On the Application of the Adomian's Decomposition Method to a Generalized Thermoelastic Infinite Medium with a Spherical Cavity in the Framework Three Different Models

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Abstract: A mathematical model is elaborated for a thermoelastic infinite body with a spherical cavity. A generalized set of governing equations is formulated in the context of three different models of thermoelasticity: the Biot model, also known as "coupled thermoelasticity" model; the Lord-Shulman model, also referred to as "generalized thermoelasticity with one-relaxation time" approach; and the Green-Lindsay model, also called "generalized thermoelasticity with two-relaxation times" approach. The Adomian's decomposition method is used to solve the related mathematical problem. The bounding plane of the cavity is subjected to harmonic thermal loading with zero heat flux and strain. Numerical results for the temperature, radial stress, strain, and displacement are represented graphically. It is shown that the angular thermal load and the relaxation times have significant effects on all the studied fields.

Keywords: Adomian's decomposition method, generalized thermoelasticity, relaxation time, iteration method.

Nomenclature

λ , μ	Lame's constants
ρ	Density
C_E	Specific heat at constant strain
α_T	Coefficient of linear thermal expansion
γ	$= (3\lambda+2\mu)\,\alpha_T$
t	Time
T	Temperature
T_o	Reference temperature

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θ $= (T - T_o)$; Temperature increment such that $|\theta| / T_0 << 1$

σ_{ij} Components of the stress tensor

e_{ij} Components of strain tensor

u_i Components of the displacement vector

F_i Body force vector

K Thermal conductivity

τ_0, υ Relaxation times

1 Introduction

Biot constructed the coupled thermoelasticity model (CTE), in which the heat conduction is a parabolic type partial differential equation, which leads to infinite velocity of the thermal wave [Biot (1956)]. To fix this paradox, generalized thermoelasticity theory has been introduced by Lord and Shulman (L-S) by using the definition of the second sound phenomena [Lord and Shulman (1967)]. This definition leads to heat conduction of parabolic type partial differential equation, which generates the finite velocity of the thermal wave. The Green and Lindsay (G-L) theory suggests two relaxation times, and both the energy equation and the equation of motion have been modified [Green and Lindsay (1972)]. Many mathematical models of the infinite body with a spherical cavity in the context of different types of thermoelasticity models have been solved and published [Youssef (2005a, 2006, 2009); Youssef and El-Bary (2014)]. The mixed initial boundary value problem for a dipolar body in the context of the thermoelastic theory was proposed by Matin et al. [Matin and Öchsner (2017)]. Marin studied the asymptotic of total energy for the solutions of the mixed initial boundary value problem within the context of the thermoelasticity of dipolar bodies [Marin (1997)].

Recently, much attention has been devoted to numerical methods that do not require the discretization of time-space variables, and to the linearization of the nonlinear equations [Sweilam (2007)].

The Adomian method is a decomposition method that solves linear and nonlinear partial and ordinary differential equations [Admoian, Cherruault and Abbaouui (1996); Adomian (1988)]. This method offers computable, accurate, convergent solutions to linear and nonlinear partial and ordinary differential equations. The solution can be verified to any degree of approximation. Recently, the Adomian decomposition approach has been applied to obtain formal solutions for a wide class of partial and ordinary differential equations [Ciarlet, Erell and Felix (2016); Duz (2017); El-Sayed and Kaya (2004); Górecki and Zaczyk (2016); Kaya and El-Sayed (2003); Kaya and Inan (2005); Kaya and Yokus (2005); Lesnic (2002, 2005); Li, Licheng, Rustam et al. (2017); Mustafa (2005); Vadasz and Olek (2000)]. The Adomian method has been used to solve different mathematical models of the mechanical interaction of the immune system with viruses, antigens, bacteria, or tumor cells, which had been modelled as systems of nonlinear partial differential equations by the ADM [Adomian, Cherruault and Abbaoui (1996)].

Adomian's decomposition method (ADM) separates the differential equations into linear and nonlinear parts, inverts the highest-order derivative in both sides, and obtains the successive terms of the solution using recurrent relation [Lesnic (2005) and Sweilam (2007)]. Many modifications have been made to the method to enhance the accuracy or to expand the applications of the original method by many authors [Kaya and E Inan (2005), Lesnic (2002); Vadasz and Olek (2000)]. Recently, the decomposition method has been used in fractional partial differential equations [Gejji and Jafari (2005); Ray and Bera (2005); Shawagfeh (2002)].

This work introduces, for the first time, the use of an Adomian's method for solving the problem of thermoelasticity in the context of spherical co-ordinates under three different models of thermoelasticity. The solution will be based on Adomian's decomposition method. The numerical results will be calculated and represented in figures to stand on the influence of the functionally graded parameter on the temperature increment, the strain, the stress, and the displacement.

