# Intelligent Spectrum Detection Model Based on Compressed Sensing in Cognitive Radio Network

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Abstract: In view of the uncertainty of the status of primary users in cognitive networks and the fact that the random detection strategy cannot guarantee cognitive users to accurately find available channels, this paper proposes a joint random detection strategy using the idle cognitive users in cognitive wireless networks. After adding idle cognitive users for detection, the compressed sensing model is employed to describe the number of available channels obtained by the cognitive base station to derive the detection performance of the cognitive network at this time. Both theoretical analysis and simulation results show that using idle cognitive users can reduce service delay and improve the throughput of cognitive networks. After considering the time occupied by cognitive users to report detection information, the optimal participation number of idle cognitive users in joint detection is obtained through the optimization algorithm.

**Keywords:** Cognitive wireless network, compressed sensing, intelligent frequency spectrum detection, random detection.

#### 1 Introduction

Cognitive radio technology gives more users access to the free spectrum in wireless networks, thereby greatly increasing spectrum utilization. One of the important problems is how to identify accessible channels without affecting the transmission of primary users. The primary user channel must be detected as idle before it can be used by the user. Therefore, the cognitive system must quickly and reliably detect the use of the main user channel, so as to obtain more free channel information. The cooperation between cognitive users can not only improve the detection performance of cognitive network, but also obtain more channel information of primary users. Liu et al. [Liu, Kuang, Cai et al. (2014a)] proposed stochastic perception strategy and consultative perception strategy, and analyzed the performance of cognitive network by cross-layer method. Sung et al. [Sung, Lee, Kim et al. (2006a)] proposes MAC protocol based on partially observable compressed sensing decision process, which can make good use of idle spectrum resources.

However, each cognitive user is required to be equipped with multiple perceptrons and

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the complexity is high, so it is difficult to achieve. Giorgio et al. [Giorgio, Ferraioli, Tarantino et al. (2004a)] proposed an optimal access strategy with the primary user model as continuous time markov model. Polkowski et al. [Polkowski, Larghi, Weynand et al. (2012)] proposed a detection strategy based on semi-markov prediction model. Guo et al. [Guo, Zhang, Klein et al. (2010a)] proposed a simple multi-channel detection sequence model without prior information. None of this considers the resources of idle users in the cognitive network. Kingham et al. [Kingham, D'Angelica and Jarnagin (2012a)]; Smiley et al. [Smiley, Whittaker, Gourlie et al. (2005a)] introduces idle cognitive users into spectral detection, thereby improving the detection performance of cognitive system.

In this paper, under the condition that there are idle cognitive users in the cognitive network, based on the random detection strategy proposed in Liu et al. [Liu, Kuang, Cai et al. (2014b)], the access delay and throughput after random detection of idle cognitive users in the cognitive network are analyzed. At the same time, considering that there are a large number of idle cognitive users, the cognitive user report detection information will generate reporting overhead. At this time, the addition of too many idle cognitive users will lead to the reduction of transmission time, thereby reducing the throughput of the system. Through the analysis of the throughput of the cognitive network in the joint compression perceptual detection, the throughput of the cognitive network is obtained.

## 2 The cognitive network system model for idle cognitive users

Considering that there is a cognitive base station and part of cognitive users in the single-hop cognitive network, there is a primary user network next to it, and the primary user communicates through the authorized frequency band. In cognitive networks, cognitive users do not always have data to transmit. At the current moment, some cognitive users who have data to transmit are called active cognitive users (active cognitive users). In addition, some cognitive users have no data to transmit, but are in the range of joint detection. These cognitive users are called idle cognitive users (idle cognitive users), as shown in Fig. 1.

