

An Integrated Model for Tension Stiffening and Reinforcement Corrosion of RC Flexural Members

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Abstract: An integrated model is proposed to describe tension stiffening in reinforced concrete (RC) flexural members that are undergoing uniform corrosion of reinforcement. The tension stiffening model is taken as base to incorporate the effects of reinforcement corrosion. The model is developed in two steps. In the first step, tension stiffening of concrete is modelled using an exponential stress-strain curve defined as function of a decay parameter. Modular ratio and reinforcement ratio are considered in the definition of the decay parameter. In the second step, the effects of uniform corrosion of reinforcement are integrated with the tension stiffening model. For this, fractional reduction in bar diameter is included as an additional parameter in the model. Later, global damage indicator of a structure is defined based on the secant stiffness calculated from its nonlinear load-displacement response. Performance of the proposed model is demonstrated through numerical studies using two RC beams whose details as well as responses are available in literature. Subsequently, the calculated values of damage indicator are shown useful to quantify reduction in load-carrying capacity of the beams. Based on such quantification, effect of assumed corrosion rates of reinforcement bar on capacity loss of the beam is studied in detail.

Keywords: Reinforced concrete structures; Cracking; Tension stiffening; Reinforcement corrosion; Decay parameter; Global damage indicator

List of Symbols

ϵ_{sm}	average strain of embedded reinforcement
n_b	no. of bars
A_s	area of steel

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ϵ_{cm}	average strain in concrete
A_c	concrete area
N_{cr}	cracking load
ϵ_{icr}	cracking strain
ϵ_y	strain at which reinforcement yields
ϵ_{bc}	strain in concrete before cracking
ϵ_{ac}	strain in concrete after cracking
E_c^{cr}	effective modulus of concrete after cracking
E_{sef}	effective modulus of elasticity of steel
E_c	elastic modulus of concrete
E_s	elastic modulus of steel
D_0	initial diameter of reinforcing bar
$K_{initial}$	initial elastic stiffness
N_y	tensile load at yield point
f_c'	maximum compressive strength of concrete
f_{ct}	maximum tensile strength of concrete
n	modular ratio (E_s/E_c)
β_1	parameter for considering bond characteristics of reinforcing bar
β_2	parameter for considering nature of load
K_c	secant stiffness
K_{ult}	secant stiffness at ultimate load
σ_{scr}	stress in steel at which concrete cracks
N	tensile force
ϵ_{cu}	ultimate strain in concrete
k_s	uniform rate of corrosion
ρ	A_s/A_c
σ_s	stress in steel
σ_{ct}	stress in concrete after cracking
D_n	diameter ratio
d	decay parameter
DI	Global damage indicator
t	age in years
D_1	diameter of rebar after corrosion

1 Introduction

Most of the tension stiffening models available in literature assume perfect bond of reinforcing steel with concrete and modify constitutive relation of either the steel or concrete to evaluate the nonlinear response of a cracked reinforced concrete

(RC) structure. For example, models proposed by Choi and Cheung (1994), C.E.B. (1990) modify constitutive relation of reinforcing steel whereas models proposed by Lin and Scordelis (1975), Collins and Vecchio (1986), Stevens et al. (1987), Massicote et al. (1990) redefine the constitutive equation of concrete. Some simple tension stiffening models make use of exponential decay curve to describe tensile stress-strain law of cracked concrete (Stramandinoli and Rovere, 2008; Gupta and Maestrini 1990).

Reinforcement corrosion is one of the major influencing factors for performance degradation of RC structural members (Smitha et al. 2011). Age related degradation of RC structural members due to corrosion causes reduction in tension stiffening and affect structural capacity and can also influence failure path of a structure. Several parameters such as gross dimension of member, reinforcement bar diameter, reinforcement ratio, concrete elastic modulus, etc. are known to influence tension stiffening in a member (Stramandinoli and Rovere 2008). Apart from this, corrosion of reinforcement also reduces the tension stiffening effect (Otieno et al. 2011). An effective method to numerically evaluate nonlinear response of RC structural members is by using a suitable tension stiffening model which can also incorporate the effects of uniform corrosion of reinforcement.

Corrosion is simply modelled as loss in cross-sectional area of reinforcing bars (Lee et al. 1998, Yoon et al. 2000, Coronelli and Gambarova (2004). To quantify degradation in structural stiffness due to corrosion, Kraetzig et al. (2000) have used a model proposed, Sarja and Vesikari (1996) to describe the reduction in cross-sectional area of reinforcing bar depending on the rate of corrosion. The model proposed, Kraetzig et al. (2000) has been found to perform well when local effect of reinforcement corrosion is mapped with global performance of the structure. Local damage and deterioration at material point are mapped onto the structural level by means of a damage indicator. Numerical value of damage indicator is used to identify the state of a structure with respect to its virgin condition or failure.

Investigations have been carried out in the past to show the significance of incorporating corrosion in a tension stiffening model (Shayanfar et al. 2007) Such models, generally, incorporate reinforcement corrosion through modifying bond strength based on experimental results (Cabrera 1996, Lee et al. 2002, Chung et al. 2004). Wang and Liu (2004a, 2004b) have proposed a model for bond strength of corroded bar in cracked concrete. Models meant for including the effect of corrosion need use of fictitious elements between concrete and reinforcement to represent bond-slip behaviour. Recently, Shayanfer and Safiey (2008) have reaffirmed that tension stiffening models can be a medium to incorporate the effects of corrosion on structural performance. They have further developed a semi-analytical model for tension stiffening by including bond-slip behaviour modified as a result of re-

inforcement corrosion. Main advantage of using this model is elimination of the fictitious element between reinforcing bar and neighbouring concrete. Even though the model (Shayanfer and Safiey 2008) shows superior performance compared to other available models, it includes a set of parameters, which are not commonly used in related literature, to define corrosion rate. Hence scope exists to develop a generalized model for nonlinear finite element analysis of RC structural members by incorporating the combined effects of tension stiffening and reinforcement corrosion.

