

# Prediction and Abnormity Assertion on EMU Brake Pad Based on Multivariate Integrated Random Walk

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To better solve the issue with abnormal failure of electric motor unit (EMU) brake pad resulted from various random factors in the ever-changing operating environment, in this paper, a new evaluation method of performance prediction and abnormality decision is proposed based on the Multivariate integrated random walk (MIRW) model. In this method, the state space model of the EMU brake pad performance degradation is firstly established. And then based on the observed data, the brake pad performance degradation trend is extracted by the fixed interval forward - backward smoothing algorithm. Based on it, the future degradation state can be predicted by Kalman predictor. Based on the obtained state estimation values, abnormal failure tolerance range (AFTR) can be determined applying Grubbs criterion to judge whether the brake pad is being in abnormal state before reaching the final failure or not as a new sample appears. In addition, the cumulative failure probability of the brake pad is estimated in the degradation process. Finally, the thickness data of a certain type of EMU brake pad is applied to predict the future degradation state and determine the abnormal condition, and the result shows that the proposed method is more efficient and accurate.

Keywords: EMU; brake pad; Abnormal failure; Multivariate integrated random walk; Kalman filter; fixed-interval smoothing algorithm; Grubbs criterion

## 1. INTRODUCTION

In past several years, China high-speed railway undergoes a quite rapid development, the main lines of which is extended towards remote regions and the speed of the existed lines is also increased, constantly, which proposes higher requirements for China railway safety and reliability. In northwest China, climatic condition is harsh more, say, high temperature, high cold, high altitude, and strong sandstorm weather, which generates a serious impact on the performance degradation of electric motor unit (EMU) devices [1]. Moreover, the running speed of high-speed railway is constantly refreshed, such that higher safety demand is required on the braking performance of EMU [2].

Brake pad is one of the key components of braking system, the reliability and safety of which possesses quite significant impacts on EMU operation. The quantity of brake pad installed

in each EMU is very large, for example, CRH2A with four-tractor four-trailer possesses 192 brake pads in all [3], whereas CRH380BL with eight-tractor eight-trailer has 320 brake pads [4]. According to the current maintenance schedule in China, the daily maintenance cycle of EMU is two days alone, which generates the workload heavier. In addition, the initial cost of the brake pad is quite high and its replacement rate is the most frequent for the maintenance of all the EMU components [5]. With EMU running continuously, the performance degradation of the brake pad is constantly accumulated. Although it does not fail at that time, the failure probability is increasing gradually. Hence, it is necessary to investigate the dynamic performance degradation level of brake pads to ensure safe operation of EMU braking systems before it is finally replaced, which is the problem to expect to be solved for EMU industry.

Train condition monitoring system is proposed in [6], where the brake pad wear monitoring system (BWMS) can monitor the

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condition of brake pads, automatically. But the assembling site of the brake pad is hidden, and the system is vulnerable to the external environment interference so that the image capture is difficult. In [5], the damage recording of the EMU main components operation and repair process is collected and analyzed under the particular environment in China, and then based on the running kilometrage of EMU, the life-expectancy of trailer brake pad is estimated. In [7], a life prediction model is established based on the gray system theory, and the collected thickness data is used to predict the life cycle of the brake pad. In [8], under the condition that the relevant factors affecting brake pad wear are considered, machine learning algorithm is used to establish the estimation model of the brake pad wear amount for the remaining useful life prediction. However, the studies above are lack of the consideration of the randomness and uncertainty in the degradation process, and abnormal failure working condition of the brake pad is ignored.

In [9], the mean value of device degradation in the future is directly predicted using the Kalman filter based on the historical status and data of the device itself. But the data is not preprocessed, as a result, an obvious prediction error is then generated. In [10], in order to deal with the limitations and mislead results that most reliability assess of the circuit connector is in accordance with certain standards, the physical failure model and accelerated degradation test are combined to obtain the reliability estimation of the electrical connector with the data from the accelerated degradation test.

It can be seen from the above that the observational or experimental data of the device degradation are directly used to predict the performance reliability, but during the actual operating, it is inevitable for the equipment to suffer from various random factors from the running and serving environment, such that the measurement data containing noise.

Hence, to evaluate the reliability of brake pad accurately, it is necessary to implement preprocess on the measurement data to obtain an accurate estimate of the degradation level. In this paper, the characteristics of multivariate integrated random walk (MIRW) model are applied [11]. Based on MIRW, the state space model of device performance degradation is established, and then Kalman fixed-interval smoothing algorithm is firstly applied to extract the trend of device performance degradation. Further, Kalman filter is used to predict and estimate the degradation state in future. In addition, according to the state estimation obtained, the abnormal failure tolerance range(AFTR) is determined to judge that whether the abnormal failure occurs in the device degradation process before it reaches a critical degradation failure threshold or not. Based on the obtained results above and the failure probability of device in the degradation process, the cumulative failure probability is estimated and the dynamic reliability is analyzed. Finally, brake pad thickness data of a certain type of EMU is applied to verify the effectiveness of the method proposed.

## 2. MODELLING

Under normal circumstances, many factors, say, random background noise, the seasonal changes, and sudden interference, and etc, will affect the reality of performance degradation data

acquired by the monitoring devices [12]. Therefore, to obtain the true trend and characteristics of the device degradation, it is necessary to implement preprocessing on the observed data of performance degradation.

