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Multi-Objective Optimization of Slow Moving Inventory System Using Cuckoo Search

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ABSTRACT
This paper focuses on the development of a multi-objective lot size–reorder point backorder inventory model for a slow moving item. The three objectives are the minimization of (1) the total annual relevant cost, (2) the expected number of stocked out units incurred annually and (3) the expected frequency of stockout occasions annually. Laplace distribution is used to model the variability of lead time demand. The multi-objective Cuckoo Search (MOCS) algorithm is proposed to solve the model. Pareto curves are generated between cost and service levels for decision-makers. A numerical problem is considered on a slow moving item to illustrate the results. Furthermore, the performance of the MOCS algorithm is evaluated in comparison to multi-objective particle swarm optimization (MOPSO) using metrics, such as error ratio, maximum spread and spacing.

KEYWORDS
MOCS; Slow moving; Lead time; Inventory; Laplace

1. Introduction

The long-term survival and growth of a firm depends on sound inventory management. An inventory is classified by annual dollar usage through a two-tier A-B-C classification system. In the A-B-C classification system, A refers to the most important items, B signifies items that have intermediate importance items and C denotes the least important items (Peterson & Silver, 1979). Furthermore, an inventory can be sub-classified by average usage during the lead time in terms of fast moving and slow moving. A significant portion of a firm's inventory consists of slow moving items. Peterson and Silver (1979) classified slow moving items as those with an average demand during the lead time below 10 units. They further recommended Poisson or Laplace probability distribution for modelling the variability of lead time demand for slow moving items. They also suggested using Laplace distribution over Poisson for slow moving items, when average demand is not reasonably close to the standard deviation of forecast errors (i.e., average demand is not within 10 percent of the standard deviation of forecast errors). Laplace distribution (is a double exponential probability distribution) was first used by Presutti and Trepp (1970) to model the variability of lead time demand in a continuous review inventory system. Later research by like Archibald, Silver, and Peterson (1974); Peterson and Silver (1979); Chang, Chung, and Yang (2001); Nahmias (2009); and Muckstadt and Sapra (2009) used this approach for slow moving inventory items.

The above authors focused solely on the minimization of total relevant inventory cost. Multi-objective optimization is preferred over single-objective optimization, as it provides more flexibility to an inventory decision-maker in terms of choosing from among a number of optimal solutions and circumventing erroneous assumptions about shortage cost. There are several papers on multi-objective optimization for inventory systems, which consider Normal probability distribution for modelling the variability of lead time demand, such as Agrell (1995); Tsou (2008, 2009); Moslemi and Zandieh (2011), Park and Kyung (2014), and Srivastav and Agrawal (2015a, 2015b, 2016). Normal probability distribution is recommended for fast moving items (Peterson & Silver, 1979). It cannot be used for slow moving items as it has a short tail in comparison to Laplace distribution and is not suitable for modelling low-volume lead time demand. Thus, the use of Normal distribution for a slow moving item can result in significant errors. Therefore, in this paper, we have formulated the multi-objective inventory model for slow moving items by using Laplace distribution to model the variability of lead time demand, as well as solved the relevant problematic by using Cuckoo Search optimization. The Cuckoo Search algorithm was developed by Yang and Deb (2009). They later developed the MOCS algorithm to solve design problems (Yang & Deb, 2013). As per our knowledge, there are very few papers on multi-objective optimization for an inventory using the Cuckoo Search, although Srivastav and Agrawal (2015a) did use this approach to solve multi-objective optimization for fast moving inventory items.

The above literature survey shows that multi-objective inventory models have only been developed and successfully applied in relation to fast-moving items. In this paper, to optimize conflicting objectives of cost and service levels, a multi-objective inventory model is developed for a slow moving item. The shortages in the model are considered as complete backorders. The MOCS algorithm is proposed as a solution for the slow moving multi-objective model. The crowding distance is used in the algorithm to rank the non-dominated solutions. The external Pareto archive is used to retain the non-dominated solutions from the previous iteration.

