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Statistical Analysis of Fatigue Life Data of A356.2-T6 Aluminum Alloy

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Abstract: This paper presents the details of method of sample size determination to estimate the characteristic fatigue life of aluminum alloy, A356.2-T6. The characteristic fatigue life of the alloy has been estimated by assuming log normal distribution model. A step wise procedure is outlined to determine the number of specimens required at predetermined stress amplitude to estimate the fatigue life with an acceptable error at 50% probability and various confidence levels, 90%, 95% and 99%. Maximum percentage of errors has also been calculated for the above probability and confidence levels. Details of generation of S-N curve for aluminum alloy A356.2-T6 using regression analysis have been explained. From this study, the adequate sample size required for evaluating the average fatigue life of A356.2-T6 with an acceptable error at 50% probability and 90% confidence level using log normal distribution is established. In order to check the significance of regression equation, analysis of variance (ANOVA) is performed.

Keywords: Fatigue life, Log-Normal distribution, S-N Curve, Aluminum alloy

1 Introduction

An efficient structural element must have three primary attributes; namely, the ability to perform its intended function, adequate service life, and the capability of being produced at reasonable cost. Most of the structures such as nuclear containments, reactor vessels, aerospace structures, ship hulls, automotive industrial components and offshore structures are required to operate under controllable operating conditions. The environment may also be variable, regardless of the operating regime. Most of the above structures are generally subjected to fatigue loading.

In recent years, there is rapid increase in use of aluminum alloy materials for structural applications. One of the main applications of aluminum alloys is in automo-

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bile industry, especially for the manufacturing of wheels. The wheels are one of the most critical components of automobiles, which must perform their intended function to human safety. Most of the structural components made up of these aluminum alloys are subjected to fatigue loading during their service conditions. The loading may be of either constant amplitude loading or variable amplitude loading. In the present scenario, the interest to researchers is to understand the fatigue fracture behavior of these alloys under fatigue loading. Statistical evaluations [Lipson and Sheth (1973)] are important because of different distributions of the test results in aluminum samples. For safe and reliable applications of the materials in industry, their fatigue data as statistically must be known well. The statistical properties used in general related to distribution in mean strength. Log normal distribution [Lipson and Sheth (1973)] is widely used statistical model than other distributions in fatigue data evaluations from the point of variables in endurance life and strength parameters. Applications of log normal distribution include aero-space, electronics, materials and automotive industries.

Recent advances in Weibull theory have also created numerous specialized Weibull applications. Modern computing technology has made many of these techniques accessible across the engineering spectrum. Luko (1999) reviewed generally used Weibull distribution models including discussion and illustrations. Monte Carlo method was also discussed in brief. Belmonte et al. (2008) proposed a Weibull based methodology for assessing the condition of pipes based on strength characteristics obtained from small samples. Khandaker et al. (2008) applied a modified Weibull failure theory to biomaterial specimen under thermal loading. Many researchers [Gope (1994); Parida et al. (1990); Nakazawa and Kodama (1987); Gao (1984); ASTM STP 9 (1963)] developed graphical and analytical methods to evaluate the fatigue life or strength and S-N curve from a limited amount of data. Most of the analytical methods are generally based on either normal or log normal distribution. As the fatigue testing is time consuming and costly, setting of minimum sample size required to extract the statistical information is of great importance [Gope (2002); Lawless (1973); Wilks (1942)]. Gope (2002, 1999) presented a method for determination of sample size to estimate the fatigue life, confidence level and maximum acceptable error. Ramamurtyraju et al. (2007, 2008) generated S-N curve for aluminum alloy A356.2-T6 and estimated fatigue life using two parameter weibull distribution under radial fatigue load. Safety factor was suggested for reliable fatigue life estimation by conducting a parametric finite element studies. Zhao et al. (1998) carried out a statistical investigation of 23 groups of fatigue life data on Q235 steel-welded joints in terms of linear regression analysis and observed that the three-parameter Weibull distribution may give misleading results in fatigue reliability analysis because the shape parameter is often less than 1. Schijve

