INTELLIGENT AUTOMATION & SOFT COMPUTING, 2017
https://doi.org/10.1080/10798587.2017.1367146

AutoSoft®
Taylor & Francis
Taylor & Francis Group

New Multi-layer Method for Z-number Ranking using Hyperbolic Tangent Function and Convex Combination

Somayeh Ezadi[a] and Tofigh Allahviranloo[b]

[a]Department of Applied Mathematics, Hamedan Branch, Islamic Azad University, Hamedan, Iran; [b]Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

ABSTRACT
Many practical applications, under the definitive evolutionary state of the nature, the consequences of the decisions, mental states of a decision maker are required. Thus, the need is for a new concept in the analysis of decision-making. Zadeh has introduced this concept as the Z-number. Because the concept is relatively new, Z-number in fuzzy sets, hence, its basic theoretical aspects are yet undetermined. This paper presents a method for ranking Z-numbers. Hence, we propose a new method for ranking fuzzy numbers based on that of hyperbolic tangent function and convex combination. Then, using the same technique we propose a method for ranking Z-numbers.

KEYWORDS
Z-numbers; Fuzzy numbers; Ranking of fuzzy numbers; Ranking of Z-numbers; Standard deviation; Convex Combination; Hyperbolic Tangent function; Score method

1. Introduction

During recent decades, the classic fuzzy is set heavily in distributions such as; fuzzy control, fuzzy decision, optimization, forecasting and etc. (Abbasbandy & Hajjari, 2011; Allahviranloo & Saneifard, 2012; Deng & Chan, 2011; Deng, Chan, Wu, & Wang, 2011; Deng, Chen, Zhang, & Mahadevan, 2011; Deng, Jiang, & Sadiq, 2011; Deng & Liu, 2005a; Deng & Liu, 2005b; Deng, Zhu, & Liu, 2006; Dubois & Prade, 1983; Wang, Liu, et al., 2009; Yager, 1981). But the challenge we face here is that the reliability of the data provided is not well taking into consideration compared to classical fuzzy number, Z-number has more ability to represent human perceptions (Zadeh, 2011). A few ranking Z-number techniques are introduced in (Alive, Alizadeh, & Huseynov, 2015; Alive, Huseynov, Alive, et al 2015; Alive, Huseynov, & Serdaroglu, 2016; Bakar & Gegov, 2015; Jiang, Xie, Luo, & Tang, 2017; Kang, Wei, Li, & Deng, 2012a; Kang, Wei, Li, & Deng, 2012b; Mohamad, Shaharani, & Kamis, 2014). In this work, we develop a new method for ranking fuzzy numbers. By applying the method and the one described by Kang (Kang, Wei, Li and Deng, 2012b) proposes a method for ranking Z-numbers. It is defined in the base. Section 2, includes a new approach based on hyperbolic tangent function for ranking fuzzy numbers is developed. In Section 3, a multilayer method for ranking Z-numbers is generated. Then, the numerical outcomes are presented in Section 4. Finally in Section 5, conclusions will be expressed.

Definition 1. A fuzzy set is said to be convex if for $x, y \in X, \lambda \in [0,1], \mu_{\tilde{A}}\left(\lambda x + (1-\lambda)y\right) \geq \min\left[\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\right]$

Definition 2. A fuzzy number $\tilde{A} = (a, b, c, d; \omega)$ is described as any fuzzy subset of the real line R with membership function R, which processes the following properties:

(a) $\mu_{\tilde{A}}$ is a continuous mapping from R to the closed interval $[0, w]$, $0 \leq w \leq 1$.
(b) $\mu_{\tilde{A}} = 0$, for all $x \in (-\infty, a]$.
(c) $\mu_{\tilde{A}}$, is strictly increasing on [a, b].
(d) $\mu_{\tilde{A}} = \omega$, for all $x \in [b, c]$, where ω is constant and $\omega \in (0, 1]$.
(e) $\mu_{\tilde{A}}$, is strictly increasing on [c, d].
(f) $\mu_{\tilde{A}} = 0$, for all $x \in [d, \infty)$, where a, b, c, d are real numbers. We may let $a = -\infty$, or $a = b$, or $c = d$, or $d = +\infty$.

If $\omega = 1$, in (d) $\tilde{A}$ is a normal fuzzy number, and if $0 < \omega < 1$, in (d)) $\tilde{A}$ is a non-normal fuzzy number. The image (opposite) of $\tilde{A} = (a, b, c, d; \omega)$ can be given by $-\tilde{A} = (-d, -c, -b, -a; \omega)$.

Definition 3. (Hyperbolic Tangent) This function follows form $h(n) = \frac{e^n - e^{-n}}{e^n + e^{-n}}$. The output of this function values in the range $[-1, 1]$. And To express very well and very badly used. As it is in Figure 1.

2. Introducing an Algorithm for Ranking Fuzzy Numbers

We use the technique for ranking fuzzy numbers mean, and standard deviation. We use a combination of convex and the hyperbolic tangent function.

Fuzzy numbers ranking algorithm is as follows:

Step one: If $\tilde{A} = (a_{i1}, a_{i2}, a_{i3}, a_{i4}; \omega_{\tilde{A}_j})$ Then $\tilde{A}_j^* = \left(a_{i1}^*, a_{i2}^*, a_{i3}^*, a_{i4}^*; \omega_{\tilde{A}_j}\right)$ is a normal fuzzy number, *Where* $\omega_{\tilde{A}_j} \in [0, 1], a_{ij}^* = \frac{a_{ij}}{C}$, $\forall j = 1, 2, 3, 4$, $\forall i = 1, \ldots, n$. And $c = \max_{i,j}\left(a_{ij}, 1\right)$ represents the maximum value of the universe of discourse.

