

# Comparisons of MF DFA, EMD and WT by Neural Network, Mahalanobis Distance and SVM in Fault Diagnosis of Gearboxes

Jinshan Lin<sup>a\*</sup>, Chunhong Dou<sup>b</sup>, Qianqian Wang<sup>a</sup>

<sup>a</sup> School of Mechatronics and Vehicle Engineering, Weifang University, No. 5147 Dong Feng Dong Street, Weifang 261061, China

<sup>b</sup> School of Information and Control Engineering, Weifang University, No. 5147 Dong Feng Dong Street, Weifang 261061, China

\* Corresponding author: jslinmec@hotmail.com, Tel & Fax: +86 536 8785603

A method for gearbox fault diagnosis consists of feature extraction and fault identification. Many methods for feature extraction have been devised for exposing nature of vibration data of a defective gearbox. In addition, features extracted from gearbox vibration data are identified by various classifiers. However, existing literatures leave much to be desired in assessing performance of different combinatorial methods for gearbox fault diagnosis. To this end, this paper evaluated performance of several typical combinatorial methods for gearbox fault diagnosis by associating each of multifractal detrended fluctuation analysis (MF DFA), empirical mode decomposition (EMD) and wavelet transform (WT) with each of neural network (NN), Mahalanobis distance decision rules (MDDR) and support vector machine (SVM). Following this, performance of different combinatorial methods was compared using a group of gearbox vibration data containing slightly different fault patterns. The results indicate that MF DFA performs better in feature extraction of gearbox vibration data and SVM does the same in fault identification. Naturally, the method associating MF DFA with SVM shows huge potential for fault diagnosis of gearboxes. As a result, this paper can provide some useful information on construction of a method for gearbox fault diagnosis.

**Keywords:** Multifractal, detrended fluctuation analysis, support vector machine, fault diagnosis, gearbox.

## 1 Introduction

A gearbox, composed of many elements, usually plays an important part in mechanical transmission. When something goes wrong with a gearbox, vibration data from a gearbox generally display nonlinear and non-stationary properties. Consequently, fault diagnosis of gearboxes is a difficult problem, especially when fault patterns are very similar. Currently, many methods for feature extraction have been put forward for revealing nature of gearbox vibration data. Additionally, these features extracted from gearbox vibration data are identified by various classifiers. Currently, many methods for gearbox fault diagnosis have been constructed by associating a method for feature extraction with that for fault identification. Unfortunately, evaluation of performance of these methods for gearbox fault diagnosis leaves much to be desired. In this paper, performance of three typical methods for feature extraction, multifractal detrended fluctuation analysis (MF DFA), empirical mode decomposition (EMD) and wavelet transform (WT), was assessed using a group of gearbox vibration data containing very similar fault patterns. In the following, performance of three typical classifiers, neural network (NN), Mahalanobis distance decision rules (MDDR) and support vector machine (SVM) was compared. The results indicate that MF DFA performs better in feature extraction of gearbox vibration data and SVM does in classification of characteristic parameters of gearboxes. Naturally, the method associating MF DFA with SVM seemingly has high potential for fault diagnosis of gearboxes.

This paper was structured as follows. The following section outlined the principles of WT, EMD and MF DFA. In the third section, three typical classifiers, i.e. NN, MDDR and SVM, were separately formulated. In the fourth section, the effectiveness of MF DFA, EMD and WT in feature extraction of gearbox vibration data was compared by each of NN, MDDR and SVM and a discussion was set up. Finally, a conclusion was drawn in the fifth section.

## 2 Three methods for data analysis

### 2.1 Wavelet transform

Wavelet transform (WT) can be used to process non-stationary data<sup>1,2</sup>. For a signal  $x(t)$ , the continuous WT is defined below<sup>1,2</sup>.

$$x(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} \overline{\Psi\left(\frac{t-b}{a}\right)} x(t) dt \quad (1)$$

Here,  $a$  and  $b$  represent the scale and time factors, respectively, the symbol  $\Psi(t)$  indicates a wavelet basis function and the symbol  $\overline{\Psi(t)}$  stands for the conjugation of  $\Psi(t)$ . As a result, there is a distinct lack of adaptability for the analyzed signal in the WT algorithm since a prior knowledge about the basis function is required.