In this paper, a combined model is proposed for tension stiffening in RC flexural members by integrating the effects of uniform corrosion in reinforcement. Degradation of bond between concrete and reinforcing steel is taken as basis to develop the integrated model. This is conceptually expressed in Figure 1. Development of the model is carried out in two steps. In the first step, post-peak segment of the stress-strain curve of concrete is proposed as exponentially decaying to account for tension stiffening effects. Perfect bond is assumed between concrete and steel while developing this model. A decay parameter (d) is defined as a function of reinforcement ratio (ρ) and modular ratio (n). The stiffening effect of concrete between the cracks is modelled by relating the tensile load and average strain in cracked concrete. Average strain in steel is calculated by making use of suitable expressions available in Shayanfer and Safiey (2008). As second step of the model development, effects of uniform corrosion of reinforcement are incorporated into the tension stiffening model. Diameter ratio (D_n), which denotes the degree of corrosion of the reinforcing bar, is introduced into the model for this purpose. A fifth order polynomial with coefficients defined using ρ , n and D_n is proposed for the decay parameter. The resultant stress-strain curve of concrete after incorporating the effects of tension stiffening and reinforcement corrosion is defined by using the decay parameter.

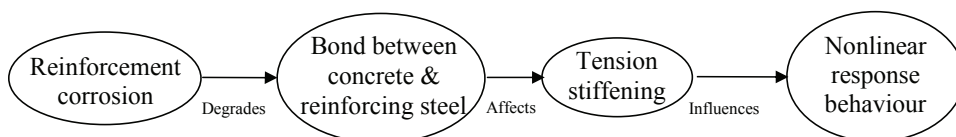


Figure 1: Interaction between reinforcement corrosion and tension stiffening

The proposed model is validated by analysing two simply supported RC beams for different corrosion rates. The computed nonlinear responses of the beams are compared in relative terms to quantify the effect of uniform corrosion on load carrying capacity of the structure. Besides this, a global damage indicator is evaluated based on secant stiffness obtained from the load-deformation response of the beams. To

the author's knowledge, the proposed integrated model is first of its kind to integrate the effects of tension stiffening with that of reinforcement corrosion to evolve a unified stress-strain curve for concrete.

2 Research Significance

Tension stiffening and reinforcement corrosion are two major phenomena that significantly influence the nonlinear behaviour of RC flexural members. An integrated model is proposed by combing the effects of reinforcement corrosion with that of the tension stiffening. The degradation of bond between concrete and reinforcing steel is taken as basis to formulate the model. A damage indicator, which is useful to quantify the reduction in load carrying capacity due to corrosion, is also proposed in the paper. Based on such quantification, effect of corrosion rates of reinforcement bar on capacity loss of the RC flexural members can be studied in detail.

3 Proposed Tension Stiffening Model

Tension stiffening in cracked concrete is modelled by considering a relationship between tensile stress and average strain of reinforcement bar. Tensile stress-strain curve of concrete similar to that reported in C.E.B (1990), (Figure 2) is taken as basis to develop the relation. In Fig. 2, σ_{scr} is stress at the instant of concrete cracking and ϵ_{sm} is the average tensile strain in the reinforcing bar. For the stress state in the initial linear segment up to formation of first crack, the strain in concrete is same as that in reinforcement. Due to this, tensile force carried by the RC member is shared by reinforcement and concrete depending on their relative stiffness. For sufficiently high tensile stress (for example, beyond the point A in Figure 2), the RC member crack intermittently thus inducing differential movement between concrete and reinforcement. A relationship is developed between the average tensile stress in steel (σ_s) and average strain in the reinforcement bar to denote the stress state beyond point A in Figure 2. The relation is given as

$$\sigma_s = E_{sef} \cdot \epsilon_{sm} \quad (1)$$

where E_{sef} is the effective modulus of elasticity of the steel bar as shown in Figure 2.

Average tensile strain ϵ_{sm} in reinforcing steel is calculated as (Eurocode 2. 1991).

$$\epsilon_{sm} = \frac{\sigma_s}{E_s} \left[1 - \beta_1 \beta_2 \left(\frac{\sigma_{scr}}{\sigma_s} \right)^2 \right] \quad (2)$$

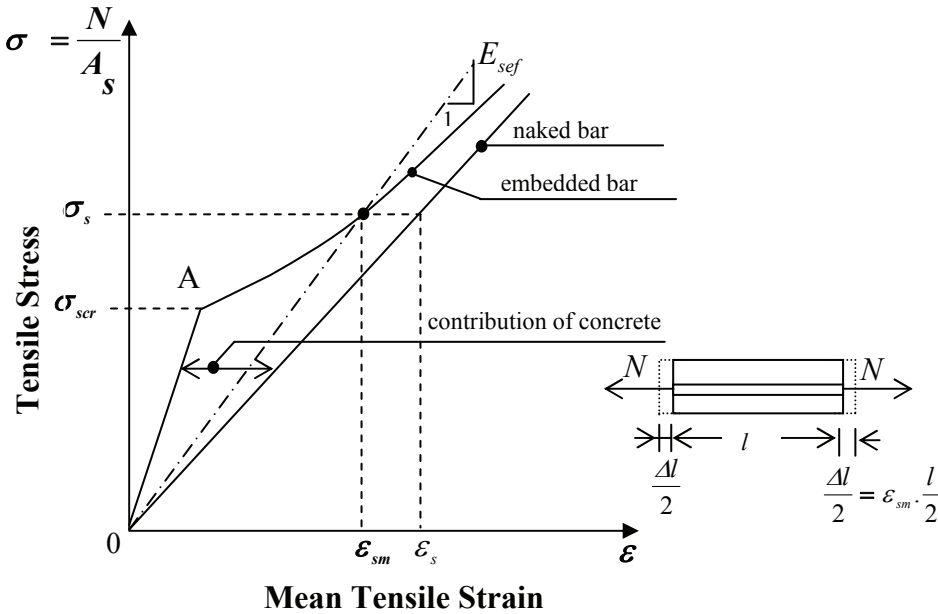


Figure 2: Representation of tension stiffening: Average tensile stress versus tensile strain

where E_s is elastic modulus of reinforcing steel, β_1 is bond coefficient that denotes the bond characteristics (= 1 for ribbed bars and 0.5 for smooth bars), and β_2 is a coefficient takes into account the nature of loads (= 1 for short term loads and 0.5 for long term or repeated loads). From Eqns. (1) and (2), effective modulus of elasticity of the steel bar is derived as

$$E_{sef} = \frac{E_s}{1 - \beta_1 \beta_2 \left(\frac{\sigma_{scr}}{\sigma_s} \right)^2} \tag{3}$$