Step two: Calculating $x_{\tilde{A}_j^*}$, the defuzzified value for each standardized generalized fuzzy number, $x_{\tilde{A}_j^*}$, using (by combining the definition of the mean value and convex combination. Convex combination can be considered for each case. We explain the procedure for a case (Other states are also similar)).

CONTACT Tofigh Allahviranloo tofigh@allahviranloo.com

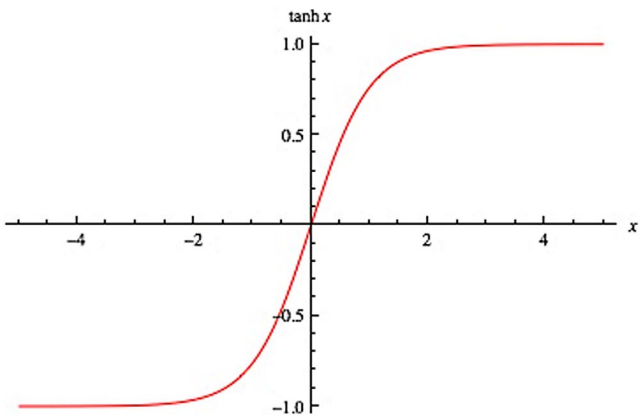

Figure 1. Shows a Hyperbolic Tangent Function.

$$x_{\tilde{A}_i^*} = \frac{\lambda\left(a_{i2}^* + a_{i3}^*\right) + (1-\lambda)(a_{i1}^* + a_{i4}^*)}{\lambda\delta_1 + (1-\lambda)\delta_2} \tag{2.1}$$

Where $\delta_1 = \delta_2 = 2,\ x_{\tilde{A}_i^*} \in [-1, 1], i = 1, \ldots, n.$

Step three: Calculate the spread $STD_{\tilde{A}_j^*}$ (A combination of convex combination definition and standard deviation)

$$STD_{\tilde{A}_i^*} = \sqrt{\frac{\lambda\left(\left(a_{i2}^* - x_{\tilde{A}_i^*}\right)^2 + \left(a_{i3}^* - x_{\tilde{A}_i^*}\right)^2\right) + (1-\lambda)(\left(a_{i1}^* - x_{\tilde{A}_i^*}\right)^2 + \left(a_{i4}^* - x_{\tilde{A}_i^*}\right)^2)}{\left(\lambda\delta_1 + (1-\lambda)\delta_2\right) - 1}} \tag{2.2}$$

Step four: Calculate the fuzzy score $(\tilde{A}_j^*)$ of each standardized generalized fuzzy number, $\tilde{A}_j^*$ denoted as

$$scor\left(\tilde{A}_i^*\right) = \frac{e^d - e^{-d}}{e^d + e^{-d}} \tag{2.3}$$

where

$$d = \left(x_{\tilde{A}_i^*} \times \omega_{\tilde{A}_i}\right) + STD_{\tilde{A}_i^*} \tag{2.4}$$

where $\mathrm{scor}(\tilde{A}_i^*) \in (-1, 1),\ i = 1, \ldots n.$ The larger value of $(\tilde{A}_i^*)$, the higher the preference of $\tilde{A}_i^*$.

Property 1. For $\tilde{A}_i^* = (0, 0, 0, 0, 0)$ clear that $x_{\tilde{A}_i^*} = 0$ so $scor(\tilde{A}_i^*) = 0$.

Property 2. For $\tilde{A}_i^* = (1, 1, 1, 1, 1)$, We have,
$x_{\tilde{A}_i^*} = 1$,
$STD(\tilde{A}_i^*) = 0$.
So $scor(\tilde{A}_i^*) = 0.76$.

Property 3. For $\tilde{A}_i^* = (-1, -1, -1, -1, 1)$, we have
$x_{\tilde{A}_i^*} = -1$,
$STD(\tilde{A}_i^*) = 0$,
$d = (-1 \times 1) + 0$.
So $scor(\tilde{A}_i^*) = \frac{e^{(-1)} - e^{-(-1)}}{e^{(-1)} + e^{-(-1)}} = -0.76$

Property 4. For $\tilde{A}_i^* = (a, a, a, a, 1)$, we have
$x_{\tilde{A}_i^*} = a$,
$STD(\tilde{A}_i^*) = 0$ and
$d = a^2$.
So $scor(\tilde{A}_i^*) = \frac{e^{(a^2)} - e^{-(a^2)}}{e^{(a^2)} + e^{-(a^2)}}$.

3. Ranking Z-numbers

Zadeh (Zadeh, 2011) proposed a subject called Z-number that has two components; $Z = (\tilde{A}, \tilde{B})$. The first component, $\tilde{A}$, is a restriction (constraint) on the values, which a real-valued uncertain variable, X, is allowed to take. The second component, $\tilde{B}$, is a measure of reliability (certainty) of the first component. Typically, $\tilde{A}$ and $\tilde{B}$ are described in a natural language.

A Z–number is represented by the following membership functions given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \le x \le a_2 \\ \omega_{\tilde{A}} & \text{if } a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{Otherwise} \end{cases}$$

And

$$\mu_{\tilde{B}}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1} & \text{if } b_1 \le x \le b_2 \\ \omega_{\tilde{B}} & \text{if } b_2 \le x \le b_3 \\ \frac{b_4-x}{b_4-b_3} & \text{if } b_3 \le x \le b_4 \\ 0 & \text{Otherwise} \end{cases}$$

where $\tilde{A} = (a_1, a_2, a_3, a_4; \omega_{\tilde{A}})$ and $\tilde{B} = (b_1, b_2, b_3, b_4; \omega_{\tilde{B}})$. For $Z = (\tilde{A},\ \tilde{B})$,

(I) Convert the $\tilde{B}$ (reliability) into crisp number by using

$$\alpha = \frac{\int_X x\mu_{\tilde{B}} d\mathrm{d}x}{\int_X \mu_{\tilde{B}} \mathrm{d}x} \tag{3.1}$$

where $\int$ denotes an algebraic integration.