2 Basic equations

The unified system of governing equations in the context of CTE, L-S and (G-L) has been constructed for a linear and homogeneous isotropic medium without any external heat source to be in the following form [Youssef and El-Bary (2014)]:

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ji} + F_i - \gamma \left(1 + \upsilon \frac{\partial}{\partial t} \right) T_{,i} = \rho \ddot{u}_i , \tag{1}$$

$$K T_{,ii} = \rho C_E \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T + T_0 \gamma \left(\frac{\partial}{\partial t} + n \tau_0 \frac{\partial^2}{\partial t^2} \right) u_{j,j} , \tag{2}$$

$$\sigma_{ij} = \mu \left(u_{i,j} + u_{j,i} \right) + \lambda u_{i,i} \delta_{ij} - \gamma \left(1 + \upsilon \frac{\partial}{\partial t} \right) (T - T_0) \delta_{ij} . \tag{3}$$

1. Putting $\tau_0 = \upsilon = 0$ for coupled thermoelasticity (Biot model).
2. Putting $n = 1$, $\upsilon = 0$ and $\tau_0 \neq 0$, for generalized thermoelasticity with one relaxation time (Lord-Shulman, i.e., the L-S model).
3. Putting $\mathrm{n} = 0$, and $\tau_0 \neq 0,\ \upsilon \neq 0$ for generalized thermoelasticity with two relaxation times (Green-Lindsay, i.e., the G-L model).

   $i, j = 1,2,3$ are the indicators of the coordinates system.

3 Formulation of the problem

Consider a perfectly conducting thermoelastic infinite body with spherical cavity that occupies the region $R \le r < \infty$. The spherical system of coordinates $\left(r,\Theta,\phi\right)$ with the z-axis lying along the axis of the cylinder will be used.

Due to symmetry, the problem is one-dimensional with all the functions considered depending on the radial distance *r* and the time *t*. It is assumed that there are no body forces and no heat sources in the medium and the surface of the cavity.

Thus, the governing one-dimensional equations of (1)-(3) in spherical coordinates take the following forms [Youssef (2005b, 2010)]:

$$\left(\lambda+2\mu\right)\frac{\partial e}{\partial r}-\gamma\frac{\partial}{\partial r}\left(1+\upsilon\frac{\partial}{\partial t}\right)T=\rho\frac{\partial^2 u}{\partial t^2}, \tag{4}$$

$$\nabla^2 T=\frac{\rho C_E}{K}\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\frac{T_0\gamma}{K}\left(\frac{\partial}{\partial t}+n\tau_0\frac{\partial^2}{\partial t^2}\right)e, \tag{5}$$

$$\sigma_{rr}=2\mu\frac{\partial u}{\partial r}+\lambda e-\gamma\left(1+\upsilon\frac{\partial}{\partial t}\right)\left(T-T_0\right), \tag{6}$$

$$\sigma_{\psi\psi}=2\mu\frac{u}{r}+\lambda e-\gamma\left(1+\upsilon\frac{\partial}{\partial t}\right)\left(T-T_0\right), \tag{7}$$

$$\sigma_{zz}=\lambda e-\gamma\left(1+\upsilon\frac{\partial}{\partial t}\right)\left(T-T_0\right), \tag{8}$$

$$\sigma_{zr}=\sigma_{\psi r}=\sigma_{zz}=0, \tag{9}$$

$$e=\frac{1}{r}\frac{\partial\left(ru\right)}{\partial r}, \tag{10}$$

where $\nabla^2=\dfrac{\partial^2}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial}{\partial r}$.

For convenience, we will use the following non-dimensional [Youssef and El-Bary (2014)]:

$$\left(r',u'\right)=c_o\eta\left(r,u\right),\ \left(t',t_0',\tau_0',\upsilon'\right)=c_o^2\eta\left(t,t_0\tau_0,\upsilon\right),\ \theta=\frac{\left(T-T_0\right)}{T_0},\ \sigma'=\frac{\sigma}{\mu},\ \eta=\frac{\rho C_E}{K},$$

$$c_o^2=\frac{\lambda+2\mu}{\rho}.$$

Eqs. (4)-(8) take the forms (the primes are suppressed for simplicity)

$$\nabla^2 e-\alpha\left(1+\upsilon\frac{\partial}{\partial t}\right)\nabla^2\theta=\frac{\partial^2 e}{\partial t^2}, \tag{11}$$

$$\nabla^2\theta=\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)\theta+\varepsilon\left(\frac{\partial}{\partial t}+n\,\tau_0\frac{\partial^2}{\partial t^2}\right)e, \tag{12}$$

$$\sigma_{rr}=\beta^2 e-2\frac{u}{r}-\alpha\,\beta^2\left(1+\upsilon\frac{\partial}{\partial t}\right)\theta, \tag{13}$$

