Computers, Materials & Continua CMC, vol.63, no.2, pp.873-890, 2020

A Novel Quantum-Behaved Particle Swarm Optimization Algorithm

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Abstract: The efficient management of ambulance routing for emergency requests is vital to save lives when a disaster occurs. Quantum-behaved Particle Swarm Optimization (QPSO) algorithm is a kind of metaheuristic algorithms applied to deal with the problem of scheduling. This paper analyzed the motion pattern of particles in a square potential well, given the position equation of the particles by solving the Schrödinger equation and proposed the Binary Correlation QPSO Algorithm Based on Square Potential Well (BC-QSPSO). In this novel algorithm, the intrinsic cognitive link between particles' experience information and group sharing information was created by using normal Copula function. After that, the control parameters chosen strategy gives through experiments. Finally, the simulation results of the test functions show that the improved algorithms outperform the original QPSO algorithm and due to the error gradient information will not be over utilized in square potential well, the particles are easy to jump out of the local optimum, the BC-QSPSO is more suitable to solve the functions with correlative variables.

Keywords: Ambulance routing problem, quantum-behaved particle swarm optimization, square potential well, convergence.

1 Introduction

Ambulance routing problem (ARP) is one of the most important Emergency Medical Services (EMS) as it plays a vital role in saving injures's lives and reducing the rate of mortality when a disaster occurs [Hu, Qing, Yu et al. (2008)]. The sensitivity of decision making in the EMS firstly attracted the attention of operations research experts who studied numerous class of problems arising in the management of EMS systems. The ARP, called emergency logistic [Aakil, Zhang, Li et al. (2015)], is about managing and scheduling the flow of ambulances to save people affected by disasters [Tlili, Harzi and Krichen (2017)]. Various metaheuristic algorithms are applied to deal with the problem of scheduling, which is an NP-hard problem [Masdari, Salehi, Jalali et al. (2017)].

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Received: 24 May 2019; Accepted: 31 August 2019.

Particle Swarm Optimization (PSO) algorithm is a recent implementation of these techniques [Wu, Chen and Yan (2014)]. The essential idea of PSO is to emulate birds' preying behaviors. Compared with other swarm intelligence algorithms, PSO algorithm proposed by Kennedy et al. [Kennedy and Eberhart (1995)] sustains the global search strategy, avoids the complex evolutionary operations and enhances the convergence capability, thus it has become a research hot spot in various research fields. Besides, successful applications of PSO found in pattern recognition, data mining and wireless sensor networks. However, problems remain to solved in PSO algorithm, one of which presented and proven by Bergh [Bergh (2002)] is that PSO is not guaranteed to converge to the globally optimized solution with probability 1. Although plenty of works [Zhang, Tang, Hua et al. (2015); Wang, Ju, Yu et al. (2018)] have been done in recent years to modify and improve PSO, the state-of-the-art shows that the improvements in PSO promised by these works are limited.

Aiming at PSO's convergence bottleneck, through the comparison between the human learning process and particles' behavior in quantum spaces, Sun [Sun (2009)] proposed the Quantum-Behaved PSO algorithm (QPSO), which leverages the aggregation tendency of the collective intelligence of the population. In this model, individuals described as particles in quantum space, which continuously iterate according to characteristics seen in human society, such as self-organization and collaboration. Theoretical proof has shown that QPSO is a globally converged algorithm. Therefore, QPSO has drawn broad attention in various applications fields and algorithm modification [Mikki and Kishk (2006); Li, Wang, Song et al. (2012); Wu, Chen and Yan (2015)].

However, experiments on various test functions solved using QPSO have revealed its shortcomings such as the inevitable premature and weak global optimization ability as do other global optimization algorithms. In order to further improve the performance of the QPSO algorithm, we propose a novel algorithm by existing research results, which is the Binary Correlation QPSO Algorithm Based on Square Potential Well, referred to as BC-QSPSO. Because of the error gradient information will not be over utilized in a square potential well and the particles are easy to jump out of the local optimum, the BC-QSPSO model outperforms the traditional QPSO algorithm regarding optimization of functions with correlative variables.

2 Binary correlation QPSO algorithm

2.1 The standard QPSO algorithm

In PSO optimization algorithm, the solution space abstracted as the birds foraging space, where each bird is abstracted as a massless and size-free particle flying at a certain speed. Each particle has a fitness value determined by the function to be optimized. According to the fitness value a random search is carried out by each particle. In every round of iteration, each particle updates itself by tracking two optimums: The first one is the optimal solution found by each particle itself, commonly referred to as the personal optimum *p*best; the other is the optimal solution found by the entire population, commonly referred to as the global optimum *g*best. For simplicity, in this article, $P_i=(p_{i1}, p_{i2}, \ldots, p_{iD})$ and $G=(p_{g1}, p_{g2}, \ldots, p_{gD})$ are used to describe personal optimum and global

optimum of particle i in the D-dimensional search space, respectively. Particle i's personal best position $p$best is determined by Eq. (1):

$$\mathrm{P}_i(t) = \begin{cases} X_i(t) & \text{if } f(X_i(t)) < f(P_i(t-1)) \\ P_i(t-1) & \text{if } f(X_i(t)) \geq f(P_i(t-1)) \end{cases} \tag{1}$$

The index g of global best position G = (p_{g1}, p_{g2}, …, p_{gD}) is determined by Eq. (2):

$$g = \arg\min_{1 \leq i \leq D} \{f(P_i(t))\} \quad g \in \{1,2,\cdots,D\} \tag{2}$$

Bergh et al. [Bergh and Engelbrecht (2002)] have demonstrated that standard PSO is not guaranteed to converge on the global optimal solution with probability 1, which is a major shortcoming of the traditional PSO. In order to achieve global convergence, from previous studies of the particles' convergence behaviors, Quantum-behaved PSO (QPSO) algorithm was proposed based on the δ potential well by assuming PSO system as a quantum space.

