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Functional Causality between Oil Prices and GDP Based on Big Data

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Abstract: This paper examines the causal relationship between oil prices and the Gross Domestic Product (GDP) in the Kingdom of Saudi Arabia. The study is carried out by a data set collected quarterly, by Saudi Arabian Monetary Authority, over a period from 1974 to 2016. We seek how a change in real crude oil price affects the GDP of KSA. Based on a new technique, we treat this data in its continuous path. Precisely, we analyze the causality between these two variables, i.e., oil prices and GDP, by using their yearly curves observed in the four quarters of each year. We discuss the causality in the sense of Granger, which requires the stationarity of the data. Thus, in the first Step, we test the stationarity by using the Monte Carlo test of a functional time series stationarity. Our main goal is treated in the second step, where we use the functional causality idea to model the co-variability between these variables. We show that the two series are not integrated; there is one causality between these two variables. All the statistical analyzes were performed using R software.

Keywords: Functional time series, functional stationarity, FAR, FARX, causality.

1 Introduction

The Kingdom of Saudi Arabia (KSA) is the 14th largest country in the world (around 2 million square kilometers) and is the second largest OPEC (Organization of the Petroleum Exporting Countries) country member. It is also the largest exporter of petroleum and possesses about 18 percent of the world's petroleum reserves. The Saudi Arabian oil and gas sector accounts for 50 percent of GDP, and 85 percent of export earnings. So, it is really necessary to determine the causal relationship between oil prices and GDP variables in order to see who is the dependent variable and who is the independent variable. The main purpose of this contribution is to analyze this relationship by using a new approach based on the recent development of modern statistics. Noting that, the progress of technological development, both regarding observation and

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computational tools, allows us to deal with big data, either functional data or continuous data. In this context, oil prices are one of the most important economic indexes, which are continuously observed. Thus, the functional treatment of the oil prices data is the more natural way than the classical approach, where we use only some annual values. As far as we are informed, there has not been any attempt so far to study the oil prices as functional data. This is the main goal of this work.

It should be noted that these questions of statistical analysis of continuous data are arising more and more in applied statistics. For a bibliographical survey or an overview on recent developments in this topic, we cite, for example, [Bongiorno, Goia, Salinelli et al. (2014); Hsing and Eubank (2015)] for general books and Attouch et al. [Attouch, Laksaci and Rafaa (2017); Kara, Laksaci, Rachdi et al. (2017); Belarbi, Chemikh and Laksaci (2018)] for some recent advances.

From an economic point of view**,** there are two main objectives for this research. The first one is to investigate the relationship between the GDP of KSA and the oil prices and to verify if the two variables have the same order of integration (order of stationarity) using unit root tests. The second objective will deal with finding the sense of causality using the new approach; functional statistics methods.

The rest of this paper is organized as follows. Section 2 deals with the literature review. In Section 3, we present the real data and the background theory related to the test of causality (unit root test, co-integration test, and Granger test). Section 4 shows the numerical results of the relationship between the variables: the GDP of KSA and the oil prices. Section 5 gives the conclusion of this work.

2 Literature review

There are many empirical studies on the relationship between economic growth and oil price in the last decades. We mention, for instance, [Hamilton (1983)] who highlighted a negative relationship between oil prices and macroeconomic activity in the United States. Later, Mork et al. [Mork (1989); Lee and Ratti (1995); Hamilton (1996)] used nonlinear transformations into the models and Granger causality tests. Their results confirm the negative relationship between oil price fluctuations and slow-down in economic activity as well as Granger causality from oil prices to economic growth before 1973 but no Granger causality from 1973 to 1994; see also [Federer (1996); Lee and Ni (2002)]. In another study, [Hooker (1996)] showed that, from 1948 to 1972, oil price variability influences GDP growth. More recently, Gounder et al. [Gounder and Barleet (2007)] have shown a direct link between net oil price shock and economic growth in New Zealand. Also, they proved that oil price shock has a substantial effect on inflation and exchange rate. We refer to Jin [Jin (2008)] for a comparative study of three countries: Japan, China, and Russia. He showed that oil price increases have a negative impact on the economic growth in Japan and China and a positive impact on economic growth of Russia. We return to Elmezouar et al. [Elmezouar, Mazri, Benzaire et al. (2014)] for a discussion on the causality between the GDP and oil prices in Algeria, and [Maghrebi, Elmezouar and Almanjahie (2018)] for a discussion on the Granger causality between the GDP and oil prices in Saudi Arabia.

