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A Z-Number Valued Regression Model and its Application

Lala M. Zeinalova[a], O. H. Huseynov[b] and P. Sharghi[c]

[a]Department of Computer Engineering, Azerbaijan State University of Oil and Industry, Baku, Azerbaijan; [b]Research Laboratory of Intelligent Control and Decision Making Systems in Industry and Economics, Azerbaijan State Oil and Industry University, Baku, Azerbaijan; [c]University of Kyrenia, Department of Maritime Management, Kyrenia, North Cyprus

ABSTRACT
Regression analysis is widely used for modeling of real-world processes in various fields. It should be noted that information relevant to real-world processes is characterized by imprecision and partial reliability. This involves combination of fuzzy and probabilistic uncertainties. Prof.. L. Zadeh introduced the concept of a Z-number as a formal construct for dealing with such information. The present state-of-the-art of regression analysis under Z-number valued information is very scarce. In this paper we consider a Z-number valued multiple regression analysis and its application to a real-world decision-making problem. The obtained results show applicability of the proposed approach.

KEYWORDS
Z-number; fuzzy number; probability measure; multiple regression; multicriteria decision-making; port selection

1. Introduction

Due to complexity of real processes, imprecise and partially reliable relevant information, real-world problems are characterized by combination of fuzzy and probabilistic uncertainties (Aliev, Pedrycz, Huseynov, & Zeinalova, 2011; De Cooman, 2000). Indeed, values of variables of interest are not imprecisely known and are usually described by using linguistic terms characterized by imprecision and vagueness (Aliev, 1994, 2013; Aliev, Alizadeh, & Guirimov, 2010; Aliev, Mamedova, & Aliev, 1993; Aliev, Pedrycz, & Huseynov, 2012; Aliev & Tserkovny, 2011). At the same time, this information cannot be completely trusted, because its sources are knowledge, intuition, and experience, which are only partially reliable.

Zadeh (2011) introduced the concept of Z-numbers to describe the uncertain information in a generalized form. A Z-number is an ordered pair of fuzzy numbers (A, B). Here A is a value of some variable and B represents an idea of certainty or other closely related concept such as sureness, confidence, reliability, strength of truth, or probability (Yager, 2012). It should be noted that in everyday decision-making, most decisions are in the form of Z-numbers. Zadeh suggests some operations for computation with Z-numbers, using the extension principle.

In Yager (2012) the author offers an illustration of a Z-valuation, showing how to make decisions and answer questions. Also an alternative formulation is used for the information contained in the Z-valuations in terms of a Dempster-Shafer belief structure that made use of type-2 fuzzy sets. In Kang, Wei, Li, & Deng (2012a) and Kang, Wei, Li, & Deng (2012b) the authors considered multi-criteria decision-making problems with Z-numbers. For purpose of decision-making, Z-numbers are converted into classical fuzzy numbers and a priority weight of each alternative is determined. In Aliev & Zeinalova (2013) two approaches to decision-making with Z-information are considered. The first approach is based on reducing of Z-numbers to classical fuzzy numbers, and generalization of expected utility approach and use of Choquet integral with an integrand represented by Z-numbers. A fuzzy measure is calculated on a base of a given Z-information. The second approach is based on direct computation with Z-numbers. To illustrate a validity of suggested approaches to decision-making with Z-information the numerical examples are used.

In Aliev, Alizadeh, & Huseynov (2015) and Aliev, Huseynov, Aliyev, & Alizadeh (2015) the authors suggested general and computationally effective approach to computations with Z-numbers.

Z-numbers have a large spectrum of potential application in solving real-world problems in decision analysis, control, optimization and other areas, which are characterized by bimodal information. One type of such problems is regression analysis problems. A series of works was suggested in the realm of fuzzy regression analysis (Aliev, Fazlollahi, & Vahidov, 2002; Kim & Bishu, 1998; Tanaka & Ishibuchi, 1992). However, fuzzy regression model-based analysis is not capable to account for partial reliability of real-world information. In Aliev et al. (2015) the authors suggest a Z-number based regression analysis and apply differential evolution optimization approach. They solve the considered problem by direct computations over Z-numbers. However, the present state-of-the-art of regression analysis under Z-number valued information is very scarce.