According to the analysis of particles' orbits in the PSO algorithm done by Clerc et al. [Clerc and Kennedy (2002)], a δ potential well can be established at the local attraction point p_i=(p_{i1}, p_{i2}, …, p_{iD}) to impact particles in the population, whose coordinate is:

$$p_{i,j}(t) = \frac{c_1 r_{1,i,j}(t) P_{i,j}(t) + c_2 r_{2,i,j}(t) G_j(t)}{c_1 r_{1,i,j}(t) + c_2 r_{2,i,j}(t)}, 1 \leq j \leq D \tag{3}$$

In this equation, $r1$ and $r2$ are random numbers independently distributed within an interval [0, 1], called random factors; $c1$ is the individual cognitive acceleration coefficient whereas $c2$ is the global cognitive acceleration coefficient.

Eq. (3) can be simplified as:

$$\mathrm{p}_{i,j}(t) = \varphi_{i,j}(t) P_{i,j}(t) + [1 - \varphi_{i,j}(t)] G_j(t) \tag{4}$$

where,

$$\varphi_{i,j}(t) = \frac{c_1 r_{1,i,j}(t)}{c_1 r_{1,i,j}(t) + c_2 r_{2,i,j}(t)} \tag{5}$$

In our assumption, particles are flying in the quantum space, thus particles' states can be described using wave function $\Psi(X,t)$. In the point of view of the theory of dynamics, the convergence process of a particle can be described as follows: A particle is continuously approaching the local attractor pi with decreasing speed, and eventually overlaps with p_i.

The steady-state of a particle in the δ potential well can be expressed using the Schrödinger equation.

$$\frac{d^2\varphi}{dX^2} + \frac{2m}{h^2}[E + \gamma\delta(X - p_i)]\varphi = 0 \tag{6}$$

By solving the above equation, the probability distribution function (PDF) in every dimension can be obtained for each particle.

$$\varphi(X_{ij}) = \frac{1}{\sqrt{L}} e^{-|X_{ij} - p_{ij}| / L} \tag{7}$$

Moreover, the position-updating equation of every particle in each generation in QPSO can be deduced as:

$$X_{i,j}(t+1)=p_{i,j}(t)\pm\frac{L_{i,j}(t)}{2}\ln\left[1/u_{i,j}(t)\right] \tag{8}$$

where $u_{i,j}(t)$~U(0,1), $L_{i,j}(t)$ is the length of the potential well. $L_{i,j}(t)$ can be evaluated using the following equation:

$$L_{i,j}(t)=2\alpha\cdot\left|C_j(t)-X_{i,j}(t)\right| \tag{9}$$

where, $C(t)$ is the average of all personal best positions, known as the gravity position of the population, and is evaluated as follows:

$$C(t)=(C_1(t),C_2(t),\cdots C_n(t))=\frac{1}{N}\sum_{i=1}^{N}P_i(t)$$

$$C(t)=\left(\frac{1}{N}\sum_{i=1}^{N}P_{i,1}(t),\frac{1}{N}\sum_{i=1}^{N}P_{i,2}(t),\cdots\frac{1}{N}\sum_{i=1}^{N}P_{i,N}(t)\right) \tag{10}$$

Therefore, a particle's position evolution equation in (6) is finally defined as follows:

$$X_{i,j}(t+1)=p_{i,j}(t)\pm\alpha\cdot\left|C_j(t)-X_{i,j}(t)\right|\ln\left[\frac{1}{u_{i,j}(t)}\right] \tag{11}$$

In the above equation, α is referred to as contraction-expansion coefficient, which is the only parameter in this algorithm except for the population size and iteration count. In the iteration process convergence performance is controlled by fine-tuning α. The value of α can be fixed or decreased linearly.

2.2 Binary correlation QPSO algorithm

The coordinate formula of the potential well center which is denoted as $p_i=(p_{i1}, p_{i2}, \ldots, p_{iD})$ can be divided into two portions:

Ⅰ) Individual cognitive component: $\frac{c_1r1_{i,j}(t)}{c_1r1_{i,j}(t)+c_2r2_{i,j}(t)}P_{i,j}(t)$, which indicates the experiences of particle themselves;

Ⅱ) Social cognitive component: $\frac{c_2r2_{i,j}(t)}{c_1r1_{i,j}(t)+c_2r2_{i,j}(t)}G_j(t)$, which represents shared information among particle population;

Under the combined effect of two portions above, QPSO algorithm expects to find the optimal solution and adjust the position of p_i constantly in the solution space according to the sharing information and the experiences of particle themselves.

The point of p_i takes advantage of the particles personal optimum $p\text{best}$ and the global optimum $g\text{best}$ mainly depends on the acceleration factor $c1$, $c2$ and random factors $r1$, $r2$.

The coefficients $c1$, $c2$ represent the statistic weights of particles acceleration, reflecting the information exchange in the particle swarm. Setting the large $c1$ will cause the

particles to wander in the local area because of the undue reliance on their own experience, but the large $c2$ will cause particles premature and converge to local optima.

