Computers, Materials & Continua CMC, vol.63, no.1, pp.445-455, 2020

Detection of Number of Wideband Signals Based on Support Vector Machine

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Abstract: In array signal processing, number of signals is often a premise of estimating other parameters. For the sake of determining signal number in the condition of strong additive noise or a little sample data, an algorithm for detecting number of wideband signals is provided. First, technique of focusing is used for transforming signals into a same focusing subspace. Then the support vector machine (SVM) can be deduced by the information of eigenvalues and corresponding eigenvectors. At last, the signal number can be determined with the obtained decision function. Several simulations have been carried on verifying the proposed algorithm.

Keywords: Number of signals, wideband signals, support vector machine, array signal processing.

1 Introduction

Determining signal number is an important and tricky content in the signal parameter estimation, moreover, it is also applied in radar signal processing [Nielsen and Dall (2015); Ebihara, Kimura and Shimomura (2015); Takahashi, Inaba and Takahashi (2018); Oh, Ju and Nam (2016); Khabbazibasmenj, Hassanien and Vorobyov (2014)], underwater acoustic engineering [Saucan, Chonavel and Sintes (2016); Gholipour, Zakeri and Mafinezhad (2016); Lim, Boon and Reddy (2017)] and internet of things [David, Hector and Sanchez (2016); Zhang, Wang and Lu (2016); Mohamedatni, Fergani and Laheurte (2015); Han, Wan and Shu (2017)].

In general, before calculating other signals' parameters, we need to acquire their number. The early algorithm is based on hypothesis test [Schmidt (1981)], but it is often influenced by subjectivity. The information theoretic criterion [Wax and Kailath (1985)] is a kind of objective means relatively, the most famous is minimum description length (MDL) [Rissanen (1978)]. After this Gerschgorin Radii criterion (GDE) [Wu, Yang and Chen (1995)] was presented by Wu, it is appropriate for colored noise. Both of the methods above have improved the determining performance greatly. In recent years, this topic has also been studied by many other scholars: Liu et al. [Liu, Sun and Wang (2012)] estimated signal number, as well as their directions concurrently based on spatial difference with uniform linear array (ULA). Han et al. [Han and Nehorai (2013)] did the

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Received: 16 February 2019; Accepted: 18 July 2019.

same job with resampling and nested arrays, increasing the array aperture to some extent. In order to differentiate the sources which were close to one another, Liu et al. [Liu and So (2013)] designed Monte-Carlo method to determine the effective source number and distinguished the signals in the circumstance of high-level noise. Zhao et al. [Zhao, Zhang and Zhao (2011)] constructed an effective algorithm based on machine learning in 2009, he employed Gerschgorin disk algorithm by the orthotropic of the array manifold and the noise vector to construct a support vector machine (SVM), obtaining a good result. But the methods for wideband signals are still rare, a classic approach is coherent signal method (CSM) [Wang and Kaveh (1985)], which transforms the sources on the focusing point, then employs MDL to obtain the final result. All the mentioned algorithms need high SNR or many samples, or they will be invalid. Thus, there are still some troubles in practical engineering application: on one hand, time of sampling is always restricted severely, for instance, transmitted pulses of sonar is usually very narrow, there is only a little data can be utilized; on the other hand, high SNR can not be guaranteed, both of them have hindered the application of above algorithms.

This paper provides a new idea for determining wideband signal number, it is based on SVM, and the signal characteristics of received array data are fully exploited. First, technique of focusing is used for transforming signals into a same subspace. Then the SVM can be deduced by the information of eigenvalues and corresponding eigenvectors. At last, the signal number can be determined with the obtained decision function.

2 Array signal model

Signal model is represented as Fig. 1, consider that there is a ULA formed by M elements and the first one is the origin, d is the interval between sensors, then B far-field wideband signals $s_b(t)(b=1,2,\cdots,B)$ arrive at the array from directions $\theta_1,\theta_2,\cdots,\theta_B$. Assume that the observed time is T, then at time t, we can receive a serious of observed data for $t\in(0,T)$, where $x_m(t)$ is the response of the corresponding sensor at time t, c is transmission rate of the targets, $n_m(t)$ is white noise subjecting to Gaussian distribution $CN(0,\sigma^2)$.

