A Novel Method to against Quantum Noises in Quantum Teleportation

Shengyao Wu[1], Wenjie Liu[2] and Zhiguo Qu[3, *]

Abstract: In order to improve the anti-noise performance of quantum teleportation, this paper proposes a novel dynamic quantum anti-noise scheme based on the quantum teleportation which transmits single qubit state using Bell state. Considering that quantum noise only acts on the transmitted qubit, i.e., the entangled state that Alice and Bob share in advance is affected by the noise, thus affecting the final transmission result. In this paper, a method for dynamically adjusting the shared entangled state according to the noise environment is proposed. By calculating the maximum fidelity of the output state to determine the shared entangled state, which makes the quantum teleportation be affected by the noise as little as possible. This paper calculates the fidelity of teleportation under four kinds of channel noise (amplitude damping, phase damping, bit flip and depolarizing noise). The results show that the scheme has a suppression effect on phase damping, bit flip and depolarizing noise under certain conditions. When the noise intensity is larger, the optimized efficiency is better.

Keywords: Quantum noise, quantum teleportation, entangled channel, fidelity.

1 Introduction

Quantum teleportation is an important technology in quantum communication. Quantum teleportation transmits an unknown quantum bit state from one place to another through quantum measurements and some classical information. In general, quantum teleportation requires the sender and the receiver to share an entangled state as a quantum channel in advance, and the sender (called Alice) can send an arbitrary quantum bit state to the remote receiver (called Bob) through the entangled channel. The communication process was originally proposed by Bennett et al. [Bennett, Brassard, Crépeau et al. (1993)] in 1993. The scheme uses the EPR pair as a quantum channel to achieve remote transmission of a single quantum bit state. This process completes the transmission of

[1] School of Computer & Software, Nanjing University of Information Science and Technology, Nanjing, 210044, China.

[2] Jiangsu Engineering Center of Network Monitoring, Nanjing University of Information Science and Technology, Nanjing, 210044, China.

[3] Jiangsu Collaborative Innovation Center of Atmospheric Environment and Equipment Technology, Nanjing University of Information Science & Technology, Nanjing, 210044, China.

[*] Corresponding Author: Zhiguo Qu. Email: qzghhh@126.com.

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quantum information and lays a good foundation for the development of quantum information technology.

Quantum entanglement is an important resource in quantum communication systems. Due to the special correlation between the mutually entangled quantum bits, the perturbation of a certain particle in the entangled state will affect the state of other particles. Therefore, the quantum entangled state is generally shared between the sender and the receiver, then the quantum channel which can realize the remote transmission of quantum information is established [Karlsson and Bourennane (1998)]. The entangled channels always used in quantum communication include Bell state [Shi, Huang, Yang et al. (2010); Zhang, Yang, Man et al. (2005)], GHZ state [Jin, Ji, Zhang et al. (2006); Bouwmeester, Pan and Daniell (1999)], W state [Zhang, Shu and Mo (2013)], X state [Kang, Chen and Yang (2014)] and Brown state [Tao, Ming and Guang (2008)]. In addition to selecting the pure state as the quantum channel, it has been found that choosing the mixed state as the quantum channel [Werner (1989); Adhikari, Ganguly, Chakrabarty et al. (2008); Horodecki, Horodecki and Horodecki (1999); Adhikari, Majumdar, Nayak et al. (2010)] can improve the communication efficiency of the teleportation in actual quantum communication.

In the real communication process, the quantum channel is usually interfered by the quantum noise, which will affect efficiency of the quantum teleportation [Barnum, Nielsen and Schumacher (1998)]. Therefore, how to improve the fidelity of quantum states which suffers quantum noise is an urgent problem to be solved. In recent years, researchers have been working on how to resist the effects of quantum noise on quantum teleportation [Kumar and Pandey (2003); Shaw and Brun (2011); Pramanik and Majumdar (2013); Espoukeh and Pedram (2014); Liang, Liu, Feng et al. (2015); Bang, Ryu and Kaszlikowski (2018)]. In 2002, Oh et al. [Oh, Lee and Lee (2002)] calculated and analyzed the fidelity of quantum teleportation by solving the master equation. Under isotropic noise, the average fidelity is 1/2, and when the noise channel is modeled by a single Lindblad operator, the average fidelity is always higher than 2/3. In 2006, Hao et al. [Hao, Zhang and Zhu (2006)] analyzed the effect of teleportation in amplitude damping noise channel by Bloch ball representation. In 2008, Jung et al. [Jung, Hwang, Ju et al. (2008)] studied the effects of choosing GHZ state or W state as entangled channel on quantum teleportation under different noisy environment. In 2010, Hu et al. [Hu, Gu, Gong et al. (2010)] analyzed the effects of noise on the quantum teleportation scheme of two qubits using four qubit entangled channels, and studied the efficiency of utilizing parallel Bell pairs and inseparable channels with genuine multipartite entanglement. In 2013, Liang et al. [Liang, Liu, Feng et al. (2013)] studied the use of partially entangled EPR states as quantum channels to achieve quantum teleportation, and bit flip, dephasing and isotropic noise are analyzed for the protocol, respectively. In 2014, Laura et al. [Knoll, Schmiegelow and Larotonda (2014)] demonstrated through experiments that the effect of noise on quantum teleportation is consistent with theoretical results. In 2015, Fortes et al. [Fortes and Rigolin (2015)] studied the effects of four common channel noises on quantum teleportation and found that additional quantum noise or smaller entanglement can lead to higher efficiency. In 2016, Li et al. [Li, Wang, Zhang et al. (2016)] proposed to reduce the influence of quantum noise on quantum teleportation by constructing decoherence-free subspace under different noise conditions. In 2017, Wang et al. [Wang, Qu, Wang et al. (2017); Wang and Qu (2016)] studied the effect of quantum noise on the deterministic joint remote preparation of arbitrary two qubit states using GHZ as a quantum channel. Qu et al. [Qu, Wu, Wang et al. (2017)] studied the effects of quantum noise on arbitrary two-particle states of deterministic quantum remote preparation (DRSP) using different entangled channels. In 2018, Jiang [Jiang (2019)] found that under Pauli noise, different Bell channels have different effects on quantum teleportation. Sun et al. [Sun, Wu, Qu et al. (2018)] studied the effects of different quantum noises on different DRSP methods for preparing arbitrary three-particle state. The conclusions show that the same quantum noise has the same effect on different preparation schemes.

In the process of quantum information transmission, it is generally affected by four kinds of quantum channel noise, namely amplitude damping noise, phase damping noise, bit flip noise and depolarizing noise. Considering these four kinds of noisy channels, this paper proposes a new scheme to reduce the influence of quantum noise by adjusting the quantum entangled channel. In order not to consider the influence of different input quantum states on the fidelity, this paper calculates the state-independent average fidelity to analyze the relationship among the influence of quantum noise, the quantum channel and the noise factor, and enhances the quantum teleportation by adjusting the quantum entangled channel.