As the important parameters of the standard PSO algorithm, there are a lot of related studies about how to value the acceleration factors c_1 and c_2. These policies obtained some improvement of the PSO algorithm. However, they ignored the impacts of the random factors $r1$ and $r2$ on algorithm performances. The independence assumption between *r1* and *r2* in the pi formula makes the algorithm cannot distinguish the utilization of pbest and gbest. At present, there are few studies about the effects of parameters *r1* and *r2* on the algorithm. However, it is necessary to analyze the random factors in order to study further the impacts of utilization of particles' own experiences and community sharing information on the performance of the QPSO algorithm respectively.

To analyze the connection between *r1* and *r2* in QPSO, reference suggested the conception of the binary correlation factors and proposed the Binary correlation QPSO algorithm, referred as BC-QPSO algorithm. The BC-QPSO algorithm constructed the relations between *r1* and *r2* using the bivariate normal Copula function: Φρ(Φ-1(r1),Φ-1(r2), the Fréchet-Hoeffding lower bound: W(u,v)=max(u+v-1,0), the Fréchet-Hoeffding upper bound: M(u,v)=min(u,v) and the product Copula: Π(u,v)=uv. The particle's position evolution equation of BC-QPSO is finally defined as follows:

$$\begin{cases} X_{i,j}(t+1) = p_{ij} \pm \alpha \cdot \left| C_j(t) - X_{i,j}(t) \right| \cdot \ln\left[1 / u_{i,j}(t) \right], 1 \le j \le D \\ p_{ij}(t) = \dfrac{c_1 r_{1,i,j}(t) P_{i,j}(t) + c_2 r_{2,i,j}(t) G_{i,j}(t)}{c_1 r_{1,i,j}(t) + c_2 r_{2,i,j}(t)} \\ H(r_1, r_2) = C_p(r_1, r_2) \end{cases} \tag{12}$$

$$C_p(r1, r2) = \begin{cases} W(r_1, r_2), & \rho = -1 \\ \Pi(r_1, r_2), & \rho = 0 \\ M(r_1, r_2), & \rho = 1 \\ \Phi_\rho\left(\Phi^{-1}(r_1), \Phi^{-1}(r_2)\right), & -1 < \rho < 1 \text{且} \rho \ne 0 \end{cases} \tag{13}$$

where, H is the Joint distribution function of the binary correlation, factors $r1$, $r2$; C are the binary normal Copula function; ρ is the specified correlation coefficient, which is an indicator of the relevant strength and could reflect linear correlation properties between variables $r1$ and $r2$; Φ_ρ is the two-dimensional standard normal distribution function of the correlation coefficient ρ. Moreover, Φ^{-1} is an inverse function of the one-dimensional standard normal distribution function.

3 Binary correlation QPSO algorithm based on square potential well

3.1 Model establishment of the BC-QSPSO algorithm

The BC-QPSO proposed by Wu et al. [Wu, Yan and Chen (2015)] enhanced the optimization performance to some extent. The standard QPSO algorithm is based on the δ potential well model. The δ potential well can be interpreted as the extreme case of the square potential well, thus in principle, problems relevant to the δ potential well can all be

solved by getting the extremes of the solutions by means square potential well method. The finite symmetric square potential well model can also be used in the BC-QPSO algorithm.

The energy distribution of one-dimensional finite symmetric square potential well is as follows:

$$\mathrm{V}(x)=\begin{cases}0, & |x|\le W/2\\ V_0, & |x|>W/2\end{cases} \tag{14}$$

where, W is the width of the potential well and V_0 is the depth. Any particle whose energy less than $x=w/2$ is constrained inside the energy wall. Although the implementation of square potential well is easier compared with that of the d potential well, it requires much harsher solving conditions for Schrödinger. The analytical solution of the square potential well can be obtained. However, in the context of square potential well, multiple energy levels can be stimulated with regard to the relationship between the potential well width and depth. For brevity, we only consider the bound state (ground state) with minimum energy when establishing BC-QSPSO model.

Suppose that a particle with mass m and energy E moves along the X axis, the square potential well center (i.e., p_i) solved using Eq. (3) is depicted as p and the position of the particle is depicted as X, given that Y=X-p, then after the coordinate transformation the potential function is depicted as follows:

$$V(x)=\begin{cases}0, & |x|\le \mathrm{W}/2\\ V_0, & |x|>\mathrm{W}/2\end{cases} \tag{15}$$

And the Schrödinger equation of every particle outside the potential well ($|Y|>W/2$, the classic forbidden zone) is as follows:

$$\frac{d^2}{dx^2}\psi(Y)=\beta^2\psi(Y) \tag{16}$$

where,

$$\beta=\sqrt{-2mE}\ /\ \hbar$$

then, Eq. (16) can be interpreted as follows:

$$\varphi(Y)\alpha\pm e^{\pm\beta x} \tag{17}$$

Taking into consideration of the boundary condition of the ground-state wave function at |y|, the value of $\psi(Y)$ can be expressed as follows:

$$\varphi(Y)=\begin{cases}Be^{-\beta Y}, & Y>W/2\\ Be^{\beta Y}, & Y>-W/2\end{cases} \tag{18}$$

The integral constant B is yet to be determined.

Given that $|Y|<W/2$ (inside the potential well, i.e., the classically forbidden zone), the Schrödinger equation of the square potential well is as follows:

$$\frac{d^2}{dx^2}\psi(Y) = -k^2\psi(Y) \tag{19}$$

where,

$k = \sqrt{2m(E - V_0)} / \hbar$

Since particles are in the ground state, thus the wave function of the particle is as follows:

$$\varphi(Y)\alpha\ A\cos kY \quad |Y| < W/2 \tag{20}$$

The integral constant A is yet to be determined.