Referring to Figure 2, the expression for E_{sef} is valid from point A to ultimate load. Towards the development of proposed tension stiffening model, concrete under tension is assumed to behave as a linear elastic material when the tensile stress is less than the tensile strength. In cracked condition, an exponential decay curve is used to describe the stress-strain relation to incorporate tension stiffening effect. Based on Stramandinoli and Rovere (2008), stress in cracked concrete is expressed as

$$\sigma_{ct} = f_{ct} e^{-d \left(\frac{\epsilon}{\epsilon_{icr}} \right)} \tag{4}$$

where f_{ct} is tensile strength of concrete, ε_{icr} is concrete strain corresponding to the tensile strength of concrete and the non-dimensional exponent d is decay parameter whose value varies from 0 to 1. In elastic range, i.e., in the range OA in Figure 2, equilibrium conditions are invoked to determine the average strain in concrete as

$$\varepsilon_{bc} = \frac{N}{E_s A_s + E_c A_c} \quad (5)$$

where ε_{bc} is the average strain in concrete before cracking, N is the tensile force in the reinforcing bar, E_c denotes elastic modulus of concrete before cracking, A_s is the area of reinforcement and A_c is the sectional area of concrete.

Average tensile strain in reinforcing bar ε_{sm} after point A in Figure 2 is expressed as

$$\varepsilon_{sm} = \frac{\sigma_s}{E_{sef}} \quad (6)$$

Using Eqn. (3),

$$\varepsilon_{sm} = \frac{\sigma_s}{E_s} \left[1 - \beta_1 \beta_2 \left(\frac{\sigma_{scr}}{\sigma_s} \right)^2 \right] \quad (7)$$

where

$$\sigma_s = \frac{N}{A_s} \quad (8)$$

$$\sigma_{scr} = \frac{(1+n\rho) f_{ct}}{\rho} \quad (9)$$

where n is modular ratio and $\rho = A_s/A_c$.

Alternatively, tensile strain in cracked concrete is represented as (Stramandinoli and Rovere (2008))

$$\varepsilon_{ac} = \frac{N}{E_s A_s + E_c^{cr} A_c} \quad (10)$$

where E_c^{cr} is equivalent elastic modulus of concrete in cracked condition, defined as (Stramandinoli and Rovere (2008))

$$E_c^{cr} = \frac{\sigma_{ct}}{\varepsilon_{ac}} \quad (11)$$

By making use of Eqns. (7) to (11), the expression for stress in cracked concrete is derived as

$$\sigma_{ct} = \frac{E_s \rho \varepsilon_{ac}}{\sigma_s^2 - \beta_1 \beta_2 \left[\left(\frac{(1+n\rho) f_{ct}}{\rho} \right)^2 \right]^2} \left[\beta_1 \beta_2 \left[\frac{(1+n\rho) f_{ct}}{\rho} \right]^2 \right] \quad (12)$$

Eqns (4) and (12) give two different expressions for stress in cracked concrete. The main difference is in respect of the decay parameter ‘ d ’. Therefore, the objective of the present study is to derive an expression for the decay parameter d in terms of other known parameters of the problem. Based on the composition of Eqns. (4) and (12), it is proposed to derive a polynomial for the decay parameter in terms of the product of modular ratio and reinforcement ratio. A sufficiently higher order polynomial with $n\rho$ as variable is considered to satisfy the requirements of a model for analysis of RC structural members. It is proposed to simultaneously include the effects of uniform corrosion of reinforcement into such a polynomial expression for the decay parameter.

3.1 Integrated model for tension stiffening and reinforcement corrosion

Reinforcement corrosion affects the bond between concrete and reinforcing steel in a RC structural component and thus influences its nonlinear behaviour as well as load-carrying capacity. Additionally, the bond also influences the tension stiffening effects in the nonlinear behaviour of a cracked RC structural component. Therefore, an integrated model is proposed for tension stiffening and uniform corrosion of reinforcement similar to those proposed by Coronelli and Gambarova (2004), Okamura and Maekawa (1991), Maekawa et al. (2003), Auyeung et al. (2000).

The effect of corrosion is considered in the proposed model through reduction in the effective contact area of the reinforcing steel with concrete. Therefore, the quantum of material loss, which is an indicative parameter for surface area reduction, is taken as the basis to integrate the corrosion effects into the tension stiffening model. Corrosion is defined by uniform rate at which the diameter of a bar reduces with time using the parameter k_s . Accordingly, for an initial diameter of reinforcement bar, D_0 , and uniform corrosion rate, k_s , the reduced bar diameter, D_1 , after time t , can be written as

$$D_1 = (D_0 - k_s t) \quad (13)$$

In Eqn. (13), t is the active period of corrosion. The difference in bar diameter is normalized w.r.t. the original diameter to propose a non-dimensional ‘*diameter ratio*’, D_n ,

$$D_n = \frac{(D_0 - D_1)}{D_0} \quad (14)$$

The *diameter ratio* physically signifies the level of corrosion and can be used to quantify the reduction in reinforcement bar diameter. As an illustration, *nil* corrosion is denoted by $k_s=0$ and $D_n=0$ while significant corrosion over a long period

is represented by $k_{st} \approx D_0$ and $D_n \approx 1$. By combining Eqns. (13) and (14), the *diameter ratio* can be expressed as

$$D_n = \frac{k_{st}}{D_0} \quad (15)$$

As explained in the previous section, the tension stiffening model is proposed to be defined as a function of the decay parameter. A polynomial expressed as product of modular ratio and reinforcement ratio is proposed for the decay parameter. Based on the discussions presented earlier and also the similarity between tension stiffening and reinforcement corrosion in terms of influencing mechanisms, it is proposed to introduce *diameter ratio* into the expression for the decay parameter. A number of possible expressions are framed for the decay parameter based on the condition that reinforcement corrosion reduces tension stiffening effect. On the basis of extensive studies carried out using these expressions, *diameter ratio* has been included in the exponent of the polynomial terms of the decay parameter. As an example, exponent of the m^{th} order term of the polynomial is proposed as $\frac{m}{1-D_n}$. Such an expression ensures two basic requirements on the polynomial and *diameter ratio*. These requirements are

1. For *nil* corrosion, i.e., $D_n=0$ or in the presence of only the tension stiffening effect, the exponent reduces to its base value m .
2. For significant corrosion, i.e., $D_n \approx 1$, exponents of the polynomial approach infinity. Because of this, the decay parameter will assume a very large value which corresponds to negligible effect due to tension stiffening.