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The rest of the paper is organized as follows. Section 2 shows notations and assumptions used in the paper. Section 3 discusses the development of the multi-objective slow moving inventory model. Section 4 throws light on the MOCS algorithm. Section 5 considers a numerical problem and demonstrates the results. Section 6 discusses the performance evaluation of the MOCS algorithm in comparison to MOPSO. Section 7 concludes the paper.

2. Notations and Assumptions

The notations used in this paper are:

$C(Q, z_0)$	Total relevant cost (Summation of annual ordering and holding cost)
$E(Q, z_0)$	Expected number of units stocked out annually
$N(Q, z_0)$	Expected frequency of stockout occasions annually
A	Fixed ordering cost per order
D	Average annual demand
h	Holding cost per unit per unit time
B_2	Specified for fractional charge per unit short
r	Inventory carrying charge, in \$/\$/year
v	Unit variable cost, in \$/unit
Q	Order quantity
σ_x	Standard deviation of demand during lead time which follows Laplace distribution
z_0	Safety stock factor
s	Reorder point

Assumptions:

(1) A single item is considered.
(2) Demand is stochastic with average demand rate changes very little with time.
(3) Unfilled demand is completely backlogged.
(4) The lead time is known and constant.
(5) Replenishment of size Q occurs when the inventory position drops to reorder point s.
(6) Demand during lead time follows Laplace distribution.

3. Multi-objective Inventory Model for a Slow Moving Item

Motivated by the work of Chang et al. (2001) on a single-objective model for a slow moving item, a tri-objective inventory model is developed. The three objectives of the slow moving model are as follows: The first objective is the minimization of the total annual relevant cost, which is formulated in order to minimize ordering and holding cost (Equation (1)). The second objective is the minimization of the expected number of stocked-out units incurred annually (Equation (2)). The third objective is the minimization of the expected frequency of stockout occasions annually (Equation (3)).

Proposition 1: The total annual relevant cost $C(Q, z_0)$, which is the sum of ordering and holding cost is convex (see Appendix).

$$\text{Minimize } C(Q, z_0) = \frac{AD}{Q} + h\left(\frac{Q}{2} + z_0\sigma_x\right) \tag{1}$$

Proposition 2: The expected number of stocked out units incurred annually $E(Q, z_0)$ is convex (see Appendix).

$$\text{Minimize } E(Q, z_0) = \frac{D}{Q}\left(\frac{\sigma_x}{2\sqrt{2}}e^{-\sqrt{2}z_0}\right) \tag{2}$$

Proposition 3: The expected frequency of stockout occasions annually $N(Q, z_0)$ is convex (see Appendix).

$$\text{Minimize } N(Q, z_0) = \frac{D}{Q}\left(\frac{1}{2}e^{-\sqrt{2}z_0}\right) \tag{3}$$

Equation (4) ensures that the order quantity will be non-negative and not in excess of annual demand, while Equation (5) ensures that the safety stock will be positive and not greater than average annual demand.

$$0 \le Q \le D \tag{4}$$

$$0 \le z_0 \le \frac{D}{\sigma_x} \tag{5}$$

4. Multi-objective Cuckoo Search Algorithm

The Cuckoo Search is a stochastic search evolutionary algorithm. The Cuckoo Search algorithm was first proposed by Yang and Deb (2009). It is inspired by cuckoo breeding behaviour. The cuckoo lays eggs in the nests of other species with the intention that the host will look after the cuckoo's eggs as if they were the host's own. Each egg in a nest signifies a solution. A cuckoo's egg signifies a new solution. The objective is to use new and possibly superior solutions to replace an inferior solution in the nests. The probability that the host identifies the cuckoo's eggs and abandons the nests is denoted by p_a. For generating a new solution, the cuckoo search approach exhibits Levy flight distribution.