On one hand, the performance degradation of the brake pad is mainly affected by the number of passengers, and the vehicle weight, and the EMU initial braking speed, the number of braking, and as well as the specific braking operation details, which lead the brake pad to be worn constantly, such that the wear situation is different in each running. On the other hand, under the interference of various random factors, there is a certain error existing in the thickness data observed for the brake pad during the measurement. Thus the degradation process can be regarded as a stochastic dynamic process with multivariate continuously varying over time [13]. The brake pad thickness observing data can be modeled as an original structure time series, and represented by  $y(t)$ . The orthogonal decomposition of  $y(t)$  produces the trend component  $y_t(t)$  that affects the mean value of  $y(t)$  permanently, and the seasonal component  $y_s(t)$  that fluctuates around  $y(t)$  periodically, and the transient interference component  $y_p(t)$  due to the external disturbance, and as well as white noise  $e(t)$ , such that  $y(t)$  can be described by

$$y(t) = y_t(t) + y_s(t) + y_p(t) + e(t) \tag{2.1}$$

Although all components in (1.1) are potentially important, this does not mean that they all are used to fully describe the performance degradation of the brake pad. Hence, after ignoring  $y_s(t)$  and  $y_p(t)$  which does not indeed exist in the time series, the degradation process of the brake pad will be integrated in the sum of  $y_t(t)$  and  $e(t)$ , as shown by

$$y(t) = y_t(t) + e(t) \tag{2.2}$$

Since the performance tendency of brake pad can reflect the degradation processes which is characterized by a low frequency. In this paper, MIRW is used to simulate the low frequency of the brake pad degradation trend so as to build the state space model, which can be shown by

$$x(t + 1) = \Phi x(t) + \Gamma \eta(t) \tag{2.3}$$

$$y(t) = Hx(t) + v(t) \tag{2.4}$$

In (1.3),  $x(t + 1)$  expresses the state vector of the brake pad performance degradation at time  $t + 1$ , and  $x(t + 1) = \begin{bmatrix} \mu(t + 1) \\ \beta(t + 1) \end{bmatrix}$ , wherein  $\mu(t + 1)$  expresses the performance level of the brake pad degradation at time  $t + 1$ , and  $\beta(t + 1)$  indicates the degradation speed of the brake pad at time  $t + 1$ , and  $\eta(t)$  is the process noise, and  $\Phi$  and  $\Gamma$  are the state transition matrix and input matrix, respectively. In (1.4),  $y(t)$  is the observation vector of the brake pad thickness at time  $t$ , and  $H$  is the observation matrix, and  $v(t)$  is the observation noise. Assuming that  $\eta(t)$  and  $v(t)$  be uncorrelated white noise with zero mean and covariance matrix  $Q$  and  $R$  respectively, and satisfy

$$\left. \begin{aligned} E[\eta(t)] &= E[v(t)] = 0 \\ E[\eta(t_j)\eta^T(t_k)] &= \mathbf{Q}_j \delta_{jk} \\ E[v(t_j)v^T(t_k)] &= \mathbf{R}_j \delta_{jk} \\ E[\eta(t_j)v^T(t_k)] &= 0 \end{aligned} \right\} \tag{2.5}$$

where  $\delta_{jk}$  is the Kroneker coefficient, and  $\delta_{jk} = \begin{cases} 1, & t_j = t_k \\ 0, & t_j \neq t_k \end{cases}$ .

In addition, the initial state  $x(0)$  be irrelevant to  $\eta(t)$  and  $v(t)$ , and satisfy

$$E[x(0)] = \mu_0, \quad E[(x(0) - \mu_0)(x(0) - \mu_0)^T] = P_0 \quad (2.6)$$

Due to the extensive adaptability and importance of MIRW, when the observation equation is only composed with  $y_t(t)$  and  $e(t)$ , Kalman filter can be used to predict and correct it. In (1.3) and (1.4),  $\Phi$ ,  $\Gamma$ , and  $H$  owns the following form, respectively.

$$\Phi = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad H = [1 \quad 0].$$

The observed signal is usually mixed with random noise due to the uncertain factor in the actual operation and observation devices. It is very difficult to extract accurately the state value from the observed signal with the mixture of random noise. And so what we can do is just to estimate the degradation state on the basis of the observed signal. In order to make the state estimation more close to the true value, Kalman filter is applied to obtain the linear minimum variance estimation  $\hat{x}(t|t)$  of  $x(t)$  based on the observation vector  $y(t)$ , and the minimal performance indicator is expressed as [14, 15]

$$J = E[(x(t) - \hat{x}(t|t))^T(x(t) - \hat{x}(t|t))] \quad (2.7)$$

### 3. DEGRADATION STATE ESTIMATION

#### 3.1 Extracting the degradation trend

Kalman fixed interval forward - backward smoothing algorithm is a process of 'prediction - correction' continuously, and can be applied to extract the degradation trend of the brake pad. Based on the MIRW model, and considering the measurement error, this paper implements the preprocessing on the observed thickness data of the brake pad to extract the degradation trend, and performs prediction and estimation for the future degradation level.

##### 3.1.1 Backward smoothing

To estimate the optimal smoothing value of the degradation state  $\hat{x}(t|N)$ , the backward smoothing algorithm is adopted and shown by [16]

$$\hat{x}(t|N) = \hat{x}(t|t) + A_s(t) \times [\hat{x}(t+1|N) - \hat{x}(t+1|t)] \quad (3.1)$$

$$A_s(t) = P(t|t)\Phi^T P^{-1}(t+1|t) \quad (3.2)$$

$$P(t|N) = P(t|t) + A_s(t) \times [P(t+1|N) - P(t+1|t)]A_s^T(t) \quad (3.3)$$

$$\hat{x}(N|N) = \hat{x}(N|N-1) + K(N)\varepsilon(N), \quad P(N|N) = [I_n - K(N)H] P(N|N-1) \quad (3.4)$$

wherein  $\hat{x}(t|N)$  ( $t = N, N-1, \dots, 1$ ) is the fixed interval optimal smoothing estimation value, and  $A_s(t)$  is the optimal smoothing gain matrix, and  $P(t|N)$  is the error covariance matrix of optimal

smoothing and used to quantitatively evaluate the quality of the fixed interval smoothing estimation.  $\hat{x}(t|t)$  and  $P(t|t)$  may be obtained by Kalman filter described in section 3.1.2.

