

# Numerical Investigations on the Impact of Turbulent Prandtl Number and Schmidt Number on Supersonic Combustion

Yongkang Zheng<sup>1,2,\*</sup> and Chao Yan<sup>1</sup>

<sup>1</sup>National Key Laboratory of Computational Fluid Dynamics, Beihang University, Beijing, 100191, China

<sup>2</sup>State Key Laboratory of Aerodynamics, Mianyang, 621000, China

\*Corresponding Author: Yongkang Zheng. Email: zykbuaa@buaa.edu.cn

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**Abstract:** The flow field inside the combustor of a scramjet is highly complicated and the related turbulent Prandtl and Schmidt numbers have a significant impact on the effective numerical prediction of such dynamics. As in many cases researchers set these parameters on the basis of purely empirical laws, assessing their impact (via parametric numerical simulations) is a subject of great importance. In the present work, in particular, two test cases with different characteristics are selected for further evaluation of the role played by these non-dimensional numbers: Burrows-Kurkov case and DLR case. The numerical results indicate that these parameters influence ignition location. Moreover, the temperature distribution is more sensitive to them than to H<sub>2</sub>O mass fraction and velocity distributions.

**Keywords:** Turbulent Prandtl number; turbulent Schmidt number; ignition position; supersonic combustion

## 1 Introduction

For accurate predictions on scramjet engines with turbulence models, calculation requires the turbulent Prandtl number ( $Pr_t$ ) and the Schmidt number ( $Sc_t$ ) for the purpose of accounting for turbulence/chemistry interactions [1]. Eklund demonstrates that predicting  $Pr_t$  and  $Sc_t$  for turbulence models should be as part of the solution, with the assumption that low Schmidt number can lead to unstart phenomena in combustor while high Schmidt number can result in blowout phenomena for flame [2].

However, traditional turbulence models, which are applied mostly to scramjet engine simulations, only take velocity fluctuations into consideration and these two parameters as constants. Since the variance of temperature and the mixing between fuel and oxidizer are inversely proportional to  $Pr_t$  and  $Sc_t$ , respectively, these two dominant parameters have remarkable effect on the conduction and diffusion processes.

Among the researches, both variable and constant  $Pr_t$  and  $Sc_t$  are adopted as the critical factors for scramjet flow field prediction. Baurle chooses the  $Pr_t$  and the  $Sc_t$  as 0.9 and 0.5 respectively to control the turbulent mass and energy transport within the simulation of cavity flameholder using a hybrid Reynolds-averaged/large-eddy scheme [3]. Edwards sets both two parameters as 0.7 while simulating a



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three-dimensional flame/shock wave interaction problem [4]. Gao assumes these two parameters as 0.9 in supersonic turbulent combustion flows with the flamelet model [5]. As in many cases researchers set these parameters on the basis of purely empirical laws, assessing their impact is of great importance. On the other hand, Xiao proposes a strategy about variable  $Pr_t$  and  $Sc_t$  and applies it to model scramjet flows [1]. Two additional equations are introduced into the governing system to calculate  $Pr_t$  and  $Sc_t$  as a part of the solution, which avoids involving the assumed probability density function into supersonic combustion flows. Variable  $Pr_t$  is implemented in Xiao's research on heat transfer prediction for separated flows [6] and variable turbulent Schmidt Number is implemented in Large-eddy simulations by Ingenito [7]. Jiang et al. investigates some of the eddy dissipation combustion results obtained with the  $Pr_t$  and  $Sc_t$  varying from 0.25 to 0.85 and the calculated results are in good agreement with the experimental database [8]. Therefore, it can be concluded that the numerical solution is affected greatly by the selection of the  $Pr_t$  and  $Sc_t$ . For the express purpose of providing reliable evidence for further research, it is inevitable and significant to evaluate the affection of these two parameters on the flow field prediction of scramjet configuration.

In this paper, the sensitivity analysis of turbulent Prandtl number  $Pr_t$  and Schmidt number  $Sc_t$  for supersonic combustion is performed, and ignition location, temperature and combustion efficiency are focused as well. Two representative experimental cases are selected and the numerical results with different combinations of parameters are compared in detail. Some statistical analysis on the affection of the two critical factors are also performed.

## 2 Numerical Methods

A finite volume in-house code developed by the authors is utilized to simulate all the numerical cases. The compressible governing equations, including the mass, momentum, energy and species transport equations, are solved using Reynolds-averaged Navier-Stokes simulations (RANS). The finite-rate chemistry model is adopted as combustion model in this study. Main algorithms of this code are presented follows:

### 2.1 Governing Equations

The Favre-averaged conservation equations in the Cartesian coordinate are interpreted as:

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0 \quad (1)$$

Momentum equation:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \quad (2)$$

Energy equation:

$$\frac{\partial \rho E}{\partial t} + \frac{\partial \rho H u_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \left( \lambda + \frac{\mu_t C_p}{Pr_t} \right) \frac{\partial T}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \sum_{s=1}^{ns} \left( \rho D_s + \frac{\mu_t}{Sc_t} \right) h_s \frac{\partial Y_s}{\partial x_j} \right) + \frac{\partial \tau_{ij} u_i}{\partial x_j} \quad (3)$$

Species continuity equation:

$$\frac{\partial \rho Y_s}{\partial t} + \frac{\partial \rho Y_s u_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \left( \rho D_s + \frac{\mu_t}{Sc_t} \right) \frac{\partial Y_s}{\partial x_j} \right) + \dot{\omega}_s \quad (4)$$

where  $\rho$ ,  $u_j$ ,  $p$ ,  $T$  are the density, velocity component in  $j^{th}$  Cartesian, pressure and temperature, respectively.  $Y_s$ ,  $D_s$ ,  $h_s$  are the mass fraction, diffusion coefficient and enthalpy per unit mass of species  $s$ , respectively.  $\lambda$ ,  $E$  and  $H$  are the thermal conductivity coefficient, total energy per unit mass of mixture and total enthalpy per unit mass of mixture, respectively.  $\tau_{ij}$  is the viscous stress tensor defined as:

$$\tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \frac{\partial u_k}{\partial x_k} \delta_{ij} \quad (5)$$

where  $\delta_{ij}$  is the Kronecker symbol.