### 2.2 Empirical mode decomposition (EMD)

Different from WT, EMD is an adaptive method for processing non-stationary and nonlinear data<sup>3</sup>. By EMD, a signal  $x(t)$  can be adaptively decomposed into a group of components falling into different frequency bands and a trend<sup>3</sup>:

$$x(t) = \sum_{i=1}^k c_i(t) + r \quad (2)$$

Here,  $c_i(t)$  and  $r$  denote the  $i$ th component and general trend of the signal  $x(t)$ , respectively, and  $k$  represents the number of the components. In this paper, a  $k$ -dimension vector was constructed as characteristic parameters of the signal  $x(t)$ , defined as

$$e_j = c_j^2 / \sum_{i=1}^k c_i^2, j = 1, \dots, k \quad (3)$$

### 2.3 Multifractal Detrended fluctuation analysis (MF DFA)

MF DFA, as an extension of the monofractal DFA, can be applied to effectively reveal the multifractality of non-stationary time series<sup>4</sup>. The execution of MF DFA for a series  $x_k$  with the length  $N$  comprises the next five steps<sup>4</sup>:

(1) A "profile" is constructed as

$$Y(i) \equiv \sum_{k=1}^i [x_k - \langle x \rangle] \quad (4)$$

Since performed in the third step again, the detrending operation in this step is optional.

(2) The profile  $Y(i)$  is split into  $N_s = \text{int}(N/s)$  non-overlapping segments that have the same length  $s$ . However, because the length  $N$  of the series is seldom divisible by the time scale  $s$ , a small part of the profile may remain unused. To fully use these data, the same operation is again implemented from the opposite direction. In the end,  $2N_s$  data segments are got.

(3) The least-square algorithm is adopted to fit the local trend for each of

the  $2N_s$  data segments. Then the variance is defined as

$$F^2(v, s) \equiv \frac{1}{s} \sum_{i=1}^s \{Y[(v-1)s+i] - y_v(i)\}^2 \quad (5)$$

for the  $v$ th segment,  $v = 1, \dots, N_s$ , and

$$F^2(v, s) \equiv \frac{1}{s} \sum_{i=1}^s \{Y[N - (v - N_s)s + i] - y_v(i)\}^2 \quad (6)$$

for the  $v$ th segment,  $v = N_s + 1, \dots, 2N_s$ . Here,  $y_v(i)$  is the fitting polynomial for the  $v$ th segment. Since the detrending operation is to subtract the polynomial fits from the profile, DFA of different orders may yield different detrending results. As a result, by making a comparison between the results derived from DFA of different orders, the nature of the polynomial trend in the time series can be determined<sup>4</sup>.

(4) The  $q$ th-order fluctuation function  $F_q(s)$  can be acquired by calculating the average for all the  $2N_s$  segments:

$$F_q(s) \equiv \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(v, s)]^{q/2} \right\}^{1/q} \quad (7)$$

Here, any real value can be assigned to the index  $q$  except zero. For  $q = 2$ , MF DFA degenerates into the standard DFA. For a different time scale  $s$ , steps 2~4 will be repeated. Thus, the fluctuation  $F_q(s)$  can be obtained for different  $q$  and  $s$ . In addition, it must be noted that a definition of  $F_q(s)$ , which relates to the DFA order  $m$ , is accepted only for  $s \geq m + 2$  ( $m$  is the order of the polynomial fits).

(5) A power-law relation is established between  $F_q(s)$  and  $s$  for different  $q$ :

$$F_q(s) \sim s^{H(q)} \quad (8)$$

For stationary time series,  $H(2)$  is equal to the famous Hurst exponent  $H^4$ .