(2005) made a comparison between three statistical distribution functions, namely, (i) the log (N)-normal distribution function, (ii) the 3-parameter Weibull distribution function and (iii) the log (N-N_0)-normal distribution function. It was observed that second and third functions gave a good data fit of the results of 30 similar tests with a skew distribution, but it still has to be recognized that the distribution function is actually unknown. Ravi et al. (2004) used analysis of variance (ANOVA) method to predict the fatigue life of HSLA steel welds using regression analysis. Balasubramanian et al. (2000) developed a mathematical model to predict the fatigue life of Shielded Metal Arc Welded (SMAW) cruciform joints failing from toe region. ANOVA technique was applied to find out the significant factors. Mahagaonkar et al. (2009) employed design of experiment (DOE) technique in carrying out test, using an air blast type shot peening machine. An ANOVA was carried out to identify significant peening parameters.

From the above discussion, it can be observed that the studies on determination of sample size to estimate the fatigue life of aluminum alloy A356.2-T6 using log normal distribution are limited [Ramamurtyraju et al. (2007, 2008)] . Since this alloy has been extensively used in automobile wheel applications and undergo fatigue loading, it is necessary to determine the sample size. The present work focuses on sample size determination to evaluate the characteristic fatigue life of Aluminum alloy A356.2-T6 at the desired probability and confidence level. Details of generation of S-N curve for aluminum alloy A356.2-T6 have been presented.

2 Experimental Set Up

To determine the fatigue properties of aluminum alloy A356.2-T6 at actual manufacturing conditions, a test is carried out on specimen taken from the wheels. A rotary bending fatigue test is conducted according to BIS (1985) for all identical specimens that are machined from the spokes of alloy wheels at various predetermined stress amplitudes. The schematic diagram of specimen geometry and test set up are shown in Fig. 1 and Fig. 2 respectively. The wheels, from which specimens machined, are manufactured at low pressure die casting followed by T6 heat treatment process. The chemical composition of A356.2-T6 is shown in Table 1.

Table 1: Chemical composition of A356.2-T6 aluminum alloy in wt. %

Si	Fe	Cu	Mn	Mg	Zn	Ti
6.5-7.5	0.15	0.03	0.10	0.3-0.45	0.07	0.1-0.18

The following are monotonic material data for the specimens taken from finished wheels.

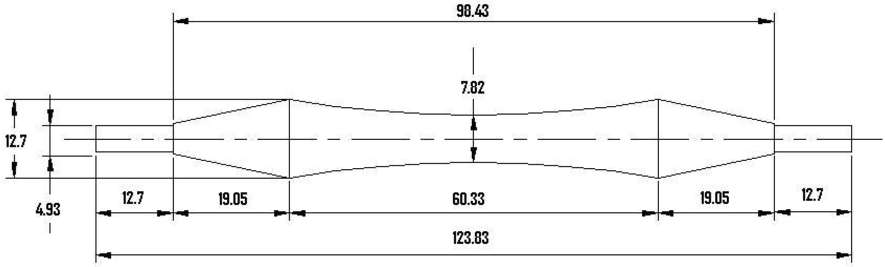

Figure 1: Rotating bending fatigue test specimen as per BIS 5075 (All dimensions are in mm)

