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On an Optimization Method Based on Z-Numbers and the Multi-Objective Evolutionary Algorithm

Dong Qiu, Rongwen Dong, Shuqiao Chen and Andi Li

College of Mathematics and Physics, Chongqing University of Posts and Telecommunications, Chongqing, P. R. China

ABSTRACT

In this paper, we research the optimization problems with multiple Z-number valued objectives. First, we convert Z-numbers to classical fuzzy numbers to simplify the calculation. A new dominance relationship of two fuzzy numbers based on the lower limit of the possibility degree is proposed. Then according to this dominance relationship, we present a multi-objective evolutionary algorithm to solve the optimization problems. Finally, a simple example is used to demonstrate the validity of the suggested algorithm.

KEYWORDS

Z-number; Multi-objective optimization; Multi-objective evolutionary algorithm

1. Introduction

Since 2011, the concept of Z-number has been introduced by L. A. Zadeh (2011); it has been widely used to describe the uncertain phenomena in the real world. The researchers' passion for studying Z-numbers has not been reduced. It is intuitive and adaptive to describe the uncertain information by Z-number in the decision problem. Based on the fact that the randomness occurs in optimization problems, Aliev et al. (2015) proposed the study of Z-number based linear programming (Z-LP) model to solve real world problems. Utilizing differential evolution optimization and the arithmetic of Z-number introduced by Aliev, Alizadeh and Huseynov (2015), they suggest the method of solving Z-LP problems. However, it is difficult to operate on two Z-numbers. A method of converting Z-number to classical fuzzy number is presented by Kang et al. (2012). In a multi-objective optimization problem, the degree of satisfaction with the objective function values plays a significance role. Due to the different weights of the objective functions for the decision-maker, F. A. Lootsma (1997) points out that we can control the computational process by weighted degrees of satisfaction. A non-dominated sorting genetic algorithm *II* (NSGA-*II*) is provided by Deb et al. (2002). Specifically, that is a fast non-dominated sorting approach with presented computational complexity. After that, Sun and Gong (2013) find a novel method of effectively solving interval multi-objective optimization problems (IMOPs). They give the definition of the lower limit of the possibility degree, which is used to describe a dominance relation of IMOPs. Then uses this dominance relationship to correct the sorting of NSGA-*II*, so as to obtain better results.

The paper is organized as follows: In the following Section, we will recall some related definitions and concepts and introduce a method of converting a Z-number to a classical fuzzy number. In the third Section, we will present a new dominance relationship of two fuzzy numbers based on the lower limit of the possibility degree. Then, a multi-objective evolutionary algorithm to solve the optimization problems with multiple Z-number valued objectives will be studied. More details of the computing process will be given in a related example. Conclusions will be made in the final Section.

2. Preliminaries

A fuzzy set $\tilde{A}$ on R is characterized by a membership function $\mu_{\tilde{A}}:\mathrm{R} \to [0,1]$. For each fuzzy set, $\tilde{A}$ the α-level set is denoted by $[\tilde{A}]^{\alpha} = \{x \in \mathrm{R}: \mu_{\tilde{A}}(x) \geq \alpha\}$ or each $\alpha \in (0,1]$. [The support of $\tilde{A}$ is defined as; $supp(\tilde{A})$, where $supp(\tilde{A}) = \{x \in \mathrm{R}: \mu_{\tilde{A}}(x) > 0\}$]. We denote the 0-level set as the closure of $supp(\tilde{A})$, i.e., $[\tilde{A}]^{0} = cl(\mathrm{supp}(\tilde{A}))$.

Definition 2.1 (Chang & Zadeh, 1996): *A fuzzy set $\tilde{A}$ is said to be a fuzzy number if the following conditions are satisfied:*

(1) $\tilde{A}$ is normal, i.e., there exists an $x_0 \in \mathrm{R}$ such that $\mu_{\tilde{A}}(x_0) = 1$;
(2) $\tilde{A}$ is an upper semi-continuous function;
(3) $\tilde{A}$ is convex, i.e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$ for all $x_1, x_2 \in \mathrm{R}$ and $\lambda \in (0,1)$;
(4) The 0-level set $[\tilde{A}]^{0}$ is compact.