Finally, we get the trend component of the brake pad performance degradation, as shown by

$$y_t(t) = y(t|t) = H\hat{x}(t|N) \quad (3.5)$$

##### 3.1.2 Forward smoothing

Kalman filtering algorithm is applied to forward smooth recursively to obtain the optimal filtering estimation value of the brake pad thickness, which is described in [14].

$$\hat{x}(t+1|t) = \Phi\hat{x}(t|t) \quad (3.6)$$

$$P(t+1|t) = \Phi P(t|t)\Phi^T + \Gamma Q \Gamma^T \quad (3.7)$$

$$\varepsilon(t+1) = y(t+1) - H\hat{x}(t+1|t) \quad (3.8)$$

$$K(t+1) = P(t+1|t)H^T [HP(t+1|t)H^T + R]^{-1} \quad (3.9)$$

$$\hat{x}(t+1|t+1) = \hat{x}(t+1|t) + K(t+1)\varepsilon(t+1) \quad (3.10)$$

$$P(t+1|t+1) = I_n - K(t+1)H P(t+1|t) \quad (3.11)$$

$$\hat{x}(0|0) = \mu_0, \quad P(0|0) = P_0 \quad (3.12)$$

wherein  $\hat{x}(t+1|t+1)$  represents the optimal filtering estimation value of the degradation state and will be used to backward smooth, and  $K(t+1)$  presents the filtering gain matrix obtained by minimizing performance indicator  $J$ , and  $K(t+1)$  and  $\varepsilon(t+1)$  are used to correct the error due to the observation noise in the estimation of  $\hat{x}(t+1|t)$ , and  $P(t+1|t)$  and are respectively the error covariance matrix of filtering and predicting, and  $I_n$  is the unit matrix.

### 3.2 Predicting the future state

Kalman predictor is applied to predict the future degradation state at time  $t+1$  based on the trend component  $y_t(t)$ . And it's given by

$$\hat{x}(t+1|t) = \Phi\hat{x}(t|t-1) + K_p(t)[y_t(t) - H\hat{x}(t|t-1)] \quad (3.13)$$

$$K_p(t) = \Phi P(t|t-1)H^T [HP(t|t-1)H^T + R]^{-1} \quad (3.14)$$

$$P(t+1|t) = \Phi\{P(t|t-1) - P(t|t-1)H^T \times [HP(t|t-1)H^T + R]^{-1} \times HP(t|t-1)\}\Phi^T + \Gamma Q \Gamma^T \quad (3.15)$$

$$\hat{x}(1|0) = \Phi\hat{x}(0|0), \quad P(1|0) = \Phi P(0|0)\Phi^T + \Gamma Q \Gamma^T \quad (3.16)$$

where  $\hat{x}(t+1|t)$  is the optimal prediction of degradation state,  $K_p(t)$  is Kalman predictor gain matrix,  $P(t+1|t)$  is the error covariance matrix of optimal prediction.

Therefore, we have the optimal thickness prediction of the brake pad at the future time  $t+1$  that we are looking forward to getting, which is shown by

$$y(t+1) = H\hat{x}(t+1|t) \quad (3.17)$$

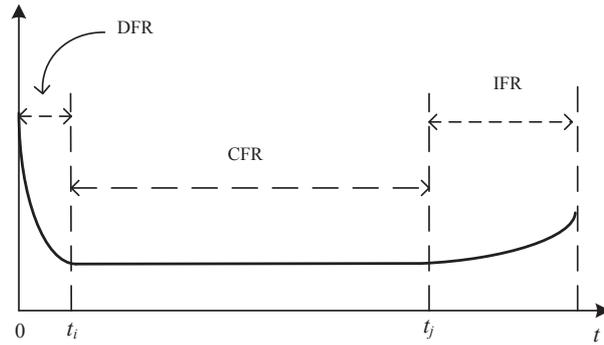


Figure 1 Failure rate curve of mechanical wear equipments.

#### 4. ABNORMAL CONDITION DETERMINATION

##### 4.1 Related work

For most of mechanical wear equipment, they roughly possess the failure rate curve as shown in Fig. 3 [17].

Seen from Fig. 1, the overall life cycle model of the device can be distributed into three stages, that is, the infant (DFR, decreasing failure rate with time), and middle age (CFR, constant failure rate with time), and ageing (IFR, increasing failure rate with time). If we ignore the infant death, and then it will start to work from middle age, up to ageing. In the middle age, the device will possess a low and basically stable failure rate, and approximates a constant. As entering into ageing, the device will possess an abruptly increasing failure rate, and finally be replaced as it reaches a predefined failure threshold beforehand [17]. In this paper, we pay attention to the failure in middle age with constant failure rate and generally following the exponential. Clearly, this just is the abnormal failure. And contrary, if the failure occurs in ageing, and then it is called as normal one.

Let the true degradation level of the device follow the exponential distribution, which can be described by

$$z(t) = z_0 e^{-\lambda t} \tag{4.1}$$

where  $z_0$  represents the initial performance value at time  $t_0$ , and  $z(t)$  is the one in  $t$ , and  $\lambda$  is the failure rate of the devices.

For time  $t_i$ , we have

$$z_i = z_0 e^{-\lambda t_i} \tag{4.2}$$

Similarly, for time  $t_{i+1}$ , we have

$$z_{i+1} = z_0 e^{-\lambda t_{i+1}} \tag{4.3}$$

And so, we easily obtain

$$\frac{z_i}{z_0} = e^{-\lambda t_i} \tag{4.4a}$$

$$\frac{z_{i+1}}{z_0} = e^{-\lambda t_{i+1}} \tag{4.4b}$$

Then make the logarithmic processing of (3.4a) and (3.4b) respectively, we get

$$\ln \frac{z_i}{z_0} = -\lambda t_i \tag{4.5a}$$

$$\ln \frac{z_{i+1}}{z_0} = -\lambda t_{i+1} \tag{4.5b}$$

From (3.5a) and (3.5b), we then obtain

$$\ln z_{i+1} - \ln z_i = -\lambda (t_{i+1} - t_i) = -\lambda \Delta t_{i+1} \tag{4.6}$$

where  $\Delta t_{i+1}$  is the  $(i + 1)$  the time interval.

Let

$$\tilde{\Delta}_{i+1} = \ln z_{i+1} - \ln z_i = -\lambda \Delta t_{i+1} \tag{4.7}$$

where  $\tilde{\Delta}_{i+1}$  is the logarithm increment of the device degradation level.