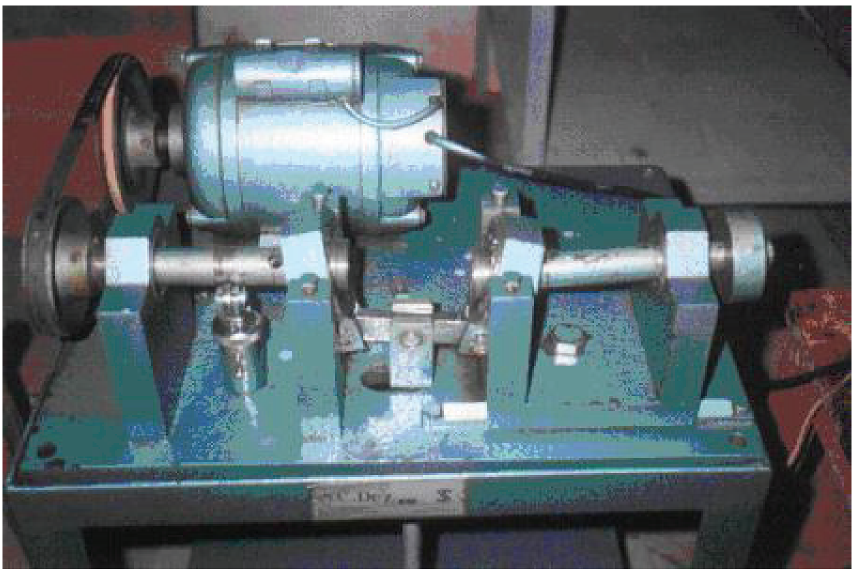

Figure 2: Typical test set-up for conducting rotating bending fatigue test

Ultimate Tensile Strength (Su): 250MPa.

Yield Strength (Sy): 230 MPa.

Elongation (e): 5%

Hardness (HB):90

Low pressure die casting process is used for manufacturing of aluminum car wheels for passenger vehicles. The molten aluminum kept in a gas tight heat insulated container flows under a mild pressure of approximately 70-100 kPa via a standpipe to escape through vent-holes and enters the die without turbulence. After solidification of the material in the die, the container is depressurized and the molten contents of the standpipe flow back into the container. Finally the wheel is then machined. The results obtained from the rotary bending fatigue test are plotted as shown in Fig. 3.

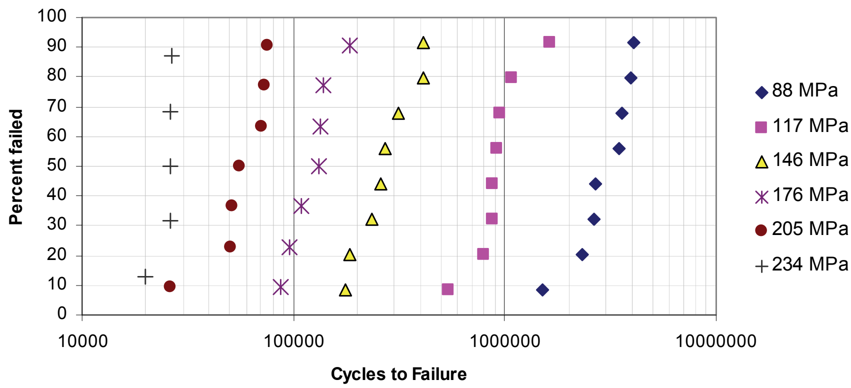

Figure 3: Scattered points of fatigue life at different stress levels

3 Method for Determination of Minimum Sample Size

From the literature [Gope (1994); Parida et al (1990); Nakazawa et al (1987); Gao (1984)], it was observed that graphical and analytical methods were used to evaluate the fatigue life with limited experimental observations. Further, it was noted that some of these methods can not be used for fatigue life prediction or strength at higher levels of probability. It is known that reliability and functionality are two of the most important requirements of engineered structures and components. It is an important requirement to find out the minimum sample size as the fatigue testing is time consuming and costly. In general, S-N plots are based on limited test data ranging from 6 to 10 specimens. It is mainly due to the availability of specimens and test time, and secondarily on the actual number needed to plot S-N curve. The aspects of testing are largely a matter of subjective choice and accumulated experience [Little et al (1975)]. A stepwise procedure is outlined to determine the number of specimens required at predetermined stress amplitude to estimate the average fatigue life within an acceptable error at 50% probability and various confidence levels. Fig. 4 shows the flow chart to be followed for determination of minimum sample size for A356.2-T6 aluminum alloy.

Initially tests were conducted on minimum of three samples [Gope (1994)]. At 50% probability, u = 0, hence the equation (1) which will be reduced to equation (2). Using equation (2), error was estimated using stepwise statistical procedure. Likewise tests were further conducted at each of the stress levels until the error estimation converges to an acceptable level.