Although the causal relationship between the GDP and the oil prices in KSA has been studied, none of the previous studies have used functional analysis for studying such a relationship. In this contribution, we will investigate such a relationship based on real data collected from reliable resources in Saudi Arabia [Saudi Arabian Monetary Authority (2016)].

3 Functional approach

3.1 Description and analysis

The functional data analysis is an emergent field of research in the mathematical statistics. In this area, the modulization of the input-output relationship between two functional variables has attracted considerable interest. The first interesting studies on this topic are given by Bosq et al. [Bosq (2000); Ramsay and Silverman (2002)]. The nonparametric treatment of these questions is much more recent, and only a few theoretical results have been obtained until now. We cite, for instance, Ferraty et al. [Ferraty, Sued and Mariela (2014)] for the kernel method and [Demongeot, Laksaci and Raffa (2017)] for the local linear method. We return to Delsol et al. [Delsol, Ferray and Vieu (2011)] on the robust regression. In this paper, we focus on the causality test between two functional variables. Indeed, according to Saumard [Saumard (2017)], the causality test in functional time series is carried out by the following algorithm:

- Step 1: Test the stationarity of the data.
- Step 2: Estimate the autoregressive operator of the model $\boldsymbol{T_n} = \boldsymbol{\rho}(\boldsymbol{T_{n-1}})$ with $\mathrm{T}_n = (\mathrm{Y}_n, \mathrm{X}_{n+1})$
- Step 3: Estimate the autoregressive operator of the model $\boldsymbol{Y_n} = \boldsymbol{\rho_1}(\boldsymbol{Y_{n-1}})$
- Step 4: Test if $\Gamma_{\sigma^2}\left(X\middle|\overline{A-Y}\right) - \Gamma_{\sigma^2}(X|A)$ is a positive definite operator and calculate the F-test to compare the variance of errors obtained by Step 2 and the variance of errors obtained by Step 3.
- Decision: If the null hypothesis accepted, then there is no effect of variable X on variable Y, else X cause Y.

3.2 Mathematical formalism of the different steps

3.2.1 Statistical test of stationarity of functional time series

Briefly, the stationarity test in functional statistics is relatively recent. It is started by [Horvath, Kokoszka and Rice (2014)]. They proposed to generalize in functional data the KPSS test of Kwiatkowski [Kwiatkowski (1992)]. The statistical test is constructed by using the fact that all stationary time series models in practice can be represented as Bernoulli shifts. Specifically, for a given functional time series variable $X_i(t)$, the null hypothesis of the stationarity test is formulated by

H_0: $X_i(t) = \mu(t) + \rho_i(t)$,

where $(\rho_i(t))$ is a sequence of Bernoulli shifts. Thus, the statistical test is defined by

$T_N = \iint Z_N^2(x,t)dxdt$, where $Z_N(x,t) = S_N(x,t) - xS_1(x,t), \quad 0 \le x,t \le 1$

with

$$S_N(x,t) = \frac{1}{N}\sum_{i=1}^{[Nx]} X_i(t) \qquad 0 \le x, t \le 1.$$

[Horvath, Kokoszka and Rice (2014)] have established the functional asymptotic normality for this statistical test which allows to determine the *p*-value of the stationarity. This statistical test is available in R-package *ftsa* by using *T_stationary.*

3.2.2 Autoregressive operator estimation

In the second Step, we aim to model our data as autoregressive Hilbertian Processes of Order 1 (ARH(l) model). The monograph of Bosq [Bosq (2000)] is the pioneering work of this kind of model. In order to help the reader to understand our approach, we recall some basic notions of the ARH(l) model.