Energy values can be determined according to the continuity of the wave function ψ and its reciprocal ψ' or $(\ln\psi)'$ at $Y = \pm W/2$, thus:

$$(\ln\cos kY)'\big|_{Y=W/2} = (\ln e^{-\beta Y})'\big|_{Y=W/2} \tag{21}$$

We can deduce that,

$$k\tan(kW/2) = \beta \tag{22}$$

The results gained from the continuity at $Y = -W/2$ is identical with Eq. (22). Let

$$kW/2 = \xi, \quad \beta W/2 = \eta \tag{23}$$

Thus, Eq. (22) changes to:

$$\xi\tan\xi = \eta \tag{24}$$

From equations above, we can conclude as follows:

$$\xi^2 + \eta^2 = mV_0W^2 / 2\hbar^2 \tag{25}$$

Eq. (25) is the transcendental algebra equations that x and h must satisfy which can be solved using numerical calculation. We denote the probability density of a particle's appearance as follows to simplify and unify the expression.

$$Q(Y) = \left|\varphi(Y)^2\right| = \begin{cases} \frac{a}{W}\cos^2\left(\frac{\xi}{W}Y\right), & |Y| \le W/2 \\ \frac{b}{W}e^{-\frac{\eta}{W}Y}, & Y > W/2 \\ \frac{b}{W}e^{\frac{\eta}{W}Y}, & Y < -W/2 \end{cases} \tag{26}$$

a, b, ξ, η are undetermined constants. According to quantum dynamics theory, $\xi < \pi$. To simplify calculation, we designate $\xi = 1, a = b$, thus Eq. (26) can be written as follows:

$$Q(Y) = |\varphi(Y)|^2 = \begin{cases} \frac{a}{W}\cos^2\left(\frac{1}{W}Y\right), & |Y| \le W/2 \\ \frac{a}{W}\cos^2\left(\frac{1}{2}\right)e^{\tan\left(\frac{1}{2}\right)-\frac{2}{W}\tan\left(\frac{1}{2}\right)Y}, & Y > W/2 \\ \frac{a}{W}\cos^2\left(\frac{1}{2}\right)e^{\tan\left(\frac{1}{2}\right)+\frac{2}{W}\tan\left(\frac{1}{2}\right)Y}, & Y < -W/2 \end{cases} \tag{27}$$

According to the above analysis, we gained the probability function of the particles surrounds p as the center in the one-dimensional symmetric square potential well with finite depth. However, in real-world applications, further study on particles precise positions in the solution space is required.

Theorem: In the BC-QSPSO model, a particle's movement inside one-dimensional finite-depth symmetric square potential well with $\boldsymbol{p}$ as the center is determined using the following function:

$$X = p \pm W \cos^{-1}\left(\sqrt{u}\right) \tag{28}$$

where W is the width of the one-dimensional finite-depth symmetric square potential well, u is a random number uniformly distributed in (0,1).

Proof: The wave function can collapse to the classic state using the Monte Carlo method. Given that u is a random number uniformly distributed in (0,1), i.e.,

$$u \sim U(0,1) \tag{29}$$

Let

$$u = \cos^2\left(\frac{1}{W}Y\right) \tag{30}$$

we can get the following equation using the reverse transformation:

$$\left|Y\right| = W \cos^{-1}\left(\sqrt{u}\right) \tag{31}$$

Since $Y = X - p$, the random equation used to measure a particle's position is as follows:

$$X = p \pm W \cos^{-1}\left(\sqrt{u}\right) \tag{32}$$

where, u is a random number uniformly distributed in (0,1), and W is the width of the one-dimensional finite-depth symmetric square potential well.

End of proof.

Definition: The algorithm that describes particles' movement principle inside the one-dimensional finite-depth symmetric square potential well is called Binary Correlation QPSO based on the square potential well model, BC-QSPSO in short.

Let Z=C-X where C is the average best position. In order to accelerate the convergence of QPSO, we adopt the approach proposed by Reference that controls potential well width with probability. In BC-QSPSO, let

$$\int_{-|Z|}^{|Z|} Q(Z) d_z > 0.5 \tag{33}$$

According to the normalization condition:

$$\int_{-\infty}^{+\infty} Q(Z) d(Z) = 1 \tag{34}$$

We conclude that $a = 0.4291$ in the Eq. (27).

The Eq. (33) is also solved as:

$$W = \frac{1.4829}{\kappa}\left|Z\right| \tag{35}$$

where $\alpha > 1$. Take $Z = C - X$ into Eq. (35), we get:

$$W = \frac{1.4829}{k} \cdot |C - X| \tag{36}$$

Taking into consideration the variation of time, the evolution equation of a particle inside a one-dimensional finite-depth symmetric square potential well in the BC-QSPSO model is as follows:

$$X(t+1) = p(t) \pm \frac{1.4829}{k} \cdot |C(t) - X(t)| \cos^{-1}\left(\sqrt{u}\right) \tag{37}$$

where $\kappa > 1$, u is a random number uniformly distributed in (0,1).