If the fatigue life, N, follows normal distribution then x= Log (N) will follow log normal distribution. Then the percentage of error can be estimated from the following relation (Gope 2002),

$$R_N = \frac{t.s\sqrt{\left(\frac{1}{n}+u^2(\psi^2-1)\right)}}{\bar{x}+u\cdot\psi\cdot s} \tag{1}$$

where

s = standard deviation of log fatigue life

$\bar{x}$ = sample mean of log fatigue life

t = student 't' value

$\psi = \sqrt{\frac{n-1}{2}}\frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}$

= Correction factor for sample standard deviation; n = sample size;

u = normal deviate corresponding to 50 % probability of failure, at 50% probability u = 0, hence the above equation will reduce to

$$R_N = t\cdot\left(\frac{s}{\bar{x}}\right)\sqrt{\frac{1}{n}} \tag{2}$$

4 Results and Discussions

4.1 Determination of minimum sample size

The error is estimated using equation 2 at various confidence levels (90%, 95% and 99%) for various predetermined stress amplitudes 88, 117, 146, 176, 205 and 234 MPa. For the evaluation of fatigue life of A356.2-T6 aluminum alloy, an acceptable error of 5% and 50% probability are considered. The variation of error with sample size for 88 MPa stress level is plotted at various confidence levels and is shown in Fig. 5. The Fig. 5 illustrates that at 90% confidence level the error is with in an acceptable value. Similar procedure is adopted for the remaining stress levels and it is observed that 90% confidence level is most reliable in evaluating the fatigue