The source term  $\dot{\omega}_s$  is given with finite rate chemical model as follows:

$$\dot{\omega}_s = M_s \sum_{j=1}^{nr} (\beta_{js} - \alpha_{js}) (R_j - R_{-j}) \quad (6)$$

where  $M_s$  is the molecular weight of the species  $s$ ,  $\alpha_{js}$  and  $\beta_{js}$  are the stoichiometric coefficients of the related species in the  $j^{th}$  reaction.  $R_j$  and  $R_{-j}$  the forward and backward net rate of the  $j^{th}$  reaction respectively:

$$R_j = K_j \prod_{s=1}^{ns} \left( \frac{\rho_s}{M_s} \right)^{\alpha_{js}}, R_{-j} = K_{-j} \prod_{s=1}^{ns} \left( \frac{\rho_s}{M_s} \right)^{\beta_{js}} \quad (7)$$

where  $K_j$  is the forward rate coefficient of the  $j^{th}$  reaction, and  $K_{-j}$  is the backward rate coefficient of the  $j^{th}$  reaction. The state equation of mixture gas is provide in the form:

$$p = R_u T \sum_{s=1}^{ns} \frac{\rho Y_s}{M_s} \quad (8)$$

where  $R_u$  is the universal gas constant,  $M_s$  is the molecular weight of species  $s$ .

The static enthalpy and the specific heat at constant pressure are considered with temperature and species dependent thermodynamic properties as follow:

$$h = \sum_{s=1}^{ns} h_s Y_s, C_p = \sum_{s=1}^{ns} C_{p,s} Y_s \quad (9)$$

where  $h_s$ ,  $C_{p,s}$  are the enthalpy and specific heat at constant pressure of species  $s$ , modeled with a polynomial function of static temperature.

For each species, thermal conductivity and molecular viscosity are calculated with Sutherlands formula. Wilkes formula [9] is then applied to calculate the mixture thermal conductivity and the mixture molecular viscosity. Besides, mass diffusivity is calculated according to the kinetic theory as follows [10]:

$$\rho D_s = \frac{1 - Y_s}{1 - X_s} \frac{\mu}{Sc} \quad (10)$$

where  $X_s$  is the mole fraction of species  $s$ .

The total energy per unit mass of mixture  $E$  and total enthalpy per unit mass of mixture  $H$  are defined as:

$$H = h + \frac{1}{2} u_j u_j, E = H - \frac{p}{\rho} \quad (11)$$

## 2.2 Turbulence Model

As shown in previous investigations, turbulence model plays very important roles in prediction of the flow field variables and the interaction between turbulence and combustion for supersonic combustion simulations. According to its good performance for less computational cost and more numerical robustness [11], Menter's shear stress transport (SST) model [12] has extensive applications in engineering and thus employed in this paper. The SST turbulence model introduces the original  $k - \omega$  model inside boundary layer region and switches into the standard  $k - \epsilon$  model outside boundary layer region as well as in free shear flows [12]. The transport equations used for SST turbulence model take the form:

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho u_j k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_j} \right] + P_k - \beta^* \rho \omega k \quad (12)$$

$$\frac{\partial \rho \omega}{\partial t} + \frac{\partial \rho u_j \omega}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_\omega \mu_t) \frac{\partial \omega}{\partial x_j} \right] + P_\omega - \beta \rho \omega^2 + 2(1 - f_1) \frac{\rho \sigma_{\omega 2}}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \quad (13)$$

where  $k$  and  $\omega$  are the turbulent kinetic energy and the specific dissipation rate of turbulence, and  $P_k$  and  $P_\omega$  denote the corresponding production term respectively.  $P_k$  and  $P_\omega$  are defined as follows:

$$P_k = \mu_t \Omega^2, P_\omega = C_\omega \rho \Omega^2 \quad (14)$$

where  $\Omega$  is the magnitude of vorticity and  $\mu_t$  is the eddy viscosity,

$$\mu_t = \frac{a_1 \rho k}{\max(a_1 \omega, f_2 \|\Omega\|)} \quad (15)$$

The constants  $\sigma_k, \sigma_\omega, \beta$  and  $C_\omega$  are calculated via

$$\begin{aligned} \phi &= f_1 \phi_1 + (1 - f_1) \phi_2, f_1 = \tanh(\Gamma_1^4) \\ \Gamma_1 &= \min \left[ \max \left( \frac{500 \mu}{\rho \omega d^2}, \frac{\sqrt{k}}{0.09 \omega d} \right), \frac{4 \rho \sigma_{\omega 2} k}{C D_{k\omega} d^2} \right] \\ C D_{k\omega} &= \max \left( 2 \frac{\rho \sigma_{\omega 2}}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, 1 \times 10^{-20} \right) \end{aligned} \quad (16)$$

where  $d$  is the nearest distance to the wall. The constant  $a_1$  is set to be 0.31.  $f_2$  takes the form

$$f_2 = \tanh(\Gamma_2^4), \Gamma_2 = \max \left( \frac{2 \sqrt{k}}{0.09 \omega d}, \frac{500 \mu}{\rho \omega d^2} \right) \quad (17)$$