In this paper we study a Z-number valued multiple regression analysis problem. We use the simplified method for computations over Z-numbers proposed in Zadeh (2011) in the considered regression analysis. A multicriteria seaport selection problem is used as an example of the proposed study.

The paper is organized as follows: In Section 2 we present some prerequisite material. In Section 3 we formulate a statement of the Z-number valued multiple regression analysis problem. In Section 4 we suggest an application of the suggested approach. Concluding comments are included in Section 5.

CONTACT Lala M. Zeinalova lzeynalova@list.ru

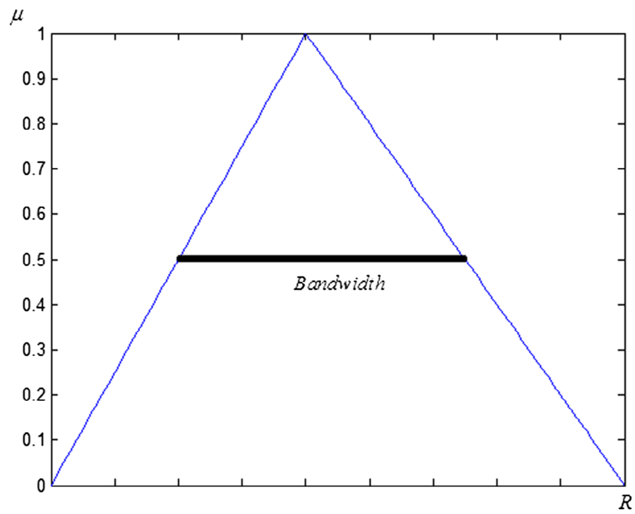

Figure. 1. Interval-valued Approximation to Triangular Fuzzy Number.

2. Preliminaries

Definition 1. Fuzzy sets. (Aliev, Fazlollahi, & Aliev, 2004). Let X be a classical set of objects, called the universe, whose generic elements are denoted x. Membership in a classical subset A of X is often viewed as a characteristic function μ_A from X to {0, 1}such that

$$\mu_A(x) = \begin{cases} 1 & iff\, x \in A \\ 0 & iff\, x \notin A \end{cases}$$

where {0, 1} is called a valuation set; 1 indicates membership while 0—non membership. If the valuation set is allowed to be in the real interval [0, 1], then A is called a fuzzy set, μ_A is the grade of membership of x in A: $\mu_A(x)$: $X \rightarrow [0, 1]$.

Definition 2. Probability measure of a continuous fuzzy number (Zadeh, 2011). Let X be a continuous random variable with pdf$p$. Let A be a continuous fuzzy number describing a possibilistic restriction on values of X. A probability measure of A denoted $P(A)$ is defined as

$$P(A) = \int_{\mathcal{R}} \mu_A(x)p(x)dx.$$

Definition 3. A Continuous Z-number (Aliev, Huseynov, Aliyev, & Alizadeh, 2015). A continuous Z-number is an ordered pair $Z = (A, B)$ where A is a continuous fuzzy number playing a role of a fuzzy constraint on values that a random variable X may take:

$$X\ is\ A$$

B is a continuous fuzzy number with a membership function μ_B:[0, 1] → [0, 1], playing a role of a fuzzy constraint on the probability measure of A: $P(A)$ *is* B.

Zadeh (2011) outlined a general approach to computation over Z-numbers. The approach represents a fusion of fuzzy arithmetic and probabilistic arithmetic and contains complex optimization and variational problems. In order to simplify operations over Z-numbers for problems where approximated solutions are acceptable, Zadeh proposed a bandwidth method. ***A bandwidth method*** **(Zadeh, 2011).** This method is based on an approximation of a fuzzy number A of a Z-number by an interval as its bandwidth $A_1^{Bandwidth}$. As a bandwidth, an α-cut can be used $A_1^{Bandwidth} = A^{\alpha=0.5}$ (Figure. 1).