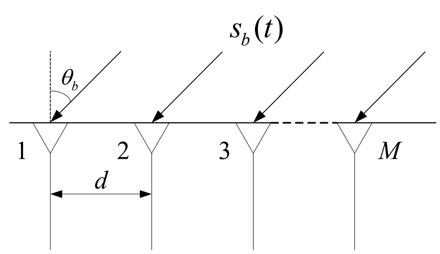

Figure 1: Signal model

$$\boldsymbol{x}(t)=\begin{bmatrix} x_1(t) \\ \vdots \\ x_m(t) \\ \vdots \\ x_M(t) \end{bmatrix}=\begin{bmatrix} \sum_{b=1}^{B} s_b(t) \\ \vdots \\ \sum_{b=1}^{B} s_b\left(t-(m-1)\frac{d}{c}\sin\theta_b\right) \\ \vdots \\ \sum_{b=1}^{B} s_b\left(t-(M-1)\frac{d}{c}\sin\theta_b\right) \end{bmatrix}+\begin{bmatrix} n_1(t) \\ \vdots \\ n_m(t) \\ \vdots \\ n_M(t) \end{bmatrix} \tag{1}$$

Define that signal frequency is limited in $[f_L, f_H]$, sampling times at every frequency is U, then the array response can be demonstrated as

$$\begin{aligned}\boldsymbol{X}(f_g) &= \boldsymbol{A}(f_g,\boldsymbol{\theta})\boldsymbol{S}(f_g)+\boldsymbol{N}(f_g) \\ &= \begin{bmatrix} x_1(f_g,1) & \cdots & x_1(f_g,u) & \cdots x_1(f_g,U) \\ \vdots & \ddots & \vdots & \vdots \\ x_m(f_g,1) & \cdots & x_m(f_g,u) & \cdots x_m(f_g,U) \\ \vdots & \ddots & \vdots & \vdots \\ x_M(f_g,1) & \cdots & x_M(f_g,u) & \cdots x_M(f_g,U) \end{bmatrix} \qquad g=1,2,\cdots,G \\ &= \left[\boldsymbol{X}(f_g,1)\cdots\boldsymbol{X}(f_g,u)\cdots\boldsymbol{X}(f_g,U)\right]\end{aligned} \tag{2}$$

Here, $f_L \le f_g \le f_H$ for $g=1,2,\cdots,G$, $\boldsymbol{X}(f_g,u)=\left[x_1(f_g,u)\cdots x_m(f_g,u)\cdots x_M(f_g,u)\right]^{\mathrm{T}}$, and $x_m(f_g,u)$ is the received data on the mth sensor of f_g at sample u, $A(f_g,\boldsymbol{\theta})=\left[\boldsymbol{a}(f_g,\theta_1),\cdots,\boldsymbol{a}(f_g,\theta_b),\cdots,\boldsymbol{a}(f_g,\theta_B)\right]$ is the array steering, where

$$\boldsymbol{a}(f_g,\theta_b)=\left[1,\cdots,\exp\left(-\mathrm{j}m2\pi f_g\frac{d}{c}\sin\theta_b\right),\cdots,\exp\left(-\mathrm{j}(M-1)2\pi f_g\frac{d}{c}\sin\theta_b\right)\right]^{\mathrm{T}} \tag{3}$$

signal $\boldsymbol{S}(f_g)$ ($g=1,2,\cdots,G$) are normal distributed and independent with $\boldsymbol{N}(f_g)$ ($g=1,2,\cdots,G$).

3 Proposed method

3.1 Focusing

The covariance of the received data at f_g is

$$\hat{\boldsymbol{R}}(f_g)=\frac{1}{U}\boldsymbol{X}(f_g)\boldsymbol{X}^{\mathrm{H}}(f_g) \qquad g=1,2,\cdots,G \tag{4}$$

Then we can obtain the focused matrix $\hat{\boldsymbol{R}}(f_0)$ by transforming the signals at every frequency bin into a single covariance at f_0 with two-sided correlation transformation (TCT) [Valaee and Kabal (1995)], here f_0 can be chosen from $f_1,\cdots,f_G$, then the eigenvalues $\lambda_1(f_0)>\cdots>\lambda_B(f_0)>\lambda_{B+1}(f_0)=\cdots=\lambda_M(f_0)$ and corresponding eigenvector

