Effects of Precise FRF Measurements for Frequency Based Substructuring

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Impedance modeling is often utilized to generate frequency response functions of an assembled system using modal characteristics of individual components. Sometimes slight inaccuracies are manifested in the characteristics of the components. Although those inaccuracies may seem minor in regards to the component for a traditional modal model, the inaccuracies can be amplified when impedance modeling is performed. Frequency based substructuring involves inversion of the frequency response functions and therefore requires considerable accuracy for the measurement, particularly in the area of anti-resonances.

For this study, some aspects of frequency based substructuring were explored, namely difficulties that might be encountered in experimental testing related to drive point measurements. The majority of studies were performed using analytical models to minimize contamination of data. The study focuses on connection point measurements where the actual location might be inaccessible due to physical constraints such as hardware, holes, etc. Hence the effect of introducing slight inaccuracies to the location of the drive point measurement was studied, along with other aspects related to the accuracy of the critical connection measurement.

Introduction

Frequency based substructuring (FBS) is one of several techniques that can be utilized to predict modal characterization of an assembled system without actually making measurements on the assembled system, but rather on the components. Details of the frequency based substructuring theory and some practical examples can be found in references 1 and 2. In order to perform frequency based substructuring, certain frequency response function (FRF) measurements are required-most notably, the drive point measurement. FRFs are needed at the locations where the structures will be connected as well as measurements relative to the desired input and output locations, as illustrated in Figure 1. However it is sometimes nearly impossible to make measurements of FRFs at certain locations; for instance it could be difficult to mount accelerometers internally on a lengthy box beam to acquire a true drive point measurement. Another issue is generated when the substructures are connected using bolts because the connection locations can be holes, bolts, or other difficult measurements. To approximate the measurement at the immeasurable locations, a common approach is to make measurements close to the locations where attachment occurs. However, depending on how far the actual measurement is from the desired measurement locations, the measured FRFs can have some dissimilarities with the desired FRFs. The differences could likely be very minute at peaks of the FRF, but are more likely to be noticeable near the anti-resonances. With inversion of the FRFs necessary in the frequency based substructuring process, those differences in the FRF anti-resonances can propagate errors to the solution.

Impetus for study

For a directed study project, frequency based substructuring was performed on two aluminum beams both analytically and experimentally. A schematic is displayed in Figure 2 to show the configuration. The test setup shown in Figure 3, involved bolting the beams together at two locations and making measurements next to the holes/rods due to the difficulty of making measurements exactly at the connection locations. During testing, there was uncertainty as to where the accelerometer and impact locations should be placed to best estimate the difficult FRFs. To approximate the measurement locations, accelerometers were placed slightly below the desired locations and impacts were made slightly above the desired locations. Although other approximations could have been made and investigated prior to testing, these points were merely intuitively chosen. When frequency based substructuring was performed using the obtained FRFs, extreme There was reason to believe that the main cause of these differences could be due to the offset of the measurements, so the following case studies were generated to study the necessity for precise FRF measurements in frequency based substructuring. The cases were executed using only analytical data to minimize data contamination and to illustrate a very basic problem devoid of any additional measurement issues.

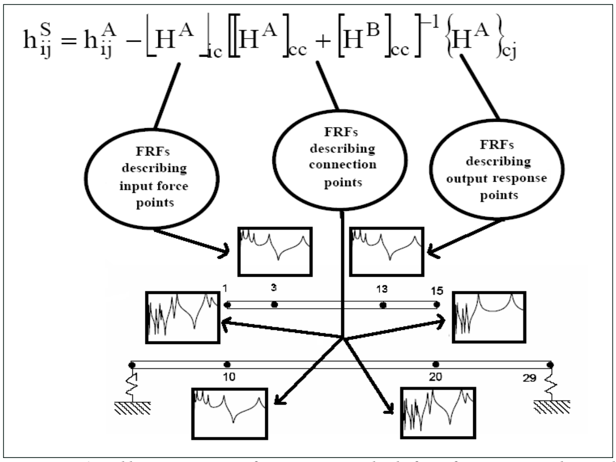


Figure 1. Illustration of FRFs needed for frequency based substructuring.