As we all know the α-level sets of a fuzzy number $\tilde{A}$ are non-empty bounded closed intervals for all $\alpha \in [0,1]$. Hence, we can write as $[\tilde{A}]^{0} = [\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)]$. And let $^{\mathrm{F}}$ be the set of all fuzzy numbers on R.

Notice that the real number $a \in \mathrm{R}$ can be embedded in F by defining a fuzzy number $\tilde{a}$ as;

$$\mu_{\tilde{a}}(x) = \begin{cases} 1, \text{ if } x = a; \\ 0, \text{otherwise.} \end{cases}$$

Definition 2.2: *Let $\tilde{A}$ be a fuzzy number, if there is a x such that $\mu_{\tilde{A}}(x) = 1$, $\tilde{A}$ is called a regular fuzzy number. In addition, a trapezoidal fuzzy number, denoted by $\tilde{A} = \langle a, b, c, d; 1\rangle$, where*

CONTACT Dong Qiu dongqiumath@163.com

$a \le b \le c \le d$, has α-level $[\tilde{A}]^{\alpha} = [a + \alpha(b-a), d - \alpha(d-c)]$, $\alpha \in [0,1]$. *And if* $b = c$, *we say that* $\tilde{A}$ *is a triangular fuzzy number. If there is no x such that* $\mu_{\tilde{A}}(x) = 1$, $\tilde{A}$ *is called an irregular fuzzy number. Furthermore, an irregular trapezoidal fuzzy number* $\tilde{A}$ *is represented by* $\tilde{A} = \langle a, b, c, d; \alpha\rangle$, *where* α *is the largest membership degree.*

Using the extension principle by Zadeh (1975), we can respectively denote the addition and scalar multiplication of any two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ as follows:

$$\mu_{\tilde{A}+\tilde{B}}(x) = \sup_{x_1,x_2:x_1+x_2=x} \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{B}}(x_2)\} \quad (1)$$

And

$$\mu_{\lambda\times\tilde{A}}(x) = \mu_{\lambda\tilde{A}}(x) = \begin{cases} \mu\left(\frac{x}{\lambda}\right), \text{ if } \lambda \neq 0\,; \\ 0, \text{ if} \lambda = 0\,. \end{cases}$$

for any $x \in$ R and $\lambda \in$ R. On the other hand, we also can get the following expressions by the α-level set of the fuzzy numbers:

$$[\tilde{A}+\tilde{B}]^{\alpha} = [\tilde{A}]^{\alpha} + [\tilde{B}]^{\alpha} = [\tilde{A}_L(\alpha) + \tilde{B}_L(\alpha), \tilde{A}_R(\alpha) + \tilde{B}_R(\alpha)] \quad (2)$$

and

$$[\lambda\tilde{A}]^{\alpha} = \lambda[\tilde{A}]^{\alpha} = \begin{cases} [\lambda\tilde{A}_L(\alpha), \lambda\tilde{A}_R(\alpha)], \text{ if } \lambda > 0\,, \\ \{0\}\text{if} \lambda = 0\,, \\ [\lambda\tilde{A}_R(\alpha), \lambda\tilde{A}_L(\alpha)], \text{ if } \lambda < 0\,, \end{cases}$$

for all $\tilde{A}, \tilde{B} \in$ F, $x \in$ R and $\lambda \in$ R.