$\boldsymbol{E}(f_0)=[\boldsymbol{e}_1(f_0),\cdots,\boldsymbol{e}_B(f_0),\boldsymbol{e}_{B+1}(f_0),\cdots,\boldsymbol{e}_M(f_0)]$ can be acquired too, here $\boldsymbol{E}_{Si}(f_0)=[\boldsymbol{e}_1(f_0),\cdots,\boldsymbol{e}_B(f_0)]$ belongs to signal subspace and $\boldsymbol{E}_{No}(f_0)=[\boldsymbol{e}_{B+1}(f_0),\cdots,\boldsymbol{e}_M(f_0)]$ corresponds to the noise one.

3.2 Weighting

Since $\boldsymbol{a}(f_0,\theta)$ and $\boldsymbol{E}_{Si}(f_0)$ both correspond to the signal subspace, $\boldsymbol{a}(f_0,\theta)$ can be expressed

$$\boldsymbol{a}(f_0,\theta)=\sum_{b=1}^{B}\beta_b(f_0)\boldsymbol{e}_b(f_0) \tag{5}$$

Here, $\beta_b(f_0)$ is the relevant coefficient, define the following variables

$$\mu_m(f_0)=\left|\boldsymbol{a}^{\mathrm{H}}(f_0,\theta)\boldsymbol{e}_m(f_0)\right|,\quad m=1,\ 2,\cdots,M \tag{6}$$

Due to the orthogonality between signal and noise subspace, we have

$$\boldsymbol{a}^{\mathrm{H}}(f_0,\theta)\boldsymbol{e}_b(f_0)=0\ ,\ b=B+1,\ \cdots,M \tag{7}$$

referencing Eq. (5), Eq. (7), Eq. (6) can be transformed as

$$\mu_m(f_0)=\left|\left(\sum_{b=1}^{B}\beta_b(f_0)\boldsymbol{e}_b(f_0)\right)^{\mathrm{H}}\boldsymbol{e}_m(f_0)\right|=\begin{cases}|\beta_m|, & m=1,\ 2,\cdots,B\\ 0, & m=B+1,\cdots,M\end{cases} \tag{8}$$

Now we can only use eigenvalues to determine signal number [Han and Nehorai (2013); Liu and So (2013)], but when the samples is small, we need to take full advantage of the eigenvectors to enhance the detecting performance, fuse $\mu_m(f_0)$ with $\varsigma_m(f_0)$

$$\xi_m(f_0)=\varsigma_m^{1/2}(f_0)\mu_m(f_0),\quad m=1,\ 2,\cdots,M-1 \tag{9}$$

Then the classifying characteristic vector $\xi(f_0)=\left[\xi_1(f_0),\cdots,\xi_m(f_0),\cdots,\xi_{M-1}(f_0)\right]^{\mathrm{T}}$ is evaluated. According to Viberg et al. [Viberg, Ottersten and Kailath (1991)], the optimal $\varsigma_m(f_0)$ $(m=1,\ 2,\cdots,M-1)$ are chosen as

$$\varsigma_m(f_0)=\frac{\left(\lambda_m(f_0)-\lambda_M(f_0)\right)^2}{\lambda_m(f_0)},\quad m=1,\ 2,\cdots,M-1 \tag{10}$$

3.3 Theory of SVM

Theory of SVM [Vapnik (1995)] can be simply described as a binary classification problem: that is to determine a hyperplane satisfying the classifying requirement through training. If the set Φ_1 and Φ_2 are linear separable, namely existing (ω,b), they satisfy the purpose of classification is to determine (ω,b) to separate Φ_1 and Φ_2 optimally. In order to avoid duplication of the hyperplane, we constrain (ω,b) as follows:

$$\omega x_n+b>0,\forall x_n\in\Phi_1\ ,\ \omega x_n+b<0,\forall x_n\in\Phi_2 \tag{11}$$

$$\min_{n=1,2,\cdots,N}\left|\omega x_n+b\right|=1 \tag{12}$$

SVM solves nonlinear classification problem by introducing kernel function a proper kernel function $K(p,q)$ can transform the nonlinear problem into a linear one in higher dimensional space, then the complexity of corresponding dual problem depends on sampling times rather than space dimensionality. The maximal classification distance is written:

$$K(p,q)=\sum_{l=1}^{L}\alpha_l z_l(p)z_l(q),\ \alpha_l\geq 0 \tag{13}$$

$$\begin{aligned}&\max W(\alpha)=\sum_{i=1}^{N}\alpha_i-\frac{1}{2}\sum_{i,j=1}^{N}\alpha_i\alpha_j y_i y_j K(x_i,x_j),\ \alpha_i\geq 0,\ i=1,2,\cdots,N\\ &\text{s.t.}\qquad \sum_{i=1}^{N}\alpha_i y_i=0\end{aligned} \tag{14}$$

Then the optimal decision function can be obtained:

$$f(x)=\operatorname{sgn}\left\{\sum_{i=1}^{N}\alpha_i y_i K(x_i,x)+b\right\} \tag{15}$$

The SVM can be deduced according to (13) and (14), where

$$K(x_i,x_j)=\exp\left(-0.5\left\|x_i-x_j\right\|^2\right) \tag{16}$$

3.4 Proposed algorithm

Here, SVM will be employed for the classification of signal and noise to determine source number, then the input information need to include characteristic of signal and noise, thus, estimation performance can be effectively improved under the circumstance of small samples or low SNR. Signals and noise is separated on the basis of the elements in the vector $\boldsymbol{\xi}(f_0)$, they contain the information not only the eigenvalues but also the eigenvectors, where the larger ones correspond to sources, and there are greater distinctions between them; While the others are smaller and close to each other, they correspond to the noise. Then as the classification characteristic, $\xi(f_0)$ can be chosen to be the input data.

$$\boldsymbol{\xi}(f_0)=\begin{bmatrix}\xi_1(f_0)\\ \xi_2(f_0)\\ \vdots\\ \xi_{M-1}(f_0)\end{bmatrix}=\begin{bmatrix}\varsigma_1^{1/2}(f_0)\mu_1(f_0)\\ \varsigma_2^{1/2}(f_0)\mu_2(f_0)\\ \vdots\\ \varsigma_{M-1}^{1/2}(f_0)\mu_{M-1}(f_0)\end{bmatrix} \tag{17}$$

When $\xi(f_0)$ is used for training, we can preset its output as $y=[1,\cdots,1,\text{-}1,\cdots,\text{-}1]$, where $[1,\cdots,1]$ represents signal vector, and the amount of 1 is *B*, it means there is *B* signal; While $[\text{-}1,\cdots,\text{-}1]$ denotes noise vector, so $\{\boldsymbol{\xi}(f_0),y\}$ is selected to be the training data. Here, we define $\omega_i=\dfrac{\xi_1(f_0)}{\sum_{i=1}^{M-1}\xi_i(f_0)}$ as the initial weighting.

The proposed algorithm has used the course of focusing, fusion and SVM, so it can be called FFSVM for short. Now let us generalize the derivation process above:

(1) Collect array signal as the training data, source number ranges from $1,2,\cdots,M-1$; SNR varies from -20 dB to 20 dB, step size is 4 dB;

(2) Decompose covariance matrix of every frequency, then focusing them on a reference frequency by CSM;

(3) Calculate classification characteristic vector $\xi(f_0)=[\varsigma_1^{1/2}(f_0)\mu_1(f_0),\cdots,\varsigma_{M-1}^{1/2}(f_0)\mu_{M-1}(f_0)]^{\mathrm{T}}$ according to eigenalues and eigenvectors;

(4) Construct SVM through (14)-(16);

(5) Take $\{\xi(f_0),y\}$ into the SVM for the training;

(6) Modify the weighting parameters in the light of training effect properly;

(7) Estimate signal number in line with the testing data.

4 Simulations

Now we will test this algorithm, some examples are provided in the following, several linear frequency modulation signals arrive at a ULA with 10 omni-directional sensors, the bandwidth is 30% of center frequency (1 GHz), signals are divided into 40 frequency bins, and $d=3\times10^8/(2\times1\times10^9)$ m. Here, FFSVM, MDL based on CSM (CSM-MDL) and GDE based on CSM (CSM-GDE) are compared with one another, 500 trials, where 300 ones are used for training and 200 ones are exploited for the testing.