(II) Add the weight of the $\tilde{B}$ to the $\tilde{A}$ (restriction). The weighted Z-number is denoted as

$$\tilde{Z}^\alpha = \left\{(x, \mu_{\tilde{A}^\alpha}(x)) | \mu_{\tilde{A}^\alpha} = \alpha\mu_{\tilde{A}}(x), x \in [0, 1]\right\} \tag{3.2}$$

Note that, α represents the weight of the reliability component of Z-number. In this work, we examine the case:

- Ordering of Z-numbers
- Ordering Batch of Z-numbers

Which are described below.

3.1. Ordering of Z-numbers

Z-numbers ranking algorithm is as follows:

Step one: For $Z = (\tilde{A}, \tilde{B})$, Calculate α using equation (3.1).

Step two: For $Z_i = (\tilde{A}_i, \tilde{B}_i)$, If $\tilde{A}_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4}; \omega_{\tilde{A}_i})$ is converted into a

Table 1. The Calculated Results by the Existing Methods for Example 1.

Methods	Z_1	Z_2
Mohamad's method (2014)	0.0774	0.0774
Bakar's method (2015)	0.0288	0.0288
Kang's method (2012)	0.3000	0.3000

Table 2. The Calculated Results by the Proposed Method for Example 1.

λ_i	Convex Combination Trapezoidal Fuzzy Numbers	$x_{\tilde{A}}$	$STD_{\tilde{A}}$	$Scor(Z_1)$	$Scor(Z_2)$
$\lambda=0$	$\lambda(a^*_{i1}+a^*_{i2})+(1-\lambda)(a^*_{i3}+a^*_{i4})$	0.4	0.14	0.2556	0.2556
$\lambda=0.5$		0.3	0.2	0.2821	0.2821
$\lambda=1$		0.2	0.14	0.1987	0.1987
$\lambda=0$	$\lambda(a^*_{i1}+a^*_{i3})+(1-\lambda)(a^*_{i2}+a^*_{i4})$	0.4	0.14	0.2556	0.2556
$\lambda=0.5$		0.3	0.2	0.2821	0.2821
$\lambda=1$		0.2	0.14	0.1987	0.1987
$\lambda=0$	$\lambda(a^*_{i1}+a^*_{i4})+(1-\lambda)(a^*_{i2}+a^*_{i3})$	0.3	0	0.0897	0.0897
$\lambda=0.5$		0.3	0.2	0.2821	0.2821
$\lambda=1$		0.3	0.28	0.3564	0.3564
$\lambda=0$	$\lambda(a^*_{i2}+a^*_{i3})+(1-\lambda)(a^*_{i1}+a^*_{i4})$	0.3	0.28	0.3564	0.3564
$\lambda=0.5$		0.3	0.2	0.2821	0.2821
$\lambda=1$		0.3	0	0.0897	0.0897
$\lambda=0$	$\lambda(a^*_{i2}+a^*_{i4})+(1-\lambda)(a^*_{i1}+a^*_{i3})$	0.2	0.31	0.3594	0.3594
$\lambda=0.5$		0.3	0.2	0.2821	0.2821
$\lambda=1$		0.4	0.31	0.4105	0.4105
$\lambda=0$	$\lambda(a^*_{i3}+a^*_{i4})+(1-\lambda)(a^*_{i1}+a^*_{i2})$	0.4	0.14	0.2556	0.2556
$\lambda=0.5$		0.3	0.2	0.2821	0.2821
$\lambda=1$		0.2	0.14	0.1987	0.1987

standardized fuzzy number, $\tilde{A}^*_i=(a^*_{i1},a^*_{i2},a^*_{i3},a^*_{i4};\omega_{\tilde{A}_i})$, where $a^*_{ij}=\frac{a_{ij}}{C}$, $\forall j=1,2,3,4,\ \forall i=1,\ \ldots,\ n.$

$\omega_{\tilde{A}_i}\in[0,1]$ and $c=\max\limits_{i,j}\left(a_{ij},1\right)$ represents the maximum value of the universe of discourse.

Step three: Calculating $x_{\tilde{A}_i}$ by (2.1). (Convex combination can be considered for each case. We explain the procedure for a case (Other states are also similar)).

Step four: Calculate the spread $STD_{\tilde{A}_i}$ by (2.2).

Step five: Calculate score $\left(Z_i\right)$, where

$$scor\left(Z_i\right)=\frac{e^D-e^{-D}}{e^D+e^{-D}} \tag{3.3}$$

$$D=\left(x_{\tilde{A}^*_i}\times|\alpha|\right)+STD_{\tilde{A}^*_i} \tag{3.4}$$

where $scor\left(Z_i\right)\in(-1,1), i=1,\ldots,n.$ The larger value of $scor\left(Z_i\right)$, the higher the preference of.

Property 1. Let $Z_i=(\tilde{A}_i,\tilde{B}_i)$, if $\tilde{A}_i=(0,0,0,0,0)$ and $\tilde{B}_i=(0,0,0,0,0)$, then $scor\left(Z_i\right)=0$.

Proof: Clear, $scor(\tilde{A}_i)=0$ and $scor(\tilde{B}_i)=0$. Obviously, $\alpha=0$, so $scor\left(Z_i\right)=0$.