For expression simplicity, let, $\alpha_3 = \frac{1.4829}{\kappa}$, then $\alpha_3 < 1.5$. Eq. (37) can be re-written as follows:

$$X(t+1) = p(t) \pm \alpha_3 |C(t) - X(t)| \cos^{-1}\left(\sqrt{u}\right) \tag{38}$$

For particle I, the attractor p in Eq. (38) can be expressed as $p_i=(p_{i,1},p_{i,2},\ldots p_{i,N})$. For every dimension, a $p_{i,j}$ centric one-dimensional finite-depth symmetric square potential well can be established, thus the evolution equation for the j-th dimension of the i-th particle is as follows:

$$X_{i,j}(t+1) = p_{i,j}(t) \pm \alpha \left| C_{i,j} - X_{i,j}(t) \right| \cos^{-1}\left(\sqrt{u_{i,j}(t)}\right) \quad u_{i,j}(t) \sim U(0,1) \tag{39}$$

Therefore, the complete evolution equation for a particle in the D-dimensional space using BC-QSPSO is as follows.

$$\begin{cases} X_{i,j}(t+1) = p_{i,j} \pm \alpha_3 \left| C_{i,j}(t) - X_{i,j}(t) \right| \cos^{-1}\left(\sqrt{u_{i,j}(t)}\right), \quad u_{i,j}(t) \sim U(0,1) \\ p_{i,j}(t) = \dfrac{c_1 r_{1,i,j}(t)}{c_1 r_{1,i,j}(t) + c_2 r_{2,i,j}(t)} P_{i,j}(t) + \dfrac{c_2 r_{2,i,j}(t)}{c_1 r_{1,i,j}(t) + c_2 r_{2,i,j}(t)} G_{i,j}(t), \quad 1 \le j \le D \\ H(r_1, r_2) = C_\rho(r_1, r_2) = \Phi_\rho\left(\Phi^{-1}(r_1), \Phi^{-1}(r_2)\right) \end{cases} \tag{40}$$

3.2 Algorithm execution process

Based on the designs and definitions discussed above, the execution process of the BC-QSPSO algorithm is as follows:

Step 1: Setting Parameters. The parameters needed to be setup includes the individual cognitive acceleration coefficient $\boldsymbol{c_1}$, the global cognitive acceleration coefficient $\boldsymbol{c_2}$, the contraction-expansion factor $\boldsymbol{\alpha_3}$, the swarm population size $\boldsymbol{N}$ and the maximum number of iterations *iter*Max or the error precision of fitness.

Step 2: Initialization of the population. For S_1, initialize the position for every particle in the solution space, that is, randomly generating $X_{i,j}(0)$ for every particle and let be the personal best position $P_{i,j}(0)=X_{i,j}(0)$, where $i \in \{1,\ldots, N_{S1}\}$, $j \in \{1,\ldots,D\}$; For S_2, initialize the position for every particle in the solution space, that is, randomly generating $X_{i,j}(0)$ for every particle and let be the personal best position $P_{i,j}(0)=X_{i,j}(0)$, where i$\in \{N_{s1}+1,\ldots, N\}$, $j \in \{1,\ldots,D\}$;

Step 3: Calculating the fitness values for all particles in S_1 and S_2. Suppose that the optimization problem to be solved is a minimization one, and then assign the position corresponding to smallest fitness value to the global best position of sub-groups respectively, i.e., $gbest_1=\{X_i \,|\min(f(X_i)), i\in\{1,\ldots, N_{S1}\}$, $gbest_2=\{X_i \,|\min(f(X_i)), i\in\{1,\ldots, N_{S2}\}$.

Step 4: Calculating the average best positions of the entire population $C(t)$ according to the Eq. (10) and evaluate the parameters $L_{i,j}(t)$ of the master and slave sub-groups respectively.

Step 5: Updating the position for particle $i(1\leq i\leq N)$, that is calculating new positions for all particles using the Eq. (17).

Step 6: Recalculating the particle i's current position $X_i(t)$ according to the objective function (t for iterations).

Step 7: Updating the personal best positions using Eq. (1) for the master and slave sub-group respectively. If $f(X_i(t))<f(P_i(t\text{-}1))$, then let be $P_i(t)=X_i(t)$; otherwise , $P_i(t)= P_i(t\text{-}1)$;

Step 8： If $i\in S_1$ and the fitness value of $P_i(t)$ is better than the fitness value of the global best position of the whole swarm $P_g(t\text{-}1)$, i.e., $f(P_i(t))< f(P_g(t\text{-}1))$, then $P_i(t)$ is saved as the global best position of master group S_1, which is denoted as $P_{gs1}(t)$; otherwise, $P_{gs1}(t)=P_g(t\text{-}1)$. If $i\in S_2$ and the fitness value of $P_i(t)$ is better than the fitness value of $P_g(t\text{-}1)$, i.e., $f(P_i(t))<f(P_g(t\text{-}1))$, then $P_i(t)$ is saved as the global best position of the slave group S_2 , which is denoted as $P_{gs2}(t)$; otherwise, $P_{gs2}(t)=P_g(t\text{-}1)$.

Step 9: Comparing the fitness value of $P_{gs1}(t)$ and $P_{gs2}(t)$, if $f(P_{gs2}(t))<f(P_{gs1}(t))$ then assign $P_{gs2}(t)$ to be the global best position $P_g(t)$, i.e., $P_g(t)=P_{gs2}(t)$; Otherwise , $P_{gs1}(t)$ is assigned to $P_g(t)$, i.e., $P_g(t)=P_{gs1}(t)$.

Step 10: Termination determination. If the maximum times of iterations *iter*Max or the error precision of fitness value are achieved, then stop the searching process and putout the results. Otherwise, let $t=t+1$ and repeat Step 3 to Step 10.

4 Experiment designs and results analysis

4.1 Experiment designs

Performance and efficiency of intelligent algorithms tend to be affected by the experiment parameter settings [Eberhart and Shi (1998)]. How to determine parameters to achieve optimal performance is in itself a very complex optimization problem. In order to obtain reasonable experiment results, the benchmark functions, including Sphere function, Rosenbrock function, Rastrigin function, Griewank function, Ackley function and Schaffer function, are adopted to choose the value of control parameters and test algorithms for performance comparison.