The two sets of coefficients  $\phi_1$  and  $\phi_2$  chosen from the original  $k - \omega$  model and  $k - \epsilon$  model respectively, are as follows:

$$\begin{aligned} \sigma_{k1} &= 0.85, \sigma_{\omega 1} = 0.5, \beta_1 = 0.075, C_{\omega 1} = 0.5332 \\ \sigma_{k2} &= 1.0, \sigma_{\omega 2} = 0.856, \beta_2 = 0.0828, C_{\omega 2} = 0.4403 \end{aligned} \quad (18)$$

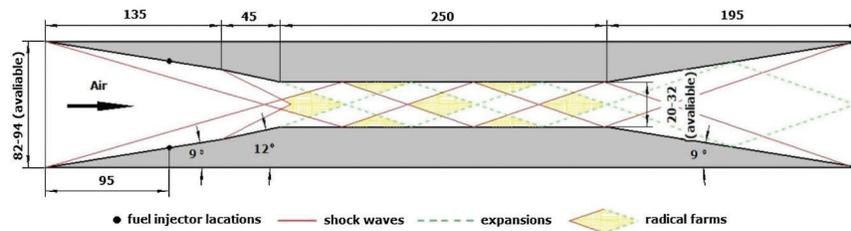
## 2.3 Discretization

The finite volume method is employed in this paper, the compressible Navier-Stokes equations are discretized on the multi-block structured grids. Based on its robustness and high resolution of stationary discontinuities, Edwards' low diffusion flux splitting scheme (LDFSS) [13] is adopted for the inviscid flux vectors. The viscous flux vectors are calculated with second order central difference scheme. The implicit LUSGS method is employed for time marching. In order to meet the computational requirements and

improve computational efficiency, Message Passing Interface (MPI) is involved for massive parallel computation. Different combinations of  $Pr_t$  and  $Sc_t$  are introduced with SST turbulence model mentioned above.

### 3 Code Validation

A test model Hyshot, performed in T4 shock tunnel facility at University of Queensland [14], is chosen to validate the above numerical methods and displayed in Fig. 1. A sequence of wind-tunnel experiments have been performed under various flow conditions and one of the experimental “fuel on” case is selected in this study. In order to reduce the computational cost, a two-dimensional configuration is employed in this paper as which used widely in previous researches. For the dimensional simplification, the injection hole is simplified to be a slot with the approximate length as 2.6 mm and the rest of the main combustor geometry stays the same.



**Figure 1:** Schematic description of the experimental scale scramjet model Hyshot [14]

#### 3.1 Geometry and Flow Conditions

The referenced flow conditions for code validation are tabulated in Tab. 1. The wall boundary conditions are set as standard no-slip and adiabatic wall. Air including 23.31% oxygen and 76.69% nitrogen is modeled instead of perfect gas model. All the numerical methods described above with a constant  $Pr_t$  as 0.7 and a constant  $Sc_t$  as 0.9 are performed in this validation case.

**Table 1:** Freestream flow conditions for the validation case

	$Ma_\infty$	$\alpha_\infty(deg)$	$T_\infty(K)$	$P_\infty(Pa)$	$Y_{H_2}$	$Y_{O_2}$	$Y_{N_2}$
Air	6.42	0.0	412.0	8958.0	0.0	0.2331	0.7669
$H_2$	1.0	54.0	300.0	64000.0	1.0	0.0	0.0

#### 3.2 Grid Sensitivity Analysis

As shown in previous studies, grid spacing in the wall normal direction is highly critical in numerical simulations for reliable results, especially in the region near solid wall. Drawing lessons from the previous numerical experience, the grid Reynolds number which determines the first cell height by controlling the grid scale [15], is introduced here:

$$Re_\Delta = \frac{\rho_\infty u_\infty \Delta}{\mu_\infty} \quad (19)$$

where  $\rho_\infty, u_\infty, \mu_\infty$  are the freestream density, velocity and viscosity respectively,  $\Delta$  denotes the first grid scale on the solid wall in the wall normal direction.

The grid convergence needs to be verified firstly to ensure the reliability of the following computations. Three different grid spacing scales with respect to empirical grid setting strategy [16], namely coarse,

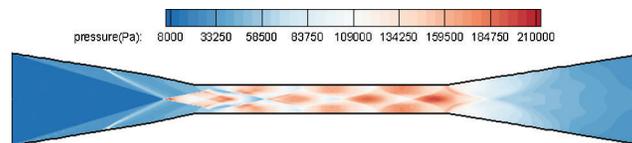
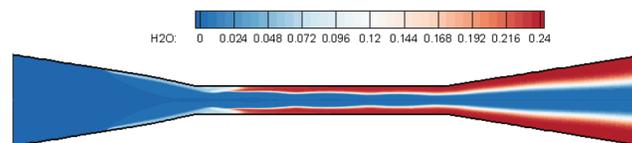
**Table 2:** Detailed information of grid system