Next step in the development of model is to propose a polynomial expression which is applicable over a range of input values for the decay parameter. After a number of trials, a fifth order polynomial is proposed for the decay parameter, d . The general form of the polynomial for the decay parameter shall be like

$$d = c_5 (n\rho)^{\frac{5}{1-D_n}} + c_4 (n\rho)^{\frac{4}{1-D_n}} + c_3 (n\rho)^{\frac{3}{1-D_n}} + c_2 (n\rho)^{\frac{2}{1-D_n}} + c_1 (n\rho)^{\frac{1}{1-D_n}} + c_0 \quad (16)$$

where the coefficients c_0 to c_5 are to be decided on proper basis.

In order to evaluate the value of the coefficients, it is required to know their valid range. In that sense, it is investigated to decide on the range of the variables involved with the decay parameter. It is expected that the value of modular ratio, n , can be as high as 10 while using low to medium strength concrete and high strength reinforcing steel. Along the same lines, maximum value for the reinforcement ratio, is taken as 0.08 (Stramandinoli and Rovere, 2008). Based on these considerations,

it has been decided that the decay parameter expressed in Eqn. (16) has to be valid for a maximum value of n equal to 0.8.

The procedure adopted to evaluate the coefficients is explained in the following:

1. The values of strains in concrete after cracking, ε_{ac} , are evaluated by using Eqn. (10) for tensile force, N , ranging from N_{cr} (tensile force carried by concrete for which it cracks) and N_y (tensile force carried by steel at the instant of reinforcement yielding).
2. For the strain values, ε_{ac} , obtained in step i, corresponding stress in cracked concrete is evaluated by using Eqn. (12).
3. By matching the stress values obtained in step ii, the Eqn. (4) is calibrated to find the value of d for a large number of closely spaced values of stress and strain.
4. Stress-strain values of cracked concrete are obtained for different values n .
5. The stress-strain values obtained for each n are used to fit a polynomial which can be used in the numerical calculations.
6. As decided in the previous section, the exponents of the polynomial are scaled by the factor $(1-D_n)$ to take into account the effect of uniform corrosion in reinforcement.

By following the above procedure, the coefficients of polynomial are evaluated and given in Table 1.

Table 1: Value of the coefficients of the polynomial

Coefficient	Value
c_5	2.1544
c_4	-6.4633
c_3	7.3411
c_2	-3.984
c_1	1.995
c_0	-0.657

Therefore, the polynomial for decay parameter shall appear as

$$d = 2.1544 (n\rho)^{\frac{5}{1-D_n}} - 6.4633 (n\rho)^{\frac{4}{1-D_n}} + 7.3411 (n\rho)^{\frac{3}{1-D_n}} - 3.984 (n\rho)^{\frac{2}{1-D_n}} + 1.1995 (n\rho)^{\frac{1}{1-D_n}} - 0.0657$$

(17)

Among the different models available for the time-dependent deterioration phenomena, the one proposed (Sarja and Vesikari 1996) has been considered to model the corrosion effect. This corrosion model is characterized by initiation time t_0 and uniform rate k_s . In a typical RC structural member, reduction in cross-section of steel, $A_s(t)$, due to corrosion is expressed as

$$A_s(t) = \frac{n_b \pi (D_0 - k_s t)^2}{4} \quad (18)$$

where n_b is the number of reinforcement bars, D_0 is the initial diameter of bar, k_s is the corrosion rate and t is the active period of the corrosion which is taken as same as the age of the structural component.

In the proposed integrated model, the *diameter ratio* D_n contributes to the effect of corrosion by modifying the tensile stress-strain relationship of concrete through the decay parameter. Variation of the decay parameter with $n?$ for different values of D_n is plotted in Figure 3. The figure indicates that decay parameter depends both on the amount of reinforcement, $n?$ s and level of reinforcement corrosion, D_n . The dependency is, however, found to be more on the amount of reinforcement. As illustration, by ignoring the corrosion effects (i.e., $D_n=0$), maximum value of the decay parameter is found to be about 0.15. For the range of nondimensional parameter D_n studied, only a marginal increase (approximately 0.025) in the value of decay parameter is observed. On the other hand, for a section with high reinforcement (i.e., $n?=0.8$), the decay parameter value is found to increase from about 0.15 for $D_n=0$ to about 0.25 for $D_n=0.16$.

3.2 Evaluation of the proposed model

The proposed integrated model is evaluated against the tension stiffening models of Gupta and Maestrini (1990) and C.E.B. (1990), using a RC structural member containing reinforcement of $n? = 0.2$. For simplicity, the evaluation is initially done without considering the influence of corrosion. The normalized stress-strain curves obtained by making use of Eqns. (4) and (14) are shown in Figure 4. In the Figure 4, ϵ_y corresponds to a strain in reinforcing bar when the structural member reaches its ultimate load. Further, the tension stiffening curves have been plotted for different age of the structure for $n? = 0.2$. It can be observed that for the condition of *nil* corrosion, the proposed curve is matches closely with that given by CEB model. It is also very clear from Figure 4 that the model predicts considerable reduction in the tension stiffening in the structural member which will definitely affect the response behaviour.