Moreover, when  $q = 0$ , the following logarithmic averaging operation is substituted for the averaging procedure in Equation (7)

$$F_0(s) \equiv \exp \left\{ \frac{1}{4N_s} \sum_{v=1}^{2N_s} \ln [F^2(v, s)] \right\} \sim s^{H(0)} \quad (9)$$

A monofractal time series typically exhibits identical scaling behavior in all the segments and thus the corresponding  $H(q)$  is independent of  $q$ ; conversely, a multifractal time series generally shows distinctly different scaling behavior in different segments and then there is a heavy dependence of  $H(q)$  on  $q$ . In addition, the mean  $F_q(s)$  for the positive and negative  $q$  will be mostly dominated by the segments  $v$  with large and small variance, respectively. As a result, the general Hurst exponents  $H(q)$  for the positive and negative  $q$  are applicable to describe the scaling behavior of the segments with large and small fluctuations, respectively<sup>4</sup>.

### 3 Three typical classifiers

#### 3.1 Neural network (NN)

NN is a nonlinear method for machine learning and pattern classification<sup>5-7</sup>. In theory, a three-layer back-propagation neural network with a large enough number of hidden-layer nodes can serve to mimic dynamical behavior of any nonlinear system<sup>5-7</sup>. However, it seems to be dealt with only empirically at present how to determine initiative and hidden-layer parameters of a neural network<sup>5-7</sup>.

#### 3.2 Mahalanobis distance decision rule (MDDR)

The Mahalanobis distance is a measure of similarities of two sets of data<sup>8</sup>. Different from the Euclidean distance, the Mahalanobis distance, which is scale-invariant, enables the correlations between data to be examined. For the

$$\text{matri } X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}, \text{ set } x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \text{ and}$$

$$\bar{x} = \frac{1}{m} \sum_{i=1}^m x_i = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)^T. \quad \text{Then, define}$$

$$C = \frac{1}{(m-1)} \sum_{i=1}^m (x_i - \bar{x})(x_i - \bar{x})^T \text{ as the covariance of the matrix } X.$$

Theoretically, the Mahalanobis distance between the sample  $y = (y_1, y_2, \dots, y_n)^T$  and the  $k$ th data group  $X_k$  that has the mean  $\bar{x}_k$  and the covariance matrix  $C_k$  is defined as

$$MD_k = \sqrt{(y - \bar{x}_k)^T C_k^{-1} (y - \bar{x}_k)}, \quad k = 1, \dots, M \quad (10)$$

$$\text{If } MD_l \leq MD_k, k = 1, \dots, l-1, l+1, \dots, M \quad (11)$$

then the sample  $y$  is classified as the  $l$ th group.

#### 3.3 Support vector machine (SVM)

SVM, a supervised method for machine learning<sup>9</sup>, has been used in a wide variety of fields, such as chaotic time series prediction<sup>10</sup>, imaging biomarker identification<sup>11</sup>, pattern classification<sup>12</sup> and fault diagnosis<sup>13</sup>. Given a training

$$\text{vector } \{x | x_i \in R^p\}_{i=1}^n \text{ and an indication vector } \{y | y_i \in \{-1, 1\}\}_{i=1}^n,$$

here the training vector contains two different categories and the element  $y_i$  in the indication vector serves to indicate which category the sample  $x_i$  in the training vector belongs to. Then, a SVM is trained to find a maximum-margin hyperplane for distinguishing the samples marked with  $y_i = -1$  from those marked with  $y_i = 1$ . If there is no hyperplane which can cleanly separate these two categories, a soft margin method can be employed to determine a hyperplane which can separate these two categories as cleanly as possible, still keeping the most maximum distance to the closest cleanly separated points<sup>14</sup>. Afterwards, an optimum balance between two objectives of a large margin and a small error penalty can be achieved by solving the next equation