Definition 1*:* A sequence $\mathbf{X} = (\mathbf{X_n}),\ \mathbf{n} \in \mathbf{Z}$ of random variables valued, in Hilbert space H, is called an autoregressive Hilbertian process of order 1 (denoted by ARH (l)) associated with (μ, ϵ, ρ) if it is stationary and such that

$$\boldsymbol{X_n} - \mu = \boldsymbol{\rho}(\boldsymbol{X_{n-1}} - \mu) + \boldsymbol{\epsilon_n}, \boldsymbol{n} \in \boldsymbol{Z}, \tag{1}$$

where $\epsilon = (\epsilon_n)$ is a Hilbert white noise, $\mu \in H$, and $\boldsymbol{\rho}$ is a linear operator from H to H. In the remainder of the paper, we put $\boldsymbol{T_n} = \boldsymbol{X_n} - \mu$, and we rewrite the Functional Autoregressive of order 1 (FAR (1)) process as follows

$$\boldsymbol{T_n} = \boldsymbol{\rho}(\boldsymbol{T_{n-1}}) + \boldsymbol{\epsilon_n}, \boldsymbol{n} \in \boldsymbol{Z}. \tag{2}$$

Now, we introduce the FAR (Functional autoregressive) model with exogenous variables. The latter is defined by cutting the variable $\boldsymbol{T_n}$ into two components: the endogenous variables $\boldsymbol{Y_n}$ (the ones we are interested in), and the exogenous variables $\boldsymbol{X_n}$ (which influence the endogenous ones). So, the one order Functional Autoregressive process with exogenous variables (FARX(1)) is defined by the following equations:

$$\boldsymbol{T_n} = \boldsymbol{\rho}(\boldsymbol{T_{n-1}}) + \boldsymbol{\epsilon_n}, \boldsymbol{n} \in \boldsymbol{Z}, \tag{3}$$

where $\boldsymbol{T_n} = \begin{pmatrix} Y_n \\ X_{n+1} \end{pmatrix}$ and $\boldsymbol{\rho} = \begin{pmatrix} \boldsymbol{\rho_A} & \boldsymbol{\rho_B} \\ \boldsymbol{\rho_C} & \boldsymbol{\rho_D} \end{pmatrix}$.

This FARX(1) model allows to check the causality of $\boldsymbol{X_n}$ over $\boldsymbol{Y_n}$. Indeed, replacing $\boldsymbol{T_n}$ and $\boldsymbol{\rho}$ in Eq. (3), we obtain $\begin{pmatrix} Y_n \\ X_{n+1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\rho_A} & \boldsymbol{\rho_B} \\ \boldsymbol{\rho_C} & \boldsymbol{\rho_D} \end{pmatrix}\begin{pmatrix} Y_{n-1} \\ X_n \end{pmatrix} + \begin{pmatrix} \boldsymbol{\epsilon_{n1}} \\ \boldsymbol{\epsilon_{n2}} \end{pmatrix}$. Thus,

$$\begin{cases} \boldsymbol{Y_n} = \boldsymbol{\rho_A}(\boldsymbol{Y_{n-1}}) + \boldsymbol{\rho_B}(\boldsymbol{X_n}) + \boldsymbol{\epsilon_{n1}} & (4) \\ X_{n+1} = \boldsymbol{\rho_C}(\boldsymbol{Y_{n-1}}) + \boldsymbol{\rho_D}(\boldsymbol{X_n}) + \boldsymbol{\epsilon_{n2}} & (5) \end{cases}$$