This article is organized as follows. In the Section 2, a brief review of the initial quantum teleportation protocol is given. Section 3 introduces four kinds of quantum channel noise, and briefly introduces the way the noise environment affects the teleportation and the way the noise influence is measured. In the Section 4, the relationship among the influence of noise, the selected quantum channel and noise intensity is studied by calculating the state-independent average fidelity in different noise environments. This article is concluded in Section 5.

2 Quantum teleportation scheme

This section will briefly review the quantum teleportation protocol [Bennett, Brassard, Crépeau et al. (1993)]. Quantum teleportation protocol can solve this problem: it is assumed there are two communicating parties Alice and Bob, they share an EPR pair in advance, Alice needs to transmit an unknown quantum state to Bob, but can only send classic messages, how to realize it?

In the ideal situation, suppose Alice wants to send Bob an arbitrary single qubit state

$$|\psi\rangle=\alpha|0\rangle+\beta|1\rangle \tag{1}$$

where α and β are unknown amplitude message and satisfy the normalized condition.

The scheme consists of the following steps:

S1) Alice and Bob share a Bell state in advance as the entangled quantum channel. Assume that the Bell state used is

$$|C\rangle=\frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right)_{AB}, \tag{2}$$

where the first particle belongs to Alice and the second particle belongs to Bob. The initial quantum system is

$$\begin{aligned}|S_0\rangle &= |\psi\rangle|C\rangle \\ &= \frac{1}{\sqrt{2}}\left[\alpha\left(|000\rangle+|011\rangle\right)+\beta\left(|100\rangle+|111\rangle\right)\right]_{A_1A_2B}.\end{aligned} \tag{3}$$

S2) Alice passes her particles through a controlled NOT (C-NOT) gate and the entire quantum system will become

$$|S_1\rangle = \frac{1}{\sqrt{2}}\left[\alpha\left(|000\rangle+|011\rangle\right)+\beta\left(|110\rangle+|101\rangle\right)\right]_{A_1A_2B} \tag{4}$$

S3) Alice lets her first particle pass a Hadamard gate, and the quantum system will become

$$\begin{aligned}|S_2\rangle &= \frac{1}{2}\left[|00\rangle\left(\alpha|0\rangle+\beta|1\rangle\right)+|01\rangle\left(\alpha|1\rangle+\beta|0\rangle\right)\right. \\ &\left.|10\rangle\left(\alpha|0\rangle-\beta|1\rangle\right)+|11\rangle\left(\alpha|1\rangle-\beta|0\rangle\right)\right]_{A_1A_2B}\end{aligned} \tag{5}$$

S4) Alice measures her two qubits under basis $\{|0\rangle,|1\rangle\}$ separately and sends the results to Bob. Bob receives the measurement results sent by Alice and performs the corresponding unitary operation on his qubit to recover the quantum state Alice wants to transmit. The relationship between the measurement result and the unitary operation is as follows.

$$\begin{aligned}&00 \Rightarrow U_{00} \equiv I \equiv \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, 01 \Rightarrow U_{01} \equiv X \equiv \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \\ &10 \Rightarrow U_{10} \equiv Z \equiv \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, 11 \Rightarrow U_{11} \equiv ZX \equiv \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\end{aligned} \tag{6}$$

Quantum teleportation subtly transfers the information of one quantum state to the other particle, realizing the remote transmission of quantum state, which has great practical value. However, in the actual transmission process, the quantum particles are easily affected by the channel noise, and the transmission efficiency will be greatly reduced. Therefore, it is an inevitable problem to analyze the interference caused by the quantum noise and study how to minimize the interference.

3 Quantum teleportation under noisy environment

In this section, we will formally analyze the influence of quantum noise on the quantum teleportation, and consider improving the anti-noise performance of the protocol by adjusting the entangled channel in the teleportation process.

3.1 Noisy channels

The dynamics of a quantum system interacting with a noisy environment can be described by operator-sum forms. In the process of quantum communication, there are four kinds of quantum noise, which are amplitude damping, phase damping, bit flipping and depolarizing noise.

3.1.1 Amplitude damping channel

Amplitude damping generally describes the energy dissipation of quantum during transmission, which has operation elements

$$E_0=\begin{pmatrix}1 & 0\\ 0 & \sqrt{1-p}\end{pmatrix}, E_1=\begin{pmatrix}0 & \sqrt{p}\\ 0 & 0\end{pmatrix} \tag{7}$$

where p is the intensity of the amplitude damping noise.

3.1.2 Phase damping channel

Phase damping is a purely quantum mechanical noise process that describes the loss of quantum information without energy dissipation. According to the unitary freedom of quantum operations, the operands are equivalent to the operands of the phase flip channel.

$$E_0=\sqrt{1-p}I=\sqrt{1-p}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, E_1=\sqrt{p}Z=\sqrt{p}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{8}$$

where p is the intensity coefficient of the phase damping noise.

3.1.3 Bit flip channel

The bit flip lets the qubit $|0\rangle$ becomes $|1\rangle$, and $|1\rangle$ becomes $|0\rangle$ with probability p. Its operands can be expressed as

$$E_0=\sqrt{1-p}I=\sqrt{1-p}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, E_1=\sqrt{p}X=\sqrt{p}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \tag{9}$$

where p denotes the intensity coefficient of the bit damping noise.

3.1.4 Depolarizing channel

Depolarization noise depolarizes a qubit with probability p, that is, a qubit gradually becomes a maximum mixed state $I/2$, and its operands can be expressed as

$$\begin{aligned}&E_0=\sqrt{1-p}I=\sqrt{1-p}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, E_1=\sqrt{\frac{p}{3}}X=\sqrt{\frac{p}{3}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},\\ &E_2=\sqrt{\frac{p}{3}}Z=\sqrt{\frac{p}{3}}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, E_3=\sqrt{\frac{p}{3}}Y=\sqrt{\frac{p}{3}}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}.\end{aligned} \tag{10}$$

where p is the noise intensity and i is the imaginary symbol.

3.2 Measuring the effects of quantum noise

The initial quantum teleportation assumes that Alice and Bob of the communication share an ideal Bell state. In the real communication process, the Bell state is generally prepared by one of the communicating parties, and then one of the particles is distributed to the other party. In this case, the transmitted qubit is inevitably affected by quantum noise. Due to the symmetry of the Bell state, it is assumed that Bell is prepared by Alice and then one of the particles is sent to Bob. Thus, the quantum teleportation circuit in a noisy environment is shown in Fig. 1.