	Coarse grid	Medium grid	Fine grid
$\Delta(\text{mm})$	0.0075	0.005	0.0025
$Re_{\Delta}$	60	40	20
Cell Number	48560	60600	72640

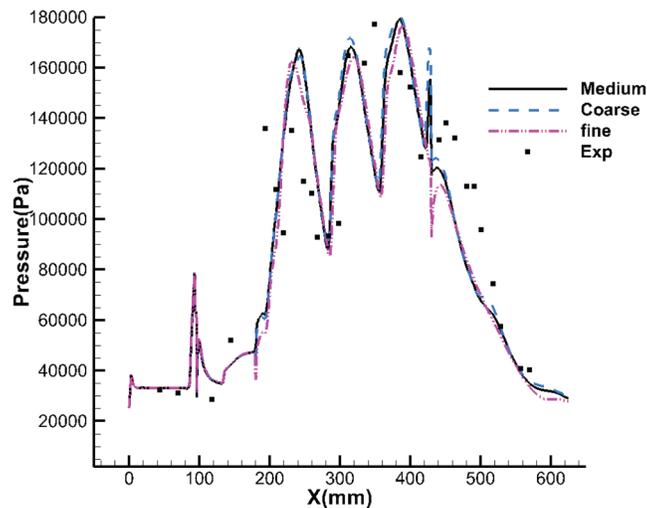
**Figure 2:** Topology of the medium grid for the two-dimensional Hyshot geometry

medium and fine grid, are employed to indicate the grid independence. The detailed grids information is tabulated in Tab. 2. and the distribution of medium grid is displayed in Fig. 2 particularly.

Fig. 3 illustrates the pressure contours distribution of the full configuration, in which the top half is mirrored by the bottom half for better observation. It can be seen that the ramp shocks impinge clearly on each other and two symmetrical strong shocks generate after interaction. Afterwards, the shocks interact with the expansion waves forming at the entrance of the combustor and two separation bubbles can be observed clearly near the throat. The separation shocks reflect between the upper wall and the lower wall and make the boundary layer separate, which brings in a shock-train like structure and some diamond structures form throughout the remaining combustor. Fig. 4 shows the  $\text{H}_2\text{O}$  contours distribution of the full configuration, which represents whether the engine can ignite successfully and whether the combustion efficiency meets the design requirement.

**Figure 3:** Pressure contours of the full configuration for the two-dimensional HyShot geometry**Figure 4:**  $\text{H}_2\text{O}$  contours of the full configuration for the two-dimensional HyShot geometry

The numerical results are quantitatively compared with the experimental data and the comparison results are illustrated in Fig. 5. The static pressure distribution along one of combustor walls is plotted. The profiles obtained from all the three different grid spacing scales match the experimental data quite well. The pressure profiles jump up and down along the combustor wall because of the shock reflections, which reveals the shock-train like structure illustrated in Fig. 1. The good consistency between the numerical results and experimental data, including the jumping positions, represents that both the position of shock wave and



**Figure 5:** Pressure distribution along the lower wall, compared with experimental data

its reflection have been numerically captured well. The qualitative and quantitative comparisons demonstrate that, the numerical approach and the strategy for grid spacing can provide reliable results for supersonic combustion simulation. Meanwhile, in consideration of numerical precision and computational cost, the medium grid is employed next to evaluate the flow field since there is no significant difference between the medium grid solution and the fine grid solution.

#### 4 Computational Details

In the present work, in particular, two test cases with different characteristics are chosen for further evaluation of the role played by these two parameters: Burrows-Kurkov case [17] and DLR case [18]. The Burrows-Kurkov case focuses on the supersonic turbulent combustion occurring inside the boundary layer. The DLR case is a famous reference case since plenty of experimental and numerical investigations have been performed. Therefore, both of the two cases are appropriate for further study.

##### 4.1 Burrows-Kurkov Case

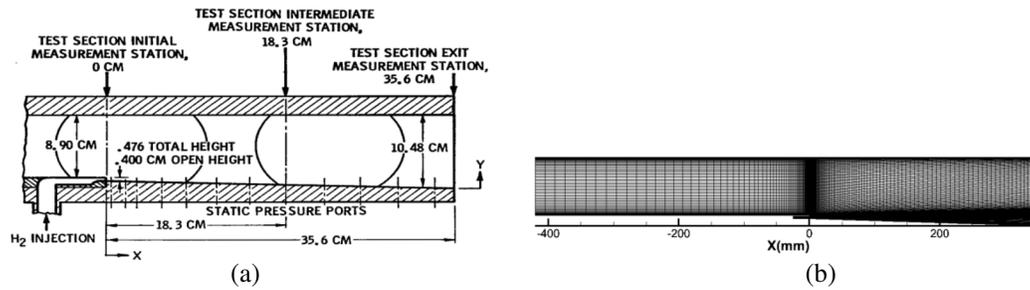
Burrows-Kurkov case [17] examines a finite-rate with hydrogen-air chemical reactions occurring in Mach 2.44 vitiated air. The turbulence/chemistry interactions play a very important role in the prediction of self-ignition process since it is strongly influenced by the wall cooling effect. The supersonic combustion near the wall is quite sensitive to the development of boundary layer. Thus, the ignition points and temperature values at the combustor outlet are investigated.

###### 4.1.1 Geometry and Flow Conditions

The schematic experiment configuration and computational grid are illustrated in Fig. 6. The minimum grid spacing on all the walls is  $5 \times 10^{-3}$  mm. Additionally, all the walls are set as isothermal with  $T_w = 300$  K. The flow conditions are listed in Tab. 3.