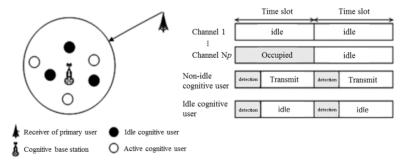


Figure 1: Cognitive network system model of idle cognitive users

Cognitive users go through detection stage and transmission stage respectively at each time slot. In the detection stage, idle cognitive users and active cognitive users are detected simultaneously, and the detection results are reported to the cognitive base station for unified processing by the cognitive base station. The cognitive base station allocates the detected idle channels to active cognitive users according to the strategy.

The existence of idle cognitive users increases the ability of cognitive networks to acquire channel information under the condition that the number of idle channels required is constant. The number of idle and active cognitive users participating in the detection is represented as  $N_i$  and  $N_a$  respectively, and the authorization spectrum is divided into  $N_p$  channels.  $\lambda$  represents the occupancy rate of the primary user, and K represents the number of authorized spectrum channels available at the current moment. Whether the authorized spectrum channel is occupied is random so that the probability that the number of available channels is P is:

$$Pr\left\{K=k\right\} = C_{N_p}^k \lambda^{N_p-k} \left(1-\lambda\right)^k \tag{1}$$

In Eq. (1),  $C_x^y$  represents the probability of taking y out of x elements without permutation.

# 3 Compressed sensing algorithm for intelligent spectrum detection

#### 3.1 Basic idea

Definition of compressible (sparse): a one-dimensional signal  $x \in \mathbb{R}^{N \times 1}$  is considered, which can be expressed linearly by  $N \times 1$   $\{\Psi_i\}_{i=1}^N$  wiki vector. To simplify the problem, assuming that the basis vector is a normal orthogonal vector, the  $N \times N$  basic matrix  $\Psi = \{\Psi_1 \mid \Psi_2 \mid ... \mid \Psi_N\}$  is used, and the signal x can be expressed as:

$$x = \sum_{i=1}^{N} s_i \psi_i \quad or \quad X = \Psi S$$
 (2)

where S is an N×1 dimensional column vector composed of projection coefficient  $s_i = \langle X, \Psi_i \rangle = \Psi_i^T X$ . Obviously, the equivalent representation of X and S are the same signal, where X is in the representation of temporal or spatial domain, S is the representation in  $\Psi$  domain.

When the signal can only be represented linearly by K basis vectors, the signal x is called K-sparse. When K<<N, signal x is called compressible [Wang, Yang and Stern (2009); Lien, Lee and Hsieh (2012a)] if the signal can be represented by a small number of large coefficients and a large number of small coefficients. Assuming that the signal is sparse, for 0<p<2 and R>0, there is:

$$\left\|s\right\|_{p} \equiv \left(\sum_{i} \left|s_{i}\right|^{p}\right)^{1/p} \le R \tag{3}$$

Compressed signal in traditional way of thinking is the way of using the orthogonal transformation, of which the coding decoding strategies are as follows: Firstly, construct the orthogonal basis matrix  $\Psi$  by coding, perform the transformation  $S = \Psi^T X$ , keep the most important K components of S and their corresponding positions. Decoding the K components back to their corresponding positions, and fill 0 in other positions to construct  $\Psi$ , finally obtain reconstructed signal by inverse transformation  $X = \Psi S$ .

Obviously, this encoding and decoding method based on Nyquist-Shanon sampling theorem has many disadvantages. 1. Compression after sampling wastes a lot of sampling

resources. If the signal length after sampling is still very long, the transformation will take a long time. 2. Since the positions of the K important components to be retained vary with the signal, this codec method is adaptive, and extra storage space needs to be allocated to retain the positions of the K important components. 3. Some of K important components may be lost in the transmission process, resulting in poor anti-interference ability. Compressed sensing theory, which has emerged in recent years, is an innovation on the traditional idea that signals can be recovered with much less sampling and measurement than traditional methods.

