# Application of Radial Basis Function Networks with Feature Selection for GDP per Capita Estimation Based on Academic Parameters

# Abdullah Erdal Tümer<sup>1\*</sup> and Aytekin Akkuş<sup>2</sup>

<sup>1</sup>Necmettin Erbakan University, Department of Computer Engineering, Campus of Köyceğiz, 42140, Konya, Turkey
 <sup>2</sup>Necmettin Erbakan University Department of Industrial Engineering, Institute Of Science, Campus of Köyceğiz, 42140, Konya, Turkey

In this work, a system based on Radial Basis Function Network was developed to estimate Gross Domestic Product per capita. The data set based on 180 academic parameters of 13 Organisation for Economic Co-operation and Development countries was used to verify the effectiveness and accuracy of the proposed method. Gross Domestic Product per capita was studied to be estimated for the first time with academic parameters in this study. The system has been optimized using feature selection method to eliminate unimportant features. Radial Basis Function network results and Radial Basis Function network with feature selection method results were compared. The findings show that the Radial Basis Function network with feature selection is 10% more successful than the Radial Basis Function results. Based on results, this methodology can be applied in applications of Gross Domestic Product per capita forecasting.

Keywords: Estimation, Radial Basic Function, Feature Selection, Gross Domestic Product per capita, Economic.

# 1. INTRODUCTION

For many years measuring the prosperity of societies has become a research object for many researchers, economists, international organizations and institutions (Ivaldi and Santagata, 2017). On the basis of research, scientists and economists have taken into account economic variables such as production, consumption, economic growth and income per capita for measuring welfare (Kuznets, 1947). Gross Domestic Product (GDP) per capita from these variables is an important indicator of economic performance. It is also a useful unit for comparing average living standards and economic well-being. GDP per capita is identified with social welfare and witnesses the "standard of living". (Van den Bergh, 2009). Economic growth must be known so that GDP per capita can be determined. Economic growth is important

both for central government (central bank, government) and for industry-level decision-making. Due to the difficulties of measuring GDP, publication periods are often delayed. This delay is an obstacle to policy makers and market participants that must be ahead of changes in the economy (Barsoum and Stankiewicz, 2015). For this reason, there is a great need for reliable estimation of gdp per capita due to economic growth. However, current forecasting studies often use economic parameters such as imports, exports, spent Money, and product amount (Feng and Zhang, 2014, Milačić et al., 2017, Sokolov-Mladenović et al., 2016). However, it is useful to consider the different possible developments since a wide range of possible factors can affect economic projections. For example, climate changes, greenhouse gas emissions, agricultural sector parameters/data, food demands. Similarly, academic parameters such as the number of academicians, produced articles and patent numbers are considered to be important. As a result of the researches carried

<sup>\*</sup>Corresponding Author. E-mail: tumer@erbakan.edu.tr

out, no studies using academic parameters have been found to predict economic growth or GDP per capita. For example, the authors (Capistrán and López-Moctezuma, 2014) also used 20 macroeconomic variables to estimate growth and inflation and GDP growth. These parameters relate to investment, production, labor markets, public finance and international trade. In (Sinclair et al., 2010), real output growth and inflation are used for real GDP estimation. In (Schumacher, 2010), the author estimated the German GDP. Estimation parameters include quarterly GDP growth, GDP expenditure components, disaggregated gross value, industrial production, received orders, labor market variables, price indices, and financial and survey variables.

In this study, GDP per capita was estimated by using some academic parameters belonging to 13 OECD countries between 1996-2015 periods. Radial Basis Function Network (RBFN) and feature selection method are used for this purpose.

In the past, very different studies have been carried out with RBFN and feature selection methods. For example, in the classification of skin cancers and mammograms (Pratiwi et al., 2015, Thirumavalavan and Jayaraman, 2016), predicted critical temperature, critical pressure and acentric factor of organic compounds (Banchero and Manna, 2018), for global solar radiation estimation (Ramedani et al., 2014), traffic volume forecast (Zhu et al., 2014), for annual electricity demand forecasting. (Yu et al., 2015), predicting the solubility of carbon dioxide in ionic liquids (Tatar et al., 2016) and many more areas (Ahmed et al., 2018, Chen, 2017, Gu et al., 2018, Shi et al., 2017, Zhang et al., 2017) have been successfully used. However, RBFN has not been used previously in GDP per capita estimates.

There are two important innovations in this work. The first is the estimation of GDP per capita by the RBFN method. The second one is the use of academic parameters for the first time in GDP per capita forecast.

The rest of the paper is organized as follows: Section 2, data collection, feature selection, the main elements of the RBFN and performance criterias are described in detail. Section 3 details the result of estimation. Conclusion is given in section 4.