$$\sigma_{\psi\psi}=\beta^2 e-2\frac{\partial u}{\partial r}-\alpha\,\beta^2\left(1+\upsilon\frac{\partial}{\partial t}\right)\theta, \tag{14}$$

$$\sigma_{zz}=\left(\beta^2-2\right)e-\alpha\,\beta^2\left(1+\upsilon\frac{\partial}{\partial t}\right)\theta, \tag{15}$$

where $\alpha=\dfrac{\gamma T_o}{\lambda+2\mu}$, $\varepsilon=\dfrac{\gamma}{\rho C_E}$, $\beta^2=\dfrac{\lambda+2\mu}{\mu}$, and $\gamma=\left(3\lambda+2\mu\right)\alpha_T$.

4 Adomian's Decomposition Method (ADM)

To apply Adomian's method, we re-write Eqs. (11) and (12) to be in the forms:

$$\frac{\partial^2 e(r,t)}{\partial r^2}=\frac{\partial^2 e(r,t)}{\partial t^2}+\alpha\left(1+\upsilon\frac{\partial}{\partial t}\right)\frac{\partial^2\theta(r,t)}{\partial r^2}+\alpha\left(1+\upsilon\frac{\partial}{\partial t}\right)\frac{1}{r}\frac{\partial\theta(r,t)}{\partial r}-\frac{1}{r}\frac{\partial e(r,t)}{\partial r}, \tag{16}$$

and

$$\frac{\partial^2\theta(r,t)}{\partial r^2}=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\theta(r,t)+\varepsilon\left(\frac{\partial}{\partial t}+n\,\tau_0\frac{\partial^2}{\partial t^2}\right)e(r,t)-\frac{1}{r}\frac{\partial\theta(r,t)}{\partial r}. \tag{17}$$

The Adomian's decomposition method usually defines the equation in an operator form by considering the highest-ordered derivative in the problem. We define the differential operator L in terms of the two derivatives that are contained in the problem [Adomian, Cherruault and Abbaoui (1996)].

Consider Eqs. (16) and (17) in the operator form as follows:

$$L_{rr}e(r,t)=L_{tt}e(r,t)+\alpha\left(1+\upsilon L_t\right)L_{rr}\theta(r,t)+\alpha\left(1+\upsilon L_t\right)\frac{1}{r}L_r\theta(r,t)-\frac{1}{r}L_r e(r,t), \tag{18}$$

$$L_{rr}\theta(r,t)=\left(L_t+\tau_0 L_{tt}\right)\theta(r,t)+\varepsilon_1\left(L_t+n\,\tau_0 L_{tt}\right)e(r,t)-\frac{1}{r}L_r\theta(r,t), \tag{19}$$

where the operators which appeared in the above equations are defined as:

$$L_t = \frac{\partial}{\partial t},\ L_{tt} = \frac{\partial^2}{\partial t^2},\ L_r = \frac{\partial}{\partial r},\ L_{rr} = \frac{\partial^2}{\partial r^2}. \tag{20}$$

Assuming that the inverse of the operator L_{rr}^{-1} exists and is taken as a definite integral with respect to r from R to r as following [Adomian, Cherruault and Abbaoui (1996)]:

$$L_{rr}^{-1} f(r) = \int_R^r \int_R^{\xi_2} f(\xi_1)\, d\xi_1\, d\xi_2,\quad L_{rr}^{-1}\left[L_{rr} f(r)\right] = f(r) - \left[f(R) + (r-R)\left(\left.\frac{\partial f(r)}{\partial r}\right|_{r=R} \right)\right]. \tag{21}$$

Thus, applying the inverse operator on both the sides of (18)-(19), we obtain

$$e(r,t) = e(R,t) + (r-R)\left(\left.\frac{\partial e(r,t)}{\partial r}\right|_{r=R} \right) + L_{rr}^{-1}\begin{bmatrix} L_{tt} e(r,t) + \alpha(1+\upsilon L_t) L_{rr}\theta(r,t) + \\ \alpha(1+\upsilon L_t)\frac{1}{r} L_r \theta(r,t) - \frac{1}{r} L_r e(r,t) \end{bmatrix}, \tag{22}$$

$$\theta(r,t) = \theta(R,t) + (r-R)\left(\left.\frac{\partial \theta(r,t)}{\partial r}\right|_{r=R} \right) + L_{rr}^{-1}\begin{bmatrix} (L_t + \tau_0 L_{tt})\theta(r,t) + \varepsilon_1 (L_t + n\tau_0 L_{tt}) e(r,t) \\ -\frac{1}{r} L_r \theta(r,t) \end{bmatrix}. \tag{23}$$