The theory relies mainly on two principles, sparsity and incoherence. As for the signals of interest, sparse means that the information rate of continuous time signal may be much lower than that suggested by its bandwidth, and the degrees of freedom dependent on discrete time signal is much less than its length. It can be said that many signals in nature are sparse or compressible to some extent. When expressed in the right base  $\Psi$ , signal can have a lot of concise expressions. Incoherence means that signal represented by  $\Psi$  incoherence can be certainly expanded in the required filed. Based on these two principles, compressed sensing theory highlights that the transformation coefficient of signal X with length of n on a set of orthogonal basis or a compact frame  $\Psi_{n\times n}$  is sparse. If an observation base  $\Phi$  ( $\Phi \in R^{m\times n}$ , m << n) not related to transform basis  $\Psi$  is used to linearly transform coefficient vector, and the observation set is yielded, the signal X can be reconstructed accurately by solving the optimization problem.

# 3.2 Compressed sensing sampling process

The sampling process of signal compression based on CS theory is presented as follows:

Step 1: Find a base or tight frame  $\Psi$ , make the signal x on  $\Psi$  sparse, and obtain the transform coefficient:  $S=\Psi^TX$ , where S is an equivalent or approximate sparse representation of X. Transform base  $\Psi$  can be selected as some widely used bases, such as wavelet and Fourier leaf base, local Fourier leaf base, etc. For the selection of orthogonal basis, see Kim et al. [Kim, Gwak and Yoh (2015); Sung, Lee, Kim et al. (2006b)]. In addition, a compact framework (atomic dictionary) can be used for sparse representation of signals, such as Giorgio et al. [Giorgio, Ferraioli, Tarantino et al. (2004b)]. These two kinds of transform basis have better directivity and anisotropy, and a small number of coefficients can effectively capture the edge contour of the image, which is superior to wavelet in the aspect of edge representation.

Step 2: Build a stable  $m \times n$  observation and measurement matrix  $\Phi$  not related to transformation  $\Psi$ , and observe S to yield the observation set  $Y = \Phi S = \Phi \Psi^T X$ . This process can be also expressed as that signal x undergoes non-self-adaptive observation through matrix  $A^{cs}$ .  $Y = A^{cs}X$  (where  $A^{cs} = \Phi \Psi^T$ , known as the CS information operator) What needs to be paid attention to is the selection of observation matrix  $\Phi$ , and important information should not be destroyed when sparse vector S is reduced from n dimension to m dimension. Restricted Isometry Property (RIP) [Guo, Zhang, Klein et al. (2010b); Kingham, D'Angelica and Jarnagin (2012b)] in compressed sensing theory is an important standard to determine whether the matrix can become a measurement matrix. For k spare vector  $S \in \mathbb{R}^N$ , if Eq. (3) is satisfied, the measurement matrix  $\Phi$  meets RIP.

$$(1-\varepsilon)\|\mathbf{s}\|_{2} \le \|\Phi\mathbf{s}\|_{2} \le (1+\varepsilon)\|\mathbf{s}\|_{2} \tag{4}$$

Compressed sensing theory has two main categories for measurement systems: the first category is random measurement system. Guo et al. [Guo, Zhang, Klein et al. (2010c); Smiley, Whittaker, Gourlie et al. (2005b)] has proved that most random matrices satisfy RIP, such as gaussian random measurement system and Bernoulli random measurement system. The work of literature shows that, in a sense, selecting random measurement is an optimal strategy for sparse matrix. Only the minimum m measurements are needed to restore the signal with sparse degree  $S \le m/\log(n/m)$ , and the constants required for analysis are all very small. The second type is the non-correlated measurement system, that is, the measurement matrix and the transform basis  $\Psi$  are unrelated, and their non-correlation can be measured by the correlation coefficient between them. The correlation coefficient  $\mu = N^{1/2} \max_{i,k} |<\Psi_i, \Phi_i>|$  between the measurement matrix  $\Phi$  and the transform basis  $\Psi$  is given in literature. The smaller the  $\mu$ , the greater the irrelevance between the measurement matrix  $\Phi$  and the transform basis  $\Psi$ , the less the number of microns is required to be measured.