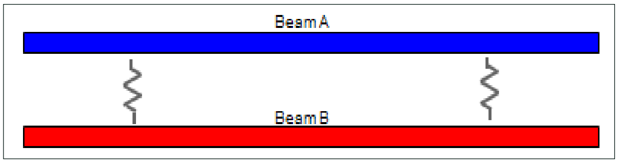


Figure 2. Basic schematic for tying two beams together.

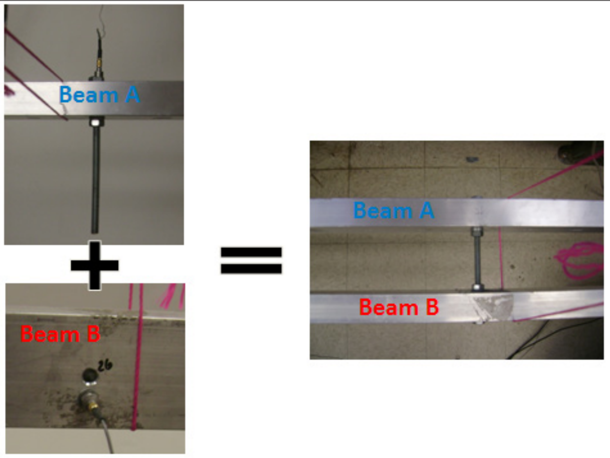


Figure 3. Experimental test setup for directed study project.

Cases studied

For this study, two simple aluminum beams were used as the test subjects to explore some possible difficulties that could be encountered when performing frequency based substructuring. The only difference between the cases was a 0.5 inch offset introduced in the FRF measurement location; all other parameters were kept constant. The cases are outlined in Figure 4, where "A" denotes location of acceleration measurement and "F" denotes location of force measurement. The first case was generated to overcome the issue of the box beam structure, while the second and third cases were generated to overcome the issue of a thru-hole for assembly at the true measurement point. The cases can be summarized as:

Case 1-Accelerometer on opposite face of box beam
Case 2-Accelerometer below point, impact above point
Case 3-Accelerometer to left of point, impact to right of point

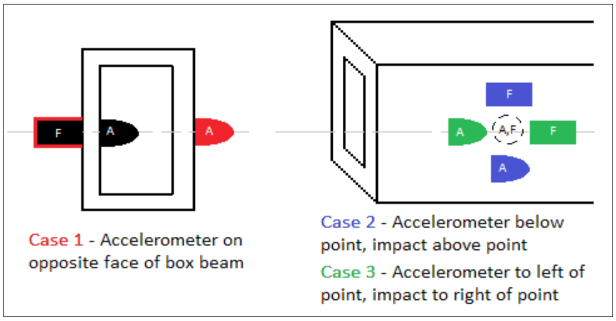
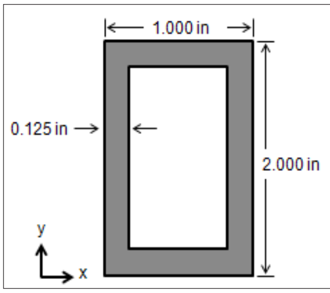


Figure 4. Comparison of drive point measurements using various impacting techniques.

Structure description & general modeling/testing performed

The test subjects for the initial project were two physical aluminum box beams, with properties displayed in Figure 5. The two box beams were assumed to be nearly identical and were tied together at two locations along the beams, as illustrated in Figure 6. For this study, the connections were only assumed to have a transverse stiffness for simplicity; this was kept constant for all cases and is an acceptable approximation because all cases involve only analytical models where the properties of the connections are dictated. However in the actual physical configuration rotation is also important, but that only further complicates the situation further and is not studied herein.



Property	Value
Length	60 in
Height	2 in
Width	1 in
Thickness	0.125 in
Elastic modulus	$9.9*10^6$ psi
Density	0.1 lbf/in^3

Figure 5. Approximate physical properties of aluminum box beams used for study.

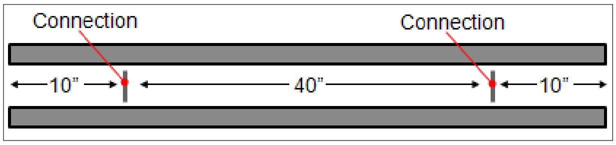
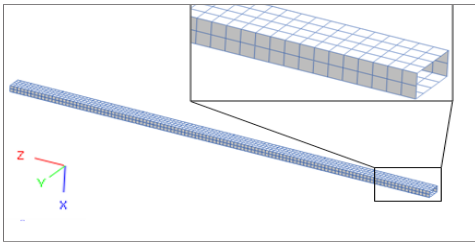


Figure 6. Schematic of initial beam attachment used in analytical and experimental studies.