$$\begin{aligned} \min_{w, b, \xi} \quad & \frac{1}{2} w^T w + C \sum_{i=1}^n \xi_i \\ \text{subject to} \quad & y_i (w^T \phi(x_i) + b) \geq 1 - \xi_i \\ & \xi_i \geq 0, i = 1, \dots, n, \end{aligned} \quad (12)$$

where  $\xi_i$  indicates a non-negative slack parameter,  $C (> 0)$  stands for a regularization parameter,  $w$  means a normal vector to the hyperplane and  $\phi(x_i)$  serves to project  $x_i$  into a higher-dimensional space. Considering that the vector  $w$  often features high dimensionality, Equation (12) is usually transformed into the next dual problem

$$\begin{aligned} \min_{\alpha} \quad & \frac{1}{2} \alpha^T Q \alpha - e^T \alpha \\ \text{subject to} \quad & y^T \alpha = 0 \\ & 0 \leq \alpha_i \leq C, \quad i = 1, \dots, n, \\ & e = [1, \dots, 1]^T \end{aligned} \quad (13)$$

Here,  $Q$  is a positive semidefinite matrix with  $Q_{ij} = y_i y_j K(x_i, x_j)$

and  $K(x_i, x_j)$  indicates a kernel function with

$K(x_i, x_j) = \phi(x_i)^T \phi(x_j)$ . Subsequently, by analyzing the above primal-dual relationship, one can obtain the optimal  $w$

$$w = \sum_{i=1}^n y_i \alpha_i \phi(x_i) \quad (14)$$

Accordingly, the decision function can be stated as

$$\text{sgn}(w^T \phi(x) + b) = \text{sgn}\left(\sum_{i=1}^n y_i \alpha_i K(x_i, x) + b\right) \quad (15)$$

Here, it must be noticed that original SVM can be used only for solving a binary classification problem<sup>15-16</sup>. Nonetheless, the multi-class classification is a frequently faced problem in real world. To address this problem, “one-against-one” and “one-against-all” approaches have been proposed<sup>15-16</sup>. In this paper, the “one-against-one” approach was employed to solve the multi-class classification problem. For the “one-against-one” approach, if there is  $p$  classes for classification, then one needs to construct  $p(p-1)/2$  binary classifiers<sup>15-16</sup>. The data from the  $k$ th and the  $l$ th classes can be separated by solving the following binary classification problem<sup>15-16</sup>

$$\min_{w^{kl}, b^{kl}, \xi_i^{kl}} \frac{1}{2} (w^{kl})^T + C \sum_i (\xi_i^{kl}) \quad (16)$$

subject to  $(w^{kl})^T \phi(x_i) + b^{kl} \geq 1 - \xi_i^{kl}$ , if  $x_i$  belongs to the  $k$ th class,  
 $(w^{kl})^T \phi(x_i) + b^{kl} \leq -1 + \xi_i^{kl}$ , if  $x_i$  belongs to the  $l$ th class,  
 $\xi_i^{ij} \geq 0$ .

Here, a voting strategy is adopted for classification: after votes are cast for a data sample using all the binary classifiers, the data sample is considered to belong to the class which wins the maximum votes. In this paper, the LIBSVM package was used for performing all the SVM operations<sup>17</sup>.

#### 4 Comparisons of MF DFA, EMD and WT by NN, MDDR and SVM

##### 4.1 Fault diagnosis of gearboxes

A gearbox experiment, whose sketch map is drawn in Figure 1, was conducted for generating desirable gearbox vibration data containing slightly different gear faults. The tooth numbers of the gears 1-4 of the gearbox in Figure 1 are 25, 40, 22 and 55, respectively. The frequency converter was employed to control an output speed of the three-phase asynchronous motor in Figure 1. In this experiment, slight-scratch, medium-scratch and broken-tooth faults were individually fed into the gear 1. Here, it must be emphasized that the slight- and medium-scratch faults are only slightly different and tough to separate. As a result, these gearbox vibration data can be used to assess a performance of an algorithm. From the housing close to the gear 1, vibration data were collected by an acceleration transducer. For each gearbox condition, thirty-five pieces of data were gathered, each with the sample frequency 16384 Hz and the length 4096 points. Afterwards, fifteen randomly selected pieces served as training data and the remaining ones testing data. In this paper, two sets of vibration data were separately gathered under two different motor running speeds: 1200 RPM (Revolutions Per Minute) and 2000 RPM.