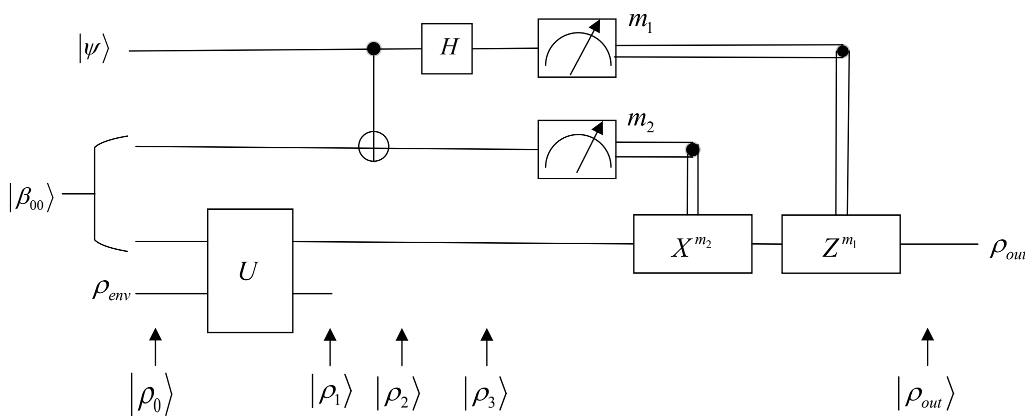


Figure 1: A single quantum bit quantum teleportation circuit, the left upper two single lines represent Alice's quantum system, and the third single line represents Bob's quantum system. The meter represents quantum measurement and the double lines represent the classical information transmission channel

In order to facilitate the process to discuss the effects of noise on quantum systems, quantum systems are generally represented as the form of density operators. Thus, the initial quantum system can be expressed as

$$\rho_0 = \frac{1}{2}\left[\alpha\left(|000\rangle + |011\rangle\right) + \beta\left(|100\rangle + |111\rangle\right)\right] \left[\alpha\left(\langle 000| + \langle 011|\right) + \beta\left(\langle 100| + \langle 111|\right)\right]_{A_1 A_2 B} \tag{11}$$

Alice sends the second particle in the Bell state to Bob, and the qubit transmitted to Bob will be affected by quantum noise. After the transfer is complete, the quantum system will become

$$\rho_1 = \sum_i E_i^B \rho_0 E_i^{B\dagger} \tag{12}$$

where E_i represents the noise operation element whose superscript represents the qubit which is suffering noise.

Alice lets her particles pass through the C-NOT gate with A_1 the control qubit and A_2 the target qubit, the quantum system becomes

$$\rho_2 = CNOT_{A_1 A_2} \rho_1 CNOT_{A_1 A_2}^{\dagger} \tag{13}$$

where CNOT represents a controlled NOT gate and the subscript represents two particles that are operated by the C-NOT gate. The C-NOT gate can be expressed as

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{14}$$

Alice lets her first qubit pass through a Hadamard gate and the entire quantum system becomes

$$\rho_3 = H_{A_1} \rho_2 H_{A_1}^{\dagger} \tag{15}$$

H represents the Hadamard gate and the subscript stands for the particles passing through the Hadamard gate. H gate can be expressed as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{16}$$

After that, Alice measures her qubits. For the sake of simplicity, Alice's measurements can be viewed as measurements using the measurement basis $\left\{ \left| M_{m_1 m_2} \right\rangle \right\}, m_1, m_1 \in \{0,1\}$.

$$M_{00} = |00\rangle\langle 00| \tag{17}$$

$$M_{01} = |01\rangle\langle 01| \tag{18}$$

$$M_{10} = |10\rangle\langle 10| \tag{19}$$

$$M_{11} = |11\rangle\langle 11| \tag{20}$$

After Alice measures her two particles, the entire system becomes

$$\tilde{\rho}_{m_1 m_2} = \frac{M_{m_1 m_2} \rho_3 M_{m_1 m_2}^{\dagger}}{tr\left(M_{m_1 m_2}^{\dagger} M_{m_1 m_2} \rho_3 \right)}. \tag{21}$$

Calculate the partial trace over Alice's particles, Bob knows that his qubit now becomes

$$\tilde{\rho}_{out} = tr_{A_1 A_2} \left(\tilde{\rho}_{m_1 m_2} \right) \tag{22}$$

In the final step of the scheme, Alice tells Bob about her measurement results through classical channel. Bob does the corresponding unitary operation on his qubit based on the classic information transmitted by Alice, so Bob finally gets the output quantum state

$$\rho_{out} = U_{m_1 m_2} \tilde{\rho}_{out} U_{m_1 m_2}^{\dagger} \tag{23}$$

For ease of calculation, the quantum bit that Alice wishes to transmit can be rewritten as follow.

$$|\psi\rangle = |\alpha| |0\rangle + |\beta| e^{ic} |1\rangle \tag{24}$$

$|\alpha|, |\beta|$ are the modulus of α, β, $c \in [0, 2\pi]$. The rewritten quantum state is as same as the original quantum state except an unimportant global phase.

Fidelity can describe the difference between two quantum states, so fidelity can be introduced as a measure of the effect of noise on quantum states. Fidelity is defined as

$$F = \langle \psi | \rho_{out} | \psi \rangle \tag{25}$$

In addition, in order to better analyze the influence of quantum noise on quantum teleportation, the average input state fidelity is defined as

$$\overline{F} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^1 F d|\alpha|^2 \, dc. \tag{26}$$

The fidelity is between 0 and 1. The greater the fidelity value, the less the effect of noise.

4 Utilizing dynamic entangled channel in noisy environments

In order to discuss the relationship between the influence of noise on quantum teleportation and the entangled channel used, it is assumed that the quantum channel used in the quantum teleportation process is an unknown two qubit state

$$|\mathrm{C}\rangle = x|00\rangle + y|11\rangle \tag{27}$$

where $x, y \in R \cap [-1,1]$ and satisfies $x^2 + y^2 = 1$. The quantum channel used is equivalent to the quantum channel in the original teleportation when $x = 1/\sqrt{2}, y = 1/\sqrt{2}$. In the following subsections, this paper will calculate the effect of noise on quantum teleportation using the channel shown in Eq. (27) as the entangled channel.

4.1 Efficiency under amplitude damping

In the case of amplitude damping noise, Alice prepares the Bell state and sends one of the qubits to Bob. Next, Alice and Bob perform the quantum teleportation protocol in the original steps. Calculations show that, Bob eventually recovers to the same quantum state when Alice measures her two particles using basis 00 or 10, and Bob recovers the same quantum state when Alice measures her two particles using basis 01 or 11. The other three kinds of noise have the same conclusion.