The Cuckoo Search has been extended by Yang and Deb (2013) to solve multi-objective problems, in which each nest has multiple eggs to represent a set of solutions. In this work, we have used the following rules for the MOCS algorithm, as presented by Yang and Deb (2013):

(1) Each cuckoo lays K eggs at a time and puts each egg in the randomly chosen nest. Egg K corresponds to the solution to the Kth objective.
(2) The best nests with high-quality eggs are used to produce the next generation.
(3) The host bird discovers the eggs laid by a cuckoo with a probability p_a (0, 1). In this case, the host birds either throw the eggs out or simply abandon the nest, such that the fraction p_a of n host nests is replaced by new nests.

Deb (2001) described that a solution (Q_1, z_{0_1}) dominates another solution (Q_2, z_{0_2}) when it satisfies the following two conditions.

First condition of dominance concept is that the solution (Q_1, z_{0_1}) is not worse than (Q_2, z_{0_2}) in all objectives, i.e.,$C^1(Q_1, z_{0_1}) \le C^2(Q_2, z_{0_2})$, $E^1(Q_1, z_{0_1}) \le E^2(Q_2, z_{0_2})$ and $N^1(Q_1, z_{0_1}) \le N^2(Q_2, z_{0_2})$.

Second, the solution (Q_1, z_{0_1}) is strictly better than (Q_2, z_{0_2}) in at least one objective, i.e., $C^1(Q_1, z_{0_1}) < C^2(Q_2, z_{0_2})$ or/and $E^1(Q_1, z_{0_1}) < E^2(Q_2, z_{0_2})$ or/and $N^1(Q_1, z_{0_1}) < N^2(Q_2, z_{0_2})$.

Motivated by Yang and Deb (2013), a MOCS algorithm is proposed to solve the multi-objective inventory problem. The

Parameters to be tuned for MOCS approach: Number of nests (n),Probability of abandoned (p_a) and Max generations

Objective functions $C^i(Q_i, z_{0_i})$, $E^i(Q_i, z_{0_i})$ and $N^i(Q_i, z_{0_i})$
Generate an initial population of n hosts nests with each having K (three) eggs
Construct Pareto Archive
while (t < Max generations)
Get cuckoo (for example, cuckoo i) randomly using Levy flight $x_i^{(t+1)} = x_i^{\,t} + \alpha L(s, \lambda)$ and evaluate the fitness $C^i(Q_i, z_{0_i})$, $E^i(Q_i, z_{0_i})$ and $N^i(Q_i, z_{0_i})$
Update Pareto Archive
Abandon a fraction (p_a) of worse nest
Generate new nests
Update the Pareto archive
end

Figure 1. Pseudo Code of Multi-objective Cuckoo Search.

algorithm is developed considering the dominance concept. An external archive is considered to store the non-dominated solutions. Crowding distance criteria is used to rank the non-dominated solutions. It increases the diversity among solutions and avoids the convergence of solutions to a single solution. The results are implemented in MATLAB 8.1. The number of nests is considered as 50. The maximum generations are considered as 2000. The pseudo code of MOCS is presented in Figure 1:

5. Results and Discussions

This section includes a numerical example to demonstrate the above results. The inventory problem is considered with the given data of a slow moving item. $D = 104$, $A = \$20$, $r = 0.24\$/\$/\text{year}$, $v = \$350$, $B_2 = 0.2$, lead time is two weeks and $\sigma_x = 1$. (Chang et al., 2001, p.393, example 1).

MOCS is used to solve the multi-objective inventory model and generate the Pareto curves. Figure 2(a) shows the Pareto curve in relation to a trade-off between cost (Equation (1)) and stockout units (Equation (2)). Figure 2(b) is generated for easy use by practitioners, so that they can choose the expedient combination of cost (Equation (1)) and fill rate (computed by Equation (2)).

Figure 2(b) also shows the comparison of results obtained by the MOCS algorithm following Chang et al., 2001; p. 393, (example 1). Chang et al. (2001) considered a single-objective cost minimization slow moving inventory model and determined the cost to be \$844.09 using a simultaneous approach. The sub-components of cost are ordering cost, holding cost and shortage cost, which equal \$292.76, \$545.69 and \$5.64, respectively. The limitation of the single-objective model is that inventory managers have to assume the shortage (backorder) cost in order to calculate inventory cost. In the numerical problem, Chang et al. (2001) assumed the backorder cost to be $B_2 = 0.2$, i.e., \$70 per unit short.