A model of the box beam was generated in FEMAP[1] using plate elements and processed in FEMTools[2] the properties of this analytical model were set equal to the properties defined in Figure 5. A screenshot of the model is shown in Figure 7 along with the beam's first 15 natural frequencies. The planar modes are highlighted because those are the modes in the direction of interest; only planar motion was considered for these studies. For each case, the appropriate FRFs of the model were synthesized in FEMTools using all modes to avoid truncation. The FRFs were synthesized up to 1300 Hz such that five modes could be observed. Frequency based substructuring was used to tie the beams together, and an FRF on one of the beams of the system was analyzed. The system FRF was chosen such that all system modes within the bandwidth would be included, and the same system FRF was evaluated for each case.



Mode	Freq [Hz]
1-6	0.00
7	85.90
8	144.49
9	234.23
10	392.44
11	451.63
12	729.84
13	753.37
14	753.95
15	1059.38

Figure 7. Three dimensional model of beam used to study effects of measurement location.

Case 1-Accelerometer on opposite face of box beam

The first case was introduced to study a possible solution to the difficulty of mounting accelerometers internally on a box beam to get a true drive point measurement. A very common way to combat this issue is to mount the accelerometers on the outer face of the box beam wall opposite from where the structure is impacted. A diagram of this approximation is shown in Figure 8a, and the resulting frequency response functions synthesized from the analytical model are shown in Figure 8b. Note that this FRF is at one of the connection degrees of freedom, as defined previously. These FRFs were identical for each beam as well as each connection degree of freedom due to symmetry. As seen, offsetting the measurement produces accurate response at peaks in the frequency response function, yet most of the anti-resonances contain substantial discrepancies. At low frequencies the FRFs are comparable, but the anti-resonance becomes farther apart as frequency increases. Note that the amplitudes at the resonances are essentially identical. Although this approximation may be acceptable for certain applications where only resonances are important (such as modal models where only shapes are used), frequency based substructuring involves inversion of FRFs where accuracy of the anti-resonances is very important.

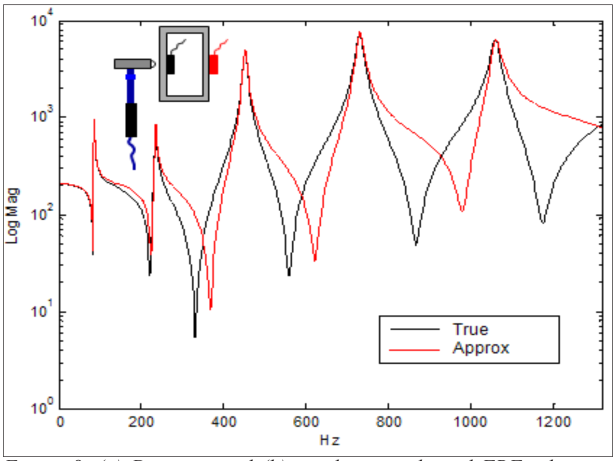


Figure 8. (a) Diagram and (b) resulting analytical FRFs showing difference between drive point measurement and having acceleration measurement on opposite wall of box beam.

Additionally, an experiment was performed to validate the characteristics observed in the analytical FRFs. An impact excitation was performed with an accelerometer mounted internally as well as an accelerometer mounted on the opposite face of the box beam, as shown in Figure 9a. Note that this measurement was performed at the end of the beam for ease of internally mounting accelerometers and therefore should not match the analytical case exactly. The resulting FRFs obtained from this experiment are shown in Figure 9b, where once again the resonances compare agreeably while the antiresonances contain noticeable discrepancies. This exercise supports the analytically discovered fact that measuring a frequency response function across a box beam is an acceptable approximation close to resonant frequencies, but a poor approximation at anti-resonances.

With experimental support of the trends being seen in the analytical FRFs, the analytical FRFs were then used to perform frequency based substructuring. All pertinent FRFs were synthesized for a true drive point measurement as well as measuring the output on the opposite box beam wall from the input. Figure 10 shows the resulting frequency response functions of the system using frequency based substructuring on both the true and approximate FRFs. In general, the resulting system FRFs do not compare very well.