###### 4.1.2 Results and Discussions

Nine individual cases are computed under various setting manner for  $Pr_t$  and  $Sc_t$  in this study. The specific parameters for each case are presented in Tab. 4. It can be seen that the ignition occurring at quite a distance downstream from the injection point, representing a typical ignition-delayed phenomenon. The ignition locations for each case are tabulated in Tab. 4. Meanwhile, the maximal temperature and  $H_2O$  mass fraction of the whole combustor obtained via numerical simulation are also



**Figure 6:** Schematic description of the experimental configuration and topology of the grid for Burrows-Kurkov geometry

**Table 3:** Freestream flow conditions for the Burrows-Kurkov case

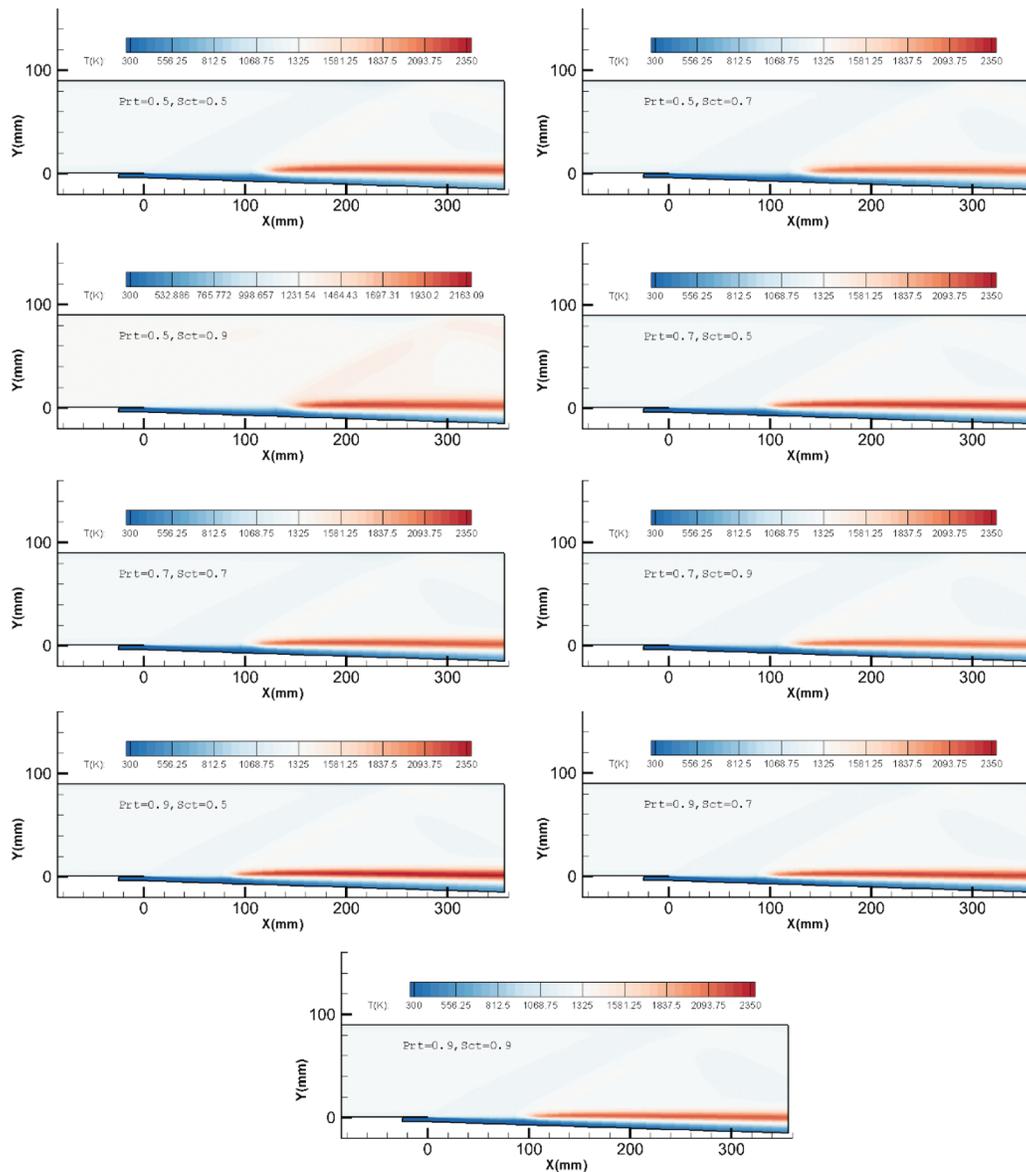
	$Ma_\infty$	$\alpha_\infty$ (deg)	$T_\infty$ (K)	$P_\infty$ (Pa)	$Y_{H_2}$	$Y_{O_2}$	$Y_{H_2O}$	$Y_{N_2}$
Air	2.44	0.0	1270.0	100000.0	0.0	0.258	0.256	0.486
$H_2$	1.0	0.0	254.0	100000.0	1.0	0.0	0.0	0.0