The multi-objective inventory model does not require the erroneous assumption of backorder cost. Therefore, to compare the simultaneous approach results with the proposed MOCS evolutionary algorithm, the shortage cost component in the total cost obtained by Chang et al. (2001) is ignored. The cost considered for comparison with Chang et al. (2001) comprise ordering and holding cost. In Chang et al. (2001) example 1, the combination of ordering and holding cost was equal to \$838.45. The Pareto curve (Figure 2(b)) shows the total annual relevant cost (comprising ordering and holding cost) on the horizontal axis and the fill rate on the vertical axis. The value corresponding to \$838.45 (total annual relevant cost) on the vertical axis is 0.99 (fill rate). This suggests that by using MOCS a much higher fill rate (0.99) is obtained in comparison to the simultaneous approach, which estimated the fill rate to be 0.94.

Another noteworthy difference between the Chang et al. (2001) single-objective cost model and the proposed multi-objective inventory model concerns a number of optimal solutions. In comparison to the Chang et al. (2001) single solution model, the large number of optimal solutions obtained by optimization of the multi-objective inventory model, via the MOCS algorithm, offers flexibility to inventory managers when choosing an expedient combination of cost and fill rate.

Figure 3(a) shows the Pareto curve in relation to a trade-off between cost (Equation (1)) and frequency of stockout situations (Equation (3)). Figure 3(b) is generated for easy use by practitioners, so that they can choose the expedient combination of cost (Equation (1)) and order service level (computed by Equation (2)). Pareto curves (Figure 3a, 3b) are applicable when shortages are measured by an event (or when the number of inventory cycles is short).

Table 1 shows the comparison of service levels (fill rate and order service level) determined for the same cost (combination of ordering and holding cost) using MOCS, as obtained through the simultaneous method by Chang et al. (2001). The results show that using MOCS, in place of the simultaneous method, increases the fill rate by 5.64%. Chang et al. (2001) only computed the fill rate in their work. For our study, we also determined the order service level for a single objective (cost optimization) using the simultaneous approach. The findings shows that order service level were almost identical in both approaches.

It is noteworthy that there are no multi-objective inventory systems for slow moving inventory items that have been reported in literatures. Therefore, a comparison is made between single objective and multi-objective inventory models. Moreover, prominent difference between single and multi-objective inventory models is the assumption of backorder cost, which is required for solving single objective inventory model. So, to make it a balance comparison between single and multi-objective inventory model, we have compared cost in both models by considering summation of ordering and holding cost.

6. Performance Evaluation of the MOCS Algorithm

The performance metrics used to evaluate the performance of the Pareto curve are error ratio, spacing and maximum spread. The performance of the MOCS algorithm is compared to another popular evolutionary algorithm, namely, MOPSO.