Let $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ be two Z-numbers and $* \in \{ + , - , /, \times \}$ be a basic arithmetic operation. In this case we can write for any operation over Z-numbers:

$$\begin{aligned}(A_1^{Bandwidth}, B_1) * (A_2^{Bandwidth}, B_2) \\ = (A_1^{Bandwidth} * A_2^{Bandwidth}, B_1 \times B_2),\end{aligned} \tag{1}$$

Where * is a binary operation and $B_1 \times B_2$ is the product of the fuzzy numbers B_1 and B_2. The bandwidth method allows approximating operations of probabilistic arithmetic within computation of Z-numbers to multiplication of fuzzy values of probability measures B_1 and B_2. This allows to significantly reduce complexity operations over Z-numbers.

3. Statement of a Problem

Let us consider a multi input-single output process characterized by Z-number valued information. Let $Z_{X_1}, Z_{X_2}, ..., Z_{X_N}$ denotes Z-numbers describing values of $X_1, X_2, ..., X_N$ and Z_Y denotes the Z-number describing the corresponding value of the output Y.

Assume a series of Z-valued observation data $Z_{X_{i,j}}, i = 1, ..., N$ and $Z_{Y,k}$, $k = 1, ..., K$ are given (K is a number of observations). It is needed to construct a Z-valued linear regression model of a process under study:

$$Z_{Y^M}(Z_{X_1}, Z_{X_2}, ..., Z_{X_N}) = Z_{C_0} + \sum_{i=1}^{N} Z_{C_i} Z_{X_i} \tag{2}$$

By using the bandwidth method (1), we will have for operation of multiplication in (2):

$Z_{C_i} \times Z_{X_i} = ((A_{C_i}^{Bandwith} \times A_{X_i}^{Bandwith}), (B_{C_i} \times B_{X_i}))$.

The operation of addition is treated analogously and the model (2) will be expressed as follows:

$$Z_{Y^M} = Z_{C_0} + Z_{C_1} \times Z_{X_1} + ... + Z_{C_N} \times Z_{X_N}$$

$$\begin{aligned}&= ((A_{C_0}^{Bandwith} + ((A_{C_1}^{Bandwith} \times A_{X_1}^{Bandwith}) \times ... \\ &\times (A_{C_N}^{Bandwith} \times A_{X_N}^{Bandwith}))), \\ &\times (B_{C_0} \times ((B_{C_1} \times B_{X_1}) \times ... \times (B_{C_N} \times B_{X_N})))\end{aligned}$$

$$(B_{C_0} \times ((B_{C_1} \times B_{X_1}) \times ... \times (B_{C_N} \times B_{X_N})))$$

Construction of regression model (2) is related to computation with a large amount of Z-numbers. In this case, the issue of computational complexity becomes very important. In order to achieve a trade-off between adequacy and computational complexity, we propose to use the bandwidth method for the arithmetic operations in (2). The problem of construction of Z_{Y^M} requires to determine Z-valued coefficients (Z-numbers) Z_{C_0}, Z_{C_i} $i = 1, ..., N$ given Z-valued input $Z_{X_i,k}$, $i = 1, ..., N$ and output data Z_{Y_j} so that the following error is minimized:

$$\sum_{k=1}^{K} \left| Z_{Y,k}^M - Z_{Y,k} \right| \rightarrow \min, \tag{3}$$

- denotes the standard subtraction of Z-numbers, || denotes the absolute value of a Z-number and Σ denotes addition of Z-numbers. These operations are handled by using the bandwidth method.

Table 1. A Z-number Valued Input-output Relationship of Seaport Evaluation.

	port services, X_1		*logistics cost*, X_2		*overall evaluation of seaport*, Y	
N	A_{X_1}	B_{X_1}	A_{X_2}	B_{X_2}	A_Y	B_Y
1	Highest	Very sure	Better than average	Almost usually	Very low	Unlikely
.						
.						
.						
	Average	Almost usually	Almost high	Very likely	Almost high	Likely

Table 2. The Values of A.