Property 2. Let $Z_i=(\tilde{A}_i,\tilde{B}_i)$, if $\tilde{A}_i=(a,a,a,a,1)$ and $\tilde{B}_i=(b,b,b,b,1)$, then

$$\forall\, b=a{:}\ scor(\tilde{A}_i)=scor(\tilde{B}_i)=\ scor\left(Z_i\right).$$

Proof: Using the formula (2.3),

$$scor\left(\tilde{A}_i\right)=\frac{e^{(a^2)}-e^{-(a^2)}}{e^{(a^2)}+e^{-(a^2)}}=scor\left(\tilde{B}_i\right).$$

and According to (2.1) Obviously, $x_{\tilde{A}^*_i}=a$ and $|\alpha|=|a|$ is the center of gravity $\tilde{B}_i$. So $STD_{\tilde{A}_i}=0$ and eventually $scor\left(Z_i\right)=\frac{e^{(a^2)}-e^{-(a^2)}}{e^{(a^2)}+e^{-(a^2)}}$.

3.2 Ordering Batch of Z-numbers

If we have $Z_{i1},Z_{i2},\ldots,Z_{in}$, So that $Z_{ij}=(\tilde{A}_i,\tilde{B}_i)$, $\forall j=1,\ \ldots,n.,\ i=1,\ldots,m.$

Table 3. A Comparison of the Proposed Method with the Existing Methods for Example 1.

Methods	Ranking
Mohamad's method (2014)	$Z_1\approx Z_2$
Bakar's method (2015)	$Z_1\approx Z_2$
Kang's method (2012)	$Z_1\approx Z_2$
The proposed method	$\mathbf{Z_1\approx Z_2}$

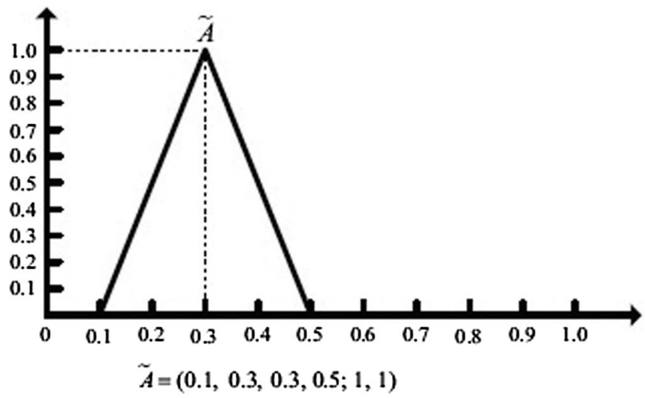

Figure 2a. $\tilde{A}_i$ of Z-numbers for Example 1.

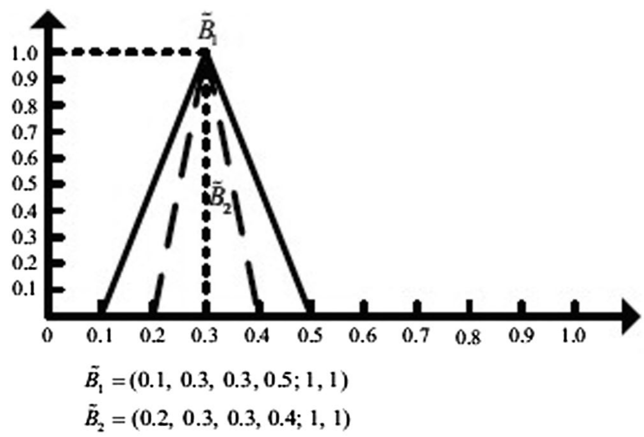

Figure 2b. $\tilde{B}_i$ of Z-numbers for Example 1.

The evaluation of each decision maker is represented as

$$Z_{ij}=\begin{bmatrix} z_{11} & \cdots & z_{1m}\\ \vdots & \ddots & \vdots\\ z_{n1} & \cdots & z_{mn}\end{bmatrix}$$

Table 4a. The Calculated Results by the Proposed Method for Scor (Z_1) Example 2, (\alpha_{1} = 0.75,).

λ_i	Convex Combination Trapezoidal Fuzzy Numbers	$x_{\tilde{A}}$	$STD_{\tilde{A}}$	$d = (x_A * \alpha_i) + STD$	$Scor(Z_1)$
$\lambda = 0$	$\lambda(a^*_{i1} + a^*_{i2}) + (1-\lambda)(a^*_{i3} + a^*_{i4})$	0.62	0.02	0.49	0.45
$\lambda = 0.5$		0.59	0.06	0.50	0.46
$\lambda = 1$		0.56	0.05	0.47	0.44
$\lambda = 0$	$\lambda(a^*_{i1} + a^*_{i3}) + (1-\lambda)(a^*_{i2} + a^*_{i4})$	0.62	0.02	0.49	0.45
$\lambda = 0.5$		0.59	0.06	0.50	0.46
$\lambda = 1$		0.56	0.05	0.47	0.44
$\lambda = 0$	$\lambda(a^*_{i1} + a^*_{i4}) + (1-\lambda)(a^*_{i2} + a^*_{i3})$	0.6	0.2	0.65	0.57
$\lambda = 0.5$		0.59	0.06	0.50	0.46
$\lambda = 1$		0.58	0.08	0.51	0.47
$\lambda = 0$	$\lambda(a^*_{i2} + a^*_{i3}) + (1-\lambda)(a^*_{i1} + a^*_{i4})$	0.58	0.08	0.51	0.47
$\lambda = 0.5$		0.59	0.06	0.50	0.46
$\lambda = 1$		0.60	0.00	0.45	0.42
$\lambda = 0$	$\lambda(a^*_{i2} + a^*_{i4}) + (1-\lambda)(a^*_{i1} + a^*_{i3})$	0.56	0.05	0.47	0.44
$\lambda = 0.5$		0.59	0.06	0.50	0.46
$\lambda = 1$		0.62	0.02	0.49	0.45
$\lambda = 0$	$\lambda(a^*_{i3} + a^*_{i4}) + (1-\lambda)(a^*_{i1} + a^*_{i2})$	0.56	0.05	0.47	0.44
$\lambda = 0.5$		0.59	0.06	0.50	0.46
$\lambda = 1$		0.62	0.02	0.49	0.45

Table 4b. The Calculated Results by the Proposed Method for Scor (Z_2) Example 2, ($\alpha_2 = 0.75$).