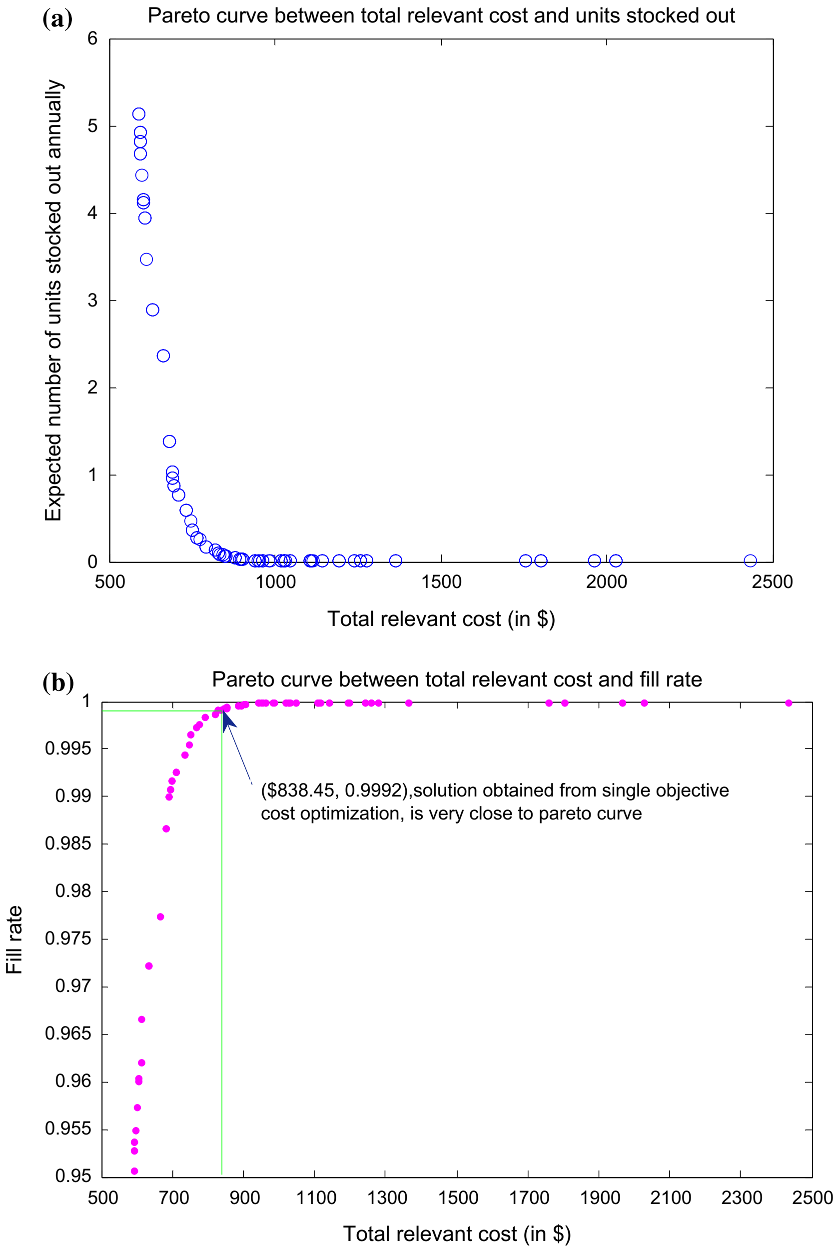

Figure 2. (a) Pareto Exchange Curve between Total Cost and Units stocked out. (b) Pareto Exchange Curve between Total Cost and Fill Rate.

Table 1. Comparison of Service Levels (Fill Rate and Order Service Level) using MOCS and Simultaneous Method.

	Single objective (Simultaneous method)	Multi-Objective Cuckoo Search	Percentage change
Fill Rate	0.9458	0.9992	5.64%
Order Service Level	0.9922	0.9920	−0.02%

6.1. Error ratio

Error ratio is defined as is ratio of a number of non-dominated solutions, which are not members of Pareto optimal set, P* to the total number of non-dominated solutions (Veldhuizen, 1999).

$$\text{Error ratio} = \frac{\sum_{i=1}^{|\tilde{A}|} e_i}{|\tilde{A}|} \begin{cases} e_i = 1, \text{ if } i \notin P^* \\ e_i = 0, \text{ otherwise} \end{cases} \tag{6}$$

$\tilde{A}$ is the non-dominated set

6.2. Spacing

Spacing is the relative distance measure between the consecutive solutions in the non-dominated set (Schott, 1995). Okabe, Jin, and Sendhoff (2003) gave below expression for the spacing metric.

$$\text{Spacing} = \sqrt{\frac{1}{|\tilde{A}|}\sum_{i=1}^{|\tilde{A}|}\left(d_i - \bar{d}\right)^2} \tag{7}$$

$$d_i = \text{minimum}_{j\in\tilde{A}\wedge j\neq i}\sum_{m=1}^{M}\left(d_i - \bar{d}\right)^2\left|f_m^i - f_m^j\right| \tag{8}$$

$\bar{d}$ is the mean of the d_i values; $\tilde{A}$ is the non-dominated set; M is number of objective functions; f_m^i is the fitness of ith solution of mth objective; f_m^j is the fitness of jth solution of mth objective.

