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Computational Approximations for Real-World Application of Epidemic Model

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Abstract: The real-world applications and analysis have a significant role in the scientific literature. For instance, mathematical modeling, computer graphics, camera, operating system, Java, disk encryption, web, streaming, and many more are the applications of real-world problems. In this case, we consider disease modeling and its computational treatment. Computational approximations have a significant role in different sciences such as behavioral, social, physical, and biological sciences. But the well-known techniques that are widely used in the literature have many problems. These methods are not consistent with the physical nature and even violate the actual behavior of the continuous model. The structural properties needed for such disciplines, as dynamical consistency, positivity, and boundedness, are the critical requirements of the models in these fields. We studied the transmission of Lassa fever dynamically and numerically. The model's positivity, boundedness, reproduction number, equilibria, and local stability are investigated in dynamical analysis. In numerical analysis, we developed some explicit and implicit methods. Unfortunately, explicit methods like Euler and Runge Kutta are time-dependent and violate the physical properties of the disease. Then, the proposed implicit method for the said model, the non-standard finite difference, is independent of the time step, dynamically consistent, positive, and bounded. In the end, a comparison of methods is presented.

Keywords: Lassa fever disease; epidemic model; computational approximations; convergence analysis

1 Literature Survey

Lassa fever is an intense hemorrhagic illness caused by the Lassa virus (member of the arenavirus). Lassa virus carries a rat, which is very common in West Africa. It is also known as a zoonosis, which

means that disease spreads from animal to human. People usually become infected with Lassa fever because of food and household items' exposure to urine of infected Mastomys rats. Its symptoms are varied and include cardiac, neurological, and pulmonary problems. In some West Africa, this disease is endemic to the rodent population. It is most common in Liberia, Sierra Leone, Guinea, and Nigeria. Lassa fever is transmitted by playing, touching, and cutting up a rat's dead body. In 2018, Usman et al. analyzed the Lassa fever virus infection for the transmission dynamics of qualitative and analytic activities of a mathematical model [1]. In 2020, Peter et al. studied Lassa fever's dynamics, and the solution model stayed and verified boundedness and positivity of basic properties [2]. In 2017, Olayiwola et al. analyzed in the lowliest endemic countries people with an ultimate risk of infection and need for constant investigation develops commanding in the endemic region like Africa with Nigeria at the essential attention will help in no small processes in scheming the scourge [3]. Woyessa et al. investigated the private and public health facilities, control interventions, and prevent infection when feverish patients avoid nosocomial infections [4]. In 2013, Ajayi et al. have studied that Epidemic contained rejoinder schemes and testing to control exertions comprised fright between health staffs, insufficient/poor quality of defensive things, insufficient extra preparation, and poor local laboratory capability [5]. In 2019, Ilori et al. analyzed the activity supporting improving planned for patient care and LLC, emerging infectious diseases, and Medscape through applied epidemiological characteristics and clarifying factors associated with mortality [6]. Adewuyi et al. studied the disease have the probable of actuality organize an infectious menace that essential be controlled by way of biological weapon and currently, vaccine of Lassa fever no available and some natural problems occurs for development of vaccines so prevention the way by control the rodent [7]. Iroezindu et al. analyzed Lassa fever spread; challenges, letdown to use proper defensive tackle, stigmatization of associates, and absence of a purpose-built isolation facility [8]. In 2019, Amodu et al. have studied Lassa fever as an acute disease of scarceness, high endemicity by way of cooperated environmentally-friendly sanitation, and relics susceptible populations to community health problems in Nigeria. Public meeting defense for attractive prevention strategy remains particular sanitation [9]. In 2019, Kangbai et al. highlighted that seasonal epidemics use an effective treatment to make a stratagem, which can control procedures of Lassa fever prevention and control the connection between humans to rodents [10]. Makinde et al. investigated Lassa fever to identify when a nonconformity toward the prestige quo has happened [11]. In 2020, Zhao et al. analyzed quantify of this impact in Nigeria's presence of Lassa fever significance measure the connotation among local precipitation and infection reproduction number which facts has probable elect applied as a bad sign for Lassa fever epidemics [12]. In 2017, Obabiyi et al. developed a mathematical model for transmission of Lassa fever dynamics with the behavior of susceptible humans, recovered humans divided the population into two parts such as human populations, and rodent population by using the positivity, bounded theorem, and suggested the stability hygiene of environment [13]. In 2018, Akpede et al. studied the necessities for achievement and enduring capacity of the control exertions in Nigeria and the sub-region. In wholly these, the Nigerian administration with NCDC necessity carries a huge responsibility for the organization, supply deployment, and support. If necessary, even persuade sub-regional administrations addicted to action. In addition, there should be expected through determined action [14]. In 2019, Mazzola et al. explored the Diagnostics necessary for acknowledging and controlling epidemics of LASV, unique prevailing and genetically various mediators of VHF, that use scenarios with different performance requirements for text complexity, sensitivity, specificity, and development time [15]. In 2019, Nwafor et al. examined the Lassa fever outbreak in Nigeria; the Health maintenance workers necessity, take a high index of doubt of the infection and follow IPC measures even though provided that maintenance for all patients. Explaining health maintenance workers by the new strategies mentioned above is also significant to reduce the menace of nosocomial transmission of Lassa fever [16]. In 2018, Shehu et al. studied that the Occurrence of rural to urban change of clinical and epidemiological reduced the Lassa fever cases during 2016 for morbidity and mortality [17]. In 2007, Ogbu et al. discussed the