Now, we will decompose the unknown functions $\theta(r,t)$ and $e(r,t)$ by a sum of components defined by the following series:

$$e(r,t) = \sum_{k=0}^{\infty} e_k(r,t) = e_0 + \sum_{k=1}^{\infty} e_k(r,t), \tag{24}$$

$$\theta(r,t) = \sum_{k=0}^{\infty} \theta_k(r,t) = \theta_0 + \sum_{k=1}^{\infty} \theta_k(r,t). \tag{25}$$

The zero-components are defined by the terms that arise from the boundary conditions on the surface of the cavity $r = R$, which give

$$e_0 = e(R,t) + (r-R)\left(\left.\frac{\partial e(r,t)}{\partial r}\right|_{r=R} \right), \tag{26}$$

$$\theta_0 = \theta(R,t) + (r-R)\left(\left.\frac{\partial \theta(r,t)}{\partial r}\right|_{r=R} \right), \tag{27}$$

Substituting from Eqs. (24)-(27) in Eqs. (22) and (23), we obtain

$$e(r,t) = e(R,t) + (r-R)\left(\left.\frac{\partial e(r,t)}{\partial r}\right|_{r=R}\right) + L_{rr}^{-1}\begin{bmatrix} L_{tt}\sum_{k=0}^{\infty} e_k(r,t) + \alpha(1+\upsilon L_t) L_{rr}\sum_{k=0}^{\infty}\theta_k(r,t) + \\ \alpha(1+\upsilon L_t)\frac{1}{r}L_r\sum_{k=0}^{\infty}\theta_k(r,t) - \frac{1}{r}L_r\sum_{k=0}^{\infty} e_k(r,t) \end{bmatrix}, \tag{28}$$

$$\theta(r,t) = \theta(R,t) + (r-R)\left(\left.\frac{\partial \theta(r,t)}{\partial r}\right|_{r=R}\right) + L_{rr}^{-1}\begin{bmatrix} (L_t + \tau_0 L_{tt})\sum_{k=0}^{\infty}\theta_k(r,t) + \\ \varepsilon_1(L_t + n\tau_0 L_{tt})\sum_{k=0}^{\infty} e_k(r,t) \\ -\frac{1}{r}L_r\sum_{k=0}^{\infty}\theta_k(r,t) \end{bmatrix}. \tag{29}$$

We obtain these components by $e_k(r,t)$ and $\theta_k(r,t)$, which are the recursive formulas [Adomian, Cherruault and Abbaoui (1996)]:

$$e_{k+1}(r,t) = L_{rr}^{-1}\begin{bmatrix} L_{tt}e_k(r,t) + \alpha(1+\upsilon L_t)L_{rr}\theta_k(r,t) + \\ \alpha(1+\upsilon L_t)\frac{1}{r}L_r\theta_k(r,t) - \frac{1}{r}L_r e_k(r,t) \end{bmatrix}, \quad k \ge 0, \tag{30}$$

$$\theta_{k+1}(r,t) = L_{rr}^{-1}\begin{bmatrix} (L_t + \tau_0 L_{tt})\theta_k(r,t) + \varepsilon_1(L_t + n\tau_0 L_{tt})e_k(r,t) \\ -\frac{1}{r}L_r\theta_k(r,t) \end{bmatrix}, \ k \ge 0. \tag{31}$$

We assume that the surface of the cavity $r = R$ is thermally loaded by harmonic heat with zero strain and heat flux.

Hence, we have:

$$\theta(0,t) = \theta^0 \sin(\omega t), \quad \left.\frac{\partial \theta(r,t)}{\partial r}\right|_{r=R} = 0, \tag{32}$$

$$e(0,t) = 0, \qquad \left.\frac{\partial e(r,t)}{\partial r}\right|_{r=R} = 0, \tag{33}$$

where θ^0 is constant and ω is the angular thermal load and assumed to be constant. Thus, we have

$$\theta_0 = \theta^0 \sin(\omega t), \quad e_0 = 0. \tag{34}$$

Substituting from Eq. (36) into Eqs. (30) and (31), we obtain the complete iteration formulas.

The first components of the iteration take the forms:

$$e_1(r,t)=0\ , \tag{35}$$

$$\theta_1(r,t)=\frac{\omega}{2}\left(cos(\omega t)-\omega\tau_0\,sin(\omega t)\right)(r-R)^2. \tag{36}$$

The remaining components of the iteration formulas (30) and (31) have been calculated by using MAPLE 17. Moreover, the decomposition series solutions (30) and (31) converge very rapidly in physical problems, and this convergence has been investigated by Kaya et al. [Kaya and Inan (2005); Kaya and Yokus (2005); Lesnic (2002, 2005)]. In an algorithmic form, the ADM can be expressed and implemented in linear generalized magneto-thermoelasticity models with the suitable value for the tolerance $Tol=10^{-6}$ and k is the iteration index, as follows:

Algorithm

1- Compute the initial approximations $\theta_0=\theta(0,t)$ and $e_0=e(0,t)$.