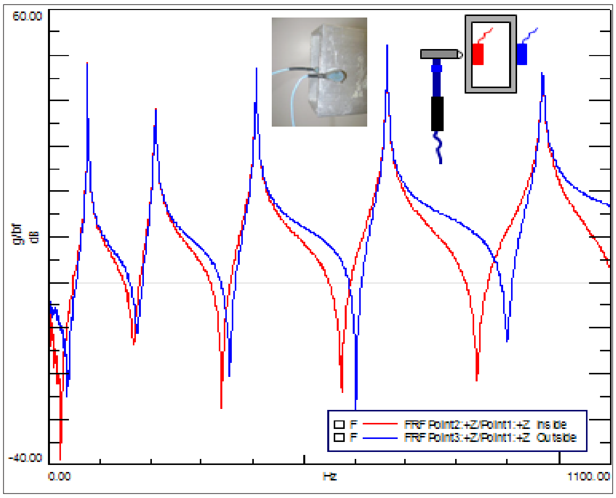


Figure 9. (a) Diagram and (b) resulting experimental FRFs obtained from placing accelerometer on opposite wall from impact and reference drive point measurement.

Looking closer, the resulting FRFs seem to compare agreeably up to about 250 Hz, which is also where the discrepancies in the individual substructures' FRFs become noticeable. This shows that only the regions where the measurements of the individual substructures are accurate will have accuracy in a system FRF after frequency based substructuring is performed. When a certain frequency range of an FRF of a substructure is initially discrepant from the true measurement, that discrepancy will be propagated to the system results. Although the differences in an FRF from measuring acceleration on the opposite wall of a box beam may seem insignificant, the approximation is not acceptable for those frequency ranges of a FBS system model.

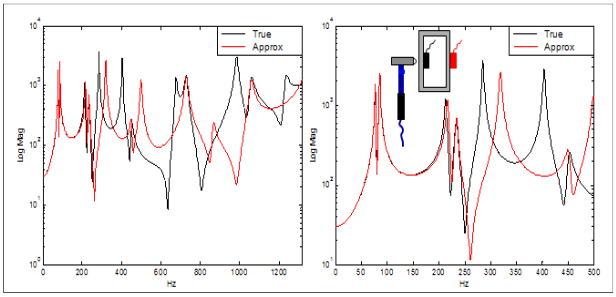


Figure 10. Comparison of FBS between drive point measurement and having acceleration measurement on opposite wall of box beam for (a) up to 1300 Hz and (b) up to 500 Hz.

In addition, a peak picking technique was used to estimate the natural frequencies of the system after performing frequency based substructuring on both the true FRFs and the estimates outlined for this case. As seen in Table 1, some of the system frequencies matched reasonably well, while others did not. The first four frequencies were obtained accurately, yet some frequencies were estimated with up to a 26 percent difference from the true system frequencies. Also the results from the approximated FRFs did not predict the sixth mode of the true system. While some frequencies were predicted acceptably, there were also several errors predicted with this approximation.

Case 2-Accelerometer below point, impact above point

The second and third cases were generated to address another common measurement difficulty-hole at the locations where the substructures would be connected. Initially the measurement was approximated above/below the actual point, as illustrated in Figure 11a, because those points would move in line with the actual point for bending about the weak axis, which was the focus of this study.

However as shown in Figure 11b, the resulting analytical FRF was not as similar to the drive point measurement as expected. Once again the resonances compare reasonably, but the anti-resonances contain major discrepancies. For this case, there were even differences between the two FRFs at frequencies lower than the first resonant frequency.

Table 1. Frequency comparison for FBS using true measurements versus approximations for Case 1.

Mode	Frequency [Hz] TRUE	Frequency [Hz] Case 1	% Diff
1	76.90	76.69	0.27
2	85.69	85.69	0.00
3	213.1	216.8	-1.74
4	234.4	234.4	0.00
5	285.6	319.3	-11.80
6	403.6	-	-
7	453.4	449.7	0.82
8	677.5	498.8	26.38
9	728.7	729.5	-0.11
10	985.8	868.6	11.89
11	1063.0	1061.0	0.19

Furthermore, due to the off-axis nature of the setup, slight effects of a torsion mode were observed, as seen around 800 Hz in the FRF.