Step 3: Reconstruct the signal x, which is different from the linear perception problem of Nyquist theory. Since the number of observations m is far less than the signal length n, reconstruction is faced with the problem of solving an under-determined system of equations. When signal x is sparse or compressible, the problem of solving under-determined equations can be converted into the minimum 0 norm problem, as shown in Eq. (4):

$$\min \left\| \Psi^T X \right\|_0 \quad s.t. \quad A^{cs} X = \Phi \Psi^T X = Y \tag{5}$$

However, according to statistical theory and combinatorial optimization theory, combinatorial optimization is a NP problem. When N is very large, it cannot be effectively implemented numerically, and its anti-noise ability is very poor. Candes et al. have demonstrated that when the Restricted Isometry Property (RIP) is satisfied by the measurement matrix  $\Phi$ , the combination optimization problem (also known as  $l_0$  constraint optimization problem) can be converted to convex optimization problem that is numerically easy to deal with  $l_1$  constraint:

$$\min \left\| \Psi^T X \right\|_{1} \quad s.t. \quad A^{cs} X = \Phi \Psi^T X = Y \tag{6}$$

In addition, there are some other methods to reconstruct the signal, such as: 1) reduce the  $l_0$  norm to  $l_p$  norm; 2) after introducing the sparse property through prior distribution, the Bayesian method is used to achieve signal sparse reconstruction; 3) use heuristic algorithms, e.g., relying on the belief-propagation and message propagation techniques of graph model and coding theory to transfer technology.

# 3.3 $l_1$ normal convex optimization algorithm

There are primarily two types of sparse reconstruction models based on  $l_1$  normal convex optimization algorithm:

$$(LS_{\tau}) \quad \min_{x} \|y - \Phi x\|_{2} \quad s.t. \quad \|x\|_{1} \le \tau$$

$$(BP_{\sigma}) \quad \min_{x} \|x\|_{1} \quad s.t. \quad \|y - \Phi x\|_{2} \le \sigma$$

$$(7)$$

where (LS $^{\tau}$ ) denotes LASSO (Least Absolute Shrinkage and Selection Operator) and (BP $^{\sigma}$ ) (Basis Pursuit Denoise, BPDN). When there is no noise, the model degenerates into a single basis tracking (BP) problem. When dealing with practical problems, the approximate estimation of noise level  $\sigma$  can be obtained by analyzing the measurement conditions of the system. In contrast, it is very difficult to estimate the  $l_1$ 's value  $\tau$  of the original signal priori, so the study of solving (BP) problem is of more practical significance. However, the (LS) problem can be used as an intermediate means to solve the (BP) problem.

When solving constraint optimization problems, e.g., (LS  $^{\tau}$ ) and (BP  $^{\sigma}$ ), constraint conditions can be converted into penalty terms to construct non-constraint optimization problems. That is:

$$(QP_{\lambda}) \quad \min_{x} \|y - \Phi x\|_{2}^{2} + \lambda \|x\|_{1}$$
 (8)

Actually, the control parameter  $\lambda$  in (QP $^{\lambda}$ ) problem can be considered as the Lagrange multiplier in solving constrained optimization problem (LS $^{\tau}$ ) and (BP $^{\sigma}$ ).

Therefore, if the parameters  $\sigma$ ,  $\lambda$  and  $\tau$  are selected properly, the solutions of the above three problems are consistent. The (QP) problem is a second-order cone program, which can be solved using interior point method. Kingham et al. [Kingham, D'Angelica and Jarnagin (2012)] compared various recovery algorithms and summarizes  $l_1$  convex optimization algorithms, e.g., LASSO or BPDN. By solving Eq. (8), the best balance between the complexity and precision of sparse computation can be provided.