Definition 2.3 (Zadeh, 2011): *A Z-number has two components, denoted as* $Z = (\tilde{A}, \tilde{B})$, *which describes a value of a real-valued uncertain variable X. The first component,* $\tilde{A}$, *is a restriction on X, which is allowed to take. The second component,* $\tilde{B}$, *is a measure of reliability of* $\tilde{A}$. *Let Z denote the set of all Z-numbers.*

Definition 2.4: *If the fuzzy numbers* $\tilde{A}$ *and* $\tilde{B}$ *of a Z-number are discrete fuzzy numbers, we say that Z-number is a discrete Z-number. Similarly, if* $\tilde{A}$ *and* $\tilde{B}$ *are continuous fuzzy numbers, we say that Z-number is a continuous Z-number.*

Definition 2.5: *Let* $Z = (\tilde{A}, \tilde{B})$ *be a Z-number and* $\lambda \in$ R, *the scalar multiplication* λZ *can be defined* $\lambda Z = \lambda(\tilde{A}, \tilde{B}) = (\lambda\tilde{A}, \lambda\tilde{B})$.

Definition 2.6: *Consider the following optimization problems with multiple Z-numbered valued objective functions:*

$$(\textbf{ZMOP}) \min f(X) = (f^{(1)}(X), f^{(2)}(X), \ldots, f^{(r)}(X))$$
$$\text{subject to} X \in \Omega$$

where the objective functions $f^{(k)} f^{(k)}: \Omega \in$ R$^n \to$ Z *are Z-number value functions for* $k = 1, \cdots, r$, *and* $\Omega \in$ R^n *is said to be the feasible set.*

Now we will introduce a method proposed by Deb et al. (2002) of converting a Z-number to a regular fuzzy number. Let $Z = (\tilde{A}, \tilde{B})$ be a Z-number where $\tilde{A}$ is a trapezoid fuzzy number and $\tilde{B}$ is a triangular fuzzy number. Furthermore, denoted $\mu_{\tilde{A}}: T \subseteq$ R $\to [0,1]$ and $\mu_{\tilde{B}}: T \subseteq$ R $\to [0,1]$ are the membership functions of fuzzy number $\tilde{A}$ and $\tilde{B}$, respectively. The detailed steps are as follows:

Step 1 Transform the second part of Z-number into a crisp number with the centroid method. Consider

$$\alpha = \frac{\int x\mu_{\tilde{B}}(x)\mathrm{d}x}{\int \mu_{\tilde{B}}(x)\mathrm{d}x}, \quad (3)$$

where R denotes an algebraic integration.

Step 2 Impose the weight of the second part of Z-number upon the first part. The weighted Z-number can be defined by Z^{α} and its membership function is $\mu_{\tilde{A}^{\alpha}}(x)$ such that $\mu_{\tilde{A}^{\alpha}}(x) = \alpha\mu_{\tilde{A}}(x)$ for $x \in T$.

Step 3 Convert the irregular fuzzy number to a regular fuzzy number. The converted regular fuzzy number is;

$$\mu_{\tilde{Z}}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), \; x \in \sqrt{\alpha}T. \quad (4).$$

It is noteworthy that a Z-number Z has a corresponding regular fuzzy number $\tilde{Z}$ by the above method.

3. Multi-objective Evolutionary Algorithm

We need to solve the problem of comparing two Z-numbers in the process of solving **ZMOP**. There is a ranking method of discrete Z-numbers in (Aliev & Huseynov, 2014). In this section, we will suggest an approach to ranking of continuous Z-numbers, then introduce a multi-objective evolutionary algorithm (MOEA) to solve **ZMOP**. It is well known that directly comparing Z-numbers is very complex, we can convert a Z-number to a regular fuzzy number in the first place. Then we define a new relationship of two fuzzy numbers.