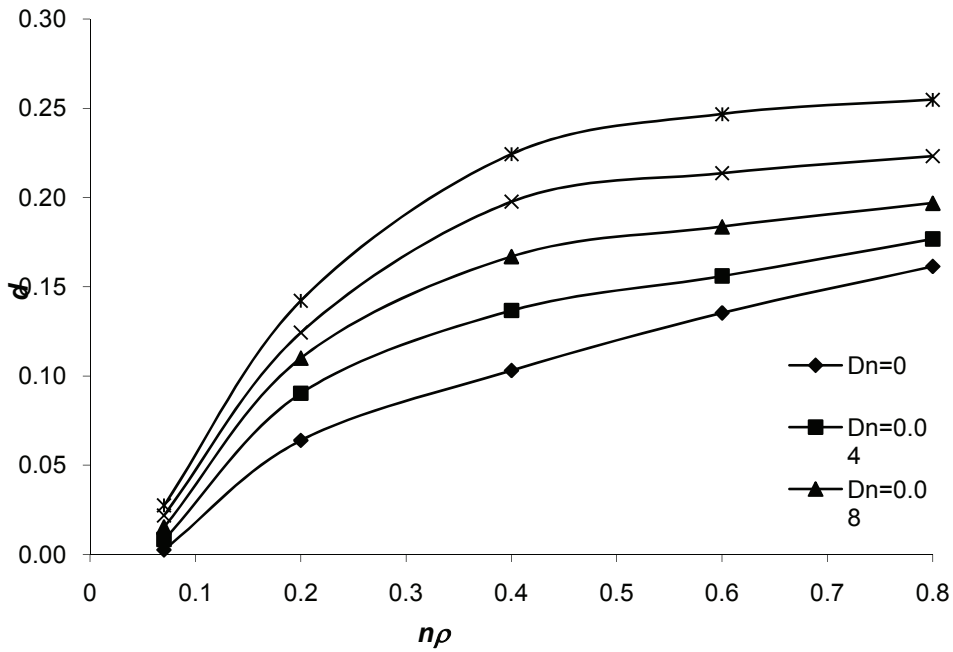


Figure 3: Variation of decay parameter wrt $n\rho$

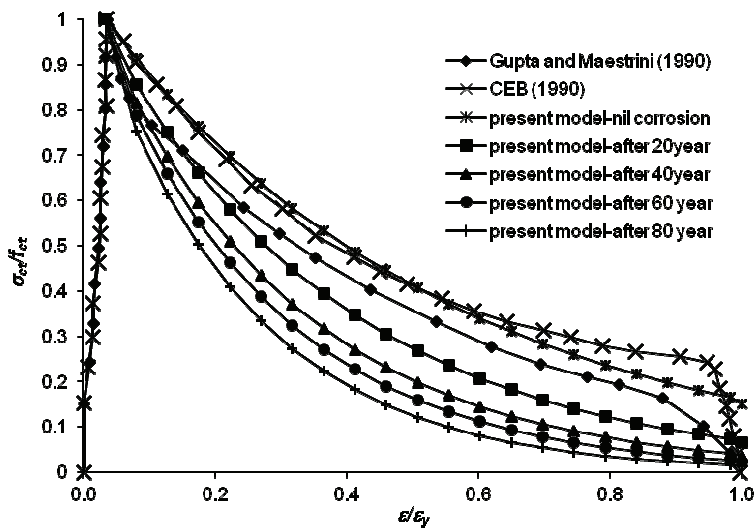


Figure 4: Comparison of proposed tension stiffening model for $n = 0.2$

4 Global Damage Indicator

During service, members of a typical RC structure undergo progressive cracking which can be qualitatively described using a damage indicator. Such numerical quantification of damage helps to gain an understanding of the current (or degraded) state of the structure w.r.t. its original state. A simple means to quantify damage is by comparing current value of a derived response quantity with its initial value. Secant stiffness of the structure estimated from the load-deformation response is a global parameter that indicates the state of the structure. Any reduction in stiffness can be interpreted as a measure of damage due to various reasons which act at local level. Within the context of ideal conditions studied using the proposed model, the reasons can be pin-pointed to reinforcement corrosion which reflects through modification in the value of decay parameter.

In this background, values of a damage indicator are calculated using the load-deformation response of the example RC structures analysed. The damage indicator (DI) is expressed as Kratzig et al. (2000)

$$DI = 1 - \frac{K_c}{K_{initial}} \quad (19)$$

where $K_{initial}$ is the initial secant stiffness in the uncracked state, K_c is the current secant stiffness at which the damage indicator is to be evaluated. Hence the damage indicators represented by Eqn. (19) can be used to study the effect of corrosion at structural level. Both the secant stiffness values are to be calculated from the load-deformation response. When $DI=0$, the structure is considered to be in the undamaged state. As far as the effect of corrosion is considered, any reduction in reinforcement due to corrosion will eventually lead to decrease in structural stiffness. The effect is more pronounced in cracked members where reinforcement is known to play relatively larger role in deciding the structural response.

5 Numerical Examples

Two example RC beams have been analysed by using the proposed model to trace their load-deformation response. Analysis of the first beam is taken as validation of the proposed model. Analysing another RC beam serves the objective to gain better understanding of the effect of reinforcement corrosion on static load-deformation response and capacity of the beam by using this model. For this, the beam has been analysed for different assumed rates of corrosion and the results of different cases are compared relatively.

5.1 Nonlinear response of a RC beam

A RC beam with 2.0 m effective span and simply supported end conditions is analysed for its load-deformation response. The beam has a cross section of 200 mm x 250 mm. Details of the geometry and reinforcement of the beam are shown in Figure 5. The beam has been analysed for two different rates of corrosion for which it has experimentally been studied by Lee et al. (2000). Two-point loading on the beam is shown in Figure 5. Concrete and reinforcement properties of the beam as reported (Lee et al. 2000) are available in Table 2. Details of the beam notation used and assumed rates of corrosion are given in Table 3.

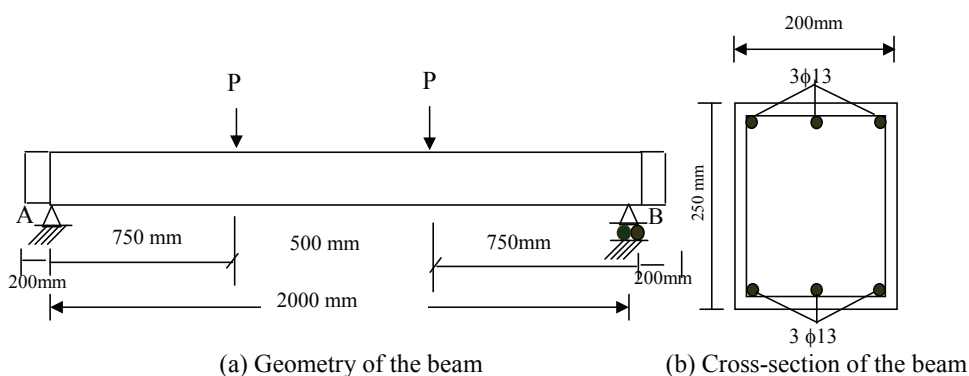


Figure 5: Details of RC beam

Table 2: Material Properties of RC beam

Concrete				Reinforcement		
f'_c (MPa)	E_c (MPa)	f_{ct} (MPa)	ϵ_{cu}	ϵ_{su}	f_y (MPa)	E_s (MPa)
70.1	38500	3.67	0.004	0.15	415.0	2×10^5

Table 3: Rates of Corrosion in RC beam

Beam Notation	BCD2	BCD3
D_n (%)	7.9	25.3

Longitudinal symmetry of the beam is taken into account in developing the finite element model. Accordingly, only half the length of the beam is used in the analysis. The chosen segment of the beam is modelled with 10 quadrilateral multi-layered finite elements. Each finite element is geometrically described by 5 layers

across thickness, 4 nodes and 5 degrees of freedom (dof) per node. Reinforcement is smeared into selected concrete layers. Such a representation is decided based on author's previous experience (Smitha et al. 2009) in analysing similar structures. The proposed integrated model is implemented as a new material in the nonlinear finite element package FINEART (2006). Analysis is controlled through displacement imposed loading on the beam. Load is continuously applied in small increments till instability is encountered in the computation. By this, the complete nonlinear load-deformation response of the beam has been captured in the analysis.