#### 2. METHODOLOGY

#### 2.1 Data Collection – Input Parameter

GDP per capita estimation is not investigated in literature thus there is a need to build a reliable GDP per capita estimation model. The main aim of this paper is to overcome high nonlinearity of the GDP per capita forecasting by applying the ANN with RBF and feature selection method. To accomplish this, a network structure is built with input parameters education level, number of paper per capita, researcher per employed, research & development expenditure as the percentage of GDP and number of patents per capita. In this study, data from 13 different countries within time period 1996 to 2015 are used. Total number of data is 180. Table 1 shows input and output parameters which are used in this study with detailed statistical. In all, 135 data are used for training and remaining 45 data are used for simulation. All data is distributed randomly for training and simulation to get better results from RBF networks. As it can be seen from Table1, data for each parameter can be approximated with normal

Table 1 Input parameters and summary statistics.

Input Name	Min	Max	Mean	Std. Dev.
Education level	8.34	52.97	29.95	11.28
Number of paper per capita	73.05	3404.86	1463.81	829.88
Researcher per em- ployed	0.7	13.74	6.91	2.92
R&D Expenditure	0.35	4.28	1.93	0.92
Number of patents per capita	4.31	3279.25	654.01	915.48

distribution.

Education level, researcher per employed and R&D Expenditure (% of GDP) data has been taken from OECD open data web site (https://data.oecd.org/) (Oecd.org, 2019). Number of paper per capita is calculated by using data taken from Scimagorj web site (http://www.scimagojr.com/countryrank.php) (Scimagojr.com, 2019). Number of patents per capita is determined by using data taken from World Bank open data web site (https://data.worldbank.org/) (Worldbank.org, 2019).

## 2.2 Feature Selection

In machine learning and statistical feature selection, a model is constructed when the effects of the input parameters that affect the result are determined and the ineffective ones are eliminated. Feature selection is also called variable selection. The main bearing of the implementation of feature selection techniques is to remove unnecessary or irrelevant features in the data set without causing any loss of information (Khan and Baig, 2015). Feature selection methods should not be confused with feature extraction. While feature selection gives a subset of features, feature extraction produces new features from existing features (Erguzel et al., 2015, Xu et al., 2014). Proper selection of features is important for the inductive learner to improve learning speed, generalization capacity, or model simplicity. In addition, working with fewer features also has other benefits such as reduced cost of measurement and better understanding of the problem area. Some situations can prevent feature selection, such as the presence of a crescent number or irrelevant features, noise in the data, redundancy and interaction between attributes.

In this study, firstly GDP per capita was investigated to be estimated by means of RBFN method with 5 different academic parameters. Then, the properties of the parameters used for estimating GDP per capita were analyzed by the feature selection method in order to find the most useful parameters. The software used for analysis is WEKA data mining software. An attribute evaluator named "CorrelationAttributeEval" and the factors affecting GDP per capita of the input parameters are determined according to ranked by importance. "CorrelationAttributeEval" evaluates an attribute value by measuring the correlation between it and the parameters that affect it. The results are shown in Table 2.

According to Table 2, the educational level is the most effective parameter, while the effect of number of patents per capita seems almost irrelevant. In the first model developed, all of the

Table 2 Input parameters and ranked attributes.

Input Parameter	Rate
Education level	0.7517
Number of paper per capita	0.6983
Researcher per employed	0.6792
R&D Expenditure (% of GDP)	0.5782
Number of patents per capita	0.0833

input parameters were used, while in the second model the number of patents per capita parameter was eliminated. The results are compared with the preferred RMSE and  $R^2$  performance criteria in a number of studies (Eusterwiemann et al., 2018, Hibbing et al., 2018, Tumer et al., 2017).

### 2.3 Radial Basic Function Network

RBF were firstly introduced into the literature by Broomhead and Lowe in 1988 (Broomhead and Lowe, 1988. The architectural structure of radial base function networks is shown in Fig 1. It consists of 3 layers: input, output and hidden layer. The input layer is the layer to which the inputs apply. The outputs of the input layer are connected directly to the non-linear processing units (neurons) in the hidden layer. The neurons in the hidden layer pass their applied inputs through the radial base function and generate a numerical value. These values are multiplied by linear weights and applied to the output layer. The weighted inputs arriving at the output layer are summed to produce a numerical result.

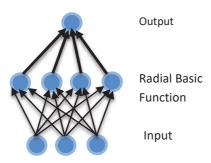


Figure 1 The architecture of RBNN.

In RBFN models, many function types can be used as activation function. Linear, Cubic, Gaussian, Multi-Quadratic, Inverse Multi-Quadratic functions are some of them. In this study, the most preferred Gaussian function (Turnbull and Elkan, 2005) is used in the literature such as stochastic computing (Ji et al., 2015), classification (Li et al., 2016) (Cheruku et al., 2017), control (ul Amin et al., 2017), image processing (Rashidi et al., 2016), modeling (Wu et al., 2016), (Yun et al., 2018), exc.. Its formula is as follows:

$$\varphi(x_i) = \exp\left(-\frac{||x_i - c_j||^2}{2 \cdot \sigma_j^2}\right) \tag{1}$$

Where xi is represent points in the multidimensional real space and cj is the center of the function and  $\sigma$  is the dispersion of the Gaussian function.

From Eq. (1), it is clear that  $\varphi(x_i)$  can obtain the maximum value when xi = cj. And  $\varphi(x_i)$  decreases with the increase of ||xi - cj||. Therefore, only the input vector x that is near the center of the radial basis function can activate the nodes of the hidden layer (Zhao et al., 2015).