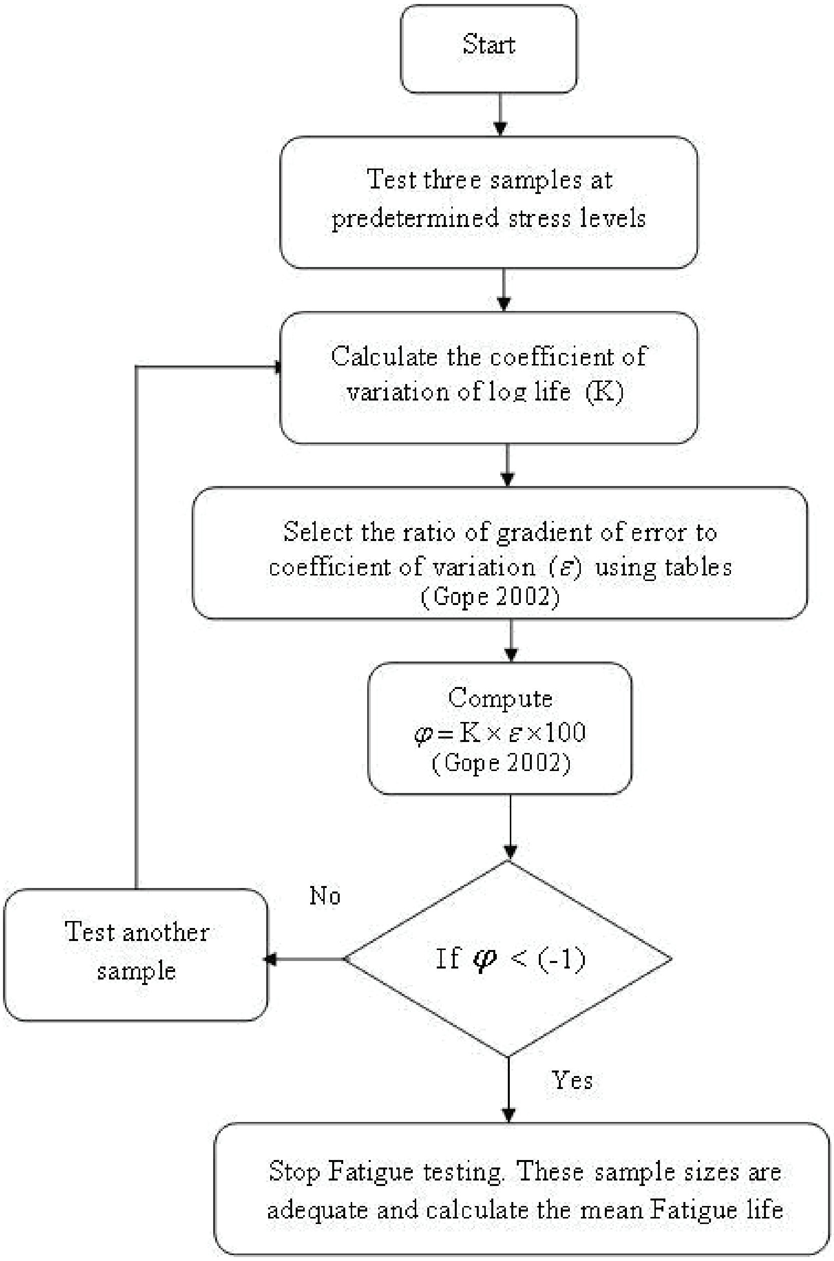

Figure 4: Flow chart for determination of minimum sample size

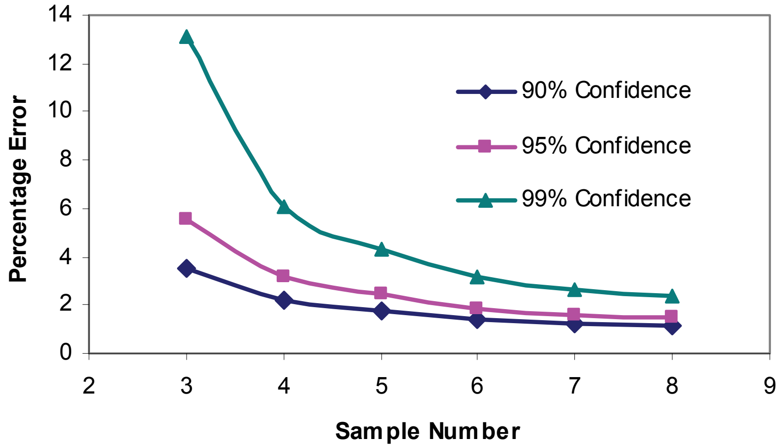

Figure 5: Variation of error with sample size at 50% probability for 88 MPa stress level

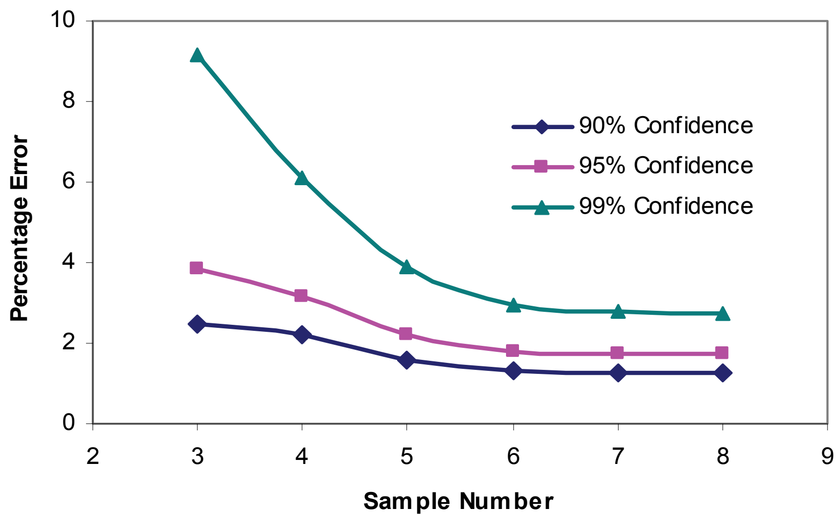

Figure 6: Variation of error with sample size at 50% probability for 146 MPa stress level