Figure 2 displays the vibration data under the motor running speed 1200 RPM. To start with, MF DFA was adopted to analyze these gearbox vibration data and the results are demonstrated in Figure 3. As demonstrated in Figure 3, the multifractal spectra for the normal and broken-tooth conditions clearly differ from those for the scratch conditions in the positions of extreme points of the multifractal spectra. Also, the multifractal spectra for the slight- and medium-scratch conditions are different only in the shapes of the multifractal spectra. Consequently, the shapes and positions of the multifractal spectra allow a separation between different gearbox conditions. With the capabilities to almost determine the shapes and positions of the multifractal spectrum, five characteristic parameters:  $\alpha_{\max}$ ,

$f(\alpha_{\max})$ ,  $\alpha_{\text{ext}}$ ,  $\alpha_{\min}$  and  $f(\alpha_{\min})$ , corresponding to coordinates of the left-end, right-end and extreme points of the multifractal spectrum, were extracted for describing a gearbox condition. Moreover, to benchmark the performance of MF DFA, EMD and WT were separately applied to research these gearbox vibration data. According to the results derived from EMD, the vibration

data corresponding to each gearbox condition were uniformly decomposed into eleven components by each of EMD and WT. Additionally, in light of Equation (3), an eleven-dimension characteristic vector was built for each of EMD and WT. Afterwards, NN, MDDR and SVM were separately employed to classify the characteristic parameters derived from each of MF DFA, EMD and WT. Consequently, comparisons of performances of MF DFA, EMD and WT are shown in Table 1. As shown in Table 1, although using fewer characteristic parameters for describing gearbox conditions, MF DFA seemingly gives a better performance than each of EMD and WT in feature extraction of gearbox vibration data. In addition, Table 1 reports that SVM performs better than each of NN and MDDR in classification of characteristic parameters. It means that the method associating MF DFA with SVM seems to show high potential for fault diagnosis of the gearbox.

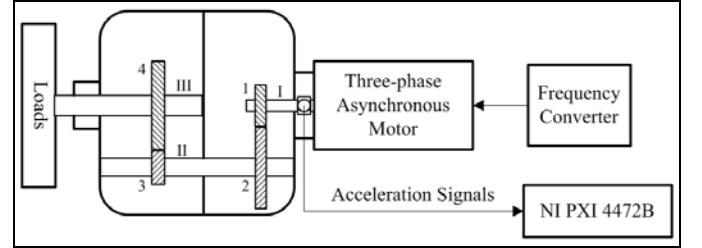


Figure 1. A sketch map of gearbox experiment table.

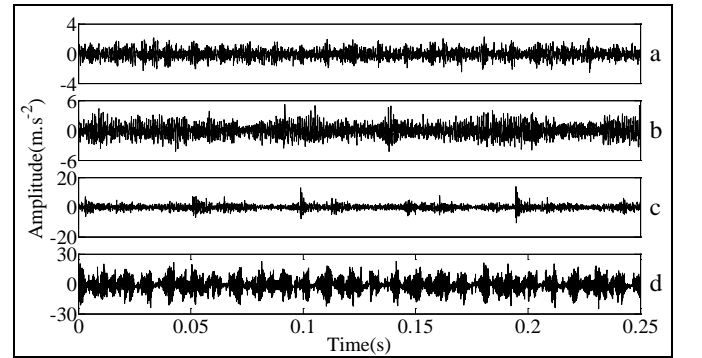


Figure 2. Four types of gearbox vibration data under the motor running speed 1200RPM, (a)-(d) correspond to normal, slight-scratch, medium-scratch and broken-tooth conditions, respectively.