In the first example, the model is shown as Fig. 1, signals are respectively from 6°, 14°, 21°, 29° and 37° synchronously, background of Gaussian white noise (GWN), Fig. 2 shows the accuracy versus SNR when number of samples is 60, while Fig. 3 presents that of number of samples when SNR is 6 dB.

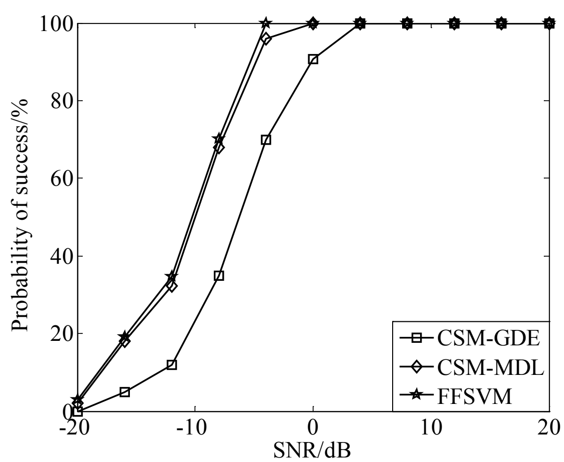

Figure 2: Accuracy *vs.* SNR for uncorrelated signals in GWN

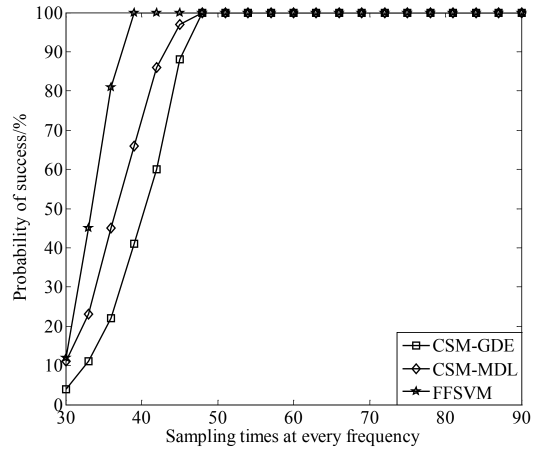

Figure 3: Accuracy *vs.* number of samples for uncorrelated signals in GWN

It can be seen in Fig. 2, all the accuracy of the three algorithms are raising versus the increasing of SNR, and FFSVM performs better than CSM-GDE and CSM-MDL, it can achieve 100% if SNR is -4 dB, while CSM-GDE and CSM-MDL can reach 100% when SNR is 4 dB and 0 dB respectively. And we see from Fig. 3, when the samples are small, FFSVM is also better than the other two algorithms, it can achieve 100% when the sample number is 39, while CSM-GDE and CSM-MDL reach 100% when their sample numbers reach 48.

For this example, we investigate the performance under the circumstances of colored noise, suppose the noise model in this section and [Stoica and Cedervall (1997)] obey the same distribution, other conditions are the same with the first example, then Fig. 4 shows the accuracy versus SNR when samples at every frequency is 60, while Fig. 5 presents that of samples at every frequency when SNR is 6 dB.