Remark 1

Notice that, only the operators $\boldsymbol{\rho_A}$ and $\boldsymbol{\rho_B}$ of Eq. (5) intervene in the causality test of X_n over Y_n. In addition, to test the causality of Y_n over X_n, we use the same arguments as those of the previous case, and we write

$$T_n^* = \boldsymbol{\rho}^*(T_{n-1}^*) + \boldsymbol{\epsilon^*_n}, \boldsymbol{n} \in \boldsymbol{Z}, \tag{6}$$

with $T_n^* = \begin{pmatrix} X_n \\ Y_{n+1} \end{pmatrix}$ and $\boldsymbol{\rho}^* = \begin{pmatrix} \boldsymbol{\rho^*}_{\boldsymbol{A}} & \boldsymbol{\rho^*}_{\boldsymbol{B}} \\ \boldsymbol{\rho^*}_{\boldsymbol{C}} & \boldsymbol{\rho^*}_{\boldsymbol{D}} \end{pmatrix}$. The matrix formula of Eq. (7) is

$$\begin{pmatrix} X_n \\ Y_{n+1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\rho^*}_{\boldsymbol{A}} & \boldsymbol{\rho^*}_{\boldsymbol{B}} \\ \boldsymbol{\rho^*}_{\boldsymbol{C}} & \boldsymbol{\rho^*}_{\boldsymbol{D}} \end{pmatrix} \begin{pmatrix} X_{n-1} \\ Y_n \end{pmatrix} + \begin{pmatrix} \boldsymbol{\epsilon^*}_{\boldsymbol{n1}} \\ \boldsymbol{\epsilon^*}_{\boldsymbol{n2}} \end{pmatrix}.$$

Therefore, we have

$$\begin{cases} \boldsymbol{X_n} = \boldsymbol{\rho^*}_{\boldsymbol{A}}(\boldsymbol{X_{n-1}}) + \boldsymbol{\rho^*}_{\boldsymbol{B}}(\boldsymbol{Y_n}) + \boldsymbol{\epsilon^*}_{\boldsymbol{n1}} & (7) \\ Y_{n+1} = \boldsymbol{\rho^*}_{\boldsymbol{C}}(\boldsymbol{X_{n-1}}) + \boldsymbol{\rho^*}_{\boldsymbol{D}}(\boldsymbol{Y_n}) + \boldsymbol{\epsilon^*}_{\boldsymbol{n2}} & (8) \end{cases}$$

Remark 2

Once again, similarly to the previous case the operators $\boldsymbol{\rho^*}_{\boldsymbol{C}}$, $\boldsymbol{\rho^*}_{\boldsymbol{D}}$ of Eq. (8) does not intervene in the testing of the causality of Y_n over X_n. To the end, let's note that the estimation of these operators is available in the ***far, R-package,*** by using the routine ***far*** or ***farx***.

3.2.3 Functional causality test

We point out that the functional causality test is more recent. The first results were obtained by Saumard [Saumard (2017)], who generalizes the Granger causality test that usually employed in multivariate statistics. In the rest of this paragraph, we present the conjecture of Saumard [Saumard (2017)] for the functional version of the causality. Indeed, let $\{\boldsymbol{X_t},\ t \in Z\}$ and let $\{\boldsymbol{Y_t},\ t \in Z\}$ be two time series. Let A_t be all the information in the universe accumulated since time t-1, and let $A_t - \boldsymbol{X_t}$ denote all information apart from the specified series $\boldsymbol{X_t}$. Furthermore, put $\overline{U}_t = \{U_{t-s}, s = 1,2,\dots,+\infty\}$ for U a stationary process. Let B an information set and $P(Y_t|B)$ the best linear predictor where:

$\varepsilon(Y_t|B) = Y_t - P(Y_t|B)$

$\sigma^2(Y_t|B) = E[\varepsilon(Y_t|B)^2]$.

From [Granger (1969)] definition, the variable $\boldsymbol{X}$ causes the variable $\boldsymbol{Y}$ iff for at least one value of t satisfies $\sigma^2(Y_t|A_t) < \sigma^2(Y_t|\overline{A_t - \boldsymbol{X_t}})$. The functional version of this definition is as follows.