Specially, as time intervals are taken equal, we then have

$$\tilde{\Delta}_{i+1} = -\lambda \Delta t \tag{4.8}$$

On the other hand, let  $\Delta_i$  be the observed logarithm increment of the degradation level between two-adjacent non-overlapping time intervals, and  $y_i$  is the observed quantity, then we have

$$\Delta_{i+1} = \ln y_{i+1} - \ln y_i \tag{4.9}$$

And so,

$$E[\Delta_{i+1}] = \tilde{\Delta}_{i+1} = -\lambda \Delta t \tag{4.10}$$

Let us define the performance degradation sequence as

$$\Delta = [\Delta_1, \Delta_2, \dots, \Delta_n] \tag{4.11}$$

Clearly, each performance degradation quantity possesses same the expected value and follows same distribution, which leads to

$$\begin{aligned} \lim_{k \rightarrow \infty} F_k(\Delta) &= \lim_{k \rightarrow \infty} P \left\{ \frac{\Delta_i - E(\Delta_i)}{\sigma} \leq \Delta \right\} \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\Delta} e^{-\frac{t^2}{2}} dt = \Phi(\Delta) \end{aligned} \tag{4.12}$$

where  $F_k(\Delta)$  is the distribution of  $\Delta$ , and  $\Phi(\Delta)$  is standard normal distribution function, and  $\sigma$  is the finite variance, which can be estimated by

$$\sigma = E(s) = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (\Delta_i - \bar{\Delta})^2} \tag{4.13}$$

where  $s$  is the sample variance, and  $\bar{\Delta}$  is the estimated value of  $E(\Delta_i)$ , namely, the average value of samples.

As a matter of fact, let  $Y_1, Y_2, \dots, Y_n$  be the observing sequence with  $E[|\bar{\Delta}|] < \infty$ , and then we can say

$$Z_n = E[\bar{\Delta}|Y_1, Y_2, \dots, Y_n], n \geq 1 \tag{4.14}$$

is a martingale, and as Doob martingale [18]. Hence

$$\begin{aligned} E[Z_{n+1}|Y_1, Y_2, \dots, Y_n] &= E[E[\bar{\Delta}|Y_1, Y_2, \dots, Y_{n+1}]|Y_1, Y_2, \dots, Y_n] \\ &= E[\bar{\Delta}|Y_1, Y_2, \dots, Y_n] \\ &= Z_n \end{aligned} \tag{4.15}$$

The formula shows that  $\bar{\Delta}$  can be revised constantly as new observing value arrives at, but  $E[|\bar{\Delta}|]$  remain unchanged.

### 4.1.1 Grubbs criterion

The Grubbs principle is adopted to exclude the bad observational value  $\Delta_i$  corresponding to the desired value  $E(\Delta_i)$  when the number times of observation is smaller ( $n < 20$  times)[19]. Let  $v_i$  be the residual error and shown by

$$v_i = \Delta_i - \bar{\Delta} \tag{4.16}$$

According to the Grubbs criterion, if  $|v_i| > G\sigma$ , the  $\Delta_i$  should be excluded.  $G$  is Grubbs coefficient and  $\sigma$  is variance of observation data. For instance, as  $G$  is taken by 2.78 for  $n = 17$ , we have 99% reason to believe that the sample should be excluded.

### 4.1.2 Distance judgement law

The distance judgment is to determine that one unknown sample belongs to which known population by computing the distances between the sample and each population [20]. In this paper, Mahalanobis Distance is adopted. Let  $G$  be the  $p$ -dimensional population with the mathematical expectation  $\mu$  and the covariance matrix  $s$ , and then Mahalanobis distance from the  $p$ -dimensional sample vector  $x$  to the population  $G$  is defined by

$$d(x, G) = \left[ (x - \mu)^T s^{-1} (x - \mu) \right]^{\frac{1}{2}} \tag{4.17}$$

Assume that  $G_1$  and  $G_2$  are two different populations with the different mathematical expectation  $\mu_1$  and  $\mu_2$ , and the same covariance matrix  $s$ , and the Mahalanobis distances from  $x$  to  $G_1$  and  $G_2$  be respectively defined as  $d(x, G_1)$  and  $d(x, G_2)$ . And thus the discriminant criterion is expressed as follows

$$\begin{cases} x \in G_1, & \text{if } d(x, G_1) \leq d(x, G_2). \\ x \in G_2, & \text{if } d(x, G_1) > d(x, G_2). \end{cases} \tag{4.18}$$

The square difference of  $d(x, G_1)$  and  $d(x, G_2)$  is shown

$$\begin{aligned} d^2(x, G_2) - d^2(x, G_1) &= (x - \mu_2)^T s^{-1} (x - \mu_2) - (x - \mu_1)^T s^{-1} (x - \mu_1) \\ &= 2(x - \bar{\mu})^T s^{-1} (\mu_1 - \mu_2) \end{aligned} \tag{4.19}$$

where  $\bar{\mu} = \frac{1}{2}(\mu_1 + \mu_2)$ .

Let  $W(x) = (x - \bar{\mu})^T s^{-1} (\mu_1 - \mu_2)$ , then (3.16) is simplified as

$$\begin{cases} x \in G_1, & \text{if } W(x) \geq 0. \\ x \in G_2, & \text{if } W(x) < 0. \end{cases} \tag{4.20}$$

## 4.2 AFTR determination

Statistical process monitoring technology is applied in anomaly detection of industrial process based on data [21]. The device degradation increments obtained from the device itself are different in different time intervals, which basically reflect the deterioration characteristics of the device. Therefore, the reasonable and reliable AFTR is available by statistical analysis for equipment performance degradation data. It is necessary to get a quantitative indicator which can determine whether there is abnormal working condition or not.