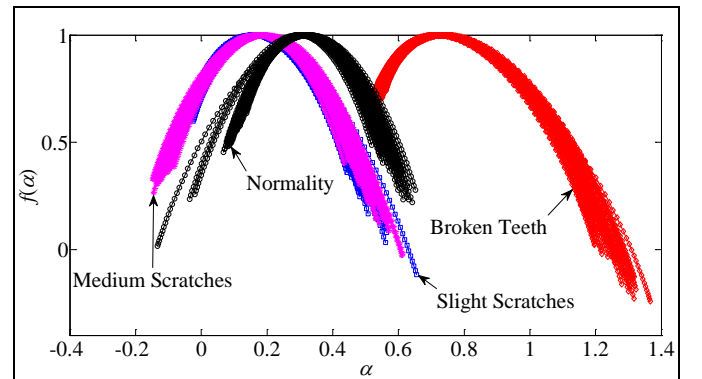


Figure 3. Multifractal spectra of four types of gearbox vibration data under the motor running speed 1200 RPM.

Subsequently, Figure 4 reveals gearbox vibration data under the motor running speed 2000RPM. To begin with, MF DFA was made use of processing these gearbox vibration data and the results are revealed in Figure 5. As revealed in Figure 5, the multifractal spectra for different gearbox conditions have markedly different positions.

Therefore, the shapes and positions of the multifractal spectra enable different gearbox conditions to be discriminated. Afterwards, the same five characteristic parameters as those extracted in the above example served as characteristic parameters of these gearbox vibration data. Also, to further benchmark the performance of MF DFA, EMD and WT were separately employed to study these gearbox vibration data. According to the results

derived from EMD, these gearbox vibration data were uniformly decomposed into twelve components by each of EMD and WT.

Table 1. Comparisons of MF DFA, EMD and WT by NN, MDDR and SVM in fault diagnosis of gearboxes for the motor running speed 1200 RPM.

Algorithms	The number of characteristic parameters	Success rates of fault diagnosis (%)		
		NN	MDDR	SVM
MF DFA	5	97.50	97.50	100.00
EMD	11	97.50	93.75	100.00
WT	11	27.50	30.00	35.00

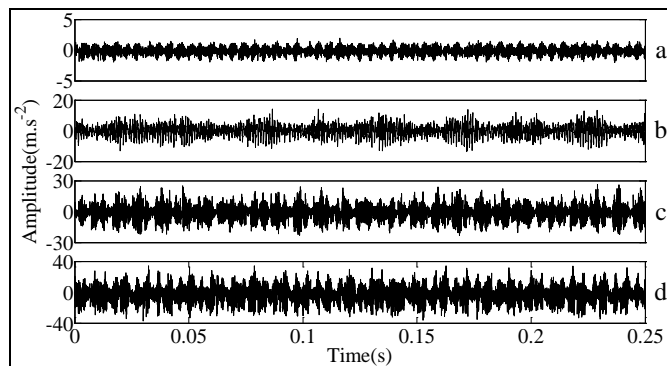


Figure 4. Four types of gearbox vibration data under the motor running speed 2000 RPM, (a)-(d) correspond to normal, slight-scratch, medium-scratch and broken-tooth conditions, respectively.

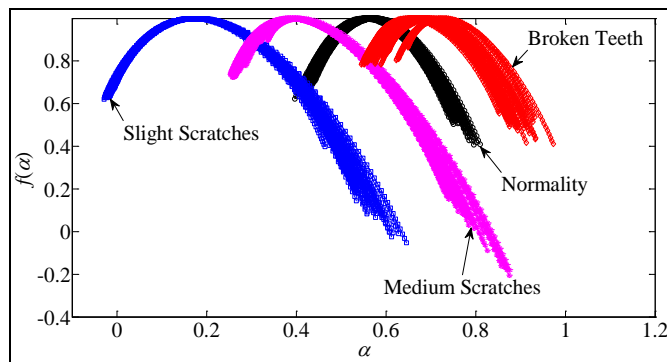


Figure 5. Multifractal spectra of four types of gearbox vibration data under the motor running speed 2000 RPM.

