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Article

Image Denoising Based on the Asymmetric Gaussian Mixture Model

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Abstract: In recent years, image restoration has become a huge subject, and finite hybrid model has been widely used in image denoising because of its easy modeling and strong explanatory results. The gaussian mixture model is the most common one. The existing image denoising methods usually assume that each component of the natural image is subject to the gaussian mixture model (GMM). However, this approach is not entirely reasonable. It is well known that most natural images are complex and their distribution is not entirely gaussian. As a result, there are still many problems that GMM cannot solve. This paper tries to improve the finite mixture model and introduces the asymmetric gaussian mixture model into it. Since the asymmetric gaussian mixture model can simulate the asymmetric distribution on the basis of the gaussian mixture model, it is more consistent with the natural image data, so the denoising effect of the natural complex image is better. We carried out image denoising experiments under different noise scales and types, and found that the asymmetric gaussian mixture model has better denoising effect and performance.

Keywords: Gaussian mixture model; asymmetric; EPLL denoising model; image denoising

1 Introduction

Images are the main means for human beings to acquire and distinguish information in the objective world. However, due to the lighting ability of the imaging equipment, the moving of the shooting target and the blur caused by artificial dithering, the image degradation is inevitable. As a basic research problem in computer vision and image processing, how to reconstruct original image from degraded image is always a hot topic.

The essence of image denoising is an ill posed inverse problem in mathematics. Generally, the image denoising process can be described by formula (1):

$$Y = X + v \tag{1}$$

where X is an image matrix. v is a gaussian white noise. We use the finite mixture model to restore X. Before denoising, we need to learn some prior information contained in natural images. The previous researches found that similar image is widespread in natural images. They put forward a method called nonlocal average filtering. Since then, the method based on image self similarity has been widely used in image denoising. At the same time, due to the complexity of the image as a whole, Niknejad et al. [1] proposed that the search of similar image blocks can be limited to a small window to achieve better results than global clustering, that is, the prior learning effect of small image blocks is better. Therefore, before starting our work, we need to extract overlapping patches in the image.

Earlier, the gaussian mixture model was proposed in 1886 by Canadian astronomer Simon Newcomb. The gaussian mixture model is widely used because of its short training time and easy implementation. However, the algorithm based on EPLL (Expected Patch Log Likelihood) [2] makes use of the external statistical characteristics of image Patch to achieve better denoising effect, which has great research significance. Subsequently, Zoran proposed a calculation method to solve the EPLL model, which became the "semi quadratic splitting" optimization method [3]. The asymmetric gaussian distribution was proposed by Kato in 2002 [4]. He changed the feature of the gaussian distribution to be symmetric distribution with an asymmetric model. Wang also proposed SURE (Stein's unrisk estimator) algorithm [5], which improved the denoising ability of the image in the edge area. However, although many experts and scholars have made many improvements in the application of gaussian mixture model in image denoising [6], they still admit that the probability distribution of their model is symmetric. Image data, on the other hand, is complex and not necessarily completely symmetrical. In this case, we need a more general mixed distribution to model data of different shapes, especially asymmetric data. Therefore, the asymmetric gaussian mixture model is introduced into the gaussian mixture model. It can simulate both symmetric distribution and asymmetric distribution.

This paper is organized as follows: in Section 2, we review the gaussian mixture model and the algorithm of EPLL. In Section 3, we introduce the asymmetric gaussian mixture model completely, illustrate the asymmetric gaussian mixture model and its benefits and give the way to match patches and the specific solution method based on EPLL algorithm. In Section 4, we give the relevant experiments and results. In Section 5, we make some conclusions about our method as well as perspectives about the future directions.