λ_i	Convex Combination Trapezoidal Fuzzy Numbers	$x_{\tilde{A}}$	$STD_{\tilde{A}}$	$d = (x_A * \alpha_i) + STD$	$Scor(Z_2)$
$\lambda = 0$	$\lambda(a^*_{i1} + a^*_{i2}) + (1-\lambda)(a^*_{i3} + a^*_{i4})$	0.30	0.14	0.36	0.35
$\lambda = 0.5$		0.25	0.23	0.42	0.39
$\lambda = 1$		0.20	0.28	0.43	0.40
$\lambda = 0$	$\lambda(a^*_{i1} + a^*_{i3}) + (1-\lambda)(a^*_{i2} + a^*_{i4})$	0.30	0.14	0.36	0.35
$\lambda = 0.5$		0.25	0.23	0.42	0.39
$\lambda = 1$		0.20	0.28	0.43	0.40
$\lambda = 0$	$\lambda(a^*_{i1} + a^*_{i4}) + (1-\lambda)(a^*_{i2} + a^*_{i3})$	0.04	0	0.03	0.02
$\lambda = 0.5$		0.07	0.10	0.16	0.15
$\lambda = 1$		0.1	0.14	0.21	0.21
$\lambda = 0$	$\lambda(a^*_{i2} + a^*_{i3}) + (1-\lambda)(a^*_{i1} + a^*_{i4})$	0.1	0.14	0.21	0.21
$\lambda = 0.5$		0.07	0.10	0.16	0.15
$\lambda = 1$		0.04	0	0.03	0.02
$\lambda = 0$	$\lambda(a^*_{i2} + a^*_{i4}) + (1-\lambda)(a^*_{i1} + a^*_{i3})$	0.02	0.02	0.04	0.04
$\lambda = 0.5$		0.07	0.10	0.16	0.15
$\lambda = 1$		0.12	0.11	0.20	0.20
$\lambda = 0$	$\lambda(a^*_{i3} + a^*_{i4}) + (1-\lambda)(a^*_{i1} + a^*_{i2})$	0.02	0.02	0.04	0.04
$\lambda = 0.5$		0.07	0.10	0.16	0.15
$\lambda = 1$		0.12	0.11	0.20	0.20

So $\forall \lambda_i$ Result $Z_1 > Z_2$.

The evaluation of each decision maker needs to be aggregated so that a single outcome or result is attained. So the aggregation of Z_i is given as.

$$Z_1 + Z_2 + \ldots + Z_n = \left(x_{\tilde{D}_k}, \tilde{R}\right) \tag{3.5}$$

Where $\tilde{R} = \min(\tilde{B}_1, \tilde{B}_2, \ldots, \tilde{B}_n)$, $x_{\tilde{D}_k}$ is the relation (3.9).

Z-numbers ranking algorithm is as follows:

Step one: First, we specify minimum value between $\tilde{B}_1, \ldots, \tilde{B}_n$. We define the $\tilde{R} = \min(\tilde{B}_1, \tilde{B}_2, \ldots, \tilde{B}_n)$ So $\tilde{R} = \min(\tilde{r}_1, \tilde{r}_2, \tilde{r}_3, \tilde{r}_4)$.

Step two: Convert the $\tilde{R}$ (reliability) into crisp number by using

$$\alpha^* = \frac{\int x\mu_{\tilde{R}} dx}{\int \mu_{\tilde{R}} dx} \tag{3.6}$$

So,

$$\alpha^* = \frac{\int_{r_1}^{r_2} x(x - r_1)dx + \int_{r_2}^{r_3} x dx + \int_{r_3}^{r_4} x(r_4 - x)dx}{\int_{r_1}^{r_2} (x - r_1)dx + \int_{r_2}^{r_3} dx + \int_{r_3}^{r_4} (r_4 - x)dx}$$

Step three:

We put

$$\tilde{D}_k = (\tilde{d}_{1k}, \tilde{d}_{2k}, \tilde{d}_{3k}, \tilde{d}_{4k}; \alpha^*) \tag{3.7}$$

Table 5. Linguistic Terms for Restrictions and Reliability for Example 3.

Linguistic terms	The Severity of Loss of the Sub-component, *Wi*	Reliability, *Ri*
Absolutely low(AL)	(0.0, 0.0,0.12)	(0.0, 0.0,0.12)
Very-low(VL)	(0.0,0.12,0.24)	(0.0,0.12,0.24)
Low(L)	(0.12,0.24,0.36)	(0.12,0.24,0.36)
Fairly low(FL)	(0.24,0.36,0.48)	(0.24,0.36,0.48)
Medium(M)	(0.36,0.48,0.60)	(0.36,0.48,0.60)
Fairly high(FH)	(0.48,0.6,0.72)	(0.48,0.6,0.72)
High(H)	(0.6,0.72,0.84)	(0.6,0.72,0.84)
Very high(VH)	(0.72,0.84,0.96)	(0.72,0.84,0.96)
Absolutely high(AH)	(0.84,1.00,1.00)	(0.84,1.00,1.00)

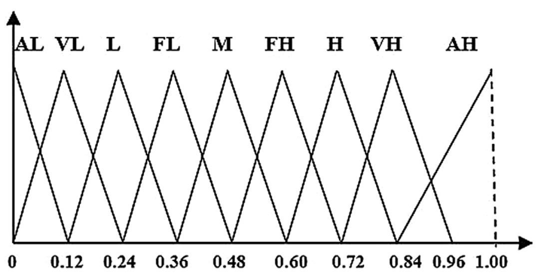

Figure 3. Linguistic Terms Representation for Example 3.