$$\min_{x} \frac{1}{2} \|y - \Phi x\|_{2}^{2} + \lambda \|x\|_{1} \tag{9}$$

Iterative thresholding algorithm is often employed in sparse optimization algorithms. Some iterative threshold algorithms are used to solve LASSO problems, which can reduce the computation amount of each iteration during the iterative process. In such a way, LASSO can be used to solve high-dimensional problems [Giorgio, Ferraioli, Tarantino et al. (2004c)]. Among iterative threshold techniques, iterative shrinkage algorithm can effectively solve convex optimization problem, including IHT, GraDeS, PCD (parallel coordinate descent) as well as FISTA (fast-iterative-shrinkage thresholding algorithm). Since IHT and GraDes algorithms use negative gradient as the search direction, i.e., Landweber iteration, these algorithms have low efficiency in execution.

### 3.4 Compressed sensing model and Xampling

Compressed sensing originates from the work of Professor E. J. Cands, Chinese mathematician Tao Zhexuan and Professor Donoho. The theory can sample sparse signals at a low rate far below the Nyquist sampling frequency, and effectively recover the original signals using the low-speed sampling results.

A N-dimensional vector  $\mathbf{x}$ , if it has a sparse representation on a set of bases  $\mathbf{\Psi}$ , that is:

$$x = \Psi_S$$
 (10)

where there are only K non-zero values ins, it is said that x is K sparse on  $\Psi$ .

Compressive sensing theory points out that the weights of K $\leq$ M $\ll$ N measurements of X can be weighted heavily.

Let X be represented by a set of measurements as follows:

$$y = \Phi x \tag{11}$$

Among them  $\Phi$ ,  $R_{MN}$  is a measurement matrix and Y is a set of linear measurement values of X.

According to the condition of  $K \le M \le N$ , compression sensing theory mainly solves the problem of under-sampling strip.

The reconstruction of signal x is essentially a morbid problem. Inverse problem, but a priori condition for signal sparsity provides M sampling values Y restores the possibility of X. In fact, signal x is the solution following  $l_0$  minimization problem.

$$\hat{\mathbf{s}} = \operatorname{argmin} \|\mathbf{s}\|_{0} \quad \text{st. } \mathbf{y} = \mathbf{\Phi} \mathbf{x} = \mathbf{\Phi} \mathbf{\Psi} \mathbf{s} \tag{12}$$

Because  $l_0$  norm minimization is a NP-hard problem, it is difficult to solve in practice. Donoho, Cands and others have proved that when sampling matrix satisfies the constrained isometric condition, x can be obtained by solving the following  $l_1$  norm minimization problem.

$$\hat{\mathbf{s}} = \operatorname{argmin} \|\mathbf{s}\|_{I_1} \quad \text{st. } \mathbf{y} = \mathbf{\Phi} \mathbf{x} = \mathbf{\Phi} \mathbf{\Psi} \mathbf{s} \tag{13}$$

It is also difficult to construct a fixed sampling matrix directly to satisfy the constrained equidistant condition. At present, the main methods are to use uniform spherical matrix, Gauss random matrix and Bernoulli random matrix. To solve the optimization problem shown in formula (13), optimization algorithms and various greedy tracking algorithms can be used.

The above theory is mainly aimed at discrete signals to realize the compression of analog signals.

Including Random Filtering (RD), Analog Information Conversion (AIC), Modulated Broadband Conversion (MWC) and Xampling are proposed. There are many schemes, among which the Xampling structure proposed by Dr. Mishali is applicable. Because of its multi-band sparse signal and simpler hardware implementation scheme, it has been accepted and widely attracted attention. In cognitive radio spectrum sensing, due to authorization the household spectrum utilization is low and the signal model facing the system can be modeled as multi-band. Because of the sparse signal, the Xampling structure is chosen as the sampling front-end of this algorithm.