situation of Lassa fever in the sub-region of West Africa and suggested strategies for socioeconomic behavior that control the shortage of health care system [18]. In 2020, Tewogbola et al. analyzed the overview and discussed the main reasons it damaged the human population and recommended the control measure of Lassa fever [19]. In 2014, Ajayi et al. reported a case of 59 years that recovers without taking a vaccine such as ribavirin. The symptoms of this disease increase day by day because few people do not use the main precautions to control Lassa fever [20]. Some well-known numerical models related to diseases are studied [21–26].

2 Formulation of Lassa Fever Model

The variables and parameters are described of the lassa fever model as follows: $S_H(t)$: denoted as the susceptible class at any time t, $I_H(t)$: characterized as the infectious class at any time t, $R_H(t)$: characterized as the recovered class at any time t, $S_R(t)$: characterized as the susceptible rodent vectors at any time t, $I_R(t)$: characterized as the infectious rodent vectors at any time t, $N_H(t)$: characterized as whole humans' population at any time t, $m = \frac{N_R}{N_H}$: characterized as the number of infectious rodent vectors by the human host, α_1: described as the rate at which contagious rodent vectors and a susceptible class of humans interact with each other, α_2: defined as the force of infection, α_3: defined as the rate at which sensitive rodent vectors and an infectious class of humans interact with each other, τ_c: denoted the speed at which infectious human hosts comply with the drug, τ_{nc}: indicated the rate at which infectious human hosts do not comply with the drug, r_c: denoted the rate at which infectious human hosts are educated to adhere to the medication, δ: indicated the rate of mortality of an infectious class, γ: indicated the rate at which humans may lose their immunity. The leading equations of the model are as follows:

$$\frac{dS_H(t)}{dt} = \Lambda_H - \frac{\alpha_1\alpha_2 S_H(t)I_R(t)}{N_H} + \gamma R_H(t) + \tau_{nc} I_H(t) - \mu_H S_H(t), t \geq 0. \tag{1}$$

$$\frac{dI_H(t)}{dt} = \frac{\alpha_1\alpha_2 S_H(t)I_R(t)}{N_H} - \tau_c I_H(t) - r_c I_H(t) - \tau_{nc} I_H(t) - \delta I_H(t) - \mu_H I_H(t), t \geq 0. \tag{2}$$

$$\frac{dR_H(t)}{dt} = \tau_c I_H(t) + r_c I_H(t) - \gamma R_H(t) - \mu_H R_H(t), t \geq 0. \tag{3}$$

$$\frac{dS_R(t)}{dt} = \Lambda_R - \frac{\alpha_1\alpha_3 S_R(t)I_H(t)}{N_H} - \mu_R S_R(t), t \geq 0. \tag{4}$$

$$\frac{dI_R(t)}{dt} = \frac{\alpha_1\alpha_3 S_R(t)I_H(t)}{N_H} - \mu_R I_R(t), t \geq 0. \tag{5}$$

where $S_H(0) \geq 0, I_H(0) \geq 0, R_H(0) \geq 0, S_R(0) \geq 0, I_R(0) \geq 0$.