Definition 2: (functional causality)

We say that Y is causing X if $\Gamma_{\sigma^2(X|\overline{A-Y})} - \Gamma_{\sigma^2(X|A)}$ *is a positive definite operator, where* Γ *is defined by:*

$$\forall\ \cup\ \boldsymbol{\epsilon L^2}([\boldsymbol{0},\boldsymbol{1}]),\quad \Gamma_U = \boldsymbol{E}\left(\langle \boldsymbol{X_i} - \boldsymbol{E}(\boldsymbol{X_i}), \boldsymbol{U}\rangle\left(\boldsymbol{X_i} - \boldsymbol{E}(\boldsymbol{X_i})\right)\right).$$

To the best of our knowledge, the R-routine of the functional causality test does not exist yet. However, since this study is the first implementation of this statistical test, we coded the functional causality test in R. The code of this function can be obtained on request.

4 Empirical study

4.1 Testing the stationarity of the data:

The above algorithm is used now for testing the causality between oil prices and GDP data viewed as functional variables. The data is plotted in Figs. 1 and 2.

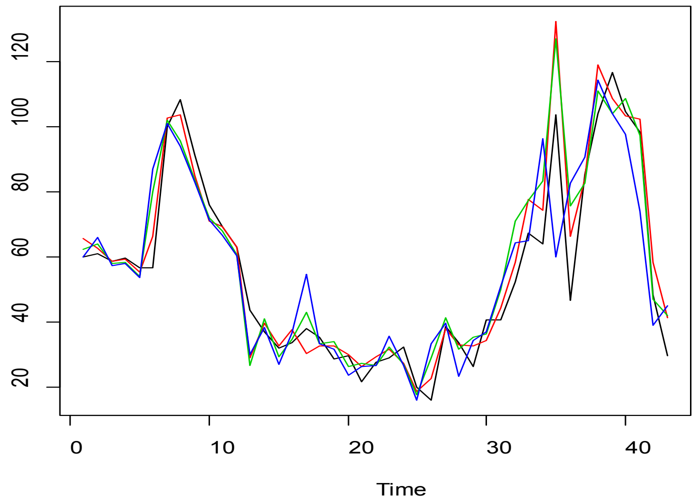

Figure 1: The GDP of saudi arabia for each quarter from 1974 to 2016

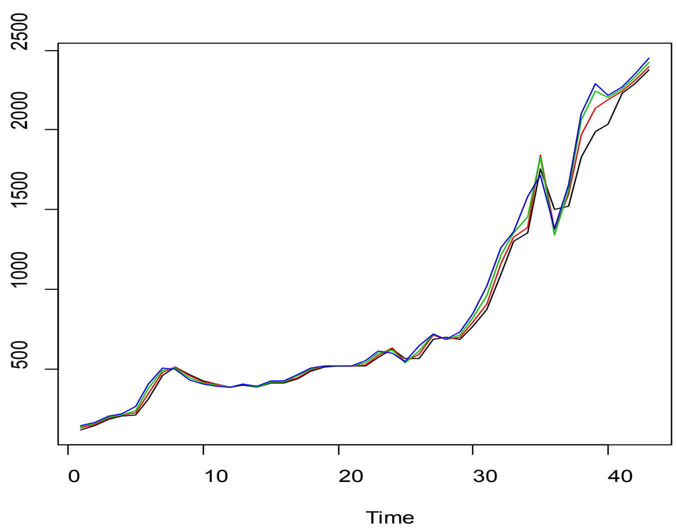

Figure 2: Oil price of saudi arabia for each quarter from 1974 to 2016

We point out that, among the advantages of the functional treatment of this data are:

(1) It is a global analysis of the relationship between oil prices and GDP data.

(2) It keeps the statistical characteristics of the data (homogeneity or heterogeneity, stationarity, cointegration, …)

(3) It keeps the functional path of the data by taking into account the effect of the time variability of the data.

(4) It gives permanent control of this relationship.

(5) It is faster than the multivariate treatment and allows us to consider extensive size data (in this approach, we use an Saudi Arabian Monetary Authority data which are collected quarterly from 1974 to 2016, while for the multivariate case, we consider only the last decade).