2- Use the calculated values of $\theta_k(r,t)$ and $e_k(r,t)$ to compute $\theta_{k+1}(r,t)$ and $e_{k+1}(r,t)$ from (30) and (31).

3- If $max\left|\theta_{k+1}(r,t)-\theta_k(r,t)\right|<Tol$ and $max\left|e_{k+1}(r,t)-e_k(r,t)\right|<Tol$, stop and set $k+1=m$, otherwise continue and go back to Step 2.

4- Calculating $e(r,t)=\sum_{k=0}^{m}e_k(r,t)$ and $\theta(r,t)=\sum_{k=0}^{m}\theta_k(r,t)$.

5- Calculating the displacement from Eqs. (10) and (28) as follows:

$$u(r,t)=\frac{1}{r}\int_R^r e(\xi,t)\,d\xi=\frac{1}{r}\int_R^r\sum_{k=0}^{m}e_k(\xi,t)\,d\xi \tag{37}$$

6- Calculating the stress from the Eqs. (13), (28), and (29), as follows:

$$\sigma(r,t)=\beta^2\sum_{k=0}^{m}e_k(r,t)-2\frac{\partial}{\partial r}\left(\frac{1}{r}\int_R^r\sum_{k=0}^{m}e_k(\xi,t)\,d\xi\right)-\alpha\beta^2\left(1+\upsilon\frac{\partial}{\partial t}\right)\sum_{k=0}^{m}\theta_k(r,t). \tag{38}$$

5 Numerical results and discussion

For the numerical evaluations, the copper material has been chosen and the constants of the problem were taken as follows [Youssef (2005); Youssef and El-Bary (2016)]:

$K = 386\ W/(mK)$, $\alpha_T = 1.78\times10^{-5}\,K^{-1}$, $C_E = 383.1\ J/(kg\,K)$, $\eta = 8886.73\ \ s/m^2$, $T_0 = 293\,K$, $\mu = 3.86\times10^{10}\ N/m^2$, $\lambda = 7.76\times10^{10}\ N/m^2$, $\rho = 8954\ kg/m^3$, $\tau_0 = 0.35\times10^{-14}$, $\upsilon = 0.33\times10^{-14}$.

Thus, the following non-dimensional parameters have been obtained;

$\varepsilon_1 = 1.618$, $\nu = 0.02$, $\tau_0 = 0.05$.

We calculate the numerical solutions when the non-dimensional value of the time is $t = 2.0$, the non-dimensional value of the distance is $1.0 \le R \le 2.0$, $\omega = \pi$, and $\theta^0 = 1.0$. According to the above algorithm, we stopped the calculation on the 5th component $\theta_5(r,t)$ and $e_5(r,t)$.

Figs. 1-4 show the temperature increment, the strain, the radial stress, and the displacement distribution, respectively, with different values of angular thermal loading parameter $\omega = (\pi,\ 1.1\pi)$ under the three models of thermoelasticity; Biot, L-S, and G-L.

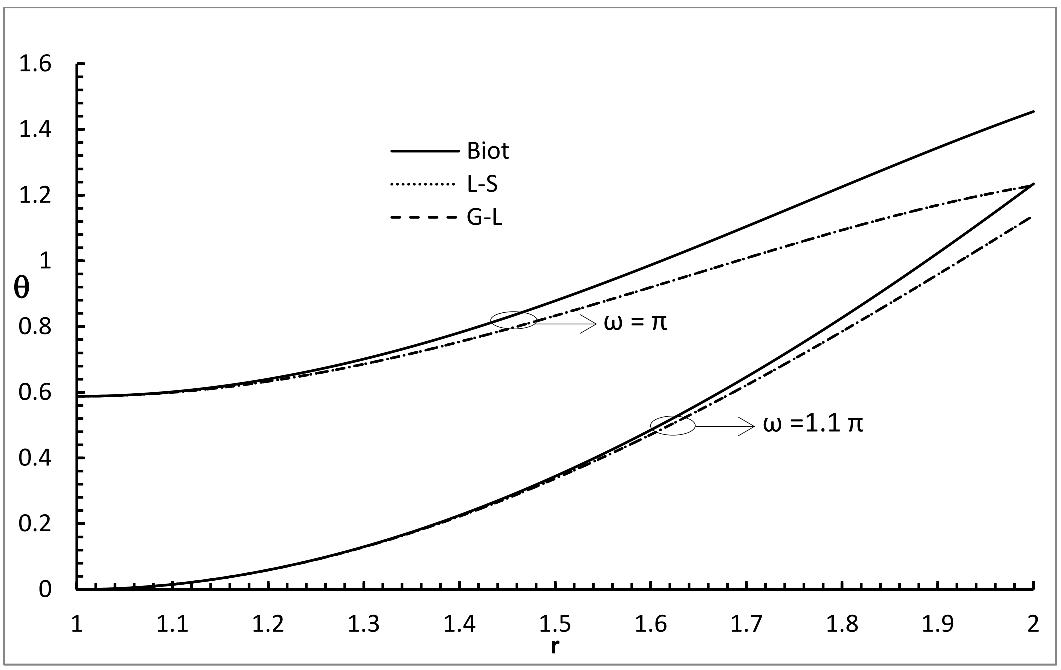

Figure 1: The temperature increment distribution with various values of angular thermal Load