1) Benchmark functions

The benchmark functions with various characteristics are a major tool for performance evaluation for evolutionary algorithms. Unimodal and multimodal problems are commonly seen in engineering projects; thus, unimodal and multimodal functions are used as testing functions in this paper. Expressions, the search range of variables, initialization range and optimal solution and optimal values are given in Tabs. 1 and 2.

Table 1: Unimodal functions

Benchmark Function	***Equation***	***Searching Range***
Sphere	$f_1(X) = \sum_{i=1}^{D} x_i^2$	$[-100, 100]^D$
Rosenbrock	$f_2(X) = [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	$[-10, 10]^D$
Benchmark Function	***Initial Range***	***Optimal and Fitness Value***
Sphere	$[-100, 50]^D$	$f_1(0, 0, \ldots, 0)=0$
Rosenbrock	$[-10, 10]^D$	$f_2(1, 1, \ldots, 1)=0$

Table 2: Multimodal function

Benchmark Function	***Equation***	***Searching Range***
Rastrigin	$f_3(X) = \sum_{i=1}^{D} (x_i^2 - 10\cos(2\pi x_i) + 10)$	$[-5.12, 5.12]^D$
Griewank	$f_4(X) = \sum_{i=1}^{D} \frac{x_i^2}{4000} - \prod_{i=1}^{D} \cos(\frac{x_i}{\sqrt{i}}) + 1$	$[-600, 600]^D$
Ackley	$f_5(X) = -20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}) - \exp(\frac{1}{D}\sum_{i=1}^{D} \cos(2\pi x_i)) + 20 + e$	$[-32.78, 32.78]^D$
Expanded Shaffer	$f_6(X) = g(x_1, x_2) + \cdots + g(x_{D-1}, x_D) + g(x_D, x_1)$ $g(x, y) = 0.5 + \frac{(\sin\sqrt{x^2 + y^2})^2 - 0.5}{(1.0 + 0.001(x^2 + y^2))^2},$	$[-100, 100]^D$
Benchmark Function	***Initial Range***	***Optimal and Fitness Value***
Rastrigin	$[-5.12, 2]^D$	$f_3(0, 0, \ldots, 0)=0$
Griewank	$[-600, 200]^D$	$f_4(0, 0, \ldots, 0)=0$
Ackley	$[-82.786, 16]^D$	$f_5(0, 0, \ldots, 0)=0$
Expanded Shaffer	$[-100, 50]^D$	$f_6(0, 0, \ldots, 0)=0$

2) Control parameters choosen strategy

α_3 is the only controllable parameter in BC-QSPSO algorithm except for the population size, the correlation coefficient and iteration times. The convergence performance of the algorithm is controlled by α_3 during the iterations so that to guarantee the balance between global search and the local search to achieve better optimization performance. The value assignment strategy for α_1 in BC-QDPSO follows that used in standard QPSO. Given that the convergence is guaranteed, this section specifies the performance of the BC-QSPSO algorithm using different value assignment strategies by solving four different standard test functions ($f1$-$f4$). The dimensions of test functions are configured as 10 and 20 while the corresponding iteration times is configured as 1000 as the maximum value. Besides, the population size is 30 when the dimension is 10 while the size is 50 when the dimension is 20. The correlation coefficient ρ of the binary correlation factor is configured as -1, 0, and 1 respectively. The above configurations are respectively applied in every test function for 30 test runs, each with 30 iterations, and the average values are studied.

In this paper, parameter α_3 in BC-QSPSO decreases linearly with the increase of iterations, i.e., as follows:

$$\alpha_3 = (a-b)\times(IterMax - t)/\, IterMax + b \tag{41}$$

where, a>b, a is the initial value of the parameter, and b is the final value of the parameter. *InterMax* is the total iterations, t is the current iteration.

Tab. 3 depicts the average optimized values of BC-QSPSO for 10 and 20 dimensions when ρ=1, ρ=0 and ρ=-1, given that the control parameter decreases linearly. Thirty tests are run for different parameter strategy.

Table 3: The optimization results of BC-QSPSO with different control parameters

BC-QSPSO		***f2***		***f3***	
		(10/30)	***(20/50)***	***(10/30)***	***(20/50)***
$\rho=1$	[1.4,0.1]	125.5476	536.9745	7.9694	**14.5262**
	[1.2,0.3]	**5.0476**	17.4405	7.0296	17.2299
	[1.0,0.5]	7.9723	**17.2969**	**6.2554**	18.7799
$\rho=0$	[1.4,0.1]	**3.9143**	**50.9379**	**6.0033**	**17.3977**
	[1.2,0.3]	62.0233	141.3957	7.1873	18.0368
	[1.0,0.5]	21.1276	262.2016	7.9968	18.9367
$\rho=-1$	[1.4,0.1]	**4.8296**	**15.3909**	**6.9709**	16.9303
	[1.2,0.3]	6.1434	17.0273	7.9597	**16.9144**
	[1.0,0.5]	91.365	23.3962	6.9859	17.9362

BC-QSPSO		***f_4***		***f_5***	
		(10/30)	***(20/50)***	***(10/30)***	***(20/50)***
$\rho = 1$	[1.4,0.1]	1.0387	0.7563	4.7339E-4	2.7855E-4
	[1.2,0.3]	0	**0**	2.7417E-8	6.9996E-5
	[1.0,0.5]	**0**	0.8654	**1.4114E-11**	**1.9866E-9**
$\rho = 0$	[1.4,0.1]	**0.4805**	0.6722	**9.0068E-5**	**0.0577**
	[1.2,0.3]	0.5329	**0.6361**	0.0056	0.2379
	[1.0,0.5]	0.5332	0.7563	0.3604	1.0822
$\rho = -1$	[1.4,0.1]	0.6498	0.8654	**2.0428E-14**	**1.1844E-6**
	[1.2,0.3]	**0.5329**	**0.7563**	2.8393E-10	8.8699E-5
	[1.0,0.5]	0.5329	0.7563	3.1358E-4	0.0234