Table 2. Comparisons of MF DFA, EMD and WT by NN, MDDR and SVM in fault diagnosis of gearboxes for the motor running speed 2000 RPM.

Algorithms	The number of characteristic parameters	Success rates of fault diagnosis (%)		
		NN	MDDR	SVM
MF DFA	5	100.00	100.00	100.00
EMD	12	82.50	91.25	95.00
WT	12	36.25	32.50	35.00

According to Equation (3), a twelve-dimension characteristic vector was constructed for each of EMD and WT. Next, NN, MDDR and SVM were separately used to classify the characteristic parameters extracted by each of MF DFA, EMD and WT and the results are demonstrated in Table 2. As demonstrated in Table 2, although using fewer characteristic parameters for characterizing gearbox conditions, MF DFA delivers a better performance than each of EMD and WT in feature extraction of gearboxes. Furthermore, Table 2 states that SVM performs better than each of NN and MDDR in classification of characteristic parameters. Accordingly, the method associating MF DFA with SVM proves reliable in fault diagnosis of gearboxes again.

## 4.2 Discussions

In this paper, MF DFA, EMD and WT were employed to probe gearbox vibration data containing slightly different gear faults. The results show that the multifractal spectrum shows a great sensitivity to minor changes of gearbox conditions and can be exploited for characterizing gearbox conditions. In addition, although using fewer characteristic parameters for characterizing gearbox conditions, MF DFA seems to produce a better performance in feature extraction of gearbox vibration data. In fact, MF DFA allows a long series to be converted into a much shorter series, which distills essence of the original series. Consequently, compared with each of EMD and WT, MF DFA has a clear advantage in exhibiting nonlinear properties of a complex gearbox system.

Also, the performances of NN, MDDR and SVM were compared. Although applied to pattern classification in a wide range of areas, the NN algorithm, using the principles of empirical risk minimization, has been confronted with many tough problems<sup>9</sup>, such as easily getting into local optimization, strict requirements for large samples and poor generalization performances. Fortunately, the SVM algorithm, using the principles of structure risk minimization, can overcome some of difficulties that the NN algorithm is facing. Consequently, SVM has an obvious advantage over each of NN and MDDR, especially when facing the situations of small samples and strong nonlinearity. Nevertheless, SVM still encounters some problems in determining a kernel function and an initiative value. By contrast, as a linear classifier, MDDR can successfully avoid some difficulties which NN and SVM run into. Accordingly, although performing slightly more poorly than SVM, MDDR seems to be more computably efficient for use. Indeed, an earlier work has demonstrated the effectiveness of MDDR in classification of rolling-bearing characteristic parameters extracted by MF DFA<sup>18</sup>. In general, SVM holds an edge over MDDR but has a more complex design and higher time cost, whereas MDDR performs slightly more poorly than SVM but looks easier for use. Hence, the method associating MF DFA with each of SVM and MDDR is recommended for fault diagnosis of gearboxes in this paper.

## 5 Conclusions


This paper adopted MF DFA, EMD and WT to explore gearbox vibration data containing slightly different gear faults and compared their performances. The comparisons indicate that the multifractal spectrum acquired by MF DFA is very sensitive to small changes of gearbox conditions. Furthermore, the effectiveness of NN, MDDR and SVM were compared. The results show that SVM performs better than each of NN and MDDR. Also, it is pointed out that MDDR is more computably efficient for use than each of NN and SVM, although delivering a slightly poorer performance than SVM. Consequently, the method associating MF DFA with each of SVM and MDDR demonstrates great potential for fault diagnosis of gearboxes.

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The author can be reached at: [jslinmec@hotmail.com](mailto:jslinmec@hotmail.com)