**Table 4:** Statistics data for all the Burrows-Kurkov cases

	$Pr_t$	$Sc_t$	Ignition(mm)	$T_{max}$ (K)	$H_2O_{max}$
case1	0.5	0.5	125.0	2210.73	0.4982
case2	0.5	0.7	140.0	2118.86	0.4996
case3	0.5	0.9	156.0	2084.03	0.5006
case4	0.7	0.5	100.0	2303.56	0.4941
case5	0.7	0.7	111.0	2193.81	0.4970
case6	0.7	0.9	125.0	2122.82	0.4971
case7	0.9	0.5	90.0	2389.18	0.4920
case8	0.9	0.7	100.0	2286.56	0.4946
case9	0.9	0.9	102.0	2190.48	0.4942

tabulated in Tab. 4. It can be seen that the self-ignition point alters between  $X = 90$  mm and  $X = 156$  mm with the variation of  $Pr_t$  and  $Sc_t$ . The lower  $Pr_t$  and the higher  $Sc_t$  bring in further back ignition location. The maximal temperature decreases along with the increase of  $Sc_t$  and seems to have no concern with  $Pr_t$ . The two parameters seemingly have little effect on the maximal  $H_2O$  mass fraction, which implies that turbulent transport process does not significantly affect the chemical reaction process within the finite rate chemical model framework.

Fig. 7 displays the temperature contours in the combustor. It can be seen obviously that the self-ignition point alters in different cases and there are significant differences in the temperature distributions among these cases. Fig. 8 presents the distributions of flow field variables together with the experimental data at the exit for all cases, including temperature and  $H_2O$  mass fraction. It also shows that the temperature distribution is more sensitive to these two parameters than the  $H_2O$  mass fraction.



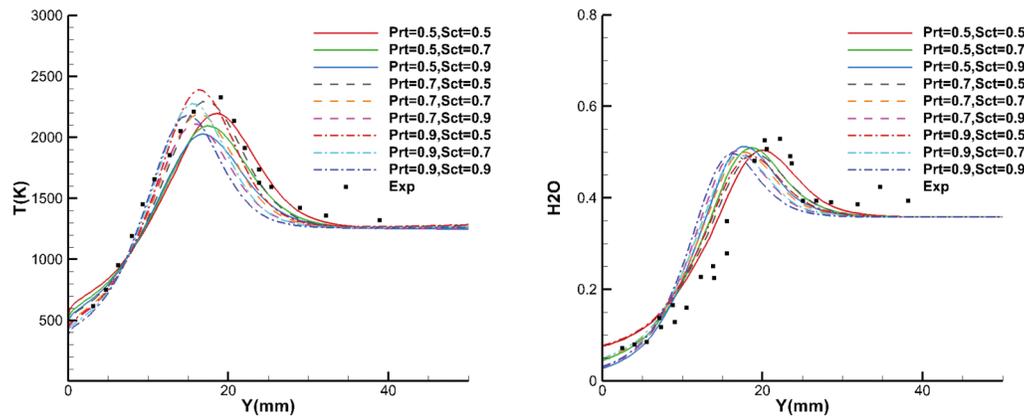
**Figure 7:** Temperature contours for all the Burrows-Kurkov cases

#### 4.2 DLR Case

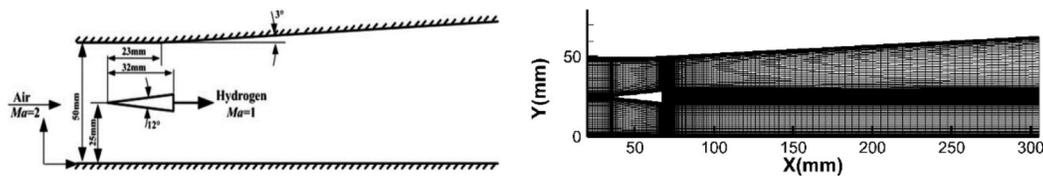
As a typical and famous supersonic combustion ramjet with strut-based injection system, the case similar to DLR’s experimental configuration has been extensively studied. In particular, the velocity and temperature distribution data are very significant and useful for comparison and validation.

##### 4.2.1 Geometry and Flow Conditions

A three-dimensional configuration of the DLR case is employed in order to retain the main properties of the experimental configuration for the RANS simulation. The full configuration with 15 spanwise holes is simplified instead by a smaller configuration with one half injector hole and half the domain between two-adjacent holes. It is considered sufficient enough for RANS simulation to involve the property of spanwise transport and mixing. The side-view schematic of the scramjet combustor and the computational grid are illustrated in Fig. 9. The minimum grid spacing on all the walls is  $5 \times 10^{-3}$  mm, except at the



**Figure 8:** Temperature (left) and  $H_2O$  (right) distributions at the exit, compared with experimental data



**Figure 9:** Schematic description of the experimental configuration and topology of the grid for DLR geometry

**Table 5:** Freestream flow conditions for the DLR case

	$Ma_\infty$	$\alpha_\infty (deg)$	$T_\infty (K)$	$P_\infty (Pa)$	$Y_{H_2}$	$Y_{O_2}$	$Y_{H_2O}$	$Y_{N_2}$
Air	2.0	0.0	340.0	100000.0	0.0	0.232	0.032	0.736
$H_2$	1.0	0.0	250.0	100000.0	1.0	0.0	0.0	0.0

injector's back plane, where the minimum spacing is increased to  $5 \times 10^{-2}$  mm. Moreover, all the combustor walls are assumed to be adiabatic in this case. The flow conditions are listed in [Tab. 5](#).