For BC-QSPSO: When ρ=1, Rosenbrock function, among other ten dimensional functions, obtained the optimal solution when the control parameter α_3 linearly decreased between [1.2, 0.3]. Other functions obtained their optimal values whenα_3 linearly decreased between [1.0, 0.5]. For 20 dimensional test functions, the control strategy that α_3 linearly decreased between [1.0, 0.5] also offered good optimization results. Given ρ=0, the linear decrement of α_3 between [1.4, 0.1] is obvious advantages where every test function acquired the optimal value except for Griewank function. Given ρ=-1, the linear decrement of α_3 between [1.4, 0.1] offered good impact over most test functions. The only exception is Griewank, whose optimal solution was obtained when α_3 linearly decreased between [1.2, 0.3], among all ten dimensional functions. Rastrigin and Griewank also acquired optimal solutions when α_3 linearly decreased between [1.2, 0.3]. In general, the optimization performance of BC-QSPSO benifites from bigger decrement range of α_3, thus we adopt the policy that α_3 linearly decreases between [1.4, 0.1].

Upon the above analysis, we conclude as follows: Better optimization performance is achieved for BC-QSPSO when the linear decrement controlling method is adopted for α_3. Bigger decrement range enables better optimization performance. The decrement range between [1.4, 0.1] for α_3 results are in good optimization performance for most test functions.

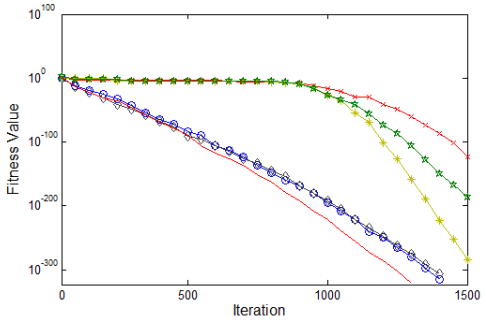
4.2 Experiments results

The parameter α decreases linearly from 1.0 to 0.5 and α3 decreases linearly from 1.4 to 0.1 in every algorithm to be tested. The dimension of the benchmark functions is set to be 20 and the maximum iterations are set to be 1000; The size of the population is set as 50, Every benchmark function is tested times independently, and the average value of each test function is evaluated at the end of 30 iterations.

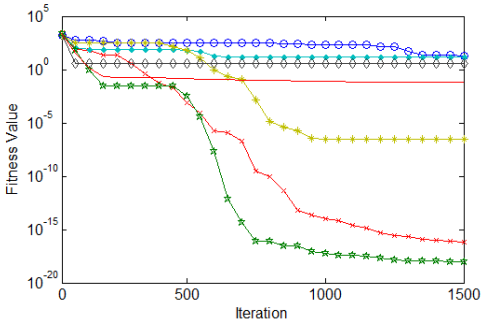
The mean fitness value of each iteration using QPSO, BC-QDPSO [Wu, Yan and Chen (2015)] and BC-QSPSO solving benchmark functions is as shown in Tab. 4.

Table 4: The results of optimization algorithms (D=20, N=50)

	Algorithms	f_1	f_2	f_3
ρ=0	BC-QDPSO	0	19.4254	19.6241
	BC-QSPSO	6.6408E-150	6.31E-17	18.2106
ρ=1	BC-QDPSO	0	15.2942	16.9480
	BC-QSPSO	2.0011E-309	2.71 E-007	17.0493
ρ=-1	BC-QDPSO	0	3.9866	**16.9143**
	BC-QSPSO	6.7054E-210	**4.49E-19**	16.9356
	QPSO	0	6.86E-2	20.9982
	Algorithms	f_4	f_5	f_6
ρ=0	BC-QDPSO	1.2972	1.7774E-4	2.7554E-6
	BC-QSPSO	**0.6209**	4.6E-3	1.0860E-4
ρ=1	BC-QDPSO	0.8654	**6.2172E-15**	5.3644E-5
	BC-QSPSO	0.7563	7.8663E-5	2.1641E-4
ρ=-1	BC-QDPSO	0.7061	5.3165E-8	**2.6695E-7**
	BC-QSPSO	0.6721	9.2707E-4	6.3021E-6
	QPSO	0.6721	6.2175E-15	3.6771E-4