Table 6. Evaluation of Sub-Components for Example 3.

Manu factory	Subcomponents	Linguistic Values of the Severity of Loss, *Wi*	Linguistic Values of the Reliability, *Ri*
C_1	A_{11}	$\tilde{W}_{11}$ = *low* (0.12,0.24,0.36)	R_{11} = *fairly − low* (0.24,0.36,0.48)
	A_{12}	$\tilde{W}_{12}$ = fairly high (0.48,0.6,0.72)	R_{12} = *medium* (0.36,0.48,0.60)
	A_{13}	$\tilde{W}_{13}$ = very low (0.0,0.12,0.24)	R_{13} = *fairly − high* (0.48,0.6,0.72)
C_{2}	A_{21}	$\tilde{W}_{21}$ = very high (0.72,0.84,0.96)	R_{21} = *very − high* (0.72,0.84,0.96)
	A_{22}	$\tilde{W}_{22}$ = fairly low (0.26,0.36,0.48)	R_{22} = *fairly − high* (0.48,0.6,0.72)
	A_{23}	$\tilde{W}_{23}$ = medium (0.36,0.48,0.60)	R_{23} = *medium* (0.36,0.48,0.60)
C_3	A_{31}	$\tilde{W}_{31}$ = absolutely high (0.84,1.00,1.00)	R_{31} = *fairly − low* (0.24,0.36,0.48)
	A_{32}	$\tilde{W}_{32}$ = absolutely low (0.0, 0.0,0.12)	R_{32} = *high* (0.6,0.72,0.84)
	A_{33}	$\tilde{W}_{33}$ = high (0.6,0.72,0.84)	R_{33} = *very − low* (0.0,0.12,0.24)

Table 7. The Calculated Results by the Proposed Method for Example 3.

$\lambda(a^*_{i1}+a^*_{i2})+(1-\lambda)(a^*_{i3}+a^*_{i4})$					$\lambda(a^*_{i1}+a^*_{i3})+(1-\lambda)(a^*_{i2}+a^*_{i4})$				
	i	$x_{\tilde{C}_i}$	$STD(\tilde{C}_i)$	$Scor(\tilde{C}_i)$		i	$x_{\tilde{C}_i}$	$STD(\tilde{C}_i)$	$Scor(\tilde{C}_i)$
$\lambda=0$	1	0.38	0.08	0.21	$\lambda=0$	1	0.38	0.08	0.21
	2	0.62	0.08	0.36		2	0.62	0.08	0.36
	3	0.61	0.05	0.12		3	0.61	0.05	0.12
Result	$\tilde{C}_2>\tilde{C}>\tilde{C}_3$				Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$			
$\lambda=1$	1	0.26	0.08	0.17	$\lambda=1$	1	0.26	0.08	0.17
	2	0.50	0.07	0.30		2	0.50	0.07	0.30
	3	0.52	0.06	0.12		3	0.52	0.06	0.12
Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$				Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$			
$\lambda=0.5$	1	0.32	0.12	0.23	$\lambda=0.5$	1	0.32	0.12	0.23
	2	0.56	0.11	0.36		2	0.56	0.11	0.36
	3	0.56	0.08	0.15		3	0.56	0.08	0.15
Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$				Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$			
$\lambda(a^*_{i1}+a^*_{i4})+(1-\lambda)(a^*_{i2}+a^*_{i3})$					$\lambda(a^*_{i2}+a^*_{i3})+(1-\lambda)(a^*_{i1}+a^*_{i4})$				
	i	$x_{\tilde{C}_i}$	$STD(\tilde{C}_i)$	$Scor(\tilde{C}_i)$		i	$x_{\tilde{C}_i}$	$STD(\tilde{C}_i)$	$Scor(\tilde{C}_i)$
$\lambda=0$	1	0.32	0	0.11	$\lambda=0$	1	0.32	0.16	0.27
	2	0.56	0	0.26		2	0.56	0.16	0.40
	3	0.57	0	0.06		3	0.56	0.12	0.18
Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$				Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$			
$\lambda=1$	1	0.32	0.16	0.27	$\lambda=1$	1	0.32	0	0.11
	2	0.56	0.16	0.40		2	0.56	0	0.26
	3	0.56	0.12	0.18		3	0.57	0	0.06
Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$				Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$			
$\lambda=0.5$	1	0.32	0.12	0.23	$\lambda=0.5$	1	0.32	0.12	0.23
	2	0.56	0.11	0.36		2	0.56	0.11	0.36
	3	0.56	0.08	0.15		3	0.56	0.08	0.15
Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$				Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$			
$\lambda(a^*_{i2}+a^*_{i4})+(1-\lambda)(a^*_{i1}+a^*_{i3})$					$\lambda(a^*_{i3}+a^*_{i4})+(1-\lambda)(a^*_{i1}+a^*_{i2})$				
	i	$x_{\tilde{C}_i}$	$STD(\tilde{C}_i)$	$Scor(\tilde{C}_i)$		i	$x_{\tilde{C}_i}$	$STD(\tilde{C}_i)$	$Scor(\tilde{C}_i)$
$\lambda=0$	1	0.26	0.08	0.17	$\lambda=0$	1	0.26	0.08	0.17
	2	0.50	0.07	0.30		2	0.50	0.07	0.30
	3	0.52	0.06	0.12		3	0.52	0.06	0.12
Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$				Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$			
$\lambda=1$	1	0.38	0.08	0.21	$\lambda=1$	1	0.38	0.08	0.21
	2	0.62	0.08	0.36		2	0.62	0.08	0.36
	3	0.61	0.05	0.12		3	0.61	0.05	0.12
Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$				Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$			
$\lambda=0.5$	1	0.32	0.12	0.23	$\lambda=0.5$	1	0.32	0.12	0.23
	2	0.56	0.11	0.36		2	0.56	0.11	0.36
	3	0.56	0.08	0.15		3	0.56	0.08	0.15
Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$				Result	$\tilde{C}_2>\tilde{C}_1>\tilde{C}_3$			

In (Mohamad et al., 2014) the results for $\tilde{C}_2\geq\tilde{C}_1\geq\tilde{C}_3$.