We can obtain the root mean square deviation according to  $\Delta_j$  before time  $t_{i+1}$  and the average value  $\bar{\Delta}$ , which is shown by

$$s_i = \sqrt{\frac{1}{i-1} \sum_{j=1}^i (\Delta_j - \bar{\Delta})^2} \tag{4.21}$$

Let  $SND$  represent the standard deviation of degradation increment, which can measure the deviation degree of the increment in each time interval compared to the average value of all increments prior to this time [22].  $SND$  is shown as follows

$$SND_i = \frac{\Delta_i - \bar{\Delta}_i}{s_i} \times 100\% \tag{4.22}$$

We know that  $SND_i$  represents the deviation degree of  $\Delta_i$  with respect to  $\bar{\Delta}_i$  and can reflect the overall fluctuation characteristics of device degradation [23]. So in order to analyze the overall degradation characteristics during the degeneration process, it is necessary to calculate the deviation degree of the increment in each time interval.  $\overline{SND}_i$  is shown by

$$\overline{SND}_i = \frac{1}{i} \sum_{j=1}^i |SND_j| \tag{4.23}$$

Thus we may use  $SND_i$  to determine whether there is abnormal failure or not in the device. Compare  $|SND_{i+1}|$  (the module value of  $SND_{i+1}$ ) to  $\overline{SND}_i$ , if the difference calculated exceeds a certain range, we can consider that there is abnormal condition in the device at time  $t_{i+1}$ . This certain range is the effective AFTR to determine.

To measure the dispersion degree of  $SND$  until time  $t_{i+1}$ , the former  $i$ -mean-square-deviation is calculated by

$$S_{SND_i} = \sqrt{\frac{1}{i-1} \sum_{j=1}^i |SND_j| - \overline{SND}_i^2} \tag{4.24}$$

In general,  $\Delta$  should not possess dramatic change during equipment normal operation. Once an abnormal condition occurs,  $SND$  will fluctuate, evidently. Therefore, let

$$SND_{SND_i} = \frac{|SND_i| - \overline{SND}_i}{S_{SND_i}} \tag{4.25}$$

Clearly,  $SND_{SND_i}$  follows  $N(0,1)$ , and represents the fluctuation degree of  $|SND_i|$  with respect to  $\overline{SND}_i$ . Based on it, according to Grubbs principle, we can define three different population  $G_1$ , and  $G_2$ , and  $G_3$  with the different mathematical expectation zero, and  $G_{SND_i}$ , and  $-G_{SND_i}$ , and as well as the same covariance 1. In fact, considering the symmetry of the distance judgement between a new sample belonging to  $G_1$  or  $G_2$ , and to  $G_1$  or

$G_3$ , it is feasible for us that only two population is involved, for instance,  $G_1$  and  $G_2$ . Thus, as a new sample  $SND_{SND_{i+1}}$  occurs, we firstly compute

$$SND_{SND_{i+1}} = \frac{|SND_{i+1}| - \overline{SND}_i}{SSND_i} \tag{4.26}$$

Then computing

$$d(SND_{SND_{i+1}}, G_1) \leq d(SND_{SND_{i+1}}, G_2) \tag{4.27}$$

We can say the new sample belongs to  $G_1$  and no abnormal failure occurs, and inversely, we can say that the new sample belongs to  $G_2$  and an abnormal failure appears.

Fig. 2 shows the algorithm flow diagram to predict and determine whether there is abnormal failure occurring, using device performance degradation data. The entire procedure of the algorithm illuminates that it can be effectively applied to predict the abnormal failure of devices, such as EMU braking pads and roulettes [24], and large-scale wind power system [25], etc, and is the basis of the follow-up work of maintenance strategy making.

### 5. RELIABILITY PREDICTION

The operation reliability of the same equipment must be different under different operating conditions and environment [26]. Reliability refers to the ability of a device or system to perform a defined function within a predetermined time under specified conditions [27]. The operation of the equipment will be affected by environmental and self-performance and other random factors, such that the reliability will also change with the running time. Dynamic reliability describes the dynamic evolution of a device or system over time and studies the effect of failure, maintenance and associated control measures on the dynamic characteristics of the equipment or system [28].

Fig. 3 shows the schematic diagram of the device performance degradation, clearly, the degraded performance value  $y_t$  is gradually decreasing with the operation of device. Although the device hasn't yet expired before reaching the critical degradation failure threshold  $y_C$ , the failure probability is increasing from the change in the shaded area.

Let  $F(t + 1|t)$  be the cumulative failure probability in the future time  $t + 1$ . And the critical failure threshold is usually defined as a constant, noted by  $y_C$ . When the thickness is less than  $y_C$ , we think that the critical degradation failure occurs and relevant maintenance measures are required [29].

For the brake pads belonging to the kind of device whose performance indicators reduce gradually with the running time,  $F(t + 1|t)$  is the estimation value of the conditional probability at time  $t + 1$  due to the continuous degradation when the current thickness estimation is less than the critical failure threshold, which can be shown by

$$F(t + 1|t) = P\{y(t + 1) \leq y_C\} = \int_{-\infty}^{y_C} f(y(t + 1)) dy \tag{5.1}$$

where  $f(y(t + 1))$  indicates the probability density function of  $y(t + 1)$  and follows normal distribution. The mean value and

variance of the degradation can be calculated using the statistical analysis in the former sections.

The reliability and the cumulative failure probability of the brake pad at time  $t + 1$  are opposite, so we have

$$R(t+1|t) = 1 - F(t + 1|t) = 1 - \int_{-\infty}^{y_C} f(y(t + 1)) dy \tag{5.2}$$

where  $R(t + 1|t)$  presents the reliability that the brake pad will normally operate at the  $(t + 1)^{th}$  time interval.

### 6. EXAMPLE

In terms of the high-speed railway EMU, the braking system determines the security and stability of the EMU. And the brake pad is one of the key parts of the braking system whose performance will directly reflect the function of the braking system. Each actual braking operation will inevitably lead to the wear of the brake pad so that the gradual performance degradation of which is accumulated continuously. Eventually, the failure occurred. Therefore, taking the thickness degradation data of a certain type of the EMU brake pad for example, the state space model of the performance degradation based on MIRW is established in this paper to extract the degradation trend. And the future degradation state is predicted and estimated. Based on the obtained state estimation values, the AFTR can be determined applying Grubbs criterion to judge whether the brake pad is being in abnormal state before reaching the final failure or not as a new sample appears. Finally, the reliability of the brake pad is predicted, which is significant to ensure that the EMU braking system operates safe and reliable.