(a) f_1



(b) f_2

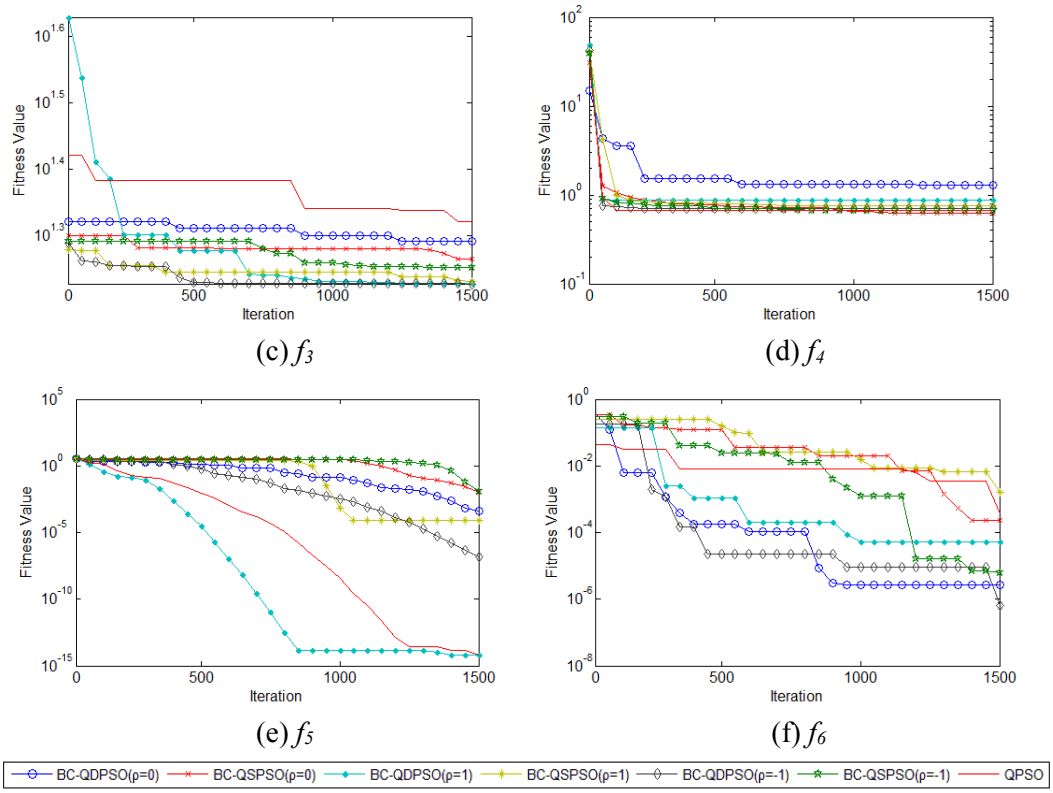


Figure 1: Convergence curves of test functions (D=20, N=50)

Fig. 1 illustrates the convergence curves of test functions using QPSO, BC-QDPSO and BC-QSPSO algorithms. From Tab. 4 and Fig. 1, We can learn that correlation-aware QPSO achieves better optimization performance compared with traditional QPSO algorithms by appropriately selecting correlation factor ρ. f_1, the Sphere function, is a unimodal function. The gradient information of Sphere function always points to the global optimal solution. The function surface is smooth as well where variables do not share mutual impacts, thus the optimization process is quite simple. Sphere function fits well in testing the optimization accuracy of various algorithms. When processing the unimodal Sphere function, all the δ potential well based QPSO algorithms managed to find the optimal solution 0.

The square potential well based BC-QSPSO provides an obvious advantage when dealing with f_2 function, i.e., the Rosenbrock function. Rastrigin function reserves a big amount of local optimal values, thus becoming one hard to be optimized globally. BC-QDPSO and BC-QSPSO offered similar optimization results. It is also as difficult to obtain the optimal solution for Griewank function as for Rosenbrock function, since the Griewank function is a non-linear violent multimodal function whose variables share strong mutual interactions. However, it is getting easier to find the optimal solution along with the increment of dimensions, where BC-QSPSO offers better performance. Ackley is a simple non-linear multimodal function due to the regular distributions around local

optimal values. For Expanded Shaffer function, the optimal value is coaxially surrounded by local optimal values. As a result, dealing with the above two functions. BC-QDPSO offered better performance than BC-QSPSO.

From the above analysis, we can see that the δ potential well based BC-QDPSO algorithm is most advantageous in solving simple unimodal functions and simple multimodal functions with regularly distributed local optimal values, due to that, for these two BC-QPSO algorithms, the probability of the appearance of a particle in the classically forbidden zone is as follows.

$$P_{\mathrm{BC-QSPSO}} < P_{\mathrm{BC-QDPSO}} \tag{42}$$

That is to say, the probability of the appearance of a particle in the classically forbidden zone in δ potential well model is higher than that in the square potential well model. Therefore, BC-QDPSO exhibits stronger mutability which that enables better optimization for unimodal functions. On the other hand, functions with the correlation between variables gain better optimization performance using the BC-QSPSO algorithm due to the even distribution of gravity around the potential well center that attracts particles. In such evenly distributed gravity, incorrect gradient information is not overly used in processing functions with variable correlations, which helps particles escape from local optimization thus better performance is provided.

5 Conclusion

This paper proposes the BC-QSPSO algorithm, i.e., the Binary Correlation QPSO based on Square Potential Well model. This approach simulated the motion of particles in the square well model and using binary normal Copula function describes the correlation between self-experience information and swarm-shared information of a particle in this potential well center. Using the correlation description, the BC-QSPSO algorithm can be deduced from the analysis of particles movement principle in the symmetric square potential well with finite depth. Various experiments and analysis are conducted in this paper to test parameter value assignment strategies, among which the linear decrement strategy is preferred. Experiments are also conducted between BC-QSPSO, BC-QDPSO and the standard QPSO algorithms for performance analysis. The analysis has revealed that the δ potential well based algorithms provide better optimization results for unimodal functions while the square potential well based BC-QSPSO algorithm offers better performance for functions where variables are somehow correlated.

Acknowledgement: This research was funded by National Key Research and Development Program of China (Grant No. 2018YFC1507005), China Postdoctoral Science Foundation (Grant No. 2018M643448), Sichuan Science and Technology Program (Grant No. 2019YFG0110) and Fundamental Research Funds for the Central Universities, Southwest Minzu University (Grant No. 2019NQN22).

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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