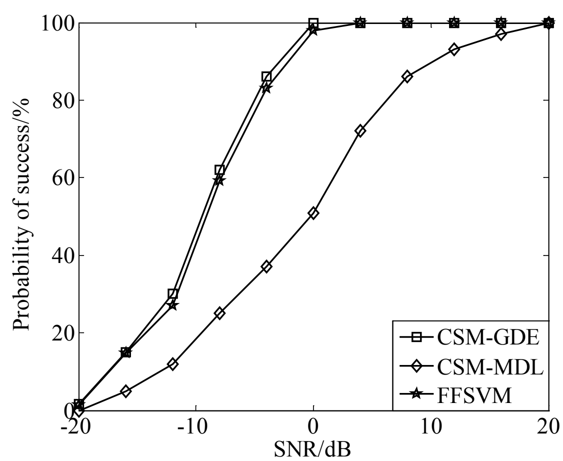

Figure 4: Accuracy *vs.* SNR for uncorrelated signals in colored noise

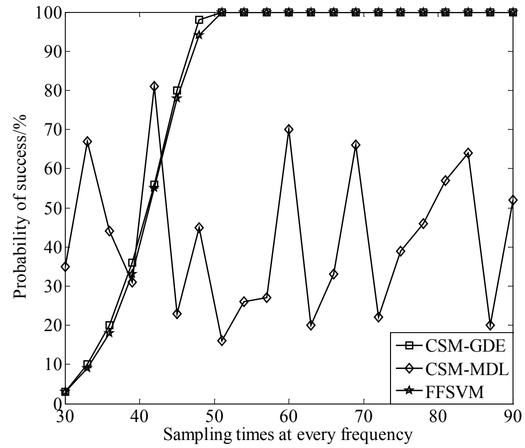

Figure 5: Accuracy *vs.* number of samples for uncorrelated signals in colored noise

From Fig. 4 we know accuracies of the three algorithms are getting better versus SNR raising, and all of them can estimate signal number successfully completely at last. And from Fig. 5 it is seen that due to the colored noise, CSM-MDL is shaking all the time, that of the CSM-GDE and FFSVM are improving, and they are nearly the same, so the proposed FFSVM can also apply to the circumstances of colored noise.

In the final example, we investigate the performance for coherent sources, suppose that signals are respectively from 6°, 14°, 21°, 29° and 37° simultaneously, other conditions are the same as the first example, then Fig. 6 and Fig. 7 have shown the results. Owing to the process of focusing, the coherence of these sources has been eliminated, so these algorithms are also appropriate for coherent signals, and Figs. 6 and 7 demonstrate that their performances are almost identical to that of uncorrelated sources.

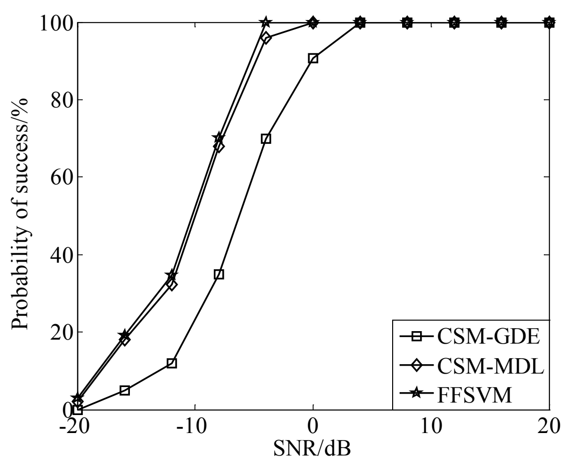

Figure 6: Accuracy *vs.* SNR for coherent signals in GWN

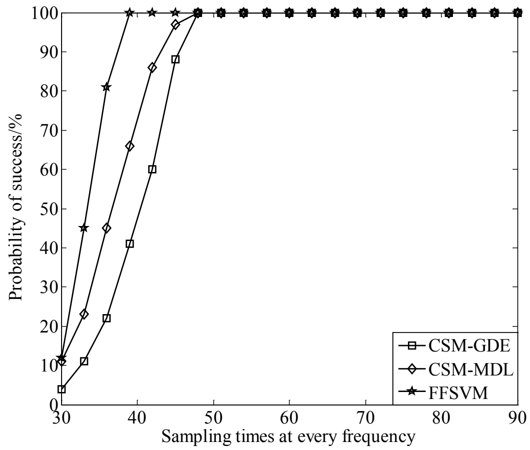

Figure 7: Accuracy *vs.* number of samples for coherent signals in GWN

5 Conclusion

This paper proposes an algorithm for detecting number of wideband signals based on SVM, after transforming the signals on the focusing frequency, we deduce the SVM according to the information of eigenvalues and their eigenvectors of received data. Then signal number can be estimated by the acquired decision function. The simulation examples have shown that the proposed algorithm performs better than that of MDL and GDE based on CSM in the condition of strong additive noise or a little sample data, and it is also suitable for colored noise.

Acknowledgement: This work was supported by the National Natural Science Foundation of China under Grant 61501176, Natural Science Foundation of Heilongjiang Province F2018025, University Nursing Program for Young Scholars with Creative Talents in Heilongjiang Province UNPYSCT-2016017, the postdoctoral scientific research developmental fund of Heilongjiang Province in 2017 LBH-Q17149.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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