2 Gaussian Mixture Model and Expected Patch Log Likelihood

2.1 Gaussian Mixture Model

Gaussian model is a commonly used variable distribution model, which plays a wide role in mathematics and statistics. The upper left of Fig. 1 depicts a single gaussian. The data that follows its distribution is concentrated in the interior of an ellipse. However, in many practical applications, data does not belong to the same property. A single gaussian has only one peak. Suppose we encounter data like that in the upper right of Fig. 1. If we assume that this set of data is generated by a gaussian distribution, we use maximum likelihood estimation to estimate the parameters of this gaussian distribution, and then we get a distribution model like that in the lower left of Fig. 1. In general, the closer you get to the center of the ellipse, the more likely you are to have a sample, but there are very few samples in the center of the ellipse with a gaussian, so it is not reasonable to assume that the sample follows a single gaussian. So someone came up with a gaussian mixture model to fit more complex data like this. It solves two gaussian models and fuses the two gaussian models into one model with a certain weight to obtain a distribution model like the bottom right of Fig. 1. Obviously, this distribution model is more reasonable than the distribution in the lower left of Fig. 1.

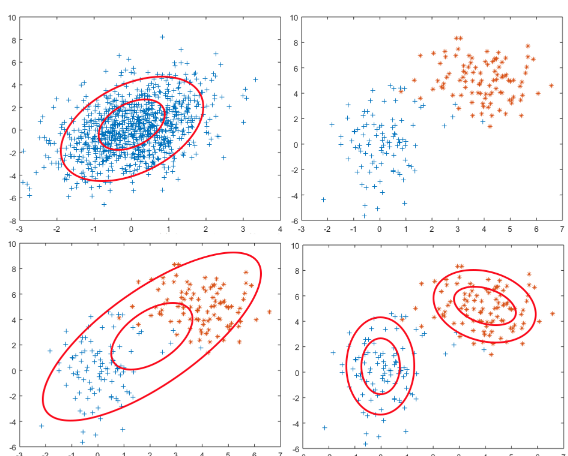

Figure 1: The solution of gaussian model

The gaussian mixture model [7–8] is a probability distribution model with the following form：

$$P(y|\theta) = \sum_{k=1}^{K} \alpha_k \phi(y|\theta_k) \tag{2}$$

where K is the number of mixing components, α_k denotes the mixing coefficient ,$\alpha_k \geq 0$, $\sum_{k=1}^{K} \alpha_k = 1$. $\phi(y|\theta_k)$ is the gaussian density,$\theta_k = (\mu_k, \sigma_k^2)$, its expression is as follows:

$$\phi(y|\theta_k) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp(-\frac{(y-\mu_k)^2}{2\sigma_k^2}) \tag{3}$$

Then, we train these parameters, use a set of clean natural image patches D, by the algorithm of the expectation maximization. The detailed algorithm can be seen in algorithm 1.

2.2 Expected Patch Log Likelihood

The basic idea of EPLL algorithm is to maximize the image patches likelihood probability and make the image patches as close to the prior distribution as possible. Theoretically, the higher the likelihood probability of a group of images under the given prior knowledge, the better the image effect restored by using this prior knowledge.

The specific model derivation process is as follows:

In the previous article, we mentioned that an image can be represented as $y = x + v$, In order to solve it, we first need to solve a maximum posteriori problem, which is expressed as follows:

$$\max_{\upsilon} P(\upsilon|\mu) = \max_{\upsilon} P(\upsilon|\mu)P(\mu) \tag{4}$$

In the prior learning, the EPLL algorithm assumes that the image patch obeys the gaussian mixture distribution. It USES EM algorithm to obtain the parameters of prior distribution. Its expression is:

$$EPLL_P(M) = \sum_i \log p(F_i M) \tag{5}$$

where F_i is a patch operator from in the image, $p(F_i M)$ is the likelihood probability. Under the given prior model, $\log p(F_i M)$ represents the logarithmic likelihood probability value of any image patch.