Fig. 2 is the basic structure of Xampling. There are m paths in it. The input signal x(t) multiplied by the periodic waveform PI(t) in the first path passes through The low-pass filter h(t) is used to obtain the sampling value Yi[n] by using low-speed A/D. Mining Sample Period T is the same as pi(t). For convenience, 1/T≥B, B is chosen. For the upper bandwidth limit of signal in multi-band frequency-domain sparse model, multi-band frequency-domain sparse signal it is shown in Fig. 3.

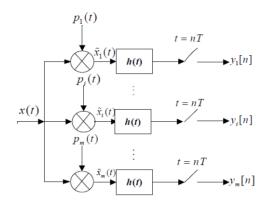


Figure 2: Xampling sampling structure

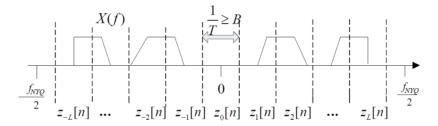


Figure 3: Multi-band sparse signal model in frequency domain

# 4 Simulation and performance analysis

Given that the total number of idle cognitive users in the system is 40, the transmission rate of each cognitive user is 1 Mbps. Parameters. Lien et al. [Lien, Lee and Hsieh (2012b)] are applied in the simulation scene, and each time slot of cognitive user is 100 ms, t=10 ms, t=10 ms.

Fig. 4 compares the throughput of the cognitive system under different numbers of primary user channels and different channel occupancy rates. It is observed that the throughput of cognitive network can be improved by adding a small amount of idle cognitive users for detection. This is because the addition of idle cognitive users enables cognitive base stations to obtain more available channels, thereby allowing active cognitive users to have more opportunities to transmit.

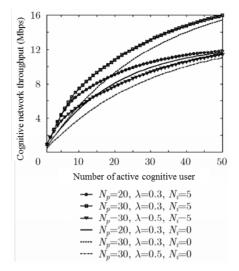


Figure 4: Cognitive network throughput

Fig. 5 compares the service delay caused by the addition of idle cognitive users when the channel status of primary users and the number of active cognitive users are different. As shown in the Figure, when the occupancy rate of primary users is high, the number of available channels in the cognitive network is small, and the system has a high service delay. As the number of idle cognitive users added to the network for detection increases, the service delay of the system is narrowed.

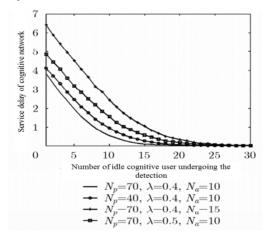
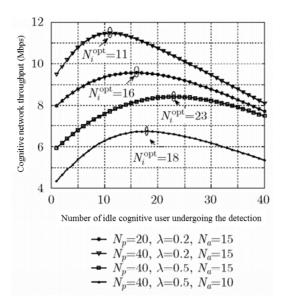


Figure 5: Service delay of cognitive network

Fig. 6 compares throughput with the number of idle cognitive users undergoing the detection under different conditions. As can be seen from the Figure, throughput is not increased when more cognitive users undergo the detection.

With the increase in active cognitive users, the number of idle cognitive users required also increases.

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**Figure 6:** Throughput changes with the number of idle cognitive users undergoing the detection

When primary users take up lower position, cognitive networks with more channels of primary users require fewer idle cognitive users. This is because most of the primary users in the network are idle, and it is easier for cognitive users to detect idle primary user channels.

When primary users take up lower position, cognitive networks containing more active cognitive users require idler cognitive users.

### **5** Conclusion

In cognitive radio networks, a method of joint detection using idle cognitive users is proposed in this paper. After using idle cognitive users to participate in joint detection, the detection ability of cognitive system is improved, thereby obtaining more available channel information.

Theoretical analysis and simulation experiments suggest that the participation of idle cognitive user in the detection can improve the throughput of cognitive network and reduce service delay. The joint detection algorithm proposed here considers the extra time brought by cognitive user reporting, so the optimal number of idle cognitive users undergoing the detection is obtained.

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**Conflicts of Interest:** The authors declare that they have no conflicts of interest to report regarding the present study.

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