2.1 Fundamental Properties of Model

We studied the feasible region, positivity, and boundedness of the model at any time, $t \geq 0$, as follows:

$$\Pi = \left\{(S_H, I_H, R_H, S_R, I_R)\epsilon R_+^5 : S_H + I_H + R_H \leq \frac{\Lambda_H}{\mu_H}, S_R + I_R \leq \frac{\Lambda_R}{\mu_R}, S_H \geq 0, I_H \geq 0, R_H \geq 0, S_R \geq 0, I_R \geq 0\right\}.$$

Lemma 1: The solutions $(S_H, I_H, R_H, S_R, I_R)\epsilon R_+^5$ of Eqs. (1)–(5) is positive at any time $t \geq 0$, with given non-negative initial conditions.

Proof: It is clear from Eqs. (1)–(5),

$\frac{dS_H}{dt}\big|_{S_H=0} = \Lambda_H + \gamma R_H + \tau_{nc} I_H \geq 0$, $\frac{dI_H}{dt}\big|_{I_H=0} = \frac{\alpha_1\alpha_2 S_H I_R}{N_H} \geq 0$, $\frac{dR_H}{dt}\big|_{R_H=0} = \tau_c I_H + r_c I_H \geq 0$, $\frac{dS_R}{dt}\big|_{S_R=0} = \Lambda_R \geq 0$, $\frac{dI_R}{dt}\big|_{I_R=0} = \frac{\alpha_1\alpha_3 S_R I_H}{N_H} \geq 0$, as desired.

Lemma 2: Forgiven any non-negative initial conditions for the solution of the system (1)–(5) is bounded if $\lim_{t\to\infty} \text{Sup } N_H(t) \le \frac{\Lambda_H}{\mu_H}$, $\lim_{t\to\infty} \text{Sup } N_R(t) \le \frac{\Lambda_R}{\mu_R}$.

Proof: let us consider the population function as follows:

$$N_H(t) = S_H + I_H + R_H, \frac{dN_H}{dt} = \frac{dS_H}{dt} + \frac{dI_H}{dt} + \frac{dR_H}{dt}, \frac{dN_H}{dt} = \Lambda_H - \mu_H N_H,$$

$$N_H(t) = A + \frac{\Lambda_H}{\mu_H}$$

By the Gronwall's inequality, we get

$$N_H(t) \le N_H(0) + \frac{\Lambda_H}{\mu_H}, t \ge 0, \lim_{t\to\infty} \text{Sup } N_R(t) \le \frac{\Lambda_H}{\mu_H}$$

$$N_R(t) = S_R + I_R, \frac{dN_R}{dt} = \frac{dS_R}{dt} + \frac{dI_R}{dt}, \frac{dN_R}{dt} = \Lambda_R - \mu_R N_R$$

$$N_R(t) = B + \frac{\Lambda_R}{\mu_R}$$

By the Gronwall's inequality, we get

$$\mathbf{N_R(t) \le N_R(0) + \frac{\Lambda_R}{\mu_R}, t \ge 0}$$

$\lim_{t\to\infty} \text{Sup } N_R(t) \le \frac{\Lambda_R}{\mu_R}$, as desired.

2.2 Steady States of Lassa fever Model

There are two steady states of Eqs. (1) to (5), as follows: disease-free equilibrium (DFE) $= (S_H, I_H, R_H, S_R.I_R) = \left(\frac{\Lambda_H}{\mu_H}, 0, 0, \frac{\Lambda_R}{\mu_R}, 0\right)$ and endemic equilibrium (EE) $= \left({S_H}^*, {I_H}^*, {R_H}^*, S_R^*, I_R^*\right)$,

$$R_H^* = \frac{(\tau_c + r_c)I_H^*}{\gamma + \mu_H} = A_1 I_H^*, A_1 = \frac{(\tau_c + r_c)}{\gamma + \mu_H}, S_H^* = \frac{\Lambda_H + \gamma A_1 I_H^* - A_2 I_H^*}{\mu_H}, A_2 = \tau_c + r_c + \delta + \mu_H, I_R^* = \frac{\alpha_1\alpha_3 S_R^* I_H^*}{\mu_R},$$

$$S_R^* = \frac{\Lambda_R}{\alpha_1\alpha_3 I_H^* + \mu_R}, I_H^* = \frac{\Lambda_H - A_4\mu_R}{A_4\alpha_1\alpha_3 - \gamma A_1 + A_2}, A_3 = \tau_c + r_c + \tau_{nc} + \delta + \mu_H, A_4 = \frac{A_3\mu_H\mu_R}{\alpha_1^2\alpha_2\alpha_3\Lambda_R}.$$