Then, for a given image Y, the EPLL image restoration model based on the given prior can be expressed as:

$$\min_x \frac{\lambda}{2} \|HM - Y\|^2 - \sum_i \log p(F_i M) \tag{6}$$

where H is the degenerate matrix and λ is the regularization parameter [9]. To solve the problem, we can solve iteratively by the method of "semi quadratic splitting". The auxiliary variable set $\{z_i\}$ is introduced, and $\{z_i\}$ is used to approximate the original image set P_iX, then the objective function becomes:

$$F_{P,\delta}(M, \{z_i\}|Y) = \frac{\lambda}{2} \|HM - Y\|^2 + \sum_i \frac{\delta}{2} (\|F_i M - z_i\|^2) - \log p(z_i) \tag{7}$$

where δ is a penalty parameter.

To solve the above equation, we can solve it by using an alternate direction method and fixing one variable at a time. The specific method is as follows:

Fixed M solution $\{z_i\}$:

$$z_i^{n+1} = \left(\Sigma_{k_{max}} + \frac{1}{\delta} I\right)^{-1} \left(\Sigma_{k_{max}} F_i M^n + \frac{1}{\delta} I \mu_{k_{max}}\right) \tag{8}$$

where $k_{max} = {}^{max}_{\;k}\ p(k|z_i)$.

Fixed $\{z_i\}$ solution M:

$$M^{n+1} = \left(\lambda H^{'}H + \delta\sum_j P_j^{'}P_j\right)^{-1}\left(\lambda H^{'}y + \delta\sum_j F_j^{'}z_j^{n+1}\right) \tag{9}$$

It is conducted by the partial derivative for M is equal to zero.

3 The AGM Model Based on EPLL

3.1 Methods to Extract Patches

Suppose an image is represented by $Y = X + v$. To restore the image X by using a finite mixture model, some prior knowledge of other general natural images must be learned. However, due to the complexity of the whole image, the priori learning effect of small image patches is better. Therefore, before we do the main work, we need to extract the overlapping patches from the whole image. Patch is low dimensional and easier to model.

For any two image patches $x_i$and x_j, their similarity can be expressed as

$$\mathrm{Sim}(x_i, x_j) = \|x_i - x_j\|_2^2 \tag{10}$$

The smaller the $\mathrm{Sim}(x_i, x_j)$ is, the more similar $x_i$and x_j are. For any given image patch x_r, Eq. (10) is used as the metric to find the K image patches which are most similar to it. After vectorization, a patch matrix X_r can be formed as

$$X_r = [x_r, x_r^1, \cdots, x_r^K] \tag{11}$$

Thus, the problem of image denoising becomes the estimation of similar patch matrices

$$X = Q + N \tag{12}$$

where Q is an ideal similar patch matrices for noise free, N is a noise matrix. The objective of denoising is to estimate Q as accurately as possible by noisy similarity patch matrix X.

3.2 Expectation Maximization Algorithm

EM (Expectation Maximization Algorithm) Algorithm was proposed by Dempster et al. in 1977 [10], which can perform MLE estimation of parameters from incomplete data sets. This method is used to process defective data, truncated data, noise and other incomplete data. EM algorithm is an algorithm to find maximum likelihood estimate or maximum posterior estimate in probability model. Its probability model relies on hidden variables that cannot be observed.

EM algorithm consists of two steps: steps E and M. It maximizes the logarithmic likelihood function of incomplete data by iteratively maximizing the expectation of logarithmic likelihood function of complete data：

E-step: estimate the expected value of the unknown parameter, and give the current parameter estimate;

M-step: re-estimate the distribution parameters to maximize the likelihood of the data and give the expected estimation of unknown variables.

By using these two steps alternately, EM algorithm gradually improves the parameters of the model, gradually increases the likelihood probability of the parameters and training samples, and finally terminates at a maximum point.

The detailed algorithm is shown in Agorithm 1.