For a practical purpose, we replace in Eqs. (5)-(7) by

X_n: the curves of the quarterly values of GDP,

Y_n: the curve of the quarterly value of oil prices.

Then, we use the routine *predict FAR* in the FAR- package in R to estimate the error for both cases. For the last step, we determine the sense of causality.

4.2 Results

4.2.1 Stationarity of GDP and oil price:

First, we test the stationarity of our data set. To do this, we use the Monte Carlo test in which the null hypothesis is the stationarity. The algorithm of this approach is applied to both variables (the GDP and oil prices). Recall that from 1974 to 2016, we dispose of 42 yearly curves for both variables. Finally, we point out that our statistical test is performed by 1000 Monte Carlo replications, and we obtained the following results.

Test of stationarity to GDP:

We used a test of stationarity of functional time series proposed by Horvath et al. [Horvath, Kokoszka and Rice (2014)], which based on the Monte Carlo distribution test such that the null hypothesis is that the series is stationary. Using this approach, the results are shown in Tab. 1.

Table 1: Monte Carlo test of stationarity of GDP

N (number of functions)	number of MC replications	*p*-value
4	1000	0.129

Note that the *p*-value equals 0.129, which is greater than 0.05. It means that we accept the null hypothesis; this means that the series (GDP) is stationary.

Test of stationarity to oil price:

Tab. 2 shows the Monte Carlo distribution test of stationarity of oil prices. Note that the *p*-value equals 0.213, which is greater than 0.05. Based on this result, we accept the null hypothesis; this means that the series (oil prices) is also stationary.

Table 2: Monte Carlo test of stationarity of oil price

N (number of functions)	number of MC replications	p-value
4	1000	0.213

From Tabs. 1 and 2, we conclude that the two functional series are stationary.

4.2.2 Construction of the FAR and FARX of the variables

As previously indicated, we denote by $\boldsymbol{X_n}$ the curve of the GDP and by $\boldsymbol{Y_n}$ the curve of the oil prices. In the FAR model described in Eq. (2), we obtain

$$\boldsymbol{Y_n} = \boldsymbol{\rho}_1(\boldsymbol{Y_{n-1}}) + \boldsymbol{\epsilon_{1n}}, \boldsymbol{n} \in \boldsymbol{Z}, \tag{9}$$

where $\boldsymbol{\rho_1} = \begin{bmatrix} 0.96 & 4.08 \\ 0.00 & 0.38 \end{bmatrix}$, and

$$\boldsymbol{X_n} = \boldsymbol{\rho}_2(\boldsymbol{X_{n-1}}) + \boldsymbol{\epsilon_{2n}}, \boldsymbol{n} \in \boldsymbol{Z}, \tag{10}$$

with $\boldsymbol{\rho}_2 = \begin{bmatrix} 0.98 & 1.78 \\ 0.00 & -0.36 \end{bmatrix}$.

While in the FARX(1), described in Eq. (5), we have

$$\begin{pmatrix} \boldsymbol{\rho_A} & \boldsymbol{\rho_B} \end{pmatrix} = \begin{pmatrix} 1.03 & 1.73 & 0.00 & 0.19 \\ 0.04 & -0.61 & 0.00 & 0.14 \\ & & & \end{pmatrix},$$

where

$$\boldsymbol{X_n} = \boldsymbol{\rho}_1(\boldsymbol{X_{n-1}}) + \boldsymbol{a}_1(\boldsymbol{Y_n}) + \boldsymbol{\epsilon_{22n}}, \boldsymbol{n} \in \boldsymbol{Z}, \tag{11}$$

Concerning the FARX model (7), we obtain

$$\begin{pmatrix} \boldsymbol{\rho^*}_{\boldsymbol{A}} & \boldsymbol{\rho^*}_{\boldsymbol{B}} \end{pmatrix} = \begin{pmatrix} 0.73 & -2.53 & -4.37 & 10.52 \\ 0.00 & 0.56 & -0.01 & 2.72 \\ & & & \end{pmatrix},$$

where

$$\boldsymbol{Y_n} = \boldsymbol{\rho^*}_{\boldsymbol{A}}(\boldsymbol{Y_{n-1}}) + \boldsymbol{\rho^*}_{\boldsymbol{B}}(\boldsymbol{X_n}) + \boldsymbol{\epsilon_{11n}}, \boldsymbol{n} \in \boldsymbol{Z},, \tag{12}$$

4.2.3 Test the sense of causality:

Does oil price cause GDP?