Fig. 4 indicates three groups of thickness data of a certain type of EMU brake pad, wherein the signal '—' presents the measurement data which are sampled every two days and obtained from CRRC (Customer-oriented Responsible Reliable Creative) Qingdao Sifang rolling stock research institute, and '\*' presents the result obtained with the evaluation method proposed in this paper, which is that the performance degradation trend of the brake pad is extracted by the fixed interval forward - backward smoothing algorithm. And the thickness estimation which is obtained just by the Kalman filter directly without extracting the degradation trend expressed by '◇'.

As we can see from Fig. 4, the brake pad is gradually worn due to the repeated braking until the final failure throughout the lifecycle. The result of the evaluation method is more consistent with the practical degradation situation than the result obtained by directly filtering, especially in the later period, the latter is rather scattered.

The covariance of the brake pad thickness obtained at each sampling time is showed in Fig. 5. It is quite clear that the MSE obtained by fixed interval forward - backward is significantly less than that the one obtained by filtering directly. And the covariance obtained by the two methods is convergent, that is to say, Kalman filtering algorithm can deal with Gaussian noise efficiently.

The above analysis suggested that the evaluation method can reduce the estimation error and improve the estimation accuracy. Actually, the brake pad may lose a predetermined braking capacity due to the various uncertain factors before reaching the

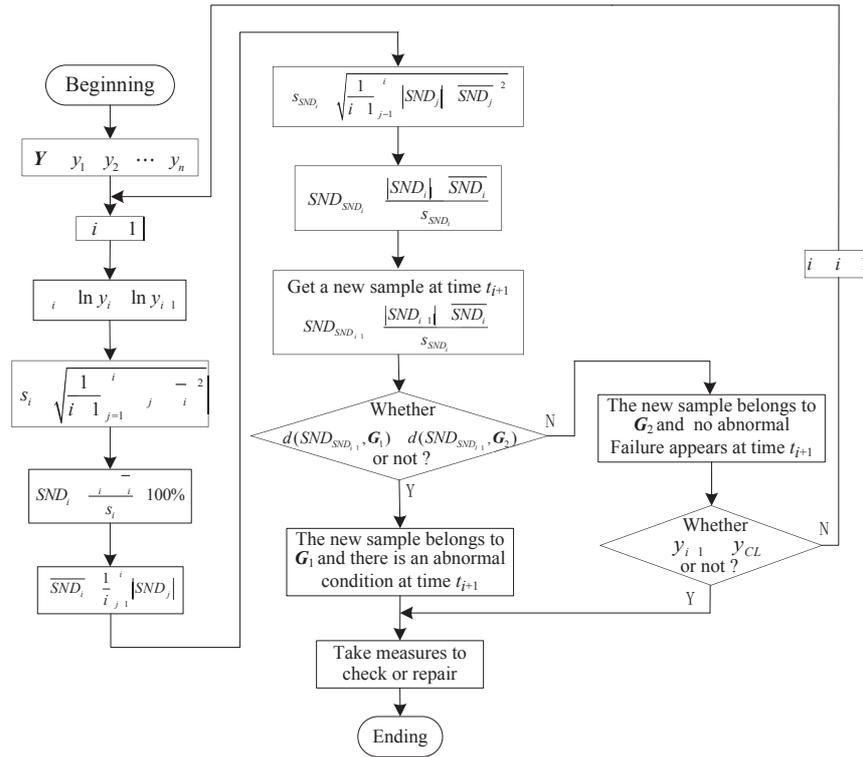


Figure 2 Flow diagram of the algorithm predicting the abnormal failure.

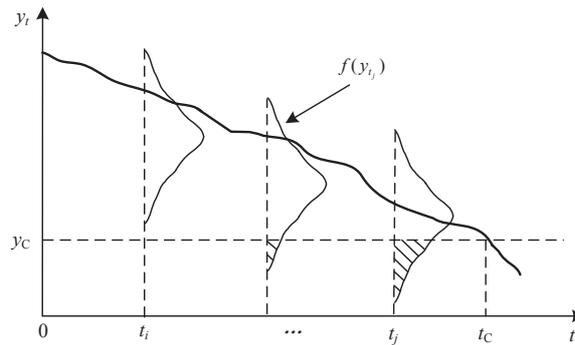


Figure 3 Degradation trend diagram.

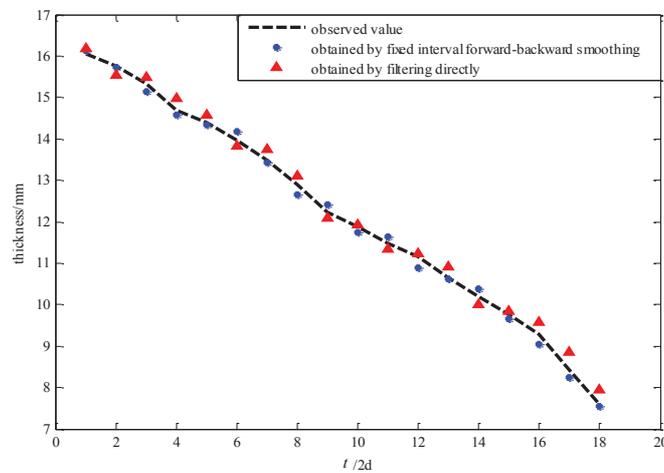


Figure 4 Thickness data of the EMU brake.