3 Reproduction Number of Lassa Fever Model

In this section, we shall find the two types of matrices like transmission and transition by assuming the disease-free equilibria in the system (1)–(5) by using the next-generation matrix method, furthermore, considering the infected classes as follows:

$$F = \begin{bmatrix} \frac{\alpha_1\alpha_2\Lambda_H}{\mu_H N_H} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha_1\alpha_2\Lambda_R}{\mu_R N_H} \end{bmatrix}, V^{-1} = \frac{\begin{bmatrix} \mu_R(\gamma+\mu_H) & 0 & 0 \\ \mu_R(\tau_c+r_c) & \mu_R(\tau_c+r_c+\tau_{nc}+\delta+\mu_H) & 0 \\ 0 & 0 & (\tau_c+r_c+\tau_{nc}+\delta+\mu_H)(\gamma+\mu_H) \end{bmatrix}}{(\tau_c+r_c+\tau_{nc}+\delta+\mu_H)(\gamma+\mu_H)\mu_R}.$$

$$FV^{-1} = \frac{1}{(\tau_c+r_c+\tau_{nc}+\delta+\mu_H)(\gamma+\mu_H)\mu_R} \begin{bmatrix} \frac{\alpha_1\alpha_2\Lambda_H}{\mu_H N_H}(\mu_R)(\gamma+\mu_H) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha_1\alpha_2\Lambda_R}{\mu_R N_H}(\tau_c+r_c+\tau_{nc}+\delta+\mu_H)(\gamma+\mu_H) \end{bmatrix}.$$

Thus, the dominant eigenvalue of the matrix is called reproduction number and denoted as follows:

$$R_0 = \frac{\frac{\alpha_1\alpha_2\Lambda_H}{\mu_H N_H}}{(\tau_c + r_c + \tau_{nc} + \delta + \mu_H)}.$$

4 Local Stability

In this section, we present two well-known theorems for stability in the sense of local. Again, consider the system (1)–(5) as function of A, B, C, D and E as follows:

$$A = \Lambda_H - \frac{\alpha_1\alpha_2 S_H I_R}{N_H} + \gamma R_H + \tau_{nc} I_H - \mu_H S_H. \tag{6}$$

$$B = \frac{\alpha_1\alpha_2 S_H I_R}{N_H} - \tau_c I_H - r_c I_H - \tau_{nc} I_H - \delta I_H - \mu_H I_H. \tag{7}$$

$$C = \tau_c I_H + r_c I_H - \gamma R_H - \mu_H R_H. \tag{8}$$

$$D = \Lambda_R - \frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R S_R. \tag{9}$$

$$E = \frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R I_R. \tag{10}$$

Theorem 1: The disease-free equilibrium is locally asymptotically stable for the system (6)–(10). If $R_0 < 1$ and otherwise unstable if $R_0 > 1$.

Proof: First, we take the partial derivates of the system (6)–(10) concerning state variables as follows:

$$\frac{\partial A}{\partial S_H} = \frac{-\alpha_1\alpha_2 I_R}{N_H} - \mu_H, \frac{\partial A}{\partial I_H} = \tau_{nc}, \frac{\partial A}{\partial R_H} = \gamma, \frac{\partial A}{\partial S_R} = 0, \frac{\partial A}{\partial I_R} = \frac{-\alpha_1\alpha_2 S_H}{N_H}.$$

$$\frac{\partial B}{\partial S_H} = \frac{\alpha_1\alpha_2 I_R}{N_H}, \frac{\partial B}{\partial I_H} = -\tau_c - r_c - \tau_{nc} - \delta - \mu_H, \frac{\partial B}{\partial R_H} = 0, \frac{\partial B}{\partial S_R} = 0.$$

$$\frac{\partial B}{\partial I_R} = \frac{\alpha_1\alpha_2 S_H}{N_H}, \frac{\partial C}{\partial S_H} = 0 \frac{\partial C}{\partial I_H} = \tau_c + r_c, \frac{\partial C}{\partial R_H} = -\gamma - \mu_H, \frac{\partial C}{\partial S_R} = 0,$$

$$\frac{\partial C}{\partial I_R} = 0, \frac{\partial D}{\partial S_H} = 0, \frac{\partial D}{\partial I_H} = \frac{-\alpha_1\alpha_3 S_R}{N_H}, \frac{\partial D}{\partial R_H} = 0, \frac{\partial D}{\partial S_R} = \frac{-\alpha_1\alpha_3 I_H}{N_H} - \mu_R, \frac{\partial D}{\partial I_R} = 0, \frac{\partial F}{\partial S_H} = 0,$$

$$\frac{\partial F}{\partial I_H} = \frac{\alpha_1\alpha_3 S_R}{N_H}, \frac{\partial F}{\partial R_H} = 0, \frac{\partial F}{\partial S_R} = \frac{\alpha_1\alpha_3 I_H}{N_H}, \frac{\partial F}{\partial I_R} = -\mu_R.$$