#### 2.4 Performance Criteria

There are several methods to evaluate the performance of ANN. Root Mean Square Error (RMSE) and Coefficient of Determination ( $\mathbb{R}^2$ ) are most widely used methods to determine performance of RBFN (Tümer and Koçer, 2017) and the other machine learning algorithms (Tasdemir et al., 2011) (Dincer et al., 2008, OZKAN and KOKLU, 2017, Saritas et al., 2009). Thus,  $\mathbb{R}^2$  and RMSE are used as performance criteria in this study. Formulation of these performance measures are displayed below.

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (Y_{\exp,i} - Y_{prd,i})^2}$$
(2)

$$R^{2} = 1 - \frac{\sum_{i=1}^{n} (Y_{prd,i} - Y_{exp,i})}{\sum_{i=1}^{n} (Y_{prd,i} - Y_{m})^{2}}$$
(3)

where Yprd is predicted data, Yexp and Ym are the measured and average of the batch study data respectively. n is the number of data.

### 3. RESULT

In this section, the academic parameters shown in Table 1 and the above methodology will be used to investigate the results of the estimation of GDP per capita. For the two-step estimate, 180 data from 13 OECD countries were used. Of the 180 data, 135 were selected for training and 45 were selected for testing (simulating) purposes. Prediction models have been developed with the WEKA program (Witten et al., 2016). WEKA is available at https://www.cs.waikato.ac.nz/ml/weka/downloading.html (Waikato.ac.nz, 2019). In the first stage, estimations are carried out using RBFN method with 5 different input parameters in Table 2. In the second stage, the unimportant/irrelevant parameters were investigated by the feature selection method. Table 2 shows the importance of the parameters as a percentage. In Table 2, 0.083 values of number of patents per capita were considered unimportant and GDP per capita was again estimated using RBFN method with 4 important academic parameters. Both estimates are compared using RMSE and R<sup>2</sup> performance criteria. Table 3 shows a comparison of the results of training and testing (before and after the application of the feature selection method) for two different estimates.

Table 3 shows that the GDP per capita can be estimated very successfully and satisfactorily according to the training and test results when the feature selection method is used with important parameters.

The graph of the values produced by the RBFN predicted values with the real values of the GDP per capita and the coefficient

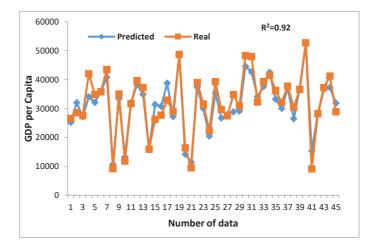


Figure 2 Simulated data and real data for GDP per capita based on feature selection.

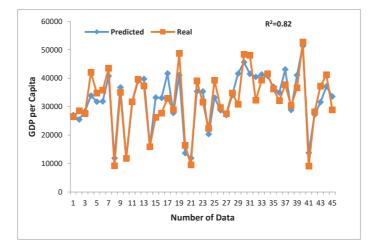


Figure 3 Simulated data and real data for GDP per capita without Feature Selection.

 Table 3 GDP per capita prediction accuracy with RBF models with feature selection and without feature selection.

	<b>RBF</b> with FS		<b>RBF</b> without FS	
	training	test	training	test
RMSE	2153.0932	2985.8336	3976.128	4523.889
$\mathbb{R}^2$	0.95	0.92	0.87	0.82

of determination, which was found as  $R^2 = 0.92$ , are shown in Fig. 2.

Similarly, Figure 3 shows the RBFN model tested using the simulated data that were not shown to the network.

According to the results, GDP per capita can be estimated with accuracy of 92%. Estimated accuracy was 82% with 5 academic parameters used for GDP per capita estimation. Number of patents per capita, one of the five academic parameters, was found to be unimportant by the feature selection method. In the second step (eliminating the unimportant parameter) 4 academic parameters and the GDP per capita estimate were carried out in the same method.

#### 4. CONCLUSION

The result is about 10% more successful prediction. This study shows that GDP per capitan can be successfully predicted by feature selection and RBFN method.

In this study, the GDP per capita forecast is explained using the feature selection approach with the RBF network. Education level, number of paper per capita, researcher per employed, research & development expenditure as the percentage of GDP and number of patents per capita were used as input for the GDP per capita forecast. These inputs were first used with the RBFN method and simulated (with the data not shown to network). The same input data were optimized with the feature selection method and re-estimated without regard to the number of patents per capita parameters determined to be unimportant. Two predicted accuracy levels were compared with RMSE and  $R^2$  performance criteria. RMSE = 4523.889 and  $R^2$  = 0.82 in the simulated results in the first model with RBFN. In RBFN model developed with input data optimized by feature selection method, RMSE = 2985.8336 and  $R^2$  = 0.92 were found. Training and test (simulation) results show that the RBFN model with parameters eliminated by feature selection method can predict GDP per capita by 10% more satisfactory than the inputs used. Feature selection and RBFN methods can be used effectively in GDP per capita estimates.

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