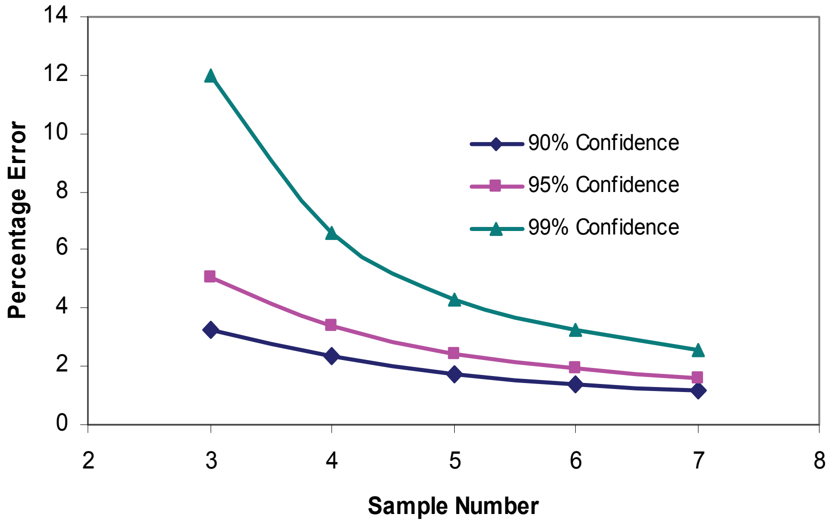

Figure 7: Variation of error with sample size at 50% probability for 176 MPa stress level

life. The variation of error with sample size for 146 MPa and 176 MPa are shown in Fig. 6 and Fig. 7 respectively.

The gradient of error (φ) at 50% probability and 90% confidence level with an acceptable error for the fatigue life data at various stress levels is calculated and is shown in Table 2. From the Table 2, it can be observed that the gradient of error (φ) for 88 MPa stress level is greater than –1.0 at n=5 which is the minimum required sample size to be tested in evaluating the fatigue life of aluminum alloy. Similarly for 117, 146, 176, 205 and 234 MPa stress levels the minimum sample size is obtained when n = 4, 5, 5, 4 and 4 respectively. The variation of gradient of error with sample size at 50% probability and 90% confidence level for different stress levels is plotted in Fig. 8 and it is observed that the sample size is independent of gradient of error when its value is greater than -1.0.

4.2 Generation of S-N curve

From the above study, it is observed that minimum sample size is obtained as n = 5, 4, 5, 5, 4 and 4 at 88, 117, 146, 175, 205 and 234 respectively which are used in evaluating the fatigue life of aluminum alloy. The fatigue life of aluminum alloy is determined by taking the average life values of minimum number of samples. The average fatigue life of aluminum alloy at different stress levels is shown in Table 3.

Table 2: Values of gradient of error at 50% probability and 90% confidence level

Sample Size (n)	Stress amplitude, MPa					
	88		117		146	
	Life	φ	Life	φ	Life	φ
1	3960264		537516		236452	
2	1518389		953019		409950	
3	2704850	-7.29968	797731	-4.83553	269709	-5.08552
4	2317776	-1.16372	878631	-0.80857	183567	-1.1623
5	3556811	-0.49775			259141	-0.45227

Sample Size (n)	Stress amplitude, MPa					
	176		205		234	
	Life	φ.	Life	φ	Life	φ
1	96453		50967		19932	
2	185900		70386		26212	
3	108392	-6.65209	74845	-4.15627	25974	-3.44178
4	87413	-1.24908	72999	-0.69824	26212	-0.57936
5	132868	-0.49745				

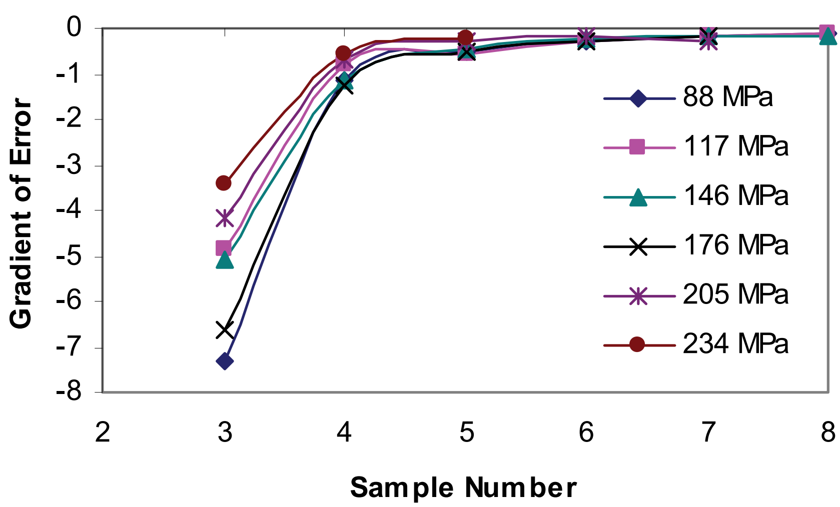

Figure 8: Variation of gradient of error with sample size at 90% confidence level