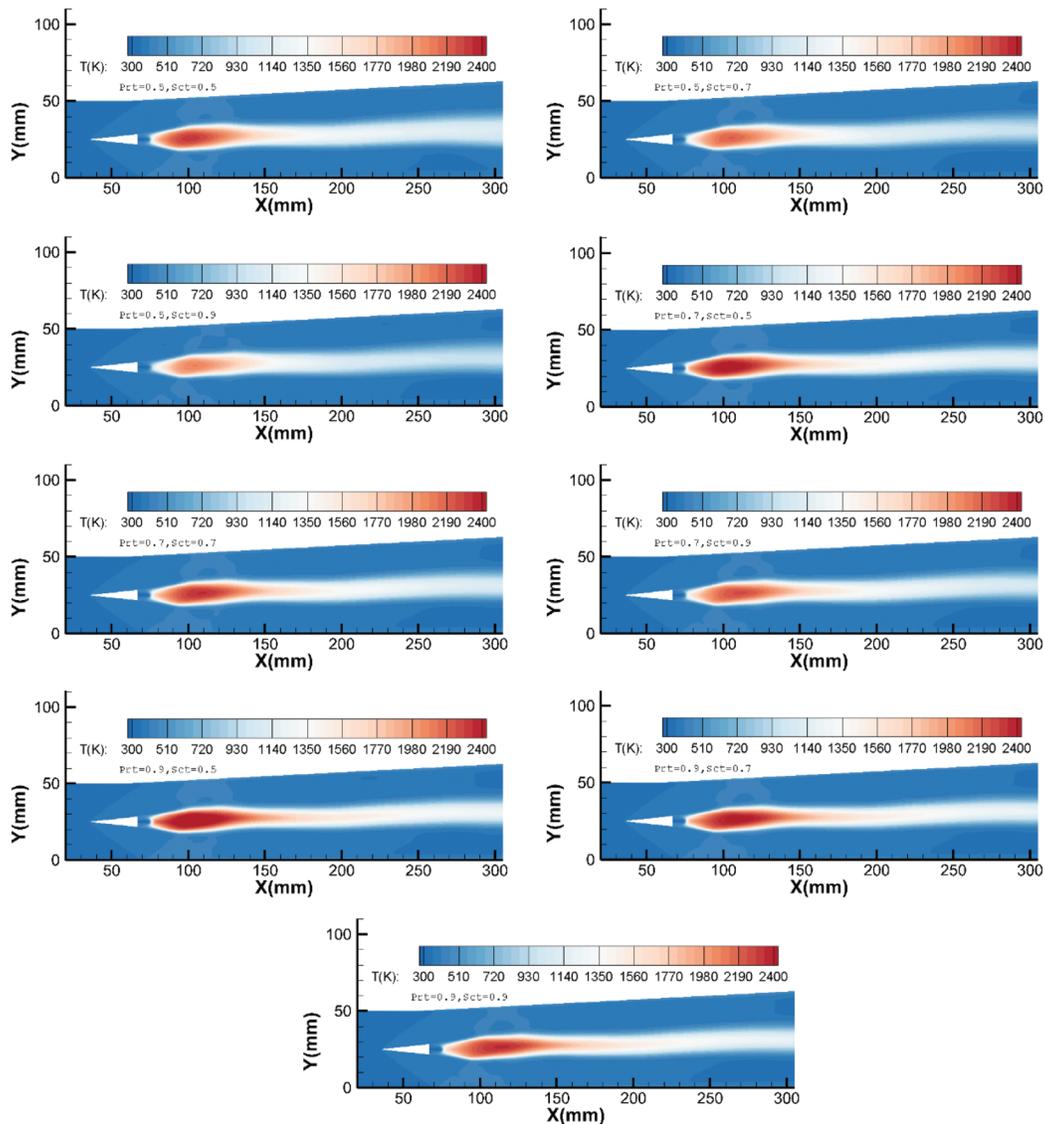
#### 4.2.2 Results and Discussions

Nine individual case simulations are carried out under the same setting manner mentioned above in Burrows-Kurkov cases. The specific setting parameters for each case are tabulated in [Tab. 6](#). It is quite different from the Burrows-Kurkov cases that ignition locations are almost indeclinable in DLR cases, which is probably because of the isothermal wall with  $T_w = 300$  K. An obvious self-ignition delay phenomenon exists inside the boundary layer, which is probably because of the strong cooling process by the wall. Meanwhile, the maximal temperature varies irregularly and seems to have no concern with the  $Pr_t$  and the  $Sc_t$ . However, the maximal  $H_2O$  mass fraction varies within such a small range, which also implies that turbulent transport has merely no significant effect on chemical reaction process.

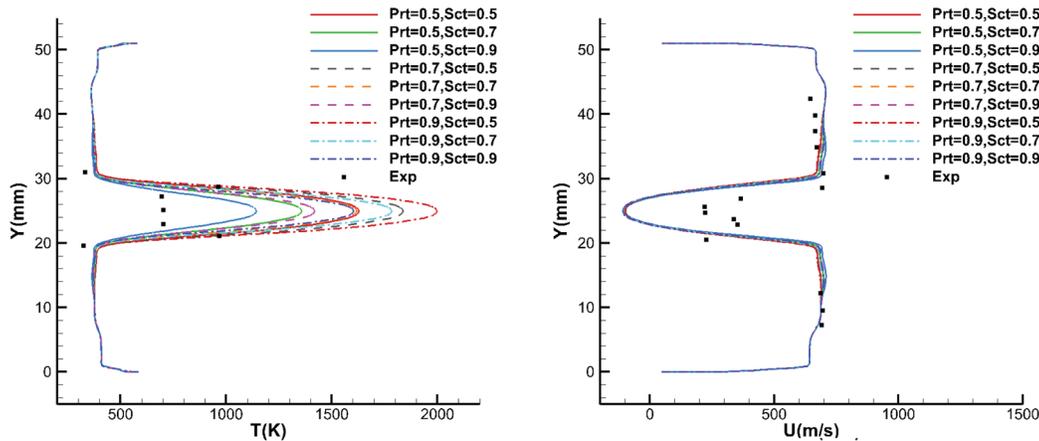
[Fig. 10](#) presents the temperature contours and it is evident that the self-ignition point stays at the position about  $X = 75$  mm. There are some visible differences of the temperature distributions between the cases. [Figs. 11](#) and [12](#) illustrate temperature distributions and velocity distributions at the cross-section  $X = 78$  mm and  $X = 125$  mm, respectively, together with the experimental data for all cases. As can be seen that the temperature distributions are more sensitive to  $Pr_t$  and  $Sc_t$  than the velocity distributions.