When Alice measures her particles under basis 00 or 10, the fidelity of quantum teleportation is

$$F_{\mathrm{AD}}^{00,10} = \frac{\left[|\alpha|^2 x + \sqrt{1-p}\,|\beta|^2 y\right]^2 + p|\alpha|^2|\beta|^2 y^2}{|\alpha|^2 x^2 + |\beta|^2 y^2}. \tag{28}$$

The average fidelity of the output state is

$$\overline{F}_{AD}^{00,10}=\frac{x^2+(1-2p)(1-x^2)-2x\sqrt{1-x^2}\sqrt{1-p}}{2(2x^2-1)^3}\Big[(2x^2-1)(4x^2-3)$$
$$+2(1-x^2)^2\ln\left(\frac{x^2}{1-x^2}\right)\Big]+\frac{2\sqrt{1-p}x\sqrt{1-x^2}-(2-3n)(1-x^2)}{(2x^2-1)^2} \tag{29}$$
$$\times\left[2x^2-1-(1-x^2)\ln\left(\frac{x^2}{1-x^2}\right)\right]+\frac{(1-p)(1-x^2)}{2x^2-1}\ln\left(\frac{x^2}{1-x^2}\right).$$

When Alice chose the 01 or 11 basis, the fidelity of quantum teleportation becomes

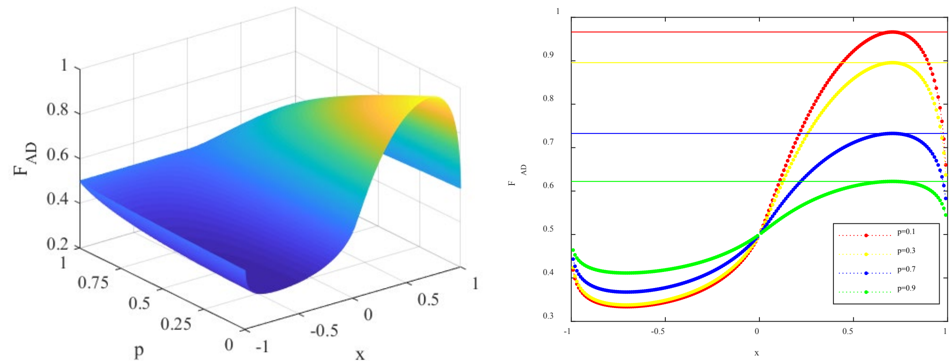
$$F_{\mathrm{AD}}^{01,11}=\frac{\left[|\alpha|^2y+\sqrt{1-p}|\beta|^2x\right]^2+p|\alpha|^2|\beta|^2y^2}{|\alpha|^2y^2+|\beta|^2x^2}. \tag{30}$$

The average fidelity of the output state is

$$\overline{F}_{AD}^{01,11}=\frac{x^2+(1-2p)(1-x^2)-2x\sqrt{1-x^2}\sqrt{1-p}}{2(1-2x^2)^3}\Big[(2x^2-1)(4x^2-1)$$
$$+2x^4\ln\left(\frac{1-x^2}{x^2}\right)\Big]+\frac{2\sqrt{1-p}x\sqrt{1-x^2}-2x^2+p(1-x^2)}{(1-2x^2)^2} \tag{31}$$
$$\times\left[1-2x^2+x^2\ln\left(\frac{x^2}{1-x^2}\right)\right]+\frac{x^2}{1-2x^2}\ln\left(\frac{1-x^2}{x^2}\right).$$

It is easy to verify that $\overline{F}_{AD}^{01,11}=\overline{F}_{AD}^{00,10}$ no matter what value x,p are. This means that no matter what measurement basis Alice uses, the average fidelity is the same. Therefore, the average fidelity of the out state is only related to the entanglement channel coefficient and the quantum noise intensity factor, regardless of which measurement basis Alice uses. For ease of discussion, the average fidelity under amplitude damping noise is expressed as $\overline{F}_{AD}$ which satisfied

$$\overline{F}_{AD}=\overline{F}_{AD}^{00,10}=\overline{F}_{AD}^{01,11}. \tag{32}$$



(a) (b)

Figure 2: a) The figure depicts a three-dimensional image of the average fidelity $\overline{F}_{AD}$ of the quantum teleportation as a function of the variables x and p. (b) The graph takes the noise intensity $p=0.1,0.3,0.7,0.9$ as an example, showing the relationship between the fidelity and the use of different quantum entangled channels (dotted line). Among them, the solid line of the same color indicates the fidelity of the original quantum teleportation ($x=1/\sqrt{2}$) in the corresponding noise environment

The curve of the average fidelity $\overline{F}_{AD}$ varies with x,p is plotted in Fig. 2. It can be seen that no matter what value the noise factor p takes, the trend of fidelity is first decreased. When the value of x is approximately -0.7, the fidelity takes a minimum value. As the value of x continues to increase, the fidelity gradually increases. When $x=1/\sqrt{2}$, the fidelity takes the maximum value. As the value of x continues to increase, the fidelity gradually decreases.

It can be seen that in the amplitude damping noise environment, regardless of the value of the noise intensity p, the fidelity of using the arbitrary quantum entangled state $x|00\rangle+y|11\rangle$ as the entangled channel is not higher than the original quantum teleportation which uses the quantum entangled channel $(|00\rangle+|11\rangle)/\sqrt{2}$. Therefore, in the amplitude damping noise environment, it is impossible to select the appropriate quantum entangled channel by adjusting the value of x to increase the fidelity of quantum teleportation.

4.2 Efficiency under phase damping

In the case of phase damping noise, when Alice measures with basis 00 or 10 , the fidelity of quantum teleportation is

$$F_{Phs}^{00,10}=\frac{(1-p)\left[|\alpha|^2 x+|\beta|^2 y\right]^2+p\left[|\alpha|^2 x-|\beta|^2 y\right]^2}{|\alpha|^2 x^2+|\beta|^2 y^2}. \tag{33}$$

The average fidelity of the output state is

$$\overline{F}_{Phs}^{00,10}=\frac{1+(4p-2)x\sqrt{1-x^2}}{2(2x^2-1)^3}\left[(2x^2-1)(4x^2-3)+2(1-x^2)^2\ln\left(\frac{x^2}{1-x^2}\right)\right]$$
$$+\frac{2(1-2p)x\sqrt{1-x^2}-2(1-x^2)}{(2x^2-1)^2}\left[2x^2-1-(1-x^2)\ln\left(\frac{x^2}{1-x^2}\right)\right] \tag{34}$$
$$+\frac{1-x^2}{2x^2-1}\ln\left(\frac{x^2}{1-x^2}\right).$$