Definition 3.1: *Let* $\tilde{A}$ *and* $\tilde{B}$ *be fuzzy numbers, we denote* $w(\tilde{A})$ and $w(\tilde{B})$ *respectively are the width of* $[\tilde{A}]^{\alpha} = [\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)]$ *and* $[\tilde{B}]^{\alpha} = [\tilde{B}_L(\alpha), \tilde{B}_R(\alpha)]$, *i.e.,*

$$w(\tilde{A}) = \tilde{A}_R(\alpha) - \tilde{A}_L(\alpha), w(\tilde{B}) = \tilde{B}_R(\alpha) - \tilde{B}_L(\alpha),$$

For each $\alpha \in [0,1]$. *We say that* $p(\tilde{A} \succcurlyeq \tilde{B})$ *is a possibility degree of* $\tilde{A} \succcurlyeq \tilde{B}$, *where*

$$p(\tilde{A} \succcurlyeq \tilde{B}) = \int_0^1 \max\left\{1 - \max\left\{\frac{\tilde{B}_R(\alpha) - \tilde{A}_L(\alpha)}{w(\tilde{A}) + w(\tilde{B})}, 0\right\}, 0\right\} \mathrm{d}\alpha$$

Proposition 1 $p(\tilde{A} \succcurlyeq \tilde{B}) + p(\ominus\tilde{B} \succcurlyeq \tilde{A}) = 1$.

Definition 3.2 (Sun & Gong, 2013): *We say that* γ *is the lower limit of the possibility degree of if* $p(\tilde{A} \succcurlyeq \tilde{B})$ *and only if* $p(\tilde{A} \succcurlyeq \tilde{B}) \ge \gamma$, *where* $\gamma \in [0.5, 1]$. *That is to say,* $\tilde{A}$ *is larger than or equal to* $\tilde{B}$ *with the possibility degree not less than* γ.

The lower limit of the possibility degree makes the comparison of fuzzy numbers more flexible and elaborate, and it overcomes the shortcoming that two fuzzy numbers cannot be comparable. Appropriately increasing the value of γ can make $\tilde{A}$ is larger than or equal to $\tilde{B}$ more credible.

Definition 3.3 (Sun & Gong, 2013): *Let* X_1 *and* X_2 *be two different solutions, we say that* X_1 *dominates* X_2 *with the possibility degree not less than* γ *if and only if for all* $i \in \{1, 2, \ldots, r\}$ *we have* $p(f^{(i)}(X_2) \succcurlyeq f^{(i)}(X_1)) \ge \gamma$ *and there is a* $k \in \{1, 2, \ldots, r\}$ *such that* $p(f^{(i)}(X_2) \succcurlyeq f^{(i)}(X_1)) > 0.5$, *denoted as* $X_1 \prec_{\gamma} X_2$. *If neither* X_1 *nor* X_2 *mutually dominates with the possibility degree not less than* γ, *then* X_1 *and* X_2 *are non-dominated with the possibility degree not less than* γ, *denoted as* $X1||_{\gamma} X2$.

Definition 3.4: *Let* $Z_1, Z_2 \in (\mathcal{Z})$ *and* $\tilde{Z}_1, \tilde{Z}_2$ *are respectively the corresponding regular fuzzy numbers by the method by Deb* *et al.* (2002). *If* $\tilde{Z}_1 \prec_\gamma \tilde{Z}_2$, *we say that* $Z_1 \prec_\gamma Z_2$; *and if* $\tilde{Z}_1 ||_\gamma \tilde{Z}_2$, *we say that* $Z_1 ||_\gamma Z_2$.

Definition 3.5 (Sun & Gong, 2013): *Let* X_γ^* *be a feasible solution of* (**ZMOP**), *i.e.,* $X_\gamma^* \in \Omega$. *We say that* X_γ^* *is a γ-Pareto optimal solution of* (**ZMOP**), *if there exists no* $X \in \Omega$ *such that* $X \prec_\gamma X_\gamma^*$; *the set of all γ-Pareto optimal solutions is called a γ-Pareto optimal set, denoted by γ-PS(f).*

Theorem 3.1 *(*Sun & Gong, 2013*): Let* $\gamma_1, \gamma_2 \in [0.5,1]$, *if* $\gamma_{1 \geq} \gamma_2$, *then* $X_{\gamma_2}^* \subseteq X_{\gamma_1}^*$.