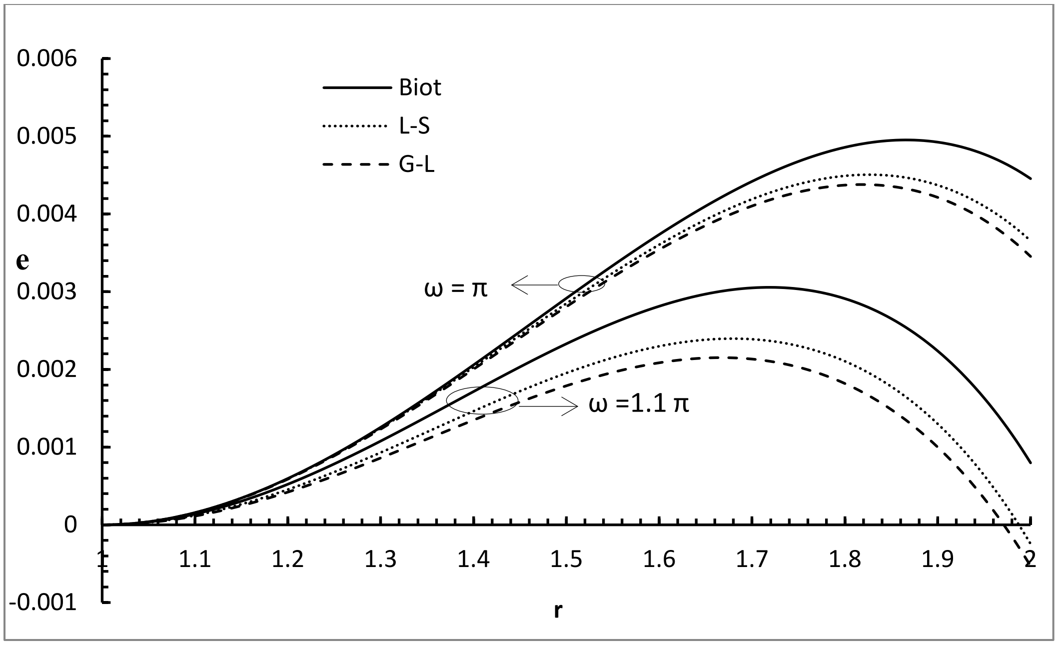

Figure 2: The strain distribution with various values of angular thermal load

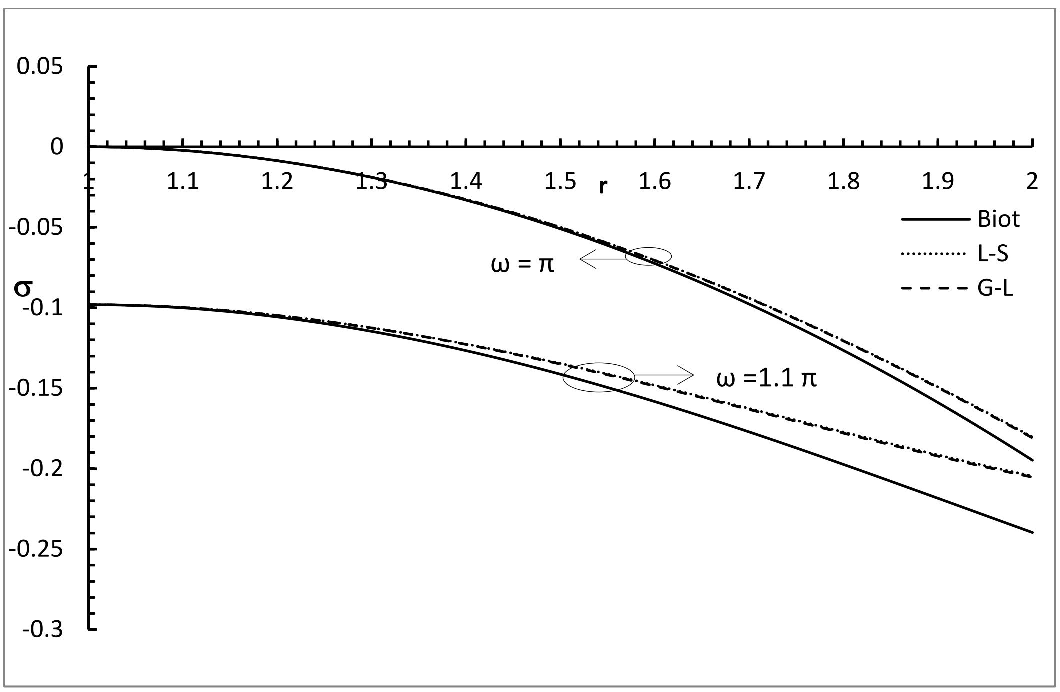

Figure 3: The stress distribution with various values of angular thermal load

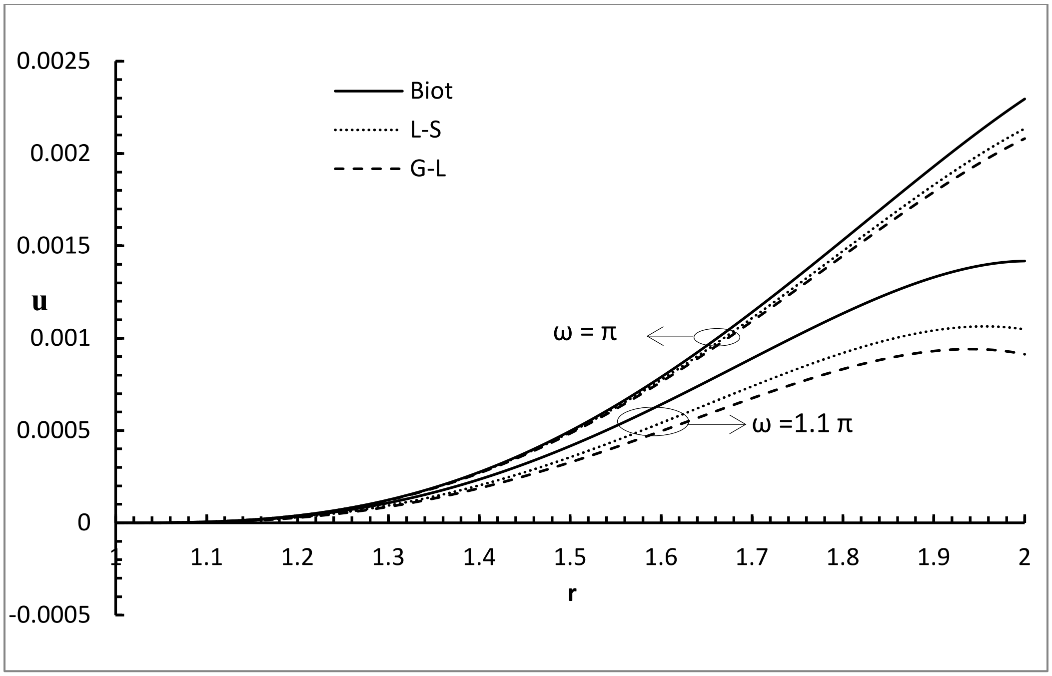

Figure 4: The displacement distribution with various values of angular thermal load

The numerical results of the L-S model and G-L model are identical almost particularly, the temperature increment distribution and the stress distribution for the different values of ω, while the strain and the displacement distributions are not.

According to the difference between the results of the Biot model and the results of the other models, the relaxation times have significant effects on all the studied functions and play a vital rule in the propagation of the thermal and mechanical waves through the thermoelastic materials.

Moreover, the angular thermal load parameter has significant impact on the temperature increment, strain, radial stress, and displacement distribution and in the propagation of the thermal and mechanical waves through the thermoelastic materials. The figures also show that, a small change in the value of the angular thermal loading parameter ω leads to significant changing in all the studied functions. When the value of the parameter ω increases, the values of the temperature increment, strain, radial stress, and displacement also increase.

Figs. 5 and 6 show the temperature increment, strain, radial stress, and displacement distribution for the L-S model with respect to the time *t* and the radial distance *r* when $\omega = \pi$ and $\omega = 2\pi$ in 3-D figures, respectively.