To examine if oil price cause GDP, we use two techniques to answer the question:

a) From Def. 1, we found that the Eigenvalues of $\Gamma_{\sigma^2}(X|\overline{A-Y}) - \Gamma_{\sigma^2}(X|A)$ are all positive, equal [27805.89, 25127.08, 24991.73, 24077.24]. This result shows that $\Gamma_{\sigma^2}(X|\overline{A-Y}) - \Gamma_{\sigma^2}(X|A)$ is a positive definite. Then, we conclude that oil price causes GDP.

b) The F test is used to test the equality of variances obtained from Eq. (9) and Eq. (12). The null hypothesis is

$H_0\colon \sigma^2_{\boldsymbol{\epsilon_{11n}}} = \sigma^2_{\boldsymbol{\epsilon_{1n}}}$

The results are shown in Tab. 3.

Table 3: Test that oil price cause GDP

Numerator df	Denominator df	F	*p*-value
41	41	5.4278	3.278e-07

Tab. 3 shows that p-value equals 3.278e-07, which is less than 0.05. Therefore, we reject the null hypotheses, and we conclude that oil price causes GDP.

Does GDP price cause oil price?

To examine if GDP cause oil price, we use two techniques to answer the question:

a) From Def. 1, we found that the Eigenvalues of $\Gamma_{\sigma^2}(\mathrm{Y}|\overline{\mathrm{A}-\mathrm{X}}) - \Gamma_{\sigma^2}(\mathrm{Y}|\mathrm{A})$ are not all positive, equal [8.430590, 7.424424, -9.012655, -69.983826]. This result shows that $\Gamma_{\sigma^2}(Y|\overline{A-X}) - \Gamma_{\sigma^2}(Y|A)$ is not a positive definite, and then we conclude that GDP does not cause oil price.

b) The F test is used to test the equality of variances obtained from Eqs. (10) and (11). The null hypothesis is

$H_0: \sigma^2_{\boldsymbol{\epsilon}_{22n}} = \sigma^2_{\boldsymbol{\epsilon}_{2n}}$

The results are shown in Tab. 4.

Table 4: Test that oil price cause GDP

Numerator df	Denominator df	F	*p*-value
41	41	0.71298	0.2828

Tab. 4 shows that p-value equals 0.2828, which is greater than 0.05. Hence, we accept the null hypotheses, and we conclude that GDP does not cause oil price.

5 Conclusion

The main objective of this study is to investigate the real relationships between oil prices and the real gross domestic product in Saudi Arabia, based on real data collected quarterly from 1974 to 2016. We used a new approach based on functional time series. This paper arrives at the following three conclusions based on the use of the functional time series theory and causality between two functional time series. First, we analyzed the characteristics of the data series using the Monte Carlo test of stationarity of a functional time series. As a result, we accept the hypothesis of stationarity for both series. Second, we estimate the errors obtained from FAR and FARX when GDP is the dependent variable, and oil price is the independent variable (the same procedure when oil price is the dependent variable and GDP is the independent variable). Finally, two techniques are used for testing the sense of causality. The first technique is to check that if $\Gamma_{\sigma^2}(X|\overline{A-Y}) - \Gamma_{\sigma^2}(X|A)$ is a positive definite, and the second technique is by using F test based on equality of the variances of the two errors obtained from FAR and FARX. We conclude that there is only one sense of causality that oil price causes GDP.

The result of this study provides a future image of the causality between GDP and oil prices in Saudi Arabia. Accordingly, the officials and decision-makers in the Saudi Arabian Monetary Authority can adopt the results of this study to face future economic changes in Saudi Arabia.

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Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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