Here, the Jacobian matrix as follows:

$$J = \begin{bmatrix} \frac{-\alpha_1\alpha_2 I_R}{N_H} - \mu_H & \tau_{nc} & \gamma & 0 & \frac{-\alpha_1\alpha_2 S_H}{N_H} \\ \frac{\alpha_1\alpha_2 I_R}{N_H} & -\tau_c - r_c - \tau_{nc} - \delta - \mu_H & 0 & 0 & \frac{\alpha_1\alpha_2 S_H}{N_H} \\ 0 & \tau_c + r_c & -\gamma - \mu_H & 0 & 0 \\ 0 & \frac{-\alpha_1\alpha_3 S_R}{N_H} & 0 & \frac{-\alpha_1\alpha_3 I_H}{N_H} - \mu_R & 0 \\ 0 & \frac{\alpha_1\alpha_3 S_R}{N_H} & 0 & \frac{\alpha_1\alpha_3 I_H}{N_H} & -\mu_R \end{bmatrix}$$

The Jacobian matrix at the disease-free equilibria of the system (6)–(10) as follows:

$$J(E_0) = J\left(\frac{\Lambda_H}{\mu_H}, 0, 0, \frac{\Lambda_R}{\mu_R}, 0\right) = \begin{bmatrix} -\mu_H & \tau_{nc} & \gamma & 0 & \frac{-\alpha_1\alpha_2\Lambda_H}{\mu_H N_H} \\ 0 & -\tau_c - r_c - \tau_{nc} - \delta - \mu_H & 0 & 0 & \frac{\alpha_1\alpha_2\Lambda_H}{\mu_H N_H} \\ 0 & \tau_c + r_c & -\gamma - \mu_H & 0 & 0 \\ 0 & \frac{-\alpha_1\alpha_3\Lambda_R}{\mu_R N_H} & 0 & 0 - \mu_R & 0 \\ 0 & \frac{\alpha_1\alpha_3\Lambda_R}{\mu_R N_H} & 0 & 0 & -\mu_R \end{bmatrix}.$$

$$|J - \lambda I| = \begin{vmatrix} -\mu_H - \lambda & \tau_{nc} & \gamma & 0 & \frac{-\alpha_1\alpha_2\Lambda_H}{\mu_H N_H} \\ 0 & (-\tau_c - r_c - \tau_{nc} - \delta - \mu_H) - \lambda & 0 & 0 & \frac{\alpha_1\alpha_2\Lambda_H}{\mu_H N_H} \\ 0 & \tau_c + r_c & -\gamma - \mu_H - \lambda & 0 & 0 \\ 0 & \frac{-\alpha_1\alpha_3\Lambda_R}{\mu_R N_H} & 0 & 0 - \mu_{R-\lambda} & 0 \\ 0 & \frac{\alpha_1\alpha_3\Lambda_R}{\mu_R N_H} & 0 & 0 & -\mu_R - \lambda \end{vmatrix} = 0.$$

$\lambda_1 = -\mu_H < 0, \lambda_2 = -\mu_R < 0, \lambda_3 = -(\gamma + \mu_H) < 0.$

$$|\mathrm{J} - \lambda I| = \begin{vmatrix} (-\tau_c - r_c - \tau_{nc} - \delta - \mu_H) - \lambda & \frac{\alpha_1\alpha_2\Lambda_H}{\mu_H N_H} \\ \frac{\alpha_1\alpha_3\Lambda_R}{\mu_R N_H} & -\mu_R - \lambda \end{vmatrix} = 0.$$

$\lambda^2 + \lambda(a_1 + \mu_R) + a_1\mu_R - a_2 = 0.$

where, $a_1 = \tau_c + r_c + \tau_{nc} + \delta + \mu_H, a_2 = \left(\frac{\alpha_1\alpha_2\Lambda_H}{\mu_H N_H}\right)\left(\frac{\alpha_1\alpha_3\Lambda_R}{\mu_R N_H}\right)$

Since all the coefficients of the polynomial are positive, therefore, by using Routh Hurwitz Criteria for 2nd order, the disease-free equilibria are locally asymptotically stable.

Theorem 2: The endemic equilibrium is locally asymptotically stable for the system (6)–(10) if $\mathrm{R}_0 > 1$.