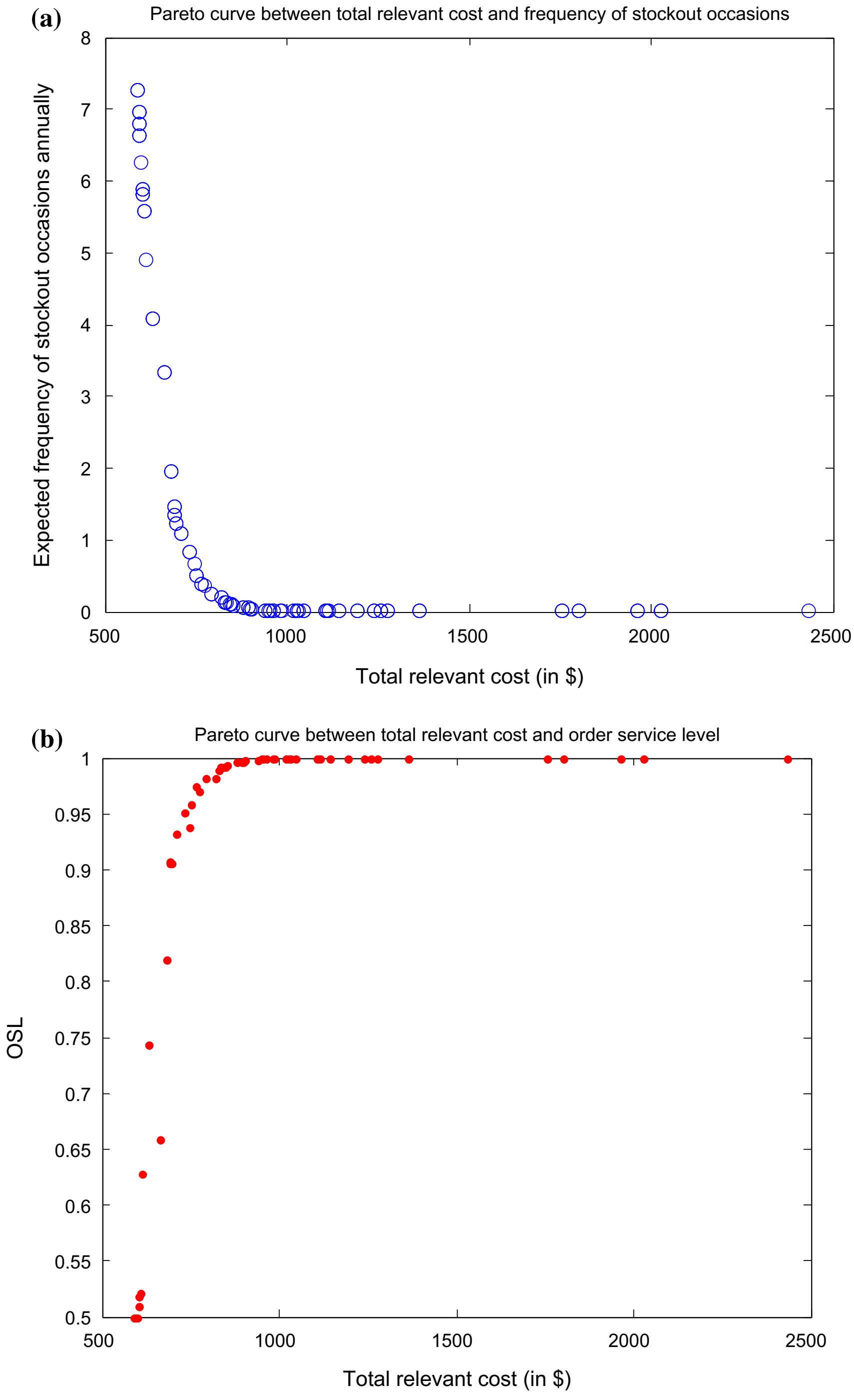

Figure 3. (a) Pareto Exchange Curve between Total Cost and Frequency of stockout occasions. (b) Pareto Exchange Curve between Total Cost and Order Service Level.