Algorithm 1: EM algorithm for Gaussian Mixture Model

Input: Training set D, the number of Guassian components K;
Initialization: GMM parameters $\{\alpha_k, \mu_k, \sigma_k^2\}$;
Repeat
  for $j = 1,2,\cdots,n$ **do**
1. (E-step) calculate posterior probability: $\gamma_{jk} = \frac{\alpha_k \phi(y_j|\theta_k)}{\sum_{k=1}^{K} \alpha_k \phi(y_j|\theta_k)}$;
  end for
  for $k = 1,2,\cdots,K$ **do**
2. (M-step) calculate new mean vectors: $\mu_k' = \frac{\sum_{j=1}^{n} \gamma_{jk} y_j}{\sum_{j=1}^{n} \gamma_{jk}}$;
   calculate new covariance matrixes: $\sigma_k^{2\,\prime} = \frac{\sum_{j=1}^{n} \gamma_{jk}(y_j - \mu_k')(y_j - \mu_k')^T}{\sum_{j=1}^{n} \gamma_{jk}}$;
   calculate new mixing weights: $\alpha_k' = \frac{\sum_{j=1}^{n} \gamma_{jk} 1}{n}$;
end for
update parameters $\{\alpha_k, \mu_k, \sigma_k^2\}$ as $\{\alpha_k', \mu_k', \sigma_k^{2\,\prime}\}$;
Until the stop condition is satisfied;

3.3 The Analysis of AGM Model and Reasons for Enhancing the Capability of Image Denoising

In the noise model, one of the most attractive methods for the estimation of noise from gaussian sources is the estimation of its kurtosis k, which is defined as the ratio of a fourth order moment to a second order moment [11]. However, if the gaussian model is extended to the asymmetric case, the asymmetric gaussian model can be estimated by relying on two second order parameters of left and right variance [12]. There is a close relationship between left and right variance and degree of deviation. The degree of deviation can be used to quantify asymmetric probability density functions.

In the process of image denoising, most of the existing methods are aimed at symmetric distribution, and they usually assume that all the components of the natural image conform to the characteristics of gaussian mixture. As is known to all, most natural images are complicated, their distribution is not gaussian distribution. Therefore, it is not appropriate to model some asymmetric data with existing methods. In order to solve this problem, we consider to replace the normal gaussian mixture model with the asymmetric gaussian mixture model, and improve the finite mixture model which assumes that the image is consistent with the characteristics of gaussian mixture. The asymmetric gaussian mixture model can simulate the asymmetric distribution, which is more consistent with the natural image data. Therefore, the denoising effect has been improved.

3.4 The AGM Model Based on EPLL

Most complex image data can be decomposed into a mixture of several distributions. We will discuss these images that follow a mixed distribution. We assume that the probability density function of AGM is

$$P(y|\theta) = \sum_{k=1}^{K} \alpha_k \phi(y|\theta_k) \tag{13}$$

Its asymmetric gaussian distribution can be expressed as

$$\phi(y|\theta_k) = \prod_{k=1}^{K} \alpha_k \frac{2}{\sqrt{2\pi}} \frac{1}{|\sigma_k^2|(r_k+1)} \times \begin{cases} \exp\left(-\frac{1}{2}(y-\mu_k)^T\Sigma^{-1}(y-\mu_k)\right) & \text{if } y > \mu_k \\ \exp\left(-\frac{1}{2r_k^2}(y-\mu_k)^T\Sigma^{-1}(y-\mu_k)\right) & \text{otherwise} \end{cases} \tag{14}$$

In the above equation, $\theta_k = (\mu_k, \sigma_k^2, \gamma_k)$ is the parameter. μ_k and σ_k^2 are the mean and the variance, γ_k is a parameter. It is used to adjust the asymmetry of the asymmetric gauss model.

Next, we will derive the algorithm flow of asymmetric gaussian mixture model in detail.