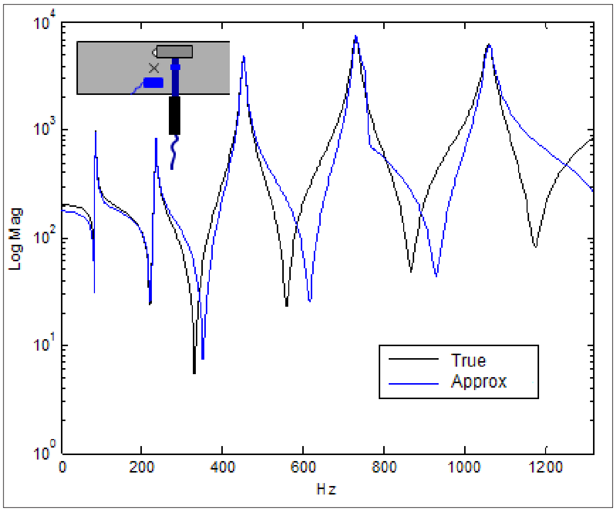


Figure 11. (a) Diagram and (b) resulting analytical FRFs showing difference between drive point mesurement and having measurement offset vertically from actual point.

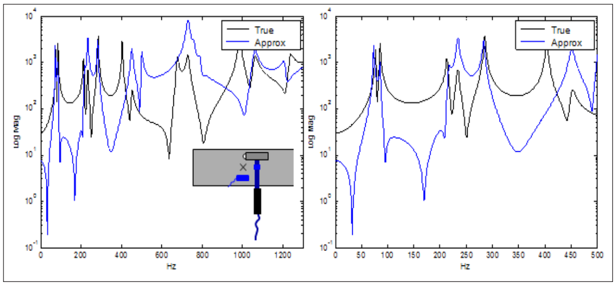


Figure 12. Comparison of FBS between drive point measurement and having measurement offset vertically from actual point for (a) up to 1300 Hz and (b) up to 500 Hz.

All pertinent FRFs were synthesized for a true drive point measurement as well as measuring the input/output above/below the true point, as detailed previously. Frequency based substructuring was performed on both data sets, with the resulting system FRFs shown in Figure 12.

Once again the resulting system FRFs do not compare very well overall. There are a few portions of the system FRFs that compare somewhat reasonably, but overall this approximation qualitatively seems worse than the first case. Because, in the original FRFs, the approximation contained differences from the true FRFs for the frequency range below the first peak, the system FRF also does not compare well for frequencies below the first peak. The irregularity noticed in the original FRF near 800 Hz was also seen to propagate through to the system FRF as another abnormality near 800 Hz. Very few portions of the system FRF match agreeably from this approximation. The approximation of taking measurements above/below a true point might seem somewhat adequate, but overall the approximation is inacceptable for use with frequency based substructuring.

Once again a peak picking technique was used to determine the frequencies of the system using both the true FRFs and the approximate FRFs, as shown in Table 2. Similar results were seen as the first case, where certain frequencies were predicted accurately while others were not. The sixth mode was also not predicted using this approximation. However this estimate caused an additional frequency to be predicted near 800 Hz, which, as mentioned previously, could possibly be attributed to the torsion mode seen in the component FRFs. Furthermore, when comparing Table 2 and Figure 12, another difference is noted; although certain frequencies were predicted accurately, such as the 234 Hz mode, there is significant difference between the amplitude of the peaks. Depending on the application, this approximation may or may not be appropriate.

Table 2. Frequency comparison for FBS using true measurements versus approximations for Case 2.

Mode	Frequency [Hz]		% Diff
	TRUE	Case 2	
1	76.90	73.97	3.81
2	85.69	85.69	0.00
3	213.1	217.5	-2.06
4	234.4	234.4	0.00
5	285.6	283.4	0.77
6	403.6	-	-
7	453.4	451.9	0.33
8	677.5	502.4	25.85
9	728.7	729.5	-0.11
10	-	758.0	-
11	985.8	787.3	18.67
12	1063.0	1061.0	0.19

Case 3-Accelerometer to left of point, impact to right of point

The third case, as mentioned previously, was also generated to combat an issue that can be presented when connection measurements are needed, but the connection location involves holes or rods. This case involved measuring FRFs to the left/right of the actual point, as outlined in Figure 13a. The measurements were kept along the centerline to avoid the issue of torsion modes appearing as seen in case 2. Synthesized FRFs of this approximation and a true drive point at the connection location are shown in Figure 13b. Note that although there are not any detectable torsion modes as in the previous case, there are still major differences between the two FRFs at the anti-resonances. The peaks of both frequency response functions do match acceptably, although the regions between peaks are noticeably different.