N	A linguistic description	A triangular fuzzy number
1	"Very low" = A_1	(0.05, 0.1, 0.15)
2	"Almost low" = A_2	(0.09, 0.15, 0.19)
3	"Low" = A_3	(0.13, 0.18, 0.23)
4	"Better than low" = A_4	(0.17, 0.23, 0.27)
5	"Almost average" = A_5	(0.21, 0.26, 0.31)
6	"Average" = A_6	(0.29, 0.35, 0.39)
7	"Better than average" = A_7	(0.36, 0.42, 0.46)
8	"Almost high" = A_8	(0.44, 0.48, 0.54)
9	"High" = A_9	(0.55, 0.61, 0.65)
10	"Better than high" = A_{10}	(0.63, 0.68, 0.73)
11	"Highest" = A_{11}	(0.7, 0.77, 0.87)

Table 3. The Values of B_i.

N	A linguistic description	A triangular fuzzy number
1	"Unlikely" = B_1	(0.08, 0.12, 0.18)
2	"Almost unlikely" = B_2	(0.16, 0.21, 0.26)
3	"Not usually" = B_3	(0.23, 0.29, 0.33)
4	"Almost usually" = B_4	(0.31, 0.36, 0.41)
5	"Usually" = B_5	(0.41, 0.45, 0.51)
6	"Not sure" = B_6	(0.48, 0.53, 0.58)
7	"Almost not sure" = B_7	(0.57, 0.61, 0.67)
8	"Likely" = B_8	(0.66, 0.71, 0.76)
9	"Very likely" = B_9	(0.75, 0.8, 0.85)
10	"Almost sure" = B_{10}	(0.81, 0.86, 0.91)
11	"Very sure" = B_{11}	(0.87, 0.92, 0.97)

Model (2) describes a general case, when all the variables and coefficients of a regression model are Z-numbers. In such case, the use of classical techniques, e.g. gradient based methods, for construction of the regression model is not suitable due to complexity of representation of derivative of a Z-valued function. Below we outline differential evolution (DE) optimization based solution approach to solve problem (2)-(3).

At the initial stage, the parameters of the DE algorithm are set, DE fitness function is defined as (3) and the population size PN is chosen. As usual, the population size is set at least ten times the number of optimization variables. As a Z-number is a pair of two fuzzy numbers, the population size is $PN = 2 \cdot 10N_{\text{var}}$, where N_{var} is the number of optimization variables. The member of population playing the role of the candidate solution is a vector of Z-valued coefficients Z_{C_0}, Z_{C_i} $i = 1, \ \dots, N$. Then the DE optimization is started.

First, a template parameter vector of dimension $2N_{var} = 2(N+1)$ is constructed for holding data of all the coefficients Z_{C_0}, Z_{C_i} $i = 1, \ \dots, N$: $u = (Z_{C_0}, Z_{C_1}, ..., Z_{C_N})$ The algorithm parameters F(mutation rate) and CR(crossover rate) are set. The standard values of these parameters are $F = 0.8$, $CR = 0.7$.

Next PN vectors of parameters are generated randomly to form a population $P = \{u_1, u_2 \dots u_{PN}\}$.

Until the termination condition is met (a predefined number of generations are reached or a required value of fitness function (3) is obtained), new parameter sets are generated. For a next vector $u_i(i=1\dots PN)$ three new different vectors u_{r1}, u_{r2}, u_{r3} are chosen randomly from P. A new trial vector $u_{new} = u_{r1} + F \cdot (u_{r2} - u_{r3})$ is generated in the population. Individual vector parameters of u_i are inherited with probability CR into the new vector u_{new}. If the fitness function (3) for u_{new} is better (lower) than that of for u_i then u_{new} replaces u_i in P. Further, the vector u_{best} with the lowest value of fitness function (3) in population P is found.

The above process is continued once the termination condition is met. Then the parameter vector u_{best} with the lowest fitness function (3) is found. Finally, we extract from u_{best} all the parameters Z_{C_0}, Z_{C_i} $i = 1\dots. N$. These parameters are substituted in (2) to provide the desired Z-number valued regression model.