When we learn the prior information, it is usually a clustering process. We need to randomly select millions of image slices from the standard natural image library, and learn the prior knowledge through a clustering process. We use the k-means algorithm to get the initialization parameters μ_k,$\alpha_k$and σ_k^2, After initialization, we use EM algorithm to train parameters. γ_k can be expressed as

$$\gamma_k^2 = \frac{\sum_{q\in A^2}\|y_q - \mu_k\|_2}{\sum_{p\in A^1}\|y_p - \mu_k\|_2} \tag{15}$$

where A^1represents the set of $y > \mu_k$and A^2 represents the set of $y > \mu_k$.

The logarithmic likelihood function is

$$Q = \sum_{n=1}^{N} \ln \sum_{k=1}^{K} (\alpha_k A(Y_n|\theta_k)) \tag{16}$$

The E-step of EM algorithm: for each k, calculating the posteriori probability. And by using θ_k and α_k,we can get the results of y.

The M-step of EM algorithm: updating the other parameters, the formulas are as follows.

$$\mu_k = \frac{\sum_{p\in A^1}\gamma^1(z_k)y_p + \sum_{q\in A^2}\gamma^2(z_k)y_q}{\sum_{p\in A^1}\gamma^1(z_k) + \sum_{q\in A^2}\gamma^2(z_k)} \tag{17}$$

$$\sigma_k^2 = \frac{\sum_{p\in A^1}\gamma^1(z_k)\times(y_p-\mu_k)(y_p-\mu_k)^T}{\sum_{p\in A^1}\gamma^1(z_k) + \sum_{q\in A^2}\gamma^2(z_k)} + \frac{\sum_{q\in A^2}\gamma^1(z_k)\times\frac{1}{\gamma_k^2}(y_p-\mu_k)(y_p-\mu_k)^T}{\sum_{p\in A^1}\gamma^1(z_k) + \sum_{q\in A^2}\gamma^2(z_k)} \tag{18}$$

$$\alpha_k = \frac{\sum_{p\in A^1}\gamma^1(z_k) + \sum_{q\in A^2}\gamma^2(z_k)}{N} \tag{19}$$

Then, we date parameter γ_k:

$$\gamma_k^{j+1} = \gamma_k^j - \frac{Q(\gamma_k^j)}{Q'(\gamma_k^j)} \tag{20}$$

where

$$\gamma^i(z_n) = \frac{\alpha_k A^i(Y_n|\theta_k)}{\sum_{k=1}^{K}(\alpha_k A^i(Y_n|\theta_k))} \qquad i = 1,2$$

$$\begin{aligned} A^1(Y_n|\theta_k) &= \frac{2}{\sqrt{2\pi}} \frac{1}{|\sigma_k^2|(\gamma_k+1)} \exp\left(-\frac{1}{2}(y-\mu_k)^T\Sigma^{-1}(y-\mu_k)\right) \\ A^2(Y_n|\theta_k) &= \frac{2}{\sqrt{2\pi}} \frac{1}{|\sigma_k^2|(\gamma_k+1)} \exp\left(-\frac{1}{2\gamma_k^2}(y-\mu_k)^T\Sigma^{-1}(y-\mu_k)\right) \end{aligned} \tag{21}$$

In the end, we can get the final solution:

$$z_i' = \begin{cases} \left(\Sigma_{k_{max}} + \frac{1}{\delta} I\right)^{-1} \left(\Sigma_{k_{max}} y + \frac{1}{\delta} I \mu_{k_{max}}\right) & \text{if } y > \mu_{k_{max}} \\ \left(\frac{\Sigma_{k_{max}}}{{\gamma_{k_{max}}}^2} + \frac{1}{\delta} I\right)^{-1} \left(\frac{\Sigma_{k_{max}}}{{\gamma_{k_{max}}}^2} y + \frac{1}{\delta} I \mu_{k_{max}}\right) & \text{otherwise} \end{cases} \tag{22}$$

4 Experiment

4.1 Method and Parameter Setting of Denoising Experiment

In this section, we carry out image denoising experiment. We change the image prior in EPLL image denoising method into an asymmetric gaussian mixture model and use the semi quadratic splitting algorithm to carry out the denoising experiment. In the denoising experiment, we added gaussian white noise of different scales to multiple images. Considering that the patches we extracted are overlapped, we calculated the mean value of each pixel in the overlapped patches. In addition, we set the value of λ to be $\frac{1}{\sigma^2}$, and set $\delta = [1\ 4\ 8\ 16\ 32]$ as in EPLL.