Where

$$\begin{cases} d_1=\frac{a_{11}+a_{21}+\ldots+a_{n1}}{n}, \\ d_2=\frac{a_{12}+a_{22}+\ldots+a_{n2}}{n}, \\ d_3=\frac{a_{12}+a_{22}+\ldots+a_{n2}}{n}, \\ d_4=\frac{a_{14}+a_{24}+\ldots+a_{n4}}{n}. \end{cases} \tag{3.8}$$

Step Four: Calculating $x_{\tilde{D}_k}$, (Convex combination can be considered for each case. We explain the procedure for a case (Other states are also similar)).

$$x_{\tilde{D}_k}=\frac{\lambda(d_{2k}+d_{3k})+(1-\lambda)(d_{1k}+d_{4k})}{\lambda\delta_1+(1-\lambda)\delta_2} \tag{3.9}$$

where $\delta_1 = \delta_2 = 2,\ x_{\tilde{D}_k} \in [-1, 1],\ k = 1,\ \ldots, n.$

Step five: Calculate the spread $STD_{\tilde{D}_k}$:

$$STD_{\tilde{D}_k} = \sqrt{\frac{\lambda\left(\left(d_{2_k} - x_{\tilde{D}_k}\right)^2 + \left(d_{3_k} - x_{\tilde{D}_k}\right)^2\right) + (1-\lambda)\left(\left(d_{1_k} - x_{\tilde{D}_k}\right)^2 + \left(d_{4_k} - x_{\tilde{D}_k}\right)^2\right)}{\left(\lambda\delta_1 + (1-\lambda)\delta_2\right) - 1}} \quad (3.10)$$

Step six: Calculate the fuzzy score $(\tilde{D}_k)$ of each standardized generalized fuzzy number, $\tilde{D}_k$ denoted as

$$scor\left(\tilde{D}_k\right) = \frac{e^d - e^{-d}}{e^d + e^{-d}} \quad (3.11)$$

Where $\hat{d} = (x_{\tilde{D}_I} \times |\alpha^*|) + STD_{\tilde{D}_k}$

Where scor$(\tilde{D}_k) \in (-1, 1),\ i = 1, \ldots, n$. The larger value of *scor* $(\tilde{D}_k)$, the higher the preference of $\tilde{D}_k$.

4. The Numerical Results

Example 1. Let $\tilde{A}(0.1,\ 0.3,\ ,\ 0.3,\ 0.5, 1)$, $\tilde{B}_1 = (0.1,\ 0.3,\ ,\ 0.3,\ 0.5, 1)$, $\tilde{B}_2 = (0.2,\ 0.3,\ ,\ 0.3,\ 0.4, 1)$.

The calculated results by the existing methods can be seen in Table 1. The Ranking for $Z_1 = (\tilde{A},\ \tilde{B}_1)$ and $Z_2 = (\tilde{A},\ \tilde{B}_2)$per convex combination can be seen in Table 2. The results of the proposed method compared with existing methods and can be seen in Table 3. We calculated,$\alpha_1 = 0.3$ for Z_1, and $\alpha_2 = 0.3$ for Z_2,

Example 2. In this example, the Z-numbers that part $\tilde{A}_i$ (see Figure 2a) and part $\tilde{B}_i$ (see Figure. 2b) rankings do not have the same values (information in Example 2 has been adopted from reference (Kang, Wei, Li and Deng, 2012b)).

$Z_1 = (0.52,\ 0.6,\ 0.64)\ (0.75,\ 1,\ 1)$, $Z_2 = (0, 0.\ 04,\ 0.2)\ (0.5,\ 0.75, 1)$.

The calculated results by the proposed method can be seen in Table 4.

Example 3. In this example, C_1 and C_2 and C_3 goal is to rank among to obtain. (information in Tables 5 and 6 has been adopted from reference (Mohamad et al., 2014)). Linguistic Terms Representation can be seen in Figure. 3. The calculated results by the proposed method can be seen in Table 7.

For C_1, we have, $\min\{R_{11}, R_{12}, R_{13}\} = R_{11}$, for reliability $\tilde{C}_1, R_{11}$ is converted to

$$\alpha_1 = \frac{\int_{0.24}^{0.36} x(x - 0.24)\mathrm{d}x + \int_{0.36}^{0.48} x(0.48 - x)\mathrm{d}x}{\int_{0.24}^{0.36}(x - 0.24)\mathrm{d}x + \int_{0.36}^{0.48}(0.48 - x)\mathrm{d}x} = 0.36$$

In a similar manner, for C_2, we have,$\min\{R_{21}, R_{22}, R_{23}\} = R_{23}$, the reliability of $\tilde{C}_2, R_{23}$ is converted to$\alpha_2 = 0.48$.

and for C_3, we have,$\min\{R_{31}, R_{32}, R_{33}\} = R_{33}$, the reliability of $\tilde{C}_3, R_{23}$ is converted to $\alpha_2 = 0.12$ (Mohamad et al., 2014). So

$$\tilde{C}_1 = (0.20, 0.32, 0.32, 0.44, 0.36),$$

$$\tilde{C}_2 = (0.45, 0.56, 0.56, 0.68, 0.48),$$

$$\tilde{C}_3 = (0.48, 0.57, 0.57, 0.65, 0.12).$$

Values $x_{\tilde{C}_i}$, $STD(_{\tilde{C}_i})$, $Scor(_{\tilde{C}_i})$ for each $\tilde{C}_i$ in the table below.