Proof: The Jacobian matrix at the endemic equilibria of the system (6)–(10) is as follows:

$$\mathrm{J}(E^*) = \mathrm{J}\left(S_H{}^*, I_H{}^*, R_H{}^*, S_R^*, I_R^*\right) = \begin{bmatrix} \frac{-\alpha_1\alpha_2 I_R^*}{N_H} - \mu_H & \tau_{nc} & \gamma & 0 & \frac{-\alpha_1\alpha_2 S_H{}^*}{N_H} \\ \frac{\alpha_1\alpha_2 I_R^*}{N_H} & -\tau_c - r_c - \tau_{nc} - \delta - \mu_H & 0 & 0 & \frac{\alpha_1\alpha_2 S_H{}^*}{N_H} \\ 0 & \tau_c + r_c & -\gamma - \mu_H & 0 & 0 \\ 0 & \frac{-\alpha_1\alpha_3 S_R^*}{N_H} & 0 & \frac{-\alpha_1\alpha_3 I_H{}^*}{N_H} - \mu_R & 0 \\ 0 & \frac{\alpha_1\alpha_3 S_R^*}{N_H} & 0 & \frac{\alpha_1\alpha_3 I_H{}^*}{N_H} & -\mu_R \end{bmatrix}.$$

$$|J - \lambda I| = \begin{vmatrix} -B_1 - \mu_H - \lambda & \tau_{nc} & \gamma & 0 & -B_2 \\ B_1 & -B_3 - \lambda & 0 & 0 & B_2 \\ 0 & B_4 & -B_5 - \lambda & 0 & 0 \\ 0 & -B_6 & 0 & -B_7 - \mu_R - \lambda & 0 \\ 0 & B_6 & 0 & B_7 & -\mu_R - \lambda \end{vmatrix} = 0.$$

where, $B_1 = \frac{\alpha_1\alpha_2 I_R^*}{N_H}, B_2 = \frac{\alpha_1\alpha_2 S_H{}^*}{N_H}, B_3 = \tau_c + r_c + \tau_{nc} + \delta + \mu_H, B_4 = \tau_c + r_c, B_5 = \gamma + \mu_H, B_6 = \frac{\alpha_1\alpha_3 S_R^*}{N_H}$, $B_7 = \frac{\alpha_1\alpha_3 I_H{}^*}{N_H}$.

$$|J-\lambda I|=(-B_1-\mu_H-\lambda)\begin{vmatrix} B_4 & -B_5-\lambda & 0 & 0 \\ -B_6 & 0 & -B_7-\mu_R-\lambda & 0 \\ B_6 & 0 & B_7 & -\mu_R-\lambda \end{vmatrix}-B_1\begin{vmatrix} B_4 & -B_5-\lambda & 0 & 0 \\ -B_6 & 0 & -B_7-\mu_R-\lambda & 0 \\ B_6 & 0 & B_7 & -\mu_R-\lambda \end{vmatrix}=0.$$

$$\begin{aligned}|J-\lambda I|=&(-B_1-\mu_H-\lambda)(B_5+\lambda)\left[(-B_3-\lambda)\begin{vmatrix}-B_7-\mu_R-\lambda & 0 \\ B_7 & -\mu_R-\lambda\end{vmatrix}+B_2\begin{vmatrix}-B_6 & -B_7-\mu_R-\lambda \\ B_6 & B_7\end{vmatrix}\right]\\ &+(-B_1)(-\gamma)\left[(B_4)\begin{vmatrix}-B_7-\mu_R-\lambda & 0 \\ B_7 & -\mu_R-\lambda\end{vmatrix}\right]\\ &+(-B_1)(B_5+\lambda)\left[\tau_{nc}\begin{vmatrix}-B_7-\mu_R-\lambda & 0 \\ B_7 & -\mu_R-\lambda\end{vmatrix}-B_2\begin{vmatrix}-B_6 & -B_7-\mu_R-\lambda \\ B_6 & B_7\end{vmatrix}\right]=0.\end{aligned}$$