4.2 Experimental Results

First of all, we show the comparison of two methods of image denoising: asymmetric gaussian mixture model and gaussian mixture model. The results of experiment are shown in Fig. 2. On the left, it is the result of EPLL. The PSNR of it is 26.53. On the right, it is the result of asymmetric model. The PSNR of it is 26.85. Through the comparison of experimental effect pictures, we can find that the denoising algorithm based on the asymmetric gaussian mixture model is better than the algorithm based on the gaussian mixture model in the denoising effect.

Figure 2: Denoising result corrupted with a noise standard deviation

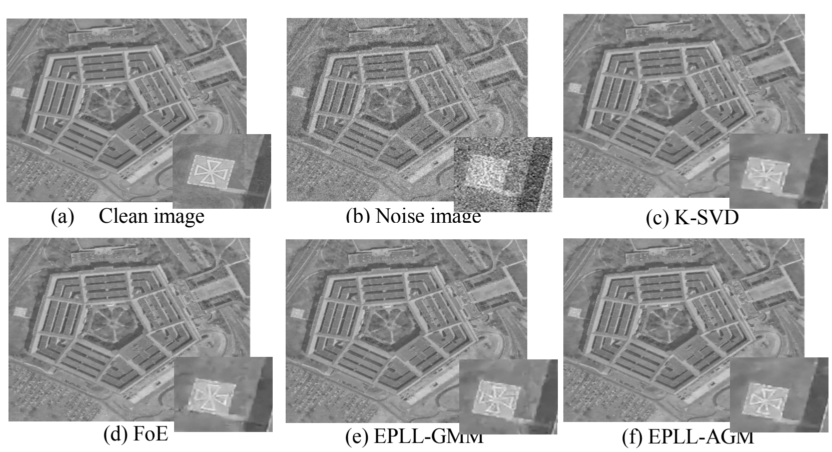

Figure 3: Comparison of the denoising result for Pentagon corrupted with a noise standard deviation

Then, in Fig. 3, we show the comparison of the denoising effect of the asymmetric gaussian mixture model and some other excellent algorithms. Where, the variance of gaussian noise is 30. Fig. 3(a) is a clean original image. Fig. 3(b) is the image with added noise. Fig. 3(c) is the image denoised by KSVD. Fig. 3(d) is the denoised image of FoE model based on markov random airport. Fig. 3(e) is the denoised image of EPLL model based on gaussian mixture model. Fig. 3(f) is the denoised image of EPLL model based on asymmetric gaussian mixture model.

In the figure, we can see that although the denoising effect of the four algorithms in the image is similar on the whole, they still have some differences in detail retention. In Fig. 3(c), most of the texture information has been lost. In Fig. 3(d), although the recovery effect is improved compared with Fig. 3(c), there are still a lot of details missing. Figs. 3(e) and 3(f) have better detail retention ability than Figs. 3(c) and 3(d), and Fig. 3(f) is better.

The more denoising results by some 512 × 512 images from the international standard test atlas are shown in Figs. 4 to 7.

