{11}$$

3.3 The DBSCAN algorithm

This is a clustering algorithm based on the density [Ester, Kriegel, Sander et al. (1996)]. Assuming that there is a group of point clusters in the data space, we first set the maximum distance ε between the data points belonging to the same clusters, determine the reachable points within the radius ε of each point, and take the points surrounded by at least *m* points in the radius ε as the core points. If a point with sufficient points surrounded, then its ε neighborhood belongs to the same clusters, otherwise set the point as noise. The process continues until all the clusters are found. Here, we use the DBSCAN to select the states having the same Br and RED (*α=2*) values (i.e., for two state A and B, A's Br equals to B's Br and A's RED equals to B's RED) which can generate the freezing phenomenon. The freezing phenomenon means that the values of quantum correlation of the quantum states do not change at the certain time interval, that is, these quantum states have the same values of quantum correlation. The different quantification methods might show different behaviors of the quantum correlation.

Firstly, we calculate the Br and RED values of the same states (total 197640). Considering the numerical computing errors, when the distance between two points is less than 0.0004, we calculate the state having the same RED and Br value and ensure that the number of samples in each cluster is more than 30. In Fig.1, it shows the result of the DBSCAN, and each point in this figure represents a cluster with more than 30 samples at the same value of Br and RED. By applying the DBSCAN, we find that the 64407 samples can generate freezing phenomena.

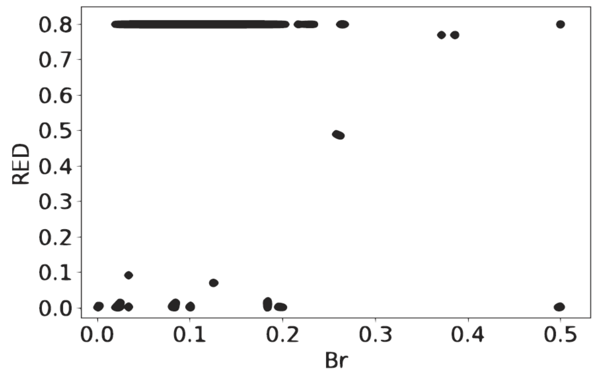

Figure 1: The result of DBSCAN algorithm

3.4 The decision tree method

Here, we use another widely used machine learning techniques for classification problems, i.e., decision tree. The classification and regression tree (CART) is one of the decision tree algorithms. It can construct a tree-structured model from data which is suitable for both classification and regression. In Fig. 2, given a sample and a well-trained decision tree, it starts from the root node and selects different branches according to its characteristics (x, y, z). This ends at leaf node with its outcome. In this case, the outcome is their classes (A, B, C) [Qu, Wu, Liu et al. (2019)].

The purpose of the decision tree is to determine whether the quantum state will be frozen. In this case, we employ the eigenvalues of the density matrix and other characteristics of a quantum state as the features, and feed them into the decision tree. Meanwhile, we also use the Gini index as the measure to select the variable at each step to best segment the project set in the decision tree. For a set of items with J classes, it is denoted as Eq. (12),

$$I_G(p) = \sum_i^J p_{i \sum_{k\neq i} p_k} = 1 - \sum_i^J p_i^2 \,, i \in (1 \dots J),\ i \in (1 \dots J) \tag{12}$$

where p_i denotes the fraction of items labeled with class i in the set, that is, $\sum_{k\neq i} p_k = 1 - p_i$. The reason why we select the Gini index as the metrics rather than information entropy is that it takes less time for calculating.

In this case, we have 197640 data samples which are different X form of the quantum state, of which 64407 can generate freezing phenomenon under Br and RED (*α=2*). We divide them into the training set (129999) and the testing sets (67639). By training the model, we can successfully divide the X form states into freezing state and non-freezing state by their eigenvalues of ρ. The accuracy of the test data is 0.98, the recall rate is 0.98, and the final score is 0.99.

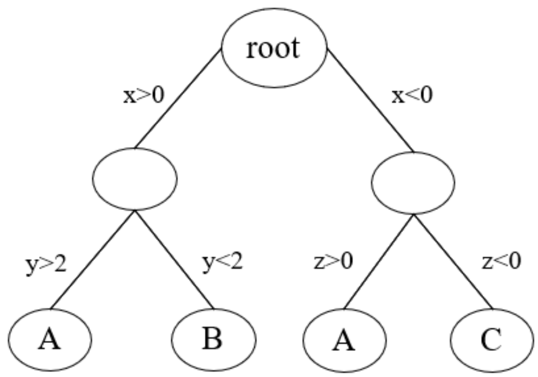

Figure 2: The decision tree workflow

In conclusion, the eigenvalues of the density matrix ρ can be used as a feature to classify the freezing states and non-freezing states.

4 Conclusion

In this paper, we study the freezing phenomenon of quantum correlation for X form quantum states. It is shown that we can use the machine learning method to classify the quantum states into two groups by the eigenvalues of ρ successfully. One is the quantum correlation with freezing phenomenon for both Rènyi discord ($\alpha = 2$) and the geometric discord (Bures distance), the other is without. Finally, our results demonstrate that the machine learning method is a useful tool to resolve the quantum correlation problems.

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