6.3. Maximum Spread

Maximum Spread is the distance between the extreme function values in the non-dominated set (Zitzler, 1999).

$$\text{Maximum spread} = \sqrt{\sum_{m=1}^{M}\left(\max_{i=1}^{|\tilde{A}|} f_m^i - \min_{i=1}^{|\tilde{A}|} f_m^i\right)^2} \quad (9)$$

$\tilde{A}$ is the non-dominated set; M is number of objective functions; $\max_{i=1}^{|\tilde{A}|} f_m^i$ is the maximum fitness among set of non-dominated solutions of mth objective; $\min_{i=1}^{|\tilde{A}|} f_m^i$ is the minimum fitness among set of non-dominated solutions of mth objective.

Table 2 shows the comparison between the performance of the MOCS algorithm and MOPSO using error ratio, spacing and maximum spread metrics. The results indicate that low values of MOCS, in comparison to MOPSO, suggest most of the non-dominated solutions in MOCS are lying on the Pareto front. The low values of spacing suggest that solutions are uniformly distributed in MOCS. The large value for the maximum spread metric value signifies the diversity among the solutions using MOCS.

Table 2. Comparison of MOCS with MOPSO.

	Error Ratio		Spacing		Maximum Spread	
	MOCS	MOPSO	MOCS	MOPSO	MOCS	MOPSO
Best Value	0.0000	0.0100	0.0015	0.0023	0.5220	0.4611
Worst Value	0.0300	0.0500	0.0139	0.0052	0.4105	0.3336
Average	0.0180	0.0320	0.0051	0.0037	0.4553	0.4006
Median	0.0200	0.0400	0.0032	0.0034	0.4352	0.4084
Std. Deviation	0.0130	0.0164	0.0051	0.0011	0.0494	0.0477

7. Conclusion

In this paper, a multi-objective backorder inventory model is developed to simultaneously optimize the conflicting objectives of cost and service levels for a slow moving item. The MOCS algorithm is used to solve a tri-objective backorder inventory model and generate Pareto exchange curves. The first exchange curve is drawn between total cost and fill rate, which provide practitioners the flexibility to function at expedient operating conditions when shortages are calculated, as per unit short. The second exchange curve occurs between total cost and order service level, which is appropriate when shortages are calculated by the number of cycles short.

It is observed that MOCS optimization shows a 5.64% increase in the fill rate, compared to single-objective optimization (simultaneous approach). The findings after computing the order service level are almost identical for both approaches. Given the number of items involved in any industry, an increase in the fill rate by 5.64% for a single item can substantially decrease cost and increase profitability in the multi-item scenario. The performance of the MOCS algorithm for optimizing a multi-objective slow moving backorder inventory model is evaluated in comparison to MOPSO, using metrics such as error ratio, spacing and maximum spread. The results show that MOCS performs well in comparison to MOPSO.

The advantage of the MOCS approach is that it can determine several non-dominated policies in a single run and generate trade-off solutions, which may be overlooked by the single-objective simultaneous method. The contribution of the work, in theory, is the development of a multi-objective inventory model for a slow moving item, along with its optimization using MOCS.

It is hopeful that the adoption of this present work on a multi-objective inventory system for a slow moving item will significantly improve existing inventory optimization. The MOCS algorithm can be used for solving combinatorial optimization location-inventory problems. The proposed multi-objective inventory model is applicable for the inventory optimization of spare parts.

There are several avenues for future work. This research could be developed in relation to a multi-item inventory system. Another possible area of research is to explore the multi-echelon supply chain. There may be other kinds of distribution that could be considered for the purpose of studying slow moving items.