Figure 4: Comparison of the denoising result of Man

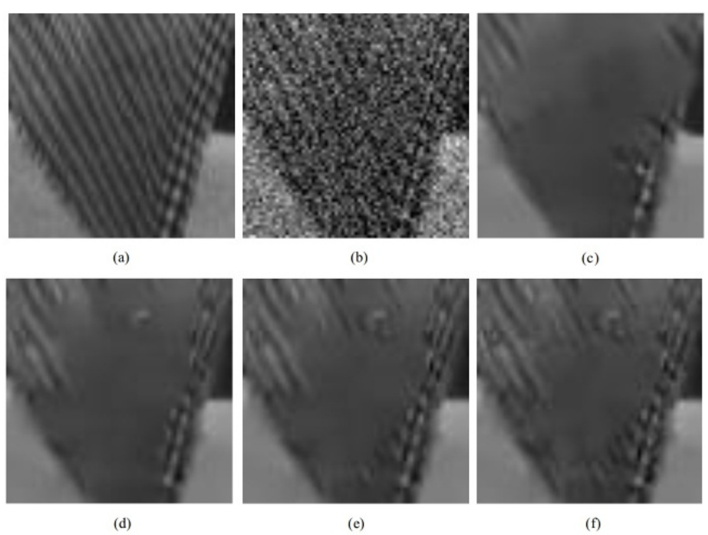

Figure 5: Comparison of the denoising result of Man (partially enlarged)

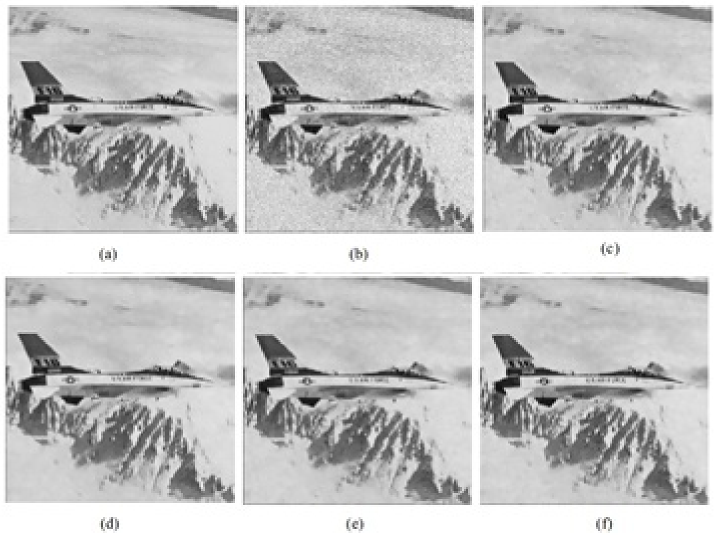

Figure 6: Comparison of the denoising result of F16

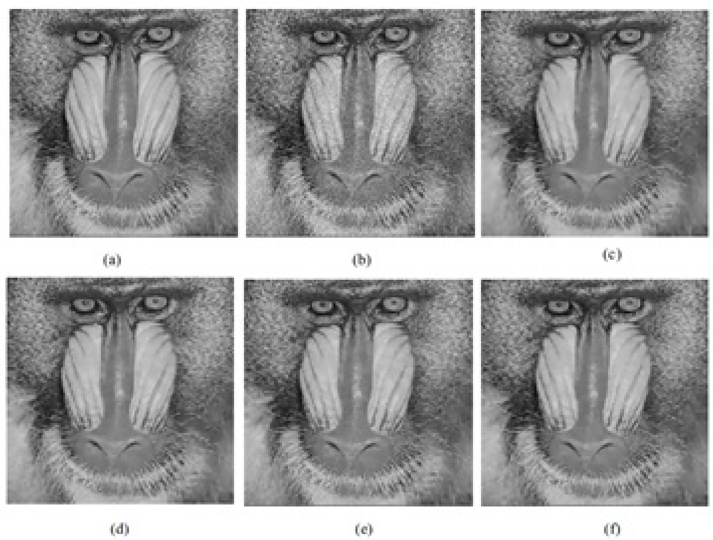

Figure 7: Comparison of the denoising result of Baboon

Of the four images above, Fig. (a) is the clear original image. Fig. (b) is the noise image with 30 gaussian noise added. Fig. (c) is the image denoised by KSVD. Fig. (d) is the image denoised with FoE. Fig. (e) is the image denoised by the gaussian mixture model and EPLL. Fig. (f) is the image denoised by the asymmetric gaussian mixture model and EPLL.