**Table 6:** Statistics data for all the DLR cases

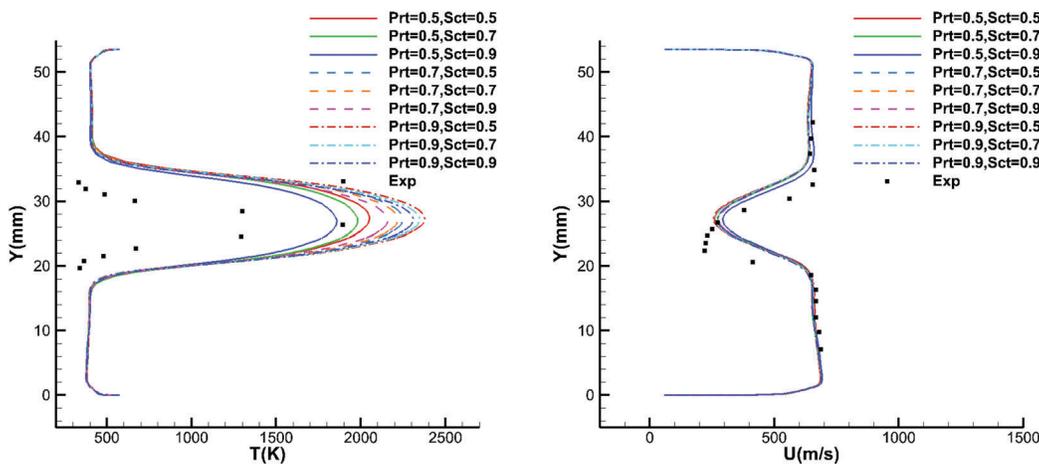
	$Pr_t$	$Sc_t$	Ignition(mm)	$T_{max}(K)$	$H_2O_{max}$
<i>case1</i>	0.5	0.5	75.0	2323.84	0.2738
<i>case2</i>	0.5	0.7	75.0	2361.53	0.2753
<i>case3</i>	0.5	0.9	75.0	2089.79	0.2766
<i>case4</i>	0.7	0.5	75.0	2475.70	0.2726
<i>case5</i>	0.7	0.7	75.0	2355.21	0.2748
<i>case6</i>	0.7	0.9	75.0	2256.60	0.2759
<i>case7</i>	0.9	0.5	75.0	2562.54	0.2715
<i>case8</i>	0.9	0.7	75.0	2453.00	0.2739
<i>case9</i>	0.9	0.9	75.0	2361.53	0.2753



**Figure 10:** Temperature contours for all the DLR cases



**Figure 11:** Temperature (left) and X-Velocity (right) distributions at the cross section  $X = 78$  mm, compared with experimental data



**Figure 12:** Temperature (left) and X-Velocity (right) distributions at the cross section  $X = 125$  mm, compared with experimental data

## 5 Conclusion

In the current study, the numerical analysis on the influence of the turbulent Prandtl number  $Pr_t$  and the Schmidt number  $Sc_t$  on the flow field prediction for scramjet configuration are conducted. Navier-Stokes equations are numerically solved with standard SST turbulence model for turbulent transport process and finite rate model for chemical reaction process. Main conclusions are summarized as follow:

1. In Burrows-Kurkov case, the lower  $Pr_t$  and the higher  $Sc_t$  bring the ignition location further back. The isothermal wall with cooling process exists inside the boundary layer can strongly delay the self-ignition process.  $H_2O$  mass fraction distribution is less sensitive to the  $Pr_t$  and the  $Sc_t$ , which implies that turbulent transport process have merely no significant effect on chemical reaction process within the finite rate chemical model framework.
2. In DLR case, ignition locations are almost indeclinable because the combustion process occurs almost in the central region of combustor. The maximal temperature varies irregularly and the maximal  $H_2O$  mass fraction varies within a small range. The  $Pr_t$  and  $Sc_t$  affect the distribution of the combustion region,

which in turn changes the temperature distribution. In addition, the intensity of the combustion remained almost constant on account of the almost unchanged maximal H<sub>2</sub>O mass fraction.

3. Temperature distribution is more sensitive than H<sub>2</sub>O mass fraction and velocity distributions. Since the  $Pr_t$  and  $Sc_t$  have no significantly effect on chemical reaction process, it implies that the heat conduction and diffusion mechanism have been greatly influenced by these two parameters via turbulent transport and mixing processes. Detailed and in-depth research for this opinion should be investigated for a further step.

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