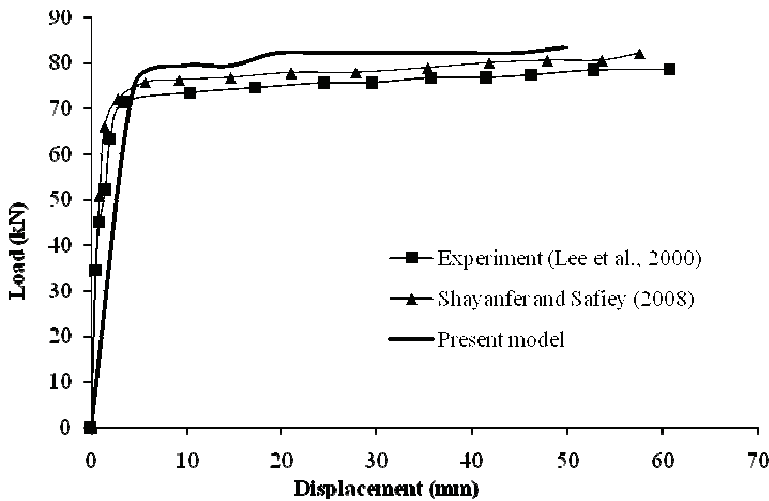


Figure 6: Load-deformation behaviour of beam BCD2

Load-deformation response of beam BCD2 (with 7.9% corrosion) is shown in Figure 6. The computed responses are compared with the values reported by Lee et al. (2000) and Shayanfar and Safiey (2008). It is found that, the present model over estimates the ultimate load by about 10%. Load-deformation response indicates flexible behaviour of the beam compared to experimental observation in the initial part. This may be due to the not accounting shear reinforcement in the present work.

The beam BCD3 which is subjected to 25.3% corrosion showed very good agreement in predicting ultimate load. The load-deformation curve obtained for beam BCD3 is shown in Figure 7. The ultimate load predicted by the present model is found to represent an error of less than 1%. Further, it can be concluded that as severity of corrosion rate increases, there is considerable reduction in the load carrying capacity and ductility of the beam.

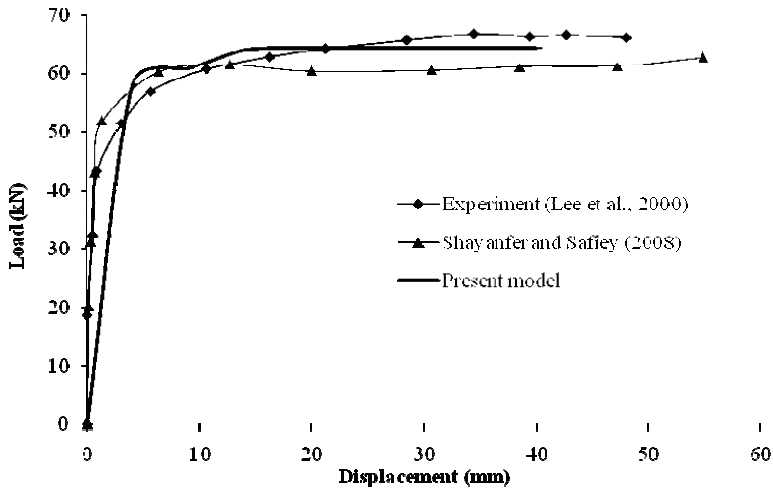


Figure 7: Load-deformation behaviour of beam BCD3

5.2 Effect of reinforcement corrosion on performance of a RC beam

Effect of reinforcement corrosion on the nonlinear performance of a RC beam is studied. The chosen beam is 3.2 m long and has a cross section of 150 mm x 300 mm. The beam has been studied experimentally (Smitha et al. 2009) for normal conditions without being interested with the effects of reinforcement corrosion. Measured responses from such an experiment are used initially to verify the proposed model. The geometric and reinforcement details of the beam are shown in Figures 8(a) and 8(b). Material properties are listed in Table 4.

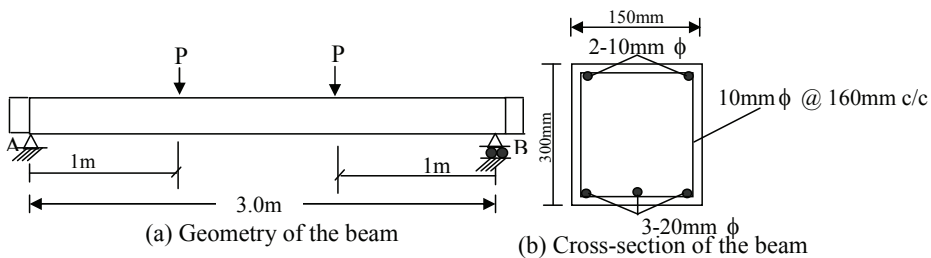


Figure 8: Details of beam

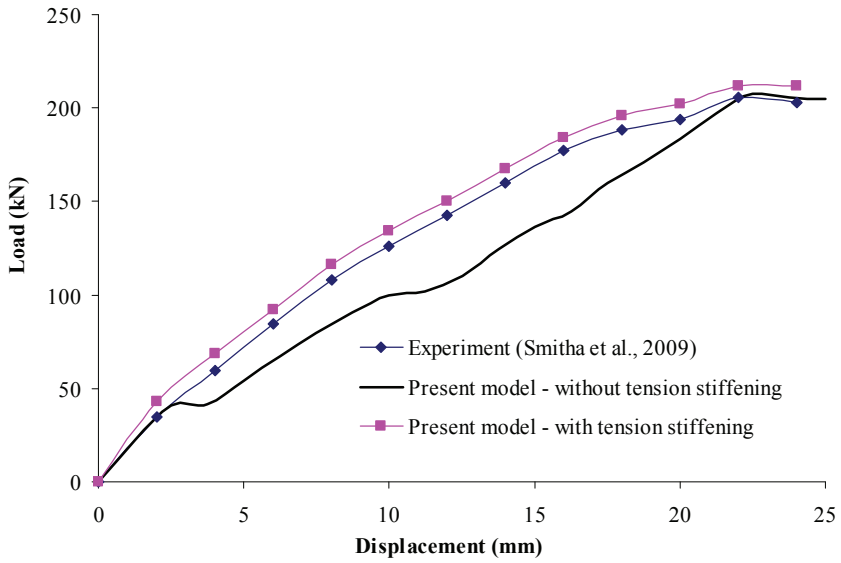
Only half the length of the beam has been modelled using 4-noded shell finite elements for analysis. Again, as in previous analysis, each finite element is made up of six concrete layers of equal thickness. Top and bottom reinforcements present