When Alice measures with base 01 or 11, the fidelity of quantum teleportation is

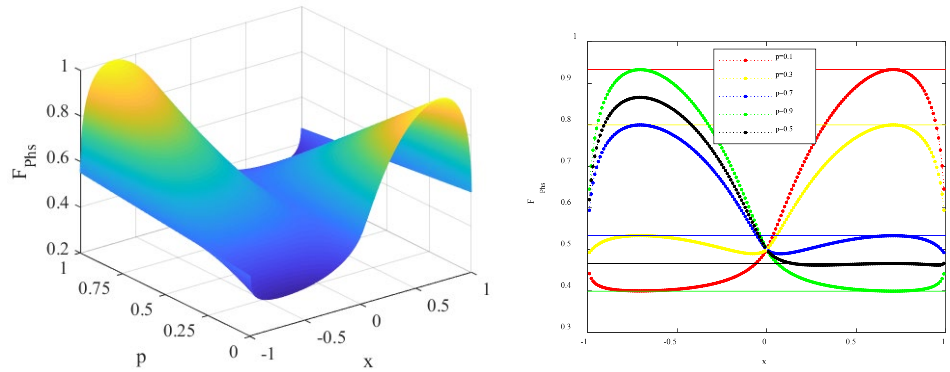
$$F_{Phs}^{01,11}=\frac{(1-p)\left[|\alpha|^2 y+|\beta|^2 x\right]^2+p\left[|\alpha|^2 y-|\beta|^2 x\right]^2}{|\alpha|^2 y^2+|\beta|^2 x^2}. \tag{35}$$

The average fidelity of the output state is

$$\overline{F}_{Phs}^{01,11}=\frac{1-2(1-2p)x\sqrt{1-x^2}}{2(1-2x^2)^3}\left[(2x^2-1)(4x^2-1)\right.$$
$$\left.+2x^4\ln\left(\frac{1-x^2}{x^2}\right)\right]+\frac{2(1-2p)x\sqrt{1-x^2}-2x^2}{(1-2x^2)^2} \tag{36}$$
$$\times\left[1-2x^2+x^2\ln\left(\frac{x^2}{1-x^2}\right)\right]+\frac{x^2}{1-2x^2}\ln\left(\frac{1-x^2}{x^2}\right).$$

Similarly, there is $\overline{F}_{Phs}^{00,10}=\overline{F}_{Phs}^{01,11}$, so the average fidelity in the phase damping environment can be expressed as $\overline{F}_{Phs}$ and satisfied

$$\overline{F}_{Phs}=\overline{F}_{Phs}^{00,10}=\overline{F}_{Phs}^{01,11} \tag{37}$$



(a) (b)

Figure 3: (a) The figure depicts a three-dimensional image of the average fidelity $\overline{F}_{Phs}$ of the quantum teleportation as a function of the variables x, p. (b) The graph takes the noise intensity p as $0.1, 0.3, 0.7, 0.9$, respectively, and shows the relationship between the fidelity and different quantum entangled channels (dotted line). Among them, the solid line of the same color indicates the fidelity of the original quantum teleportation ($x = 1/\sqrt{2}$) in the corresponding noise environment. The black line indicates the fidelity curve in a critical noise environment, and the noise intensity denotes a turning point

According to the Eq. (37), the average fidelity $\overline{F}_{Phs}$ as a function of x, p is shown in Fig. 3. It can be found that the average fidelity changes under phase damping as follows.

a) When p is less than about 0.2, fidelity decreases as x increases ($-1 \le x \le -1/\sqrt{2}$). When the value of x reaches approximately -0.7, the value of fidelity takes the minimum value. As x continues to increase ($-1/\sqrt{2} \le x \le 1/\sqrt{2}$), the fidelity gradually increases as x increases. When the value of x is taken to be about 0.7, the fidelity takes the maximum value. As x continues to increase, fidelity begins to decrease again.

b) When p is greater than about 0.2 and less than 0.5. The fidelity first increases with the increase of x, and then decreases with the increase of x. When $0 < x \le 1/\sqrt{2}$, the fidelity gradually increases with the increase of x, and the maximum value is obtained when x takes about 0.7. Then, the fidelity gradually decreases as x increases.

c) when p=0.5 and $-1 \le x \le 0$, the fidelity first increases and then decreases as x increases. Specifically, when $-1 \le x \le -1/\sqrt{2}$, the fidelity gradually increases as x increases. When x is approximately -0.7, the fidelity takes the maximum value. When $-1/\sqrt{2} \le x \le 0$, the value of fidelity gradually decreases as x increases. When $0 \le x \le 1$, the fidelity curve is symmetric with the curve at $-1 \le x \le 0$.

d) When p is greater than 0.5 and less than about 0.8, fidelity first increases as x increases ($-1 \le x \le -1/\sqrt{2}$). At this point, it can be seen that the value of the dotted line has exceeded the solid line. It should be noted that the fidelity of the quantum teleportation can be improved by adjusting the entangled channel used at this time. Then, as x continues to increase ($-1/\sqrt{2} \le x < 0$), the fidelity gradually decreases. When $-1/\sqrt{2} \le x < 0$, the fidelity increases as x continues to increase. When $x = 1/\sqrt{2}$, a second highest point is reached. The continued growth of x ($x = 1/\sqrt{2}$) will lead to a decline in fidelity.

e) When p is greater than approximately 0.8, and the x satisfies $-1/\sqrt{2} \le x \le 1/\sqrt{2}$, the fidelity increases as x increases. When $-1/\sqrt{2} \le x \le 1/\sqrt{2}$, the fidelity gradually decreases as x increases and take the minimum value at $x = 1/\sqrt{2}$. Then, the fidelity increases with the increase of x .

The following conclusions can be drawn by the variation of the fidelity with the noise intensity p and the amplitude x of the entangled channel. In the case of noise situations a) b) c), only when the original entangled state $\left(|00\rangle + |11\rangle\right)/\sqrt{2}$ is selected as the entangled channel, the efficiency of the teleportation is optimized. As the noise factor p becomes larger, that is, in a noise environment such as d) e), the case where the fidelity using other quantum entangled channels is greater than the case where the original quantum entangled channel is used occurs. This means that the fidelity of the original quantum teleportation can be improved by adjusting the quantum entanglement channel used by the teleportation. In addition, it can be seen that the fidelity curves that $p = k, k \in [0,1]$ and $p = 1-k$ have a good symmetry with respect to x=0.