4. An Example

A seaport choice problem is one of the well-known multicriteria decision-making problems (Bird & Bland, 1988; D'Este & Meyrick, 1992; De Langen, P. 2007). This problem is related to an evaluation of seaports by several performance criteria. The values of criteria are naturally obtained by using expert opinions, and, consequently are characterized by linguistic information and partial reliability. Let us consider construction of a Z-valued linear regression model (2) for overall evaluation Y of seaports (output) on the basis of *port services*, X_1, and *logistics cost*, X_2 (inputs). The regression model has the following form:

$$Z_{Y^M}(Z_{X_1}, Z_{X_2}) = Z_{C_0} + Z_{C_1}Z_{X_1} + Z_{C_2}Z_{X_2},$$

where Z_{Y^M} is the Z-valued overall evaluation of seaport, $Z_{X_1} = (A_{X_1}, B_{X_1})$ is an evaluation of seaport on the basis of X_1 and $Z_{X_2} = (A_{X_2}, B_{X_2})$ is an evaluation of seaport on the basis of X_2, $Z_{C_0} = (A_{C_0}, B_{C_0})$,$Z_{C_1} = (A_{C_1}, B_{C_1})$,$Z_{C_2} = (A_{C_2}, B_{C_2})$ are Z-valued coefficients of the model. The Z-valued coefficients $Z_{C_0} = (A_{C_0}, B_{C_0})$,$Z_{C_1} = (A_{C_1}, B_{C_1})$,$Z_{C_2} = (A_{C_2}, B_{C_2})$ can be interpreted as follows: $Z_{C_1} = (A_{C_1}, B_{C_1})$,$Z_{C_2} = (A_{C_2}, B_{C_2})$ coefficient describe the impacts of the corresponding criteria evaluations $Z_{X_1} = (A_{X_1}, B_{X_1})$ and $Z_{X_2} = (A_{X_2}, B_{X_2})$ on the overall evaluation of seaport (these can be considered as importance measures of X_1 and X_2). The coefficient $Z_{C_0} = (A_{C_0}, B_{C_0})$ is used to take into account errors of experts' estimations and some missed influence of other criteria.

A fragment of experts' opinion-based Z-number-valued information on input-output relationships in the considered problem is shown in Table 1.

The codebooks of linguistic terms of A and B parts of $Z_{X_i} = (A_{X_i}, B_{X_i})$, $i = 1, 2$ and $Z_Y = (A_Y, B_Y)$ given in Table 1, are shown in Tables 2 and 3 (triangular fuzzy numbers are used).

Let us solve the considered problem. Consider the following randomly generated initial Z-numbers $Z_{C_0} = (A_{C_0}, B_{C_0})$, $Z_{C_1} = (A_{C_1}, B_{C_1})$,$Z_{C_2} = (A_{C_2}, B_{C_2})$:

$$Z_{C_0} = (A_{C_0}, B_{C_0}) = ((0.7, 0.75, 0.8), (0.75, 0.82, 0.88)),$$

$Z_{C_1} = (A_{C_1}, B_{C_1}) = ((0.4, 0.45, 0.5), (0.5, 0.6, 0.7)),$
$Z_{C_2} = (A_{C_2}, B_{C_2}) = ((0.65, 0.7, 0.75), (0.8, 0.85, 0.9)).$

By using the bandwidth method (1), we transform these Z-numbers ($\alpha = 0.5$ cut is used as a bandwidth):

$Z_{C_0} = (A^{Bandwidth}_{C_0}, B_{C_0}) = ([0.725, 0.775], (0.75, 0.82, 0.88)),$
$Z_{C_1} = (A^{Bandwidth}_{C_1}, B_{C_1}) = ([0.425, 0.475], (0.5, 0.6, 0.7)),$
$Z_{C_2} = (A^{Bandwidth}_{C_2}, B_{C_2}) = ([0.675, 0.725], (0.8, 0.85, 0.9)).$