Table 4: Material properties

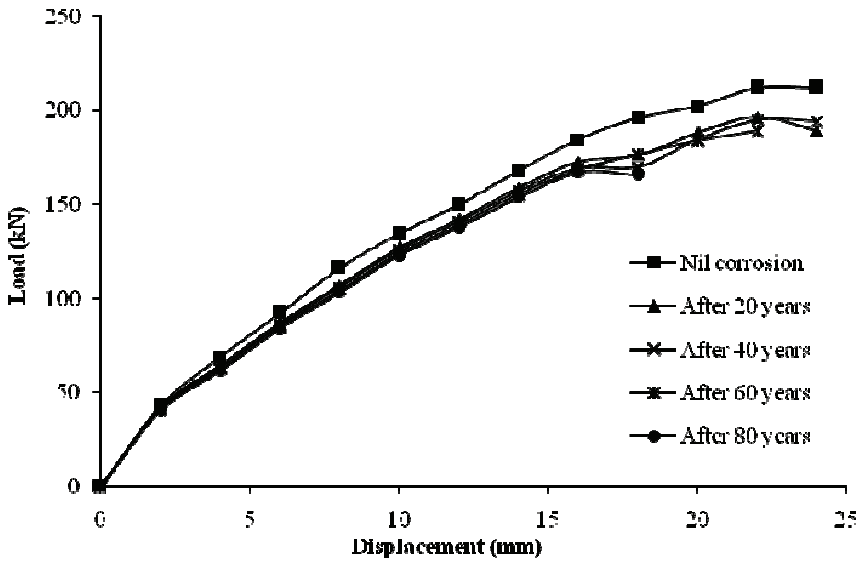
Concrete				Reinforcement	
f_c' (MPa)	E_c (MPa)	f_{ct} (MPa)	ϵ_{cu}	f_y (MPa)	E_s (MPa)
40.0	31622	4.4	0.0035	415.0	2×10^5

in the beam are smeared within two of the concrete layers. Nonlinear analysis is carried out by applying incremental load of same magnitude at the two locations shown in Figure 8(a). Nonlinear response of the beam has been obtained for two cases, with and without considering the tension stiffening. Comparison of the responses of these two cases with the experiment will yield information on the necessity for considering the tension stiffening effects in nonlinear analysis. Such a comparison is presented in Figure 9(a). As expected, the beam is found to show flexible behaviour in the case of ignoring the tension stiffening effect. The response after including the tension stiffening effects indicate significant improvement and better match with measures values both in linear as well as nonlinear regimes. In absolute terms, the finite element solution by including tension stiffening effects represents stiff behaviour of the beam compared to the measured solution. Ultimate load carrying capacity of the beam is reported as 208 kN from experiment while the corresponding value is estimated in the analysis as 212 kN.

After ensuring the appropriateness of the proposed model, the beam has been analysed for assumed condition in which the reinforcement is experiencing uniform corrosion at the rate of 0.04 mm/year. The assumed corrosion rate typically represents moderate corrosive environment. The beam is considered to be in-service for 20, 40, 60 and 80 years in such an environment characterized by uniform corrosion. This ideal condition is assumed only as a representative and in no way limits the application of the model. Further, this will perfectly serve the purpose of obtaining relative performance of the beam. The reduced diameters of the reinforcement bars at the assumed periods are calculated using Eqn. (13) and the corresponding values of decay parameter are also evaluated using Eqn. (16). The beam is analysed for nonlinear response under all the cases in similar manner as in the earlier situation. The calculated values of bar diameter and decay parameter are used in the analysis. The computed responses are compared in Figure 9(b). The comparison clearly points out to reduction in ultimate load carrying capacity as well as maximum displacement capacity with duration for which the beam is assumed to be affected by corrosion. Due to 80 years of service in the assumed ideal corrosive conditions, the beam is seen to exhibit about 40% reduction in ultimate load carrying capacity and about 30% reduction in maximum displacement capacity.



(a) with and without incorporating tension stiffening



(b) response curve for different age

Figure 9: Load versus central deflection of the beam

Damage indicator is computed by using the load-deformation response of the beam at different ages of the beam by using Eqn. 19. For each assumed age, the indicator is calculated separately based on the corresponding load-deformation response. The generated information is used to get an insight into the expected influence of reinforcement corrosion and to quantify such influence in a single term as damage indicator. The growth of damage indicator with displacement is shown in Figure 10 for loading the beam after subjecting it to uniform corrosion of rate as discussed earlier for 20, 40, 60 and 80 years. It may be noted that the values indicate damage only due to the assumed rates of uniform corrosion. During service, however, damage or degradation may result due to several reasons including to that of reinforcement corrosion. The reduction in ultimate load carrying capacities of the beam at different ages due to reinforcement corrosion is plotted in Figure 11. The plot indicates severe nonlinear relation with large reduction in the load carrying capacity at later stage.