4.3 Efficiency under bit flip

In the bit-flip noise environment, when Alice uses basis 00 or 10 , the fidelity of quantum teleportation is

$$F_{BF}^{00,10} = \frac{1}{|\alpha|^2 x^2 + |\beta|^2 y^2} \Big\{ (1-p)\left[|\alpha|^2 x + |\beta|^2 y \right]^2 + p\left(|\alpha||\beta| e^{-ic} x + |\alpha||\beta| e^{ic} y \right)\left(|\alpha||\beta| e^{ic} x + |\alpha||\beta| e^{-ic} y \right) \tag{38}$$

The average fidelity of the output state is

$$\begin{aligned} \overline{F}_{BF}^{00,10} &= \frac{1-2p+2(p-1)x\sqrt{1-x^2}}{2\left(2x^2-1\right)^3}\left[\left(2x^2-1\right)\left(4x^2-3\right) + 2\left(1-x^2\right)^2 \ln\left(\frac{x^2}{1-x^2}\right) \right] \\ &+ \frac{2(1-p)x\sqrt{1-x^2} - 2\left(1-x^2\right) + p\left(3-2x^2\right)}{\left(2x^2-1\right)^2}\left[2x^2 - 1 - \left(1-x^2\right)\ln\left(\frac{x^2}{1-x^2}\right) \right] \\ &+ \frac{(1-p)\left(1-x^2\right)}{2x^2-1}\ln\left(\frac{x^2}{1-x^2}\right) \end{aligned} \tag{39}$$

When Alice measures with basis 01 or 11, the fidelity of quantum teleportation is

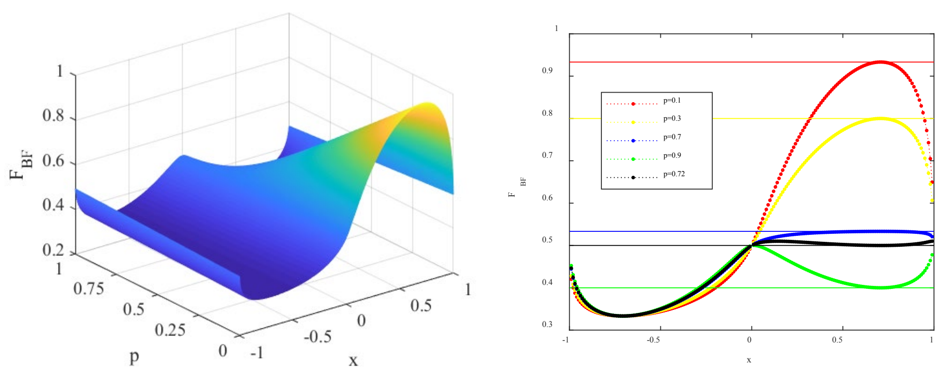
$$F_{BF}^{01,11}=\frac{1}{|\alpha|^2 y^2+|\beta|^2 x^2}\Big\{(1-p)\Big[|\alpha|^2 y+|\beta|^2 x\Big]^2$$
$$+p\left(|\alpha||\beta|e^{-ic}y+|\alpha||\beta|e^{ic}x\right)\left(|\alpha||\beta|e^{ic}y+|\alpha||\beta|e^{-ic}x\right) \tag{40}$$

The average fidelity of the output state is

$$\overline{F}_{BF}^{01,11}=\frac{1-2(1-2p)x\sqrt{1-x^2}}{2\left(1-2x^2\right)^3}\Big[\left(2x^2-1\right)\left(4x^2-1\right)$$
$$+2x^4\ln\left(\frac{1-x^2}{x^2}\right)\Big]+\frac{2(1-2p)x\sqrt{1-x^2}-2x^2}{\left(1-2x^2\right)^2} \tag{41}$$
$$\times\left[1-2x^2+x^2\ln\left(\frac{x^2}{1-x^2}\right)\right]+\frac{x^2}{1-2x^2}\ln\left(\frac{1-x^2}{x^2}\right).$$

Similarly, the average fidelities of the two measurements satisfy $\overline{F}_{Phs}^{00,10}=\overline{F}_{Phs}^{01,11}$. Thus, the average fidelity under the bit flip can be expressed as $\overline{F}_{BF}$, which satisfies

$$\overline{F}_{BF}=\overline{F}_{BF}^{00,10}=\overline{F}_{BF}^{01,11} \tag{42}$$



(a) (b)

Figure 4: (a) The figure depicts a three-dimensional image of the average fidelity of quantum teleportation as a function of variables x,p. (b) The figure takes the noise intensity p=0.1,0.3,0.7,0.9 as an example, showing the relationship between the fidelity and the use of different quantum entangled channels (dotted line). Among them, the solid line of the same color indicates the fidelity of the original teleportation ($x=1/\sqrt{2}$) in the corresponding noise environment. The black line indicates the fidelity curve in a critical noise environment, and the noise intensity indicates a turning point

The curve of the average fidelity $\overline{F}_{BF}$ with x,p is plotted in Fig. 4. As can be seen from Fig. 4, the change of fidelity in the teleportation under bit flip noise can be described as follows.

a) In the case of p is less than about 0.72, the fidelity always takes the minimum value at $x=-1/\sqrt{2}$ and the maximum value at $x=1/\sqrt{2}$. When $-1\le x\le -1/\sqrt{2}$, the fidelity decreases as x increases. When $-1/\sqrt{2}\le x\le 1/\sqrt{2}$, the fidelity gradually increases with the increase of x. Subsequently, as x continues to increase, the fidelity gradually declines.

b) when p is equal to approximately 0.72, as x increases, the fidelity gradually decreases first when $-1\le x\le -1/\sqrt{2}$, and then increases to the maximum when $-1/\sqrt{2}\le x\le 1$. Finally, the curve remains substantially stable.

c) when p is greater than about 0.72, the fidelity gradually becomes higher and then lower when $-1\le x<0$. And the fidelity at $0<x\le 1$ is also reduced first and then increased. The fidelity finally reaches the maximum when x approximately equals to zero.

Thus, under bit flip noise, when p is less than or equal to about 0.72, the fidelity can be maximized using the entangled channel in the original quantum teleportation. When p is greater than about 0.72, the partial values in $x\ne 1/\sqrt{2}$ can make the fidelity exceed the fidelity of $x=1/\sqrt{2}$. Therefore, a suitable quantum entangled channel can be selected to make the fidelity higher than that of the original quantum entangled channel, so as to improve the transmission efficiency of the quantum teleportation under noise.

4.4 Efficiency under depolarizing noise

In a depolarizing noise environment, when Alice uses basis 00 or 10 to measure her two particles, the fidelity of quantum teleportation is

$$\begin{aligned}F_D^{00,10} &= \frac{1}{|\alpha|^2 x^2+|\beta|^2 y^2}\Big\{(1-p)\left[|\alpha|^2 x+|\beta|^2 y\right]^2 \\ &+\frac{p}{3}\left(|\alpha||\beta|e^{-ic}x+|\alpha||\beta|e^{ic}y\right)\left(|\alpha||\beta|e^{ic}x+|\alpha||\beta|e^{-ic}y\right) \\ &+\frac{p}{3}\left(|\alpha|^2 x-|\beta|^2 y\right)^2+\frac{p}{3}\left(|\alpha||\beta|e^{-ic}x-|\alpha||\beta|e^{ic}y\right) \\ &\times\left(|\alpha||\beta|e^{ic}x-|\alpha||\beta|e^{-ic}y\right)\end{aligned} \tag{43}$$