For example, consider computation of Z_{Y^M} for the input values in the first row of Table 1 (the product of triangular fuzzy numbers is approximated by a triangular fuzzy number):

$$Z_{Y^M} = Z_{C_0} + Z_{C_1} \times Z_{X_1} + Z_{C_2} \times Z_{X_2}$$
$$= ((A^{Bandwith}_{C_1} \times A^{Bandwith}_{X_1}) \times (A^{Bandwith}_{C_2} \times A^{Bandwith}_{X_2}) + A^{Bandwith}_{C_0}),$$
$$((B_{C_1} \times B_{X_1}) \times (B_{C_2} \times B_{X_2})) \times B_{C_0})$$

$$= (([0.425,\ 0.475] \times [0.735, 0.82]) \times ([0.675,\ 0.725] \times [0.39, 0.44]) + [0.725,\ 0.775]), ((0.5,\ 0.6,\ 0.7) \times (0.87, 0.92, 0.97)) \times ((0.5,\ 0.6,\ 0.7) \times (0.87, 0.92, 0.97)) \times (0.75, 0.82, 0.88)) = ([1.3, 1.48], (0.08, 0.14, 0.22)).$$

Next, given this value Z_{Y^M} and output in the 1st row of Table 1, which is $Z_Y = ([0.075, 0.125], (0.08, 0.12, 0.18))$, we computed $|Z_{Y^M} - Z_Y|$:

$$|Z_Y - Z_{Y^M}| = |([1.3, 1.48] - [0.075, 1.125], (0.08, 0.14, 0.22) \times (0.08, 0.12, 0.18))|$$
$$= |([1.8, 1.4], (0.006, 0.02, 0.04))|$$
$$= ([1.8, 1.4], (0.006, 0.02, 0.04)).$$

By using the optimization technique, we solved problems (2)-(3) and found the following optimal values of $Z_{C_0} = (A_{C_0}, B_{C_0})$, $Z_{C_1} = (A_{C_1}, B_{C_1})$, $Z_{C_2} = (A_{C_2}, B_{C_2})$ that minimizes function (3):

$Z_{C_0} = (A_{C_0}, B_{C_0}) = ((0.61, 0.66, 0.72), (0.75, 0.82, 0.88)),$
$Z_{C_1} = (A_{C_1}, B_{C_1}) = ((0.21, 0.25, 0.27), (0.5, 0.6, 0.7)),$
$Z_{C_2} = (A_{C_2}, B_{C_2}) = ((0.55, 0.61, 0.65), (0.4, 0.5, 0.6)).$

5. Conclusion

An important qualitative attribute of information on which decisions are based is its reliability. Unfortunately, in almost all the existing decision theories reliability of decision relevant information is missing. In this study, we consider Z-number valued multiple regression analysis and its application. The outlined approach to decision-making brings forward a much more general framework that coincides with human-oriented assessment of imperfect information. We applied the suggested study to multicriteria decision-making in a real-world seaport selection problem. The decision relevant information in this problem is experts opinion-based and is naturally characterized by imprecision and partial reliability. The obtained results show validity of the suggested approach.

Notes on the contributors

Lala M. Zeinalova received her PhD degree from Azerbaijan State Oil Academy in 2001. She is currently an associate professor at the department of Computer Engineering in Azerbaijan State Oil and Industry University. Her current research includes decision analysis, Soft Computing, artificial intelligence and control theory.

Oleg H. Huseynov was born in Baku, Azerbaijan, 1980. Oleg H. Huseynov received the PhD degree at the Institute of Control Systems of Academy of Sciences of Azerbaijan (Baku) in 2012. His major fields of study are decision theory with imperfect information, fuzzy logic, computation with Z-numbers and stability theory.

He is a Head of the Research Laboratory of Intelligent Systems of Decision Making and Control in Industry and Economics, Azerbaijan State University of Oil and Industry. Dr. Huseynov is a regular member of the Program Committees of the International Conferences on Applications of Fuzzy Systems and Soft Computing and International Conferences on Soft Computing and Computing with Words. He has several publications in SCI and SCIE journals.

Pınar Sharghi received bachelor's degree in Maritime Management and a master's degree in MBA from Near East University, Northern Cyprus. She is currently working as an teaching staff in the Department of Maritime Management and Administration, University of Kyrenia, Northen Cyprus. Her research interests include maritime transportation, port and logistics, fuzzy set theory and Z-numbers.

Disclosure statement

No potential conflict of interest was reported by the authors.