5. Conclusions

This article introduces a method for ranking fuzzy numbers using Hyperbolic tangent function based convex combination, then the mean value and standard deviations are estimated. We then tried a multi-layered approach using the Hyperbolic tangent function to rank obtain Z-numbers. Right in the core of the first and second layer the reliability of Z-number and the mean standard deviation of Z-number constrains over combination of convex fuzzy numbers is obtained. And the third, using Hyperbolic tangent function along with the yields of the first and second layers Rating Z-numbers is computed. Ranking properties of our proposed method and some sample method is compared to other existing methods. As well as the results of examples showed that the proposed method for each mode of combining convex fuzzy numbers and per lambda same results as other methods are available.

Disclosure statement

No potential conflict of interest was reported by the authors

Notes on contributors

Somayeh Ezadi received a B.S. degree in Mathematics in 2005 from Iran Islamic Azad University, Iran, Hamedan, and her MS degree in Mathematics in 2012 in the same university. She is currently a PhD. student in Mathematics at Islamic Azad University, Iran, Hamedan. Her recent research interests numerical solution of ordinary differential equations, numerical solution of fuzzy differential equations, numerical solution of fuzzy linear regression, solve high order differential equations.

Tofigh Allahviranloo is a professor of Applied Mathematics at SRB-IAU., Iran. He received a BSc. level in Pure Mathematics, 1994, University of Tabriz, Iran and MSc level in Applied Mathematics, (Operation Research), 1999, Lahijan Branch, IAU, Iran. He received PhD in Fuzzy Differential Equations, 2002, Science & Research Branch, IAU, Tehran, Iran. He has published more than 185 articles in international journals and 9 books.

ORCID

Tofigh Allahviranloo http://orcid.org/0000-0002-6673-3560

References

Abbasbandy, S., & Hajjari, T. (2011). An improvement on centroid point method for ranking of fuzzy numbers. *Journal of Sciences (Islamic Azad University), 78*, 109-119.

Alive, R.A., Alizadeh, A.V., & Huseynov, O.H. (2015). The arithmetic of discrete Z-numbers. *Information Sciences, 290*, 134–155.

Alive, R.A., Huseynov, O.H., Alive, R.R., & Alizadeh, A.V. (2015). *The arithmetic of Z-numbers. Theory and applications*. Singapore: World Scientific.

Alive, R.A., Huseynov, O.H., & Serdaroglu, R. (2016). Ranking of Z-numbers and its application in decision making. *International Journal of Information Technology & Decision Making, 15*, 1503.

Allahviranloo, T., & Saneifard, R. (2012). Defuzzification method for ranking fuzzy numbers based on center of gravity. *Iranian Journal of Fuzzy Systems, 9*, 57–67.

Bakar, A.S.A., & Gegov, A. (2015). Multi-layer decision methodology for ranking Z-numbers. *International Journal of Computational Intelligence Systems, 8*, 395–406.

Deng, Y., & Chan, F.T.S. (2011). A new fuzzy dempster MCDM method and its application in supplier selection. *Expert Systems with Applications, 38*, 9854–9861.

Deng, Y., Chan, F.T.S., Wu, Y., & Wang, D. (2011). A new linguistic MCDM method based on multiple criterion data fusion. *Expert Systems with Applications, 38*, 6985–6993.

Deng, Y., Chen, Y.X., Zhang, Y.J., & Mahadevan, S. (2011). Fuzzy Dijkstra algorithm in shortest path problem under uncertain environment. *Applied Soft Computing*. Accepted

Deng, Y., Jiang, W., & Sadiq, R. (2011). Modeling contaminant intrusion in water distribution networks: A new similarity-based DST method. *Expert Systems with Applications, 38*, 571–578.

Deng, Y., & Liu, Q. (2005a). A TOPSIS-based centroid-index ranking method of fuzzy numbers and its application in decision-making. *Cybernetics and Systems, 36*, 581–595.

Deng, Y., & Liu, Q. (2005b). An improved optimal fuzzy information fusion method and its application in group decision. *Journal of Computer and Systems Sciences International, 44*, 531–541.

Deng, Y., Zhu, Z.F., & Liu, Q. (2006). Ranking fuzzy numbers with an area method using radius of gyration. *Computers & Mathematics with Applications, 51*, 1127–1136.

Dubois, D., & Prade, H. (1983). Ranking of fuzzy numbers in the setting of possibility theory. *Information Sciences, 30*, 183–224.

Jiang, W., Xie, C., Luo, Y.,& Tang, Y. (2017). Ranking Z-numbers with an improved ranking method for generalized fuzzy numbers. *Journal of Intelligent & Fuzzy Systems*, vol. *32*, 1931–1943.

Kang, B., Wei, D., Li, Y., & Deng, Y. (2012a). A method of converting Z-number to classical fuzzy number. *Journal of Information and Computational Science, 9*, 703–709.

Kang, B., Wei, D., Li, Y., & Deng, Y. (2012b). Decision making using Z-numbers under uncertain environment. *Journal of Computational Information Systems, 8*, 2807–2814.

Mohamad, D., Shaharani, S.A., & Kamis, N.H. (2014). A Z-number based decision making procedure with ranking fuzzy numbers method. *AIP Conference Proceedings, 1635*, 160–166.

Wang, Z.X., Liu, Y.J., Fan, Z.P., & Fenb, B. (2009). Ranking L-R fuzzy number based on deviation degree. *Information Sciences*, *179*, 2070-2077.

Yager, R. (1981). A procedure for ordering fuzzy subsets of the unit interval. *Information Sciences, 24*, 143–161.

Zadeh, L.A. (2011). A note on Z-numbers. *Information Sciences, 181*, 2923–2932.