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No potential conflict of interest was reported by the authors

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Appendix 1

Convexity of multi-objective slow moving inventory system is proven below:

Part (i) is related to the first objective of the multi-objective inventory system and presents proof of convexity for Equation (1).

(i) Convexity of the total annual relevant cost (Equation (1)) is proven.

The total annual relevant cost $C(Q, z_0)$, which is the sum of ordering and holding cost (Equation (1)), is represented as per Equation A.1.

$$C(Q, z_0) = \frac{AD}{Q} + h\left(\frac{Q}{2} + z_0\sigma_x\right) \tag{A.1}$$

The sum of ordering and holding cost, as shown in Equation A.1, is convex (Hadley & Whitin, 1963).

Part (ii) is related to the second objective of the multi-objective inventory system and presents proof of convexity for Equation 2.

(ii) Convexity of the expected number of stocked out units incurred annually (Equation (2)) is proven.

The expected number of stocked out units incurred annually $E(Q, z_0)$ (Equation (2)) is represented as per Equation A.2.

$$E(Q, z_0) = \frac{D}{Q}\left(\frac{\sigma_x}{2\sqrt{2}} e^{-\sqrt{2}z_0}\right) \tag{A.2}$$

To prove the convexity of Equation (A.2)

$$H = \begin{vmatrix} \frac{\partial^2 E(Q,z_0)}{\partial Q^2} & \frac{\partial^2 E(Q,z_0)}{\partial Q \partial Z_0} \\ \frac{\partial^2 E(Q,z_0)}{\partial Q \partial z_0} & \frac{\partial^2 E(Q,z_0)}{\partial z_0^2} \end{vmatrix} = \begin{vmatrix} \frac{D\sigma_x e^{-\sqrt{2}z_0}}{\sqrt{2}Q^3} & \frac{D\sigma_x e^{-\sqrt{2}z_0}}{2Q^2} \\ \frac{D\sigma_x e^{-\sqrt{2}z_0}}{2Q^2} & \frac{D\sigma_x e^{-\sqrt{2}z_0}}{\sqrt{2}Q} \end{vmatrix}$$

Alternatively, $H = \frac{D^2\sigma_x^2 e^{-2\sqrt{2}z_0}}{4Q^2}$ is a positive quantity, and Hessian is a positive definite, thereby proving the Equation A.2 is convex.

Part (iii) is related to the third objective of the multi-objective inventory system and presents proof of convexity for Equation 3.

(iii) Convexity of the expected frequency of stockout occasions (Equation (3)) is convex.

The expected frequency of stockout occasions annually $N(Q, z_0)$ (Equation (3)) is represented as per Equation A.3.

$$N(Q, z_0) = \frac{D}{Q}\left(\frac{1}{2} e^{-\sqrt{2}z_0}\right) \tag{A.3}$$

$$H = \begin{vmatrix} \frac{\partial^2 N(Q,z_0)}{\partial Q^2} & \frac{\partial^2 N(Q,z_0)}{\partial Q \partial Z_0} \\ \frac{\partial^2 N(Q,z_0)}{\partial Q \partial z_0} & \frac{\partial^2 N(Q,z_0)}{\partial z_0^2} \end{vmatrix} = \begin{vmatrix} \frac{De^{-\sqrt{2}z_0}}{Q^3} & \frac{De^{-\sqrt{2}z_0}}{\sqrt{2}Q^2} \\ \frac{De^{-\sqrt{2}z_0}}{\sqrt{2}Q^2} & \frac{De^{-\sqrt{2}z_0}}{Q} \end{vmatrix}$$

Alternatively, $H = \left(1 - \frac{1}{\sqrt{2}}\right)\frac{D^2 e^{-2\sqrt{2}z_0}}{Q^4}$ is a positive quantity, and Hessian is a positive definite, thereby proving the Equation A.3 is convex.

The convexities of the three objective functions are proven, confirms that they have a convex Pareto front, while all solutions lying on the Pareto front are global optimum (minimum) solutions.