The MOEA based on the lower limit of the possibility degree is obtained by associating the dominant relationship with NSGA-*II* in (Chang & Zadeh, 1996). The main idea is as follows: First we realize the population evolves by using the canonical form of NSGA-*II*. Then, substituting the dominant relationship based on the lower limit of the possibility degree for the traditional Pareto dominant relationship. When we compare different evolutionary individuals, we can get non-dominated solutions with different order values. The detailed process is as follows:

Step 1 Initialize a species $P(0)$, which size is N. Set evolutionary generation $t = 0$ and set the lower limit of the possibility degree γ.

Step 2 In order to obtain progeny population $Q(t)$ of the same magnitude, we need the tournament of size 2 for genetic manipulations (selection, crossover and mutation).

Step 3 Use symbol $R(t)$ to represent the result of combining $P(t)$ with $Q(t)$.

Step 4 Sort the individual in the population $R(t)$ by the proposed dominance relation based on the lower limit of the possibility degree. Calculate the crowded degree of individuals with the same ranking value.

Step 5 Select the dominance of top N individuals to constitute the next generation of population$P(t + 1)$.

Step 6 Judge whether the termination condition is met. If yes, put out the feasible solution X_γ^*. If not, let $t = t + 1$, and go back to step 2.

Example 3.1 *Let us consider the following optimization problem:*

$$\min f(x_1, x_2) = \left(f^{(1)}(x_1, x_2), f^{(2)}(x_1, x_2)\right)$$
$$\text{subject to } x_1, x_2 \geq 0,$$

where
$$f^{(1)}(x_1, x_2) = (Z_1 x_1 + Z_2)^2 + (Z_1 x_2 + Z_3)^2;$$
$$f^{(2)}(x_1, x_2) = (Z_1 x_1 + Z_3)^2 + (Z_1 x_2 + Z_4)^2,$$

Let $Z_1 = (\tilde{A}_1, \tilde{B}_1)$, $Z_2 = (\tilde{A}_2, \tilde{B}_2)$, $Z_3 = (\tilde{A}_3, \tilde{B}_3)$ *and* $Z_4 = (\tilde{A}_4, \tilde{B}_4)$, *where*
Apparently $\tilde{A}_1$, $\tilde{A}_2$, $\tilde{A}_3$ *and* $\tilde{A}_4$ *are trapezoid fuzzy numbers,* $\tilde{B}_1$,

$$\tilde{A}_1 = \tilde{1},\ \tilde{B}_1 = \tilde{1};$$
$$\tilde{A}_2 = \langle -4.4689, -2.2388, -2.2337, 0; 1\rangle,\ \tilde{B}_2 = \langle 0, 0.2, 0.4; 1\rangle;$$
$$\tilde{A}_3 = \langle -5.4776, -3.6555, -3.6544, -1.8275; 1\rangle,\ \tilde{B}_3 = \langle 0.1, 0.3, 0.5; 1\rangle;$$
$$\tilde{A}_4 = \langle -5.1632, -3.8710, -3.8700, -2.5825; 1\rangle,\ \tilde{B}_4 = \langle 0.5, 0.6, 0.7; 1\rangle.$$

$\tilde{B}_2$, $\tilde{B}_3$ *and* $\tilde{B}_4$ *are triangular fuzzy numbers.*

To simplify the calculation, we convert Z-numbers into regular fuzzy numbers. At the first step, we should convert the second part into a crisp number by (3)

$$\alpha_2 = \frac{\int x\mu_{\tilde{B}_2}(x)\mathrm{d}x}{\int \mu_{\tilde{B}_2}(x)\mathrm{d}x} = 0.2,\ \alpha_3 = \frac{\int x\mu_{\tilde{B}_3}(x)\mathrm{d}x}{\int \mu_{\tilde{B}_3}(x)\mathrm{d}x} = 0.3,$$
$$\alpha_4 = \frac{\int x\mu_{\tilde{B}_4}(x)\mathrm{d}x}{\int \mu_{\tilde{B}_4}(x)\mathrm{d}x} = 0.6$$