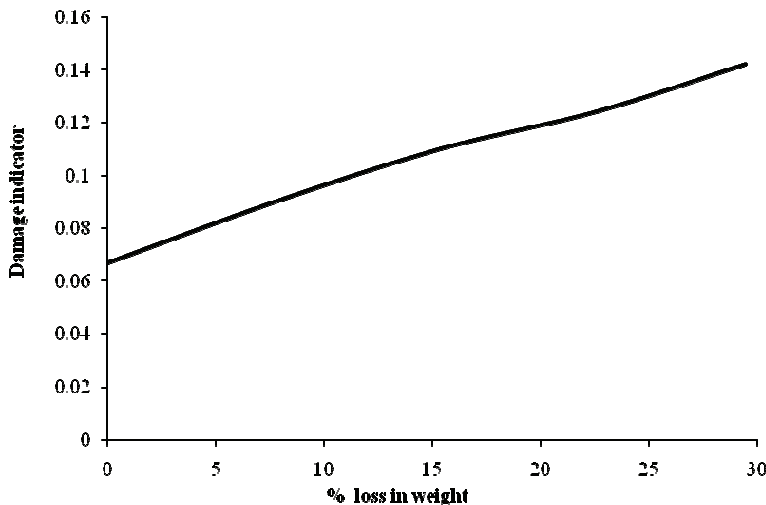


Figure 10: Growth of damage with percentage weight loss of reinforcement

To understand the nonlinear influence of reinforcement corrosion on the beam response, the damage indicator has been evaluated based on secant stiffness indicated by the set of load deformation curves given in Figure 9(b). Growth of damage indicator with applied load is plotted in Figure 12 for different ages for which the beam has been analysed earlier. The figure gives expected damage to the beam for the assumed service conditions and age. It can be seen from Figure 12 that for a damage indicator value equal to 0.3, the beam with *nil* corrosion in reinforcement has load carrying capacity of about 105 kN which reduces to about 80 kN after put

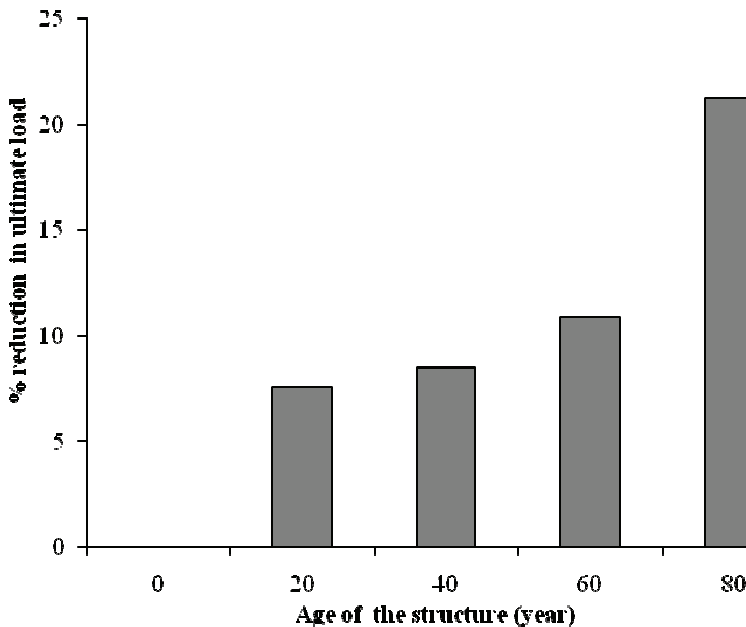


Figure 11: Loss of ultimate load carrying capacity with age of the beam

in service for a maximum of 80 years in corroded environment. For intermediate years of service, the beam shows skewed relation with significant reduction in load carrying capacity during the early years of service. The results indicate that maximum load carrying capacity of the beam is reduced by about 50 kN from a value of nearly 210 kN (25% reduction) after 80 years of reinforcement corrosion. The analysis also indicates that the maximum attainable damage indicator value of the beam is about 0.58 for 80 year old beam while in case of *nil* corrosion, the beam can reach a maximum damage indicator value of 0.62. The results confirm that the reinforcement corrosion has significant effect on strength and ductility characteristics of RC flexural members. Hence, there is a need to incorporate corrosion effects in analysis of structures. Further, damage indicator presented in this paper is capable of giving an indication about the global damage state for a particular load level.

6 Conclusions

A new tension stiffening model to is proposed to account for the effects of corrosion in nonlinear finite element analysis of RC flexural members. This model follows a simple and generalized approach for determining the influence of corrosion

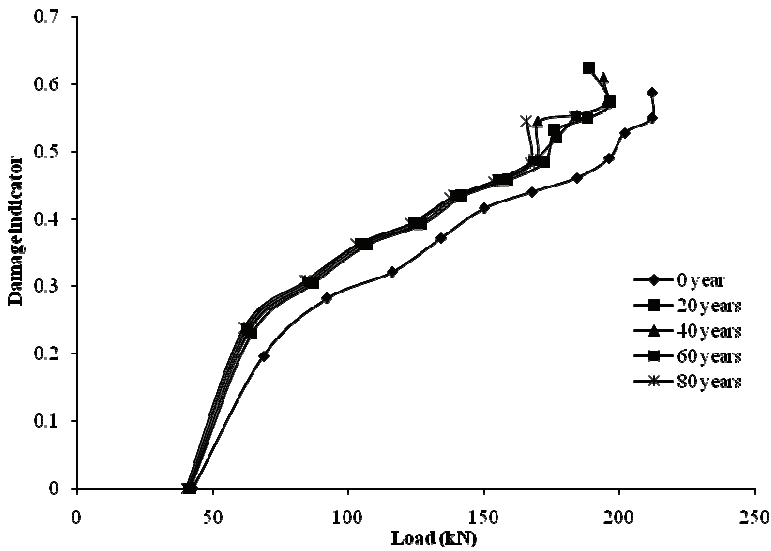


Figure 12: Growth of damage indicator with load for different ages

in response behaviour. For this, an exponential decay curve which uses a decay parameter is used to define stress-strain relation of concrete in the post-cracking stage. An expression has been derived for decay parameter based on reinforcement ratio, modular ratio and diameter ratio. In contrast to the other tension stiffening models available in literature which incorporates stiffening effect as a single numerical number from experiments, proposed model is capable of predicting the tension stiffening effect due to varies reinforcement ratio fractional loss in diameter of bar due to corrosion and modular ratio. The proposed model has been validated by tracing load-deflection response of RC flexural members for different rate of corrosion. It is observed that as rate of corrosion increases, there is considerable reduction in strength as well as ductility characteristics. Further, the effect of corrosion on structural capacity as the age of the structure progresses is also studied in detail using the proposed model. Further, a damage indicator has been arrived at from a secant stiffness based approach which helps to map the local deterioration due to corrosion on a structural level. From the analysis it is observed that there is considerable reduction in the load carrying capacity of the structure for different rate of corrosion. Also, as the age of the structure progresses, the strength as well as ductility characteristics of a structure get affected due to corrosion. Correlation with experimental data shows that the proposed analytical model is capable of providing very good estimates on ultimate load and on the response behaviour with corroding reinforcement.

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