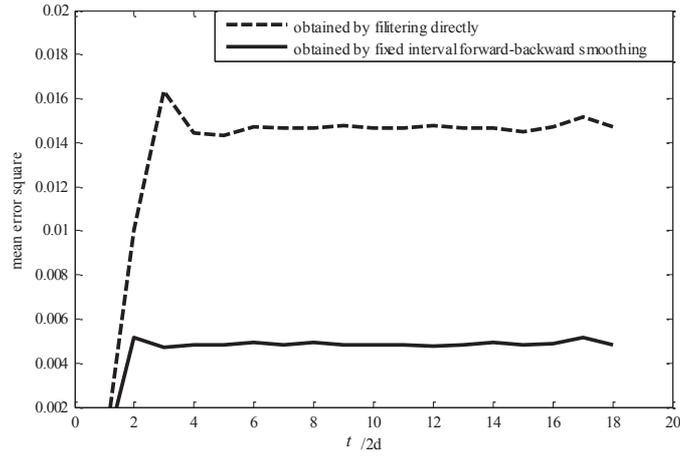


Figure 5 Comparison result of error mean square value.

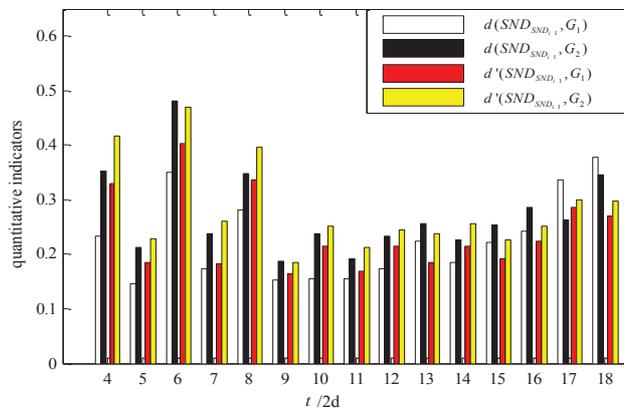


Figure 6 Judgement of abnormal failure based on distance judgement law.

Table 1 Grubbs coefficient(P=99%) [18].

|          |      |      |      |      |      |      |      |      |
|----------|------|------|------|------|------|------|------|------|
| <i>n</i> | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   |
| <i>G</i> | 1.16 | 1.49 | 1.75 | 1.94 | 2.10 | 2.22 | 2.32 | 2.41 |
| <i>n</i> | 11   | 12   | 13   | 14   | 15   | 16   | 17   |      |
| <i>G</i> | 2.48 | 2.55 | 2.61 | 2.66 | 2.70 | 2.75 | 2.78 |      |

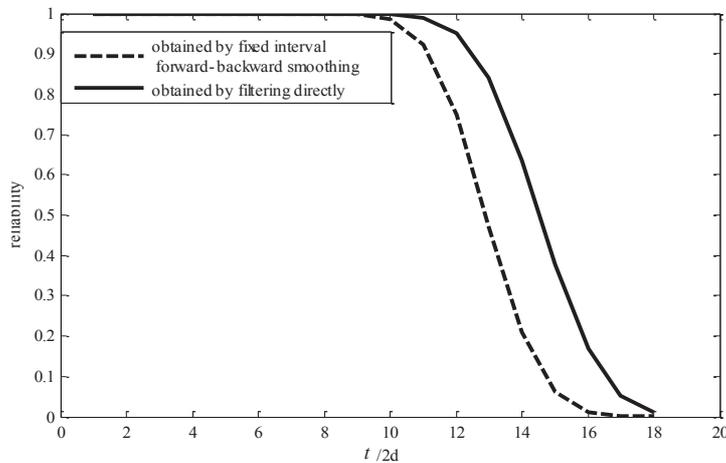


Figure 7 Reliability of the brake pad.

critical threshold. Then, based on the estimation thickness value obtained by the two methods, two kinds of abnormal failure judgment methods proposed in this paper are adopted to determine whether the abnormal failure occurs in the brake pad.

Fig. 6 indicates the comparison of the judgement of abnormal failure, where  $d(SND_{SND_{i+1}}, G_1)$  and  $d(SND_{SND_{i+1}}, G_2)$  respectively present the distance from  $|SND_{i+1}|$  to  $\overline{SND}_i$  and the AFTR at time  $t + 1$  based on the estimated value which is obtained by the fixed interval forward - backward smoothing,  $d'(SND_{SND_{i+1}}, G_1)$  and  $d'(SND_{SND_{i+1}}, G_2)$  present the ones that obtained by directly filtering, respectively. And when the Grubbs coefficient  $G$  is taken from the Table 1 which is shown as follows, we have 99% reason to believe that the sample should be excluded [18].

From Fig. 6, we can see the judgement of abnormal failure can start from the 4<sup>th</sup> sample since the initial value of  $s$  and  $SND$  may be zero. In forward-backward smoothing algorithm,  $d(SND_{SND_{i+1}}, G_1)$  does not exceed the AFTR  $d(SND_{SND_{i+1}}, G_2)$  from the 4<sup>th</sup> to the 16<sup>th</sup>, but from the 17<sup>th</sup> sampling time interval,  $d(SND_{SND_{i+1}}, G_2)$  exceeds  $d(SND_{SND_{i+1}}, G_1)$ . Hence, we judge that there is the abnormal failure occurs at the 17<sup>th</sup> sampling time interval. On the other hand, for the Kalman filtering alone,  $d'(SND_{SND_{i+1}}, G_1)$  can never exceed  $d'(SND_{SND_{i+1}}, G_2)$  in the whole process, which suggests that there is no abnormal condition.

As shown in Fig. 7, the reliability of the brake pad is then predicted based on the estimated thickness value obtained by forward - backward smoothing and filtering directly, under the current maintenance mechanism in China that  $y_C$  is formulated by 7mm, and  $f(y(t + 1))$  obeys the normal distribution and the mean value and variance have been obtained before. The reliability based on forward - backward smoothing tends to decrease gradually starting from the 10<sup>th</sup> sampling time interval, such that the reliability of the brake pad at the 16<sup>th</sup> interval approaches to 0, approximately. In another hand, this tends to decrease at the 12<sup>th</sup> sampling time interval using Kalman forward filtering alone and is still greater than 0 at the end of the lifecycle.