Table 3: Average Fatigue life of aluminum alloy at different stress levels

S. No.	No. of Samples required	Stress Level (MPa)	Average Fatigue Life (Cycles)	Log value of Average Fatigue Life
1	n = 5	88	2811618	6.448956
2	n = 4	117	791724	5.898574
3	n = 5	146	271763	5.434192
4	n = 5	176	122205	5.087090
5	n = 4	205	67299	4.828010
6	n = 4	234	24582	4.390626

S-N Curve is generated by taking the average values of fatigue log life on x-axis and stress levels on y-axis as shown in Fig. 9 and a curve fitting is performed by taking a second degree polynomial using regression analysis.

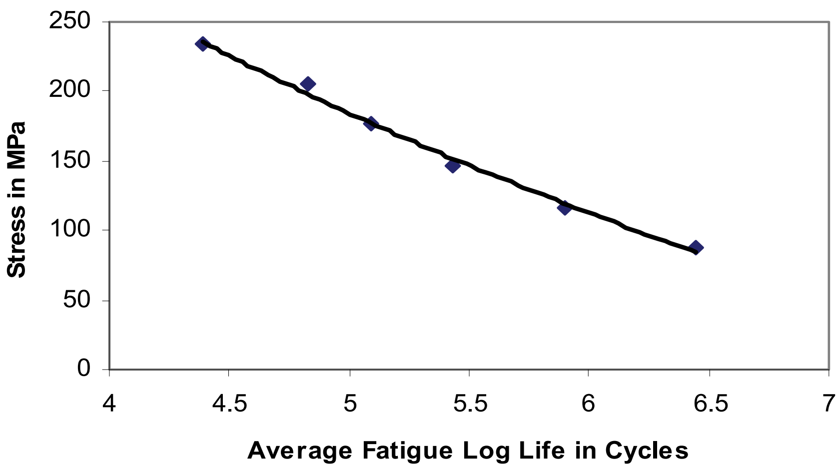

Figure 9: S-N Curve for Aluminum A356.2-T6 alloy

The mathematical relationship between stress levels and fatigue life is established and is given by equation 3. The significance of the quadratic polynomial is determined by carrying out an ANOVA [Little et al. (1975), Ravi et al. (2004), Balasubramanian et al. (2000), Mahagaonkar et al. (2009), Golinkin (1997)] for the equation 3. The R Square is 0.991, which shows that 99.1 % of the observed variability, which meant that the correlation coefficient between the stress level and average fatigue value based on the regression model is high. The analysis indicated that the regression model is highly significant because the P value is zero (P<0.05). The results of ANOVA are tabulated in Table 4.

$$Stress = 10.68N^2 - 189.0N + 861.2 \tag{3}$$

Where, N = Average Fatigue Log life in Cycles

Table 4: Results of ANOVA for regression model

Standard Deviation = 5.16261 R-Sq = 99.5% R-Sq (adj) = 99.1%					
Source	Degree of freedom	Sum of Squares	Mean Square	F ratio	P value
Regression	2	14900.0	7450.02	279.52	0.000
Error	3	80.0	26.65		
Total	5	14980.0			

5 Conclusions

The characteristic fatigue life of aluminum alloy A356.2-T6 is estimated by a log normal distribution model. Fatigue life for all the samples is obtained by conducting rotary bending fatigue test. Initially a minimum of three tests are conducted and calculated the error in estimation using a stepwise statistical procedure. Likewise tests are further conducted at each of stress levels until the error of estimation converges to an acceptable level and it is observed that percentage of error is increasing with increase of percent of confidence levels for a particular stress level. A quantitative method is presented for determination of sample size for evaluating the fatigue life of A356.2-T6 aluminum alloy with 50% probability, at various confidence levels and with a maximum acceptable error. Details of generation of S-N curve for aluminum alloy A356.2-T6 is presented. From the study, it is concluded that the estimated fatigue life is reliable and the sample size determined is adequate.

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