The average fidelity of the output state is

$$\overline{F}_D^{00,10}=\frac{1-\frac{4}{3}n+\left(\frac{8}{3}n-2\right)x\sqrt{1-x^2}}{2\left(2x^2-1\right)^3}\left[\left(2x^2-1\right)\left(4x^2-3\right)+2\left(1-x^2\right)^2\ln\left(\frac{x^2}{1-x^2}\right)\right]$$
$$+\frac{\left(2-\frac{8}{3}p\right)x\sqrt{1-x^2}+\left(2-\frac{4}{3}p\right)x^2+2n-2}{\left(2x^2-1\right)^2}\left[2x^2-1-\left(1-x^2\right)\ln\left(\frac{x^2}{1-x^2}\right)\right] \tag{44}$$
$$+\frac{\left(1-\frac{2}{3}p\right)\left(1-x^2\right)}{2x^2-1}\ln\left(\frac{x^2}{1-x^2}\right)$$

When Alice uses basis 01 or 11 to measure, the fidelity of quantum teleportation is

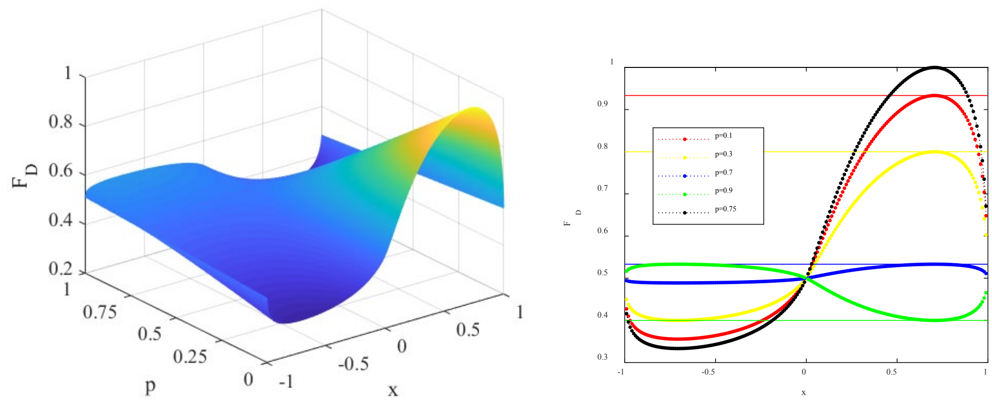
$$F_D^{01,11}=\frac{1}{|\alpha|^2x^2+|\beta|^2y^2}\Big\{(1-p)\left[|\alpha|^2y+|\beta|^2x\right]^2$$
$$+\frac{p}{3}\left(|\alpha||\beta|e^{-ic}y+|\alpha||\beta|e^{ic}x\right)\left(|\alpha||\beta|e^{ic}y+|\alpha||\beta|e^{-ic}x\right)$$
$$+\frac{p}{3}\left(|\alpha|^2y-|\beta|^2x\right)^2+\frac{p}{3}\left(|\alpha||\beta|e^{-ic}y-|\alpha||\beta|e^{ic}x\right) \tag{45}$$
$$\times\left(|\alpha||\beta|e^{ic}y-|\alpha||\beta|e^{-ic}x\right)\Big\}$$

The average fidelity of the output state will be

$$\overline{F}_D^{01,11}=\frac{\left(1-\frac{4}{3}p\right)\left(1-2x\sqrt{1-x^2}\right)}{2\left(1-2x^2\right)^3}\Big[\left(2x^2-1\right)\left(4x^2-1\right)$$
$$+2x^4\ln\left(\frac{1-x^2}{x^2}\right)\Big]+\frac{-2x^2+\frac{2}{3}p\left(2x^2+1\right)+\left(1-\frac{4}{3}p\right)2x\sqrt{1-x^2}}{\left(1-2x^2\right)^2} \tag{46}$$
$$\times\left[1-2x^2+x^2\ln\left(\frac{x^2}{1-x^2}\right)\right]+\frac{\left(1-\frac{2}{3}p\right)x^2}{1-2x^2}\ln\left(\frac{1-x^2}{x^2}\right).$$

Under depolarization noise, the average fidelity of the two measurement methods also satisfies $\overline{F}_D^{00,10}=\overline{F}_D^{01,11}$, and the average fidelity is expressed as

$$\overline{F}_D=\overline{F}_D^{00,10}=\overline{F}_D^{01,11} \tag{47}$$



(a) (b)

Figure 5: Subfigure (a) depicts three-dimensional images of the average fidelity $\overline{F}_D$ of the quantum teleportation as a function of the variable x, p. (b) The graph takes the noise intensity p as $0.1, 0.3, 0.7, 0.9$, respectively, and shows the relationship between the fidelity and the use of different quantum entangled channels (dotted line). Among them, the solid line of the same color indicates the fidelity of the original quantum entangled channel ($x = 1/\sqrt{2}$) in the corresponding noise environment. The black line indicates the fidelity curve in a critical noise environment, and the noise intensity denotes a turning point

The curve of the average fidelity $\overline{F}_D$ with x, p has been drawn in Fig. 5, the tendency of fidelity can be described as follows.

a) When p is less than about 0.75. At first, as the increases of x ($-1 \le x < -1/\sqrt{2}$), the fidelity gradually decreases, and the minimum value is obtained at $x = -1/\sqrt{2}$. When $-1/\sqrt{2} \le x \le 1/\sqrt{2}$, the fidelity gradually increases as x increases and takes the maximum at $x = 1/\sqrt{2}$.

b) When p is equal to about 0.75. Regardless of how x changes, the fidelity of quantum teleportation cannot exceed the fidelity of the original scheme ($x = 1/\sqrt{2}$).

c) When p is greater than about 0.75. The fidelity increases first as x increases. After taking the maximum value, it decreases as x increases until the minimum value is taken. Finally, the fidelity increases as x increases. The fidelity takes the maximum and minimum values at x=-0.75 and x=0.75, respectively. In this case, it can be seen that the original scheme has the worst fidelity.

Thus, by analyzing the influence of depolarizing noise on quantum teleportation, some conclusions can be drawn. When p is less than or equal to about 0.75, the fidelity always takes the maximum at $x = 1/\sqrt{2}$. When p is greater than or equal to about 0.75, the fidelity of the original quantum teleportation can be greatly improved by adjusting the entangled quantum channel.

4.5 Efficiency improvement in noisy environments

In the above, the fidelity of quantum teleportation is calculated as shown in Tab. 1 under four kinds of noise environments. The fidelities of quantum teleportation considering the noise intensity and different quantum entangled channels are analyzed. The results show that in the case of amplitude damping noise, the entangled channel of the original quantum teleportation is best and the teleportation is least affected by quantum noise. In the phase damping, bit flip and depolarizing noise environment, the efficiency of the teleportation can be improved by selecting an appropriate entangled channel.