As mentioned from the above, the thickness of the brake pad at the 17<sup>th</sup> sampling time is significantly thinner than the former ones. Moreover, the covariance estimation also shows evident fluctuation at the same time. These suggest that the abnormal failure occurs in the brake pad at that moment. In fact, the testing result on - site of the brake pad in EMU section showed that the brake pad has to be replaced at the 17<sup>th</sup> interval, otherwise, it would have a negative impact on other components in the braking system.

From the above analysis, we know that the fixed interval forward - backward smoothing can reduce the estimation error and improve the estimation accuracy of the brake pad thickness. And the judgment method of abnormal failure based on distance judgement law is more consistent with the practical testing result of the EMU brake pad and the replacement and maintenance. The above analysis proves that the evaluation method in this paper is reasonable and valid.

## 7. CONCLUSION

In this paper, the characteristics of the MIRW model are analyzed and the state space model of the performance degradation is established based on the MIRW with the consideration of the uncertainty of the environment and climate and other random factors during the brake pad's operation. In order to improve the estimation accuracy of the EMU brake pad's performance, the degradation trend is extracted with the Kalman fixed interval forward - backward smoothing algorithm and then the future degradation state is predicted and estimated with the Kalman predictor. Furthermore, AFTR is determined to judge that whether there is any abnormal failure before reaching the critical failure threshold. And the reliability of the brake pad is then predicted. Finally, the observed thickness data of a certain type of the EMU brake pad proved that it can improve the estimation accuracy of the thickness. And it is more in line with the actual degradation situation and provides the theoretical guidance for the EMU management industry safely and reliably.

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## REFERENCES

1. Han C., Operation and maintenance requirement research of Lanzhou-Xinjiang high-speed railway for CRH. Dissertation, China Academy of Railway Sciences, Beijing, 2015.
2. Peng J., EMU Traction and Braking. China Railway Publishing House, Beijing, 2013.
3. Ding J, Wang F, Zhang H, et al., Electric Locomotive Mechanical Ppart. Southwest Jiaotong University Press, Chengdu, 2010.
4. Zhang C, Yang Y, Yin F, The saving technology and application effect of EMU Brake pad. Railway Vehicles, 2015, 53(6), pp. 43-44.
5. Wang C, The damage rules and the maintenance cycle of the main components of the High-speed EMU. Beijing jiaotong University, Beijing, 2012.
6. Kuen, Tony K Y, Eddie W K, A novel intelligent train condition monitoring system coupling laser beam into image processing algorithm. Transactions Hong Kong Institution of Engineers, 2006, 13(1), pp.27-33.
7. Liang X, Research on the life prediction model of critical components of EMU based on real-time monitoring. Dissertation, Beijing jiaotong University, Beijing, 2014.
8. Liao T., Research and implementation on key technology of life prognosis for brake pads in EMU. Dissertation, Beijing jiaotong University, Beijing, 2015.

9. Qiu Y, Prediction Method of equipments reliability based on Kalman filter after a degradation failure occurrence. Dissertation, Hefei Technology University, Hefei, 2010.
10. Ren Y, Feng Q, Ye T, et al., A novel model of reliability assessment for circular electrical connectors. *IEEE Transactions on Component, Packaging and Manufacturing Technology*, 2015, 5(6), pp.755-761.
11. NG C. N., YOUNG P. C., Recursive estimation and forecasting of non-stationary time series. *Journal of Forecasting*, 1990, (9), pp.173-204.
12. Lua S, Lub H, Kolarik W, Multivariate performance reliability prediction in real-time. *Reliability Engineering and System Safety*, 2001, (72), pp.39-45.
13. Lu H, Kolarik W, Lu S, Real-time performance reliability prediction. *IEEE Transactions on Reliability*, 2001, 50(4), pp. 353-357.
14. Fu M, Deng Z, Zhang J, Kalman Filtering Theory and Its Application in Navigation System. Science Press, Beijing, 2003.
15. Deng Z, Optimal Estimation Theory with Applications - Modeling, Filtering, and Information Fusion Estimation. Harbin Institute of Technology Press, Harbin, 2005, pp. 6-126.
16. Zhou F, Lu X, Optimal Estimation Theory. High Education Press, Beijing, 2009, pp.1-92.
17. Chen Y, Reliability and Maintainability Engineering. National Defense Industry Press, Beijing, 2007, pp.21-262.
18. Sheldon M. Ross(Edition), Gong G(Translation), Stochastic Processes(2<sup>nd</sup> Edition). China Machine Press, Beijing, 2013, pp.1-203.
19. Lin D, Electrical Measurement Technology. China machine Press, Beijing, 2008, pp.17-51.
20. Mei C, Zhou J, Practical statistical analysis. Science Press, Beijing, 2002, pp.85-96.
21. Zhou D, Wei Mu, SI X, A survey on anomaly detection, life prediction and maintenance decision for industrial processes. *Acta Automatica Sinica*, 2013, 39(6), pp. 711-722.
22. Lucy Y, Michael K, Covariance control for multi-sensor systems. *IEEE Transactions on Aerospace and Electronic Systems*, 2002, 38(4), pp. 1138 -1156.
23. Fima C K, Introduction to Stochastic Calculus With Applications. Post & Telecom Press, Beijing, 2008.
24. Guo S, EMU Monitoring and Fault Diagnosis Technology. Southwest Jiaotong University Press, Chengdu, 2013.
25. Shu H, Power System Reliability-Centered Maintenance. China Machine Press, Beijing, 2010.
26. MARVIN R. System reliability theory: Models, Statistical Methods, and Applications(2<sup>nd</sup> Edition). National Defense Industry Press, Beijing, 2010.
27. He Z, Cao H, Zi Y, et al, Developments and thoughts on operational reliability assessment of mechanical equipment . *Journal of Mechanical Engineering*, 2014, 50(2), pp. 171-186.
28. Guo Y, Power System Reliability Analysis. Tsinghua University Press, Beijing, 2003.
29. Goble W M, Control systems safety evaluation & reliability. ISA, Raleigh, 2010.