The FRFs necessary for frequency based substructuring were synthesized using the horizontally offset approximations. FBS was performed using both the true measurements and the approximate measurements, with the resulting system FRFs shown in Figure 14. Once again certain portions of the resulting FRF compare somewhat adequately, but overall the system FRF is not acceptable. Similar to the previous cases, slight discrepancies in component FRFs can be transmitted to extreme inconsistencies in system FRFs after performing frequency based substructuring.

The estimated and true system frequencies were once again determined using a peak picking technique, as shown in Table 3. Yet again, certain natural frequencies were predicted with greater accuracy than others. While the other two cases only missed the prediction of one natural frequency, this estimation also caused the eighth mode to be missed. The accuracy of this estimation was slightly better than the previous two cases, with only one mode with higher than ten percent difference from the true solution. Clearly the estimations made in all three cases studied produced issues when used for frequency based substructuring.

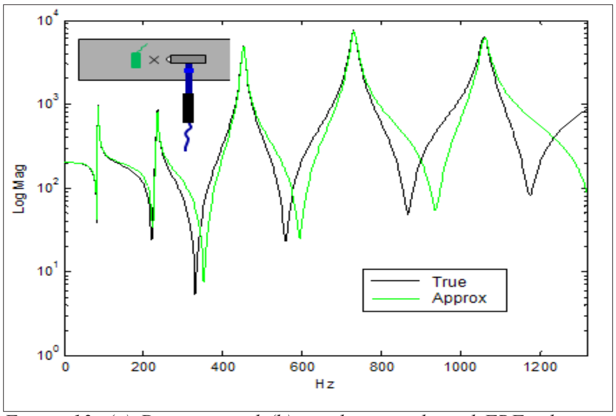


Figure 13. (a) Diagram and (b) resulting analytical FRFs showing difference between drive point measurement and having measurement offset horizontally from actual point.

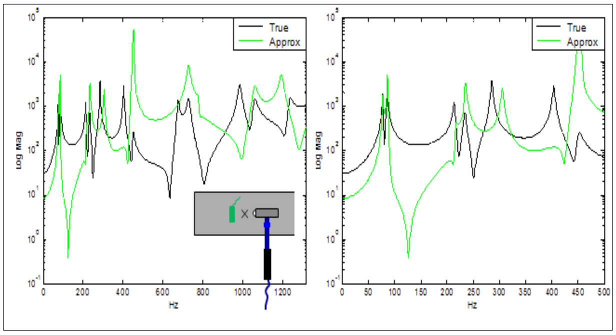


Figure 14. Comparison of FBS between drive point measurement and having measurement offset horizontally from actual point for (a) up to 1300 Hz and (b) up to 500 Hz.

Table 3. Frequency comparison for FBS using true measurements versus approximations for Case 3.

Mode	Frequency [Hz]		% Diff
	TRUE	Case 3	
1	76.90	77.63	-0.95
2	85.69	85.69	0.00
3	213.1	217.5	-2.06
4	234.4	235.1	-0.30
5	285.6	305.4	-6.93
6	403.6	-	-
7	453.4	453.4	0.00
8	677.5	-	-
9	728.7	729.5	-0.11
10	985.8	774.9	21.39
11	1063.0	1062.0	0.09

Conclusions

This paper presents some important information in regards to the need for very precisely measured frequency response functions. This is especially true for frequency based substructuring applications where the precision of the frequency response functions are of critical importance. Sometimes when certain frequency measurements are difficult to obtain, slight approximations are made. Although those approximations may not seem extremely different from the true measurements, those slight differences can have a substantial effect on the system results after performing frequency based substructuring. Several cases were explored to show how very minor effects that may not be of a severe nature for mode shape extraction, become very critical in frequency based substructuring system modeling applications. Care should be taken when dealing with those difficulties by making the best possible approximations to ensure accurate system results.

References

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