Table 1: Fidelity of quantum teleportation under quantum noise

Fidelity	Shared entangled state	
	$x\|00\rangle + y\|11\rangle$	$1/\sqrt{2}(\|00\rangle + \|11\rangle)$
Amplitude damping	Eq. (32)	$\frac{2}{3}+\frac{1}{3}\sqrt{1-p}-\frac{1}{6}p$
Phase damping	Eq. (37)	$1-\frac{2}{3}p$
Bit flip	Eq. (42)	$1-\frac{2}{3}p$
Depolarizing	Eq. (47)	$1-\frac{2}{3}p$

Next, this paper will quantitatively analyze the effect on the efficiency of quantum teleportation by using a suitable quantum entangled channel. The optimization efficiency of the proposed scheme is measured by comparing the original fidelity and the optimized fidelity under different noise intensities.

Table 2: Increased efficiency of new quantum teleportation fidelity in quantum noise environment (p is the noise intensity, F_{ini} denotes the fidelity of original teleportation, F_{opt} is the optimal fidelity, x_{opt} is amplitude of the optimal entangled channel and r is the optimization rate which is defined as $\left(F_{opt} - F_{ini}\right) / Fini$)

	Phase damping				Bit flip			
p	F_{ini}	F_{opt}	x_{opt}	r	F_{ini}	F_{opt}	x_{opt}	r
0	1	1	$x=\frac{1}{\sqrt{2}}$	0	1	1	$x=\frac{1}{\sqrt{2}}$	0
0.1	0.933	0.933	$x=\frac{1}{\sqrt{2}}$	0	0.933	0.933	$x=\frac{1}{\sqrt{2}}$	0
0.2	0.866	0.866	$x=\frac{1}{\sqrt{2}}$	0	0.866	0.866	$x=\frac{1}{\sqrt{2}}$	0
0.3	0.800	0.800	$x=\frac{1}{\sqrt{2}}$	0	0.800	0.800	$x=\frac{1}{\sqrt{2}}$	0

0.4	0.733	0.733	$x=\frac{1}{\sqrt{2}}$	0	0.733	0.733	$x=\frac{1}{\sqrt{2}}$	0
0.5	0.666	0.666	$x=\frac{1}{\sqrt{2}}$	0	0.666	0.666	$x=\frac{1}{\sqrt{2}}$	0
0.6	0.6	0.733	$x=-\frac{1}{\sqrt{2}}$	22.2%	0.6	0.6	$x=\frac{1}{\sqrt{2}}$	0
0.7	0.533	0.800	$x=-\frac{1}{\sqrt{2}}$	50.0%	0.533	0.533	$x=-\frac{1}{\sqrt{2}}$	0
0.8	0.466	0.866	$x=-\frac{1}{\sqrt{2}}$	85.7%	0.466	0.503	$x=0.05$	7.9%
0.9	0.400	0.933	$x=-\frac{1}{\sqrt{2}}$	133.3%	0.400	0.500	$x=0.01$	25.1%
1	0.333	1	$x=-\frac{1}{\sqrt{2}}$	200.0%	0.333	0.499	$x=-0.01$	49.8%

	Amplitude damping				Depolarizing			
p	F_{ini}	F_{opt}	x_{opt}	r	F_{ini}	F_{opt}	x_{opt}	r
0	1	1	$x=\frac{1}{\sqrt{2}}$	0	1	1	$x=\frac{1}{\sqrt{2}}$	0
0.1	0.966	0.966	$x=\frac{1}{\sqrt{2}}$	0	0.933	0.933	$x=\frac{1}{\sqrt{2}}$	0
0.2	0.931	0.931	$x=\frac{1}{\sqrt{2}}$	0	0.866	0.866	$x=\frac{1}{\sqrt{2}}$	0
0.3	0.895	0.895	$x=\frac{1}{\sqrt{2}}$	0	0.800	0.800	$x=\frac{1}{\sqrt{2}}$	0
0.4	0.858	0.858	$x=\frac{1}{\sqrt{2}}$	0	0.733	0.733	$x=\frac{1}{\sqrt{2}}$	0
0.5	0.819	0.819	$x=\frac{1}{\sqrt{2}}$	0	0.666	0.666	$x=\frac{1}{\sqrt{2}}$	0
0.6	0.777	0.777	$x=\frac{1}{\sqrt{2}}$	0	0.6	0.6	$x=\frac{1}{\sqrt{2}}$	0
0.7	0.732	0.732	$x=\frac{1}{\sqrt{2}}$	0	0.533	0.533	$x=\frac{1}{\sqrt{2}}$	0
0.8	0.682	0.682	$x=\frac{1}{\sqrt{2}}$	0	0.466	0.511	$x=-\frac{1}{\sqrt{2}}$	9.5%
0.9	0.622	0.622	$x=\frac{1}{\sqrt{2}}$	0	0.400	0.533	$x=-\frac{1}{\sqrt{2}}$	33.3%
1	0.500	0.500	$x=\frac{1}{\sqrt{2}}$	0	0.333	0.555	$x=-\frac{1}{\sqrt{2}}$	66.7%

Tab. 2 shows the efficiency improvement of quantum teleportation when the optimal quantum channel is selected. The results show that when the noise intensity exceeds a certain value, the fidelity of quantum teleportation can be increased by adjusting the entangled channel. When the noise intensity is 1, the fidelity improvement efficiency of the phase damping, bit flipping and depolarizing noise reaches the maximum value, and the fidelity maximum lifting efficiency can reach 200%. Although this maximum improvement effect only occurs under special conditions, this proposed method may provide help under certain circumstances.

5 Conclusions

This paper proposes a scheme to increase quantum teleportation under noise by dynamically adjusting the quantum entangled channel. The average fidelity of the quantum teleportation under noise is calculated by assuming that the quantum entangled channel is an unknown arbitrary quantum state. The average fidelity of the quantum teleportation is analyzed as a function of the quantum entanglement channel coefficient and the noise intensity. By adjusting the quantum entangled channel, the quantum teleportation in a specific noise environment achieves optimal transmission efficiency. This paper calculates the average fidelity of quantum teleportation in four quantum noise environments. The results show that the average fidelity is independent of the measurement method used by the sender (Alice) and is only related to the quantum entanglement channel and the quantum noise intensity. Through further study, it is found that the fidelity of the original quantum teleportation under amplitude damping noise is the best. Under the phase damping noise, when the noise intensity is greater than 0.5, the fidelity of the quantum teleportation can be optimized by adjusting the entangled quantum channel, and the maximum optimization efficiency can be as high as 200%. Under the bit flip noise, when the noise intensity is greater than about 0.72, it can be optimized by the scheme proposed in this paper, and the highest optimization efficiency is 49.8%. When the depolarization noise intensity is greater than about 0.75, the scheme proposed in this paper can be used with a maximum optimization efficiency of 66.7%. We hoped that the results in this paper can shed some light on the way to resist the interference of noise in quantum teleportation.

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