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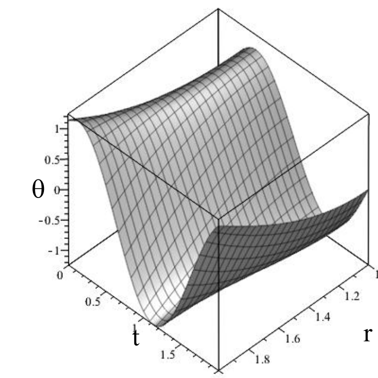
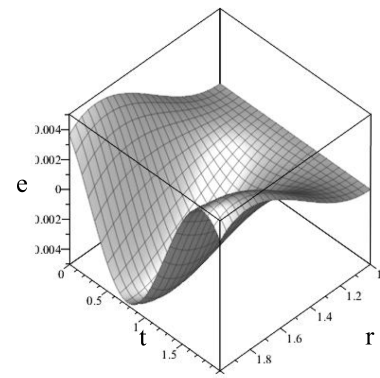
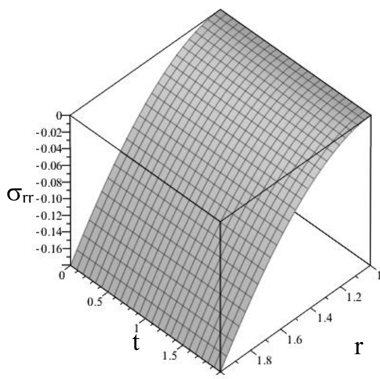


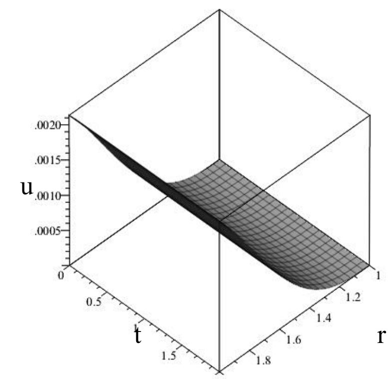
(a): Temperature increment



(b): Strain

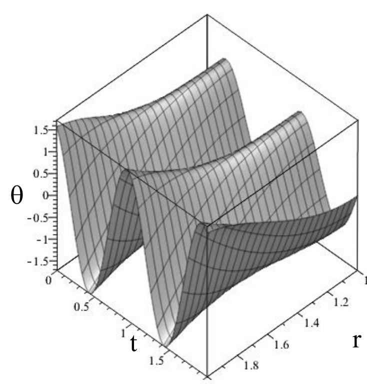


(c): Stress

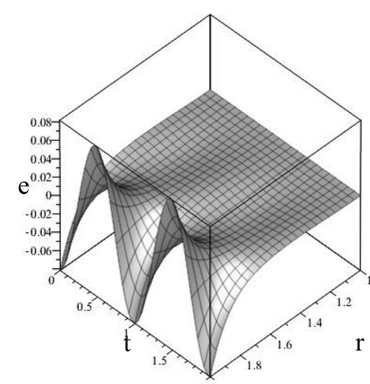


(d): Displacement

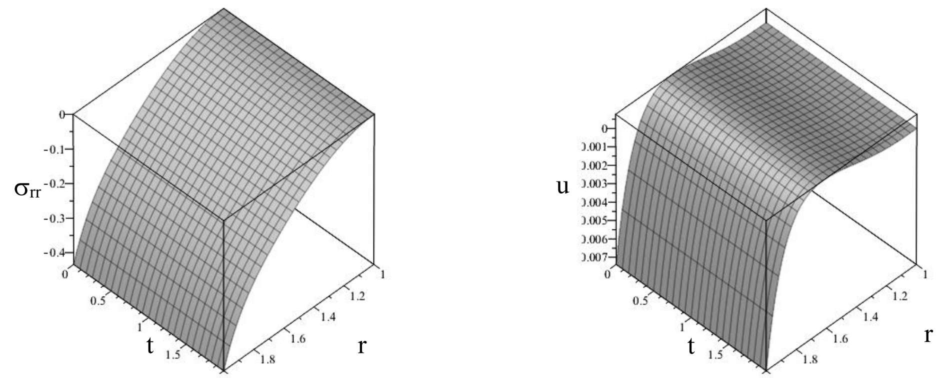
Figure 5: The studied functions for the L-S model when $\omega = \pi$



(a): Temperature increment



(b): Strain



(c): Radial stress **(d):** Displacement

Figure 6: The studied functions for the L-S model when $\omega = 2\pi$

Again, the angular thermal load parameter has significant effects on all the distributions. The number of the peak points of the increment temperature and the strain distribution increase when the value of the angular thermal load parameter increases. Finally, the temperature increment, strain, radial stress, and displacement have high values in the context of the Biot model compared to the L-S and G-L models due to the relaxation times.

6 Conclusions

A mathematical model of a thermoelastic infinite body with a spherical cavity has been constructed. A unified system of governing equations has been formulated in the context of three different models of thermoelasticity: the Biot model, the Lord-Shulman model, and the Green-Lindsay model. Adomian's decomposition method has been used when the surface of the cavity is subjected to harmonic thermal loading with zero heat flux and strain.

The numerical results show that:

- The relaxation times and the angular thermal load have significant effects on all the studied fields.
- The results from the Lord and Shulman model almost match the results obtained when applying the Green and Lindsay model.
- The temperature increment, strain, radial stress, and the displacement have higher values in the context of the Biot model compared to the L-S and G-L models.

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