$\lambda^5+\lambda^4[2\mu_R+B_3+\mu_H+B_7+B_5+B_1]+\lambda^3[B_7\mu_R+{\mu_R}^2+2B_3\mu_R+2B_5\mu_R+2B_1\mu_R+B_3B_7+B_5B_7+B_1B_7+B_3B_5+B_1B_3-B_2B_6+B_5\mu_H+B_3\mu_H+B_7\mu_H+2\mu_R\mu_H+B_1B_5-B_1\tau_{nc}]+\lambda^2[B_3B_7\mu_R+B_5B_7\mu_R+B_1B_7\mu_R+B_3{\mu_R}^2+B_5{\mu_R}^2+B_1{\mu_R}^2+2B_3B_5\mu_R+2B_1B_3\mu_R+2B_1B_5\mu_R+B_3B_5B_7+B_1B_3B_7+B_1B_5B_7+B_1B_3B_5+B_1B_4\gamma-2B_1\mu_R\tau_{nc}-B_1B_7\tau_{nc}-B_1B_5\tau_{nc}-B_1B_2B_6+B_3B_7\mu_H+B_7B_5\mu_H-2B_2B_6\mu_H+2B_3\mu_R\mu_H+2B_5\mu_R\mu_H+B_7\mu_R\mu_H-B_2B_5B_6]+\lambda[B_3B_5B_7\mu_R+B_1B_3B_7\mu_R+B_1B_5B_7\mu_R+B_3B_5{\mu_R}^2+B_5{\mu_R}^2+B_1B_5{\mu_R}^2+2B_1B_3B_5\mu_R-B_2B_6B_5\mu_R-2B_1B_2B_5B_6+2B_1B_4\gamma\mu_R-B_1B_2B_6\gamma-2B_1B_5\tau_{nc}\mu_R-B_1B_5B_7\tau_{nc}+B_7B_3B_5\mu_H+2B_3B_5\mu_R\mu_H-2B_1B_2B_7B_6-2B_1B_2B_6\mu_R+B_5B_3B_7\mu_H+B_7B_5\mu_R\mu_H+B_3B_7\mu_R\mu_H-B_2B_6\mu_R\mu_H-B_2B_6B_5\mu_H-B_1B_2B_6]+B_1B_3B_5B_7\mu_R+B_1B_3B_5{\mu_R}^2+B_1B_4B_7\gamma\mu_R+B_1B_4\gamma{\mu_R}^2-B_1B_5B_7\tau_{nc}\mu_R+B_1B_2B_6\gamma\mu_R-B_1B_5\tau_{nc}{\mu_R}^2+B_1B_3B_7B_5-2B_1B_2B_5B_6\mu_R-2B_1B_2B_5B_7B_6-B_2B_5B_6\mu_R\mu_H+B_5B_3B_7\mu_R\mu_H+B_3B_5{\mu_R}^2\mu_H=0.$

$\lambda^5a_1+\lambda^4a_2+\lambda^3a_3+\lambda^2a_4+\lambda a_5+a_6=0.$

where, $a_1=[1], a_2=-[2\mu_R+B_3+\mu_H+B_7+B_5+B_1].$

$$\begin{aligned}a_3=-\big[&B_7\mu_R+{\mu_R}^2+2B_3\mu_R+2B_5\mu_R+2B_1\mu_R+B_3B_7+B_5B_7+B_1B_7+B_3B_5+B_1B_3-B_2B_6\\&+B_5\mu_H+B_3\mu_H+B_7\mu_H+2\mu_R\mu_H+B_1B_5-B_1\tau_{nc}\big].\end{aligned}$$

$$\begin{aligned}a_4=-[&B_3B_7\mu_R+B_5B_7\mu_R+B_1B_7\mu_R+B_3{\mu_R}^2+B_5{\mu_R}^2+B_1{\mu_R}^2+2B_3B_5\mu_R+2B_1B_3\mu_R+2B_1B_5\mu_R\\&+B_3B_5B_7+B_1B_3B_7+B_1B_5B_7+B_1B_3B_5+B_1B_4\gamma-2B_1\mu_R\tau_{nc}-B_1B_7\tau_{nc}-B_1B_5\tau_{nc}\\&-B_1B_2B_6+B_3B_7\mu_H+B_7B_5\mu_H-2B_2B_6\mu_H+2B_3\mu_R\mu_H+2B_5\mu_R\mu_H+B_7\mu_R\mu_H-B_2B_5B_6].\end{aligned}$$