At the second step, add the weighted of the second part to the first part.

$$Z_2^\alpha = \langle -4.4689, -2.2388, -2.2337, 0; 0.2\rangle,$$
$$Z_3^\alpha = \langle -5.4776, -3.6555, -3.6544, -1.8275; 0.3\rangle,$$
$$Z_4^\alpha = \langle -5.1632, -3.8710, -3.8700, -2.5825; 0.6\rangle.$$

At the last step, from (4), *we convert the irregular fuzzy number to a regular fuzzy number.*

$$\begin{aligned}
\tilde{Z}_2 &= \left\langle -4.4689 \times \sqrt{0.2}, -2.2388 \times \sqrt{0.2}, -2.2337 \times \sqrt{0.2}, 0 \times \sqrt{0.2}; 1\right\rangle \\
&= \langle -1.9985, -1.0012, -0.9989, 0; 1\rangle; \\
\tilde{Z}_3 &= \left\langle -5.4776 \times \sqrt{0.3}, -3.6555 \times \sqrt{0.3}, -3.6544 \times \sqrt{0.3}, -1.8275 \times \sqrt{0.3}; 1\right\rangle \\
&= \langle -3.0001, -2.0021, -2.0015, -1.0009; 1\rangle; \\
\tilde{Z}_4 &= \left\langle -5.1632 \times \sqrt{0.6}, -3.8710 \times \sqrt{0.6}, -3.8700 \times \sqrt{0.6}, -2.5825 \times \sqrt{0.6}; 1\right\rangle \\
&= \langle -3.9994, -2.9985, -2.9977, -2.0004; 1\rangle.
\end{aligned}$$

It is easy to get $\tilde{Z}_1 = \tilde{1}$. *Therefore, in order to find the feasible solution, the objective functions can be converted to the following form on the basis of Definition* 3.4*:*

$$f^{(1)}(x_1, x_2) = (\tilde{Z}_1 x_1 + \tilde{Z}_2)^2 + (\tilde{Z}_1 x_2 + \tilde{Z}_3)^2;$$
$$f^{(2)}(x_1, x_2) = (\tilde{Z}_1 x_1 + \tilde{Z}_3)^2 + (\tilde{Z}_1 x_2 + \tilde{Z}_4)^2.$$

Apparently, we can use the MOEA to search for the feasible solution of optimization problems with fuzzy numbers. We set the lower limit of the possibility degree $\gamma = 0.8$. *In conclusion, the 0.8-Pareto optimal solution* $(4.0021, 5.9999)$ *is obtained by MOEA.*

4. Conclusion

When humans communicate information, Z-numbers can be vivid to describe uncertain information. There are some works on optimization problems based on Z-numbers. Considering the fact that calculating Z-numbers is complex, in this paper, we have introduced a method of converting a Z-number to a fuzzy number. Then, we presented a new relationship of two fuzzy numbers based on the lower limit of the possibility degree combined with NSGA-*II* to propose MOEA to solve optimization problems. An example provided in the paper has shown validity of the suggested arithmetic.

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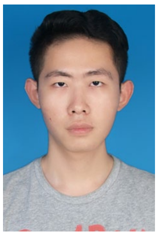
Notes on contributors

Dong Qiu received the doctor degree from University of Electronic Science and Technology of China in 2009 in Applied Mathematics and now is a professor with Chongqing University of Posts and Telecommunications (CQUPT). His research interests include non-additive measure, fuzzy analysis, approximate reasoning and natural language processing.

Rongwen Dong is studying for a master's degree at CQUPT majored in System Science. Her research focuses on Z-numbers and fuzzy optimization.

Shuqiao Chen is studying for a master's degree at CQUPT majored in System Science. Her research focuses on natural language processing.



Andi Li is studying for a bachelor's degree at CQUPT. His current research direction is the Z-numbers and fuzzy Control.

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