ORCID

Lala M. Zeinalova http://orcid.org/0000-0003-2572-4390

References

Aliev, R.A. & Tserkovny, A. (2011). Systemic approach to fuzzy logic formalization for approximate reasoning. *Information Sciences, 181*, 1045–1059.

Aliev, R.A. (1994). Fuzzy expert systems. In F. Aminzadeh and M. Jamshidi (eds) *Soft Computing* (pp. 99–108). New Jersey: Prentice-Hall.

Aliev, R.A. (2013). *Fundamentals of the fuzzy logic-based generalized theory of decisions*. New York: Springer.

Aliev, R.A., Alizadeh, A.V., & Guirimov, B.G. (2010). Unprecisiated information-based approach to decision making with imperfect information. Proccedings of the Ninth Inter. Conference on Application of Fuzzy Systems and Soft Computing. Prague, 387–397.

Aliev, R.A., Alizadeh, A.V., & Huseynov, O.H. (2015). The arithmetic of discrete Z-numbers. *Information Sciences, 290*, 134–155.

Aliev, R.A., Fazlollahi, B., & Aliev, R.R. (2004). *Soft Computing and its applications in business and economics*. Berlin, Heidelberg, New York: Springer.

Aliev, R.A., Fazlollahi, B., & Vahidov, R. (2002). Genetic algorithms-based fuzzy regression analysis. *Soft Computing, 6*, 470–475.

Aliev, R.A., Huseynov, O.H., Aliyev, R.R., & Alizadeh, A.V. (2015). *The arithmetic of Z-numbers. Theory and applications*. New York: World Scientific.

Aliev, R.A., Mamedova, G.A., & Aliev, R.R. (1993). *Fuzzy sets theory and its application*. Tabriz: Tabriz University.

Aliev, R.A., Pedrycz, W., & Huseynov, O.H. (2012). Decision theory with imprecise probabilities. *International Journal of Information Technology & Decision Making, 11*, 271–306.

Aliev, R.A., Pedrycz, W., Huseynov, O.H., & Zeinalova, L.M. (2011). Decision making with second order information granules. In *Granular computing and intelligent systems* (pp. 327–374). Berlin: Springer-Verlag, 2011.

Aliev, R.A., & Zeinalova, L.M. (2013). Decision-making under Z-information. In *Human-centric decision-making models for social sciences*, (pp. 1–20). Springer-Verlag, 2013.

Bird, J., & Bland, G. (1988). Freight forwarders speak: The Perception of Route Competition via Seaports in the European Communities Research Project. *Maritime Policy & Management, 15*, 35–55.

De Cooman, G. (2000). Precision-imprecision equivalence in a broad class of imprecise hierarchical uncertainty models. *Journal of Statistical Planning and Inference, 105*, 175–198.

D'Este, G., & Meyrick, S. (1992). Carrier selection in a RO/RO ferry trade. *Maritime Policy and Management, 19*, 115–126.

De Langen, P. (2007). Port competition and selection in contestable hinterlands; The case of Austria. *European Journal of Transport and Infrastructure Research, 7*(1), 1–14.

Kang, B., Wei, D., Li, Y., & Deng, Y. (2012a). A method of converting Z-number to classical fuzzy number. *Journal of Information & Computational Science, 9*, 703–709.

Kang, B., Wei, D., Li, Y., & Deng, Y. (2012b). Decision making using Z-numbers under uncertain environment. *Journal of Information & Computational Science, 8*, 2807–2814.

Kim, B., & Bishu, R.R. (1998). Evaluation of fuzzy linear regression models by comparing membership functions. *Fuzzy Sets and Systems, 100*, 343–352.

Tanaka, H., & Ishibuchi, H. (1992). Possibilistic regression analysis based on linear programming. In J. Kacprzyk & M. Fedrizzi (Eds.), *Fuzzy Regression Analysis* (pp. 47–60). Warsaw, and Physica, Heidelberg: Omnitech Press.

Yager, R.R. (2012). On Z-valuations using Zadeh`s Z-numbers. *International Journal of Intelligent Systems, 27*, 259–278.

Zadeh, L.A. (2011). A note on Z-numbers. *Information Sciences, 181*, 2923–2932.