In these four figures, we can find that although all four algorithms can effectively remove noise, there are still some differences between them in terms of details. For example, in Fig. 4, the texture informations of Figs. 4(c) and 4(d) are mostly deleted, while the denoising effect of Figs. 4(e) and 4(f) is better. In Fig. 5, which is a larger version of Fig. 4, we can further discover the advantages of our approach in terms of detail handling. In Fig. 6, most of the letters and numbers of the denoising image obtained by KSVD or FoE method on the aircraft have been blurred and cannot be recognized. But on the denoising effect obtained by our method, we can find that letters and Numbers can still be recognized. In Fig. 7, we can see that the denoising effect of Fig. 7(f) is obviously superior to the other three results in the edge aspect. In general, the noise effect of the algorithm presented in this paper is better than that of previous algorithms.

In addition, we used the PSNR and SSIM values to make a numerical comparison of the denoising results of multiple images. The experimental results are shown in Tab. 1.

Table 1: The PSNR and SSIM for the task of denoising on six images

Image	Noise standard variance	Metric	K-SVD	FoE	EPLL-GMM	EPLL-AGM
Barbara	σ = 10	PSNR	33.45	33.60	33.59	33.82
		SSIM	0.9223	0.9256	0.9253	0.9357
	σ = 30	PSNR	27.49	27.56	27.75	28.14
		SSIM	0.7969	0.8118	0.8169	0.8174
Man	σ = 10	PSNR	33.43	33.60	33.61	33.83
		SSIM	0.9614	0.9627	0.9619	0.9740
	σ = 30	PSNR	28.03	28.46	28.76	29.02
		SSIM	0.8483	0.8587	0.8599	0.8764
F16	σ = 10	PSNR	35.36	35.60	35.68	35.87
		SSIM	0.9634	0.9657	0.9675	0.9765
	σ = 30	PSNR	30.36	30.55	30.68	30.98
		SSIM	0.8922	0.8961	0.9062	0.9068
Peppers	σ = 10	PSNR	35.89	36.24	36.16	36.43
		SSIM	0.9580	0.9624	0.9623	0.9718
	σ = 30	PSNR	30.27	30.86	30.93	31.35
		SSIM	0.8679	0.8856	0.8863	0.8884
Pentagon	σ = 10	PSNR	33.12	33.49	33.55	33.73
		SSIM	0.9594	0.9607	0.9609	0.9739
	σ = 30	PSNR	27.92	28.22	28.65	29.01
		SSIM	0.8454	0.8539	0.8602	0.8705
Baboon	σ = 10	PSNR	30.41	30.46	30.57	30.62
		SSIM	0.9591	0.9597	0.9584	0.9704
	σ = 30	PSNR	24.45	24.69	24.60	24.97
		SSIM	0.8045	0.8219	0.8135	0.8411

It also be seen from the data values that the denoising effect based on the asymmetric gaussian mixture model is better than other algorithms.

5 Conclusion

From the perspective of statistics, when the parameter used to describe the asymmetry is set to $\gamma_k = 1$, the asymmetric gaussian distribution is the symmetric gaussian distribution. So it is of great significance to use the generalized asymmetric gaussian mixture model to replace the symmetric gaussian mixture model. This chapter tries to introduce the asymmetric gaussian mixture model into image denoising and verifies the superiority of the asymmetric gaussian mixture model in image denoising. The asymmetric gaussian mixture model can enrich the prior learning of the image and make the image to keep the detail structure more prominent. However, the estimation of the parameter γ_k of the asymmetric gaussian function introduced in this paper will produce errors, which will affect the optimal solution of the denoising model. Therefore, how to estimate the properties of γ_k will be another research difficulty. At the same time, in other areas of image restoration technology, whether the asymmetric gaussian mixture model also has the same advantages, and what kind of defects there will be remains to be further studied.

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