$$\begin{aligned}a_5=-\big[&B_3B_5B_7\mu_R+B_1B_3B_7\mu_R+B_1B_5B_7\mu_R+B_3B_5{\mu_R}^2+B_5{\mu_R}^2+B_1B_5{\mu_R}^2+2B_1B_3B_5\mu_R\\&-B_2B_6B_5\mu_R-2B_1B_2B_5B_6+2B_1B_4\gamma\mu_R-B_1B_2B_6\gamma-2B_1B_5\tau_{nc}\mu_R-B_1B_5B_7\tau_{nc}+B_7B_3B_5\mu_H\\&+2B_3B_5\mu_R\mu_H-2B_1B_2B_7B_6-2B_1B_2B_6\mu_R+B_5B_3B_7\mu_H+B_7B_5\mu_R\mu_H+B_3B_7\mu_R\mu_H-B_2B_6\mu_R\mu_H\\&-B_2B_6B_5\mu_H-B_1B_2B_6\big].\end{aligned}$$

$$\begin{aligned}a_6=-\big[&B_1B_3B_5B_7\mu_R+B_1B_3B_5\mu_R{}^2+B_1B_4B_7\gamma\mu_R+B_1B_4\gamma\mu_R{}^2-B_1B_5B_7\tau_{nc}\mu_R+B_1B_2B_6\gamma\mu_R\\&-B_1B_5\tau_{nc}\mu_R{}^2+B_1B_3B_7B_5-2B_1B_2B_5B_6\mu_R-2B_1B_2B_5B_7B_6-B_2B_5B_6\mu_R\mu_H\\&+B_5B_3B_7\mu_R\mu_H+B_3B_5\mu_R{}^2\mu_H\big].\end{aligned}$$

The Routh Hurwitz criteria of the 5th order are satisfied. Hence, the endemic equilibria are locally asymptomatically stable.

5 Computational Approximations

In this section, we present the well-known approximations like Euler, Runge Kutta, and non-standard finite difference for the system (1)–(5) as follows:

5.1 Euler Approximation

The discretization of the system (1)–(5) under the rules of the Euler approximation is as follows:

$$S_H^{n+1} = S_H^n + h[\Lambda_H - \frac{\alpha_1\alpha_2 s_H^n I_R^n}{N_H} + \gamma R_H^n + \tau_{nc} I_H^n - \mu_H S_H^n]. \tag{11}$$

$$I_H^{n+1} = I_H^n + h\left[\frac{\alpha_1\alpha_2 S_H^n I_R^n}{N_H} - \tau_c I_H^n - r_c I_H^n - \tau_{nc} I_H^n - \delta I_H^n - \mu_H I_H^n\right]. \tag{12}$$

$$R_H^{n+1} = R_H^n + h\left[\tau_c I_H^n + r_c I_H^n - \gamma R_H^n - \mu_H R_H^n\right]. \tag{13}$$

$$S_R^{n+1} = S_R^n + h\left[\Lambda_R - \frac{\alpha_1\alpha_3 S_R^n I_H^n}{N_H} - \mu_R S_R^n\right]. \tag{14}$$

$$I_R^{n+1} = I_R^n + h[\frac{\alpha_1\alpha_3 S_R^n I_H^n}{N_H} - \mu_R I_R^n]. \tag{15}$$

where h is any discretization parameter and $n \geq 0$.

5.2 Runge-Kutta Approximation

The discretization of the system (1)–(5) under the rules of the Runge Kutta approximation is as follows:

Stage#1

$K_1 = h\left[\Lambda_H - \frac{\alpha_1\alpha_2 s_H^n I_R^n}{N_H} + \gamma R_H^n + \tau_{nc} I_H^n - \mu_H S_H^n\right].$

$L_1 = h\left[\frac{\alpha_1\alpha_2 S_H^n I_R^n}{N_H} - \tau_c I_H^n - r_c I_H^n - \tau_{nc} I_H^n - \delta I_H^n - \mu_H I_H^n\right].$

$M_1 = h\left[\tau_c I_H^n + r_c I_H^n - \gamma R_H^n - \mu_H R_H^n\right].$

$O_1 = h\left[\Lambda_R - \frac{\alpha_1\alpha_3 S_R^n I_H^n}{N_H} - \mu_R S_R^n\right].$

$P_1 = h\left[\frac{\alpha_1\alpha_3 S_R^n I_H^n}{N_H} - \mu_R I_R^n\right].$

Stage#2

$K_2 = h\left[\Lambda_H - \frac{\alpha_1\alpha_2\left(S_H^n + \frac{K_1}{2}\right)\left(I_R^n + \frac{P_1}{2}\right)}{N_H} + \gamma\left(R_H^n + \frac{M_1}{2}\right) + \tau_{nc}\left(I_H^n + \frac{L_1}{2}\right) - \mu_H\left(S_H^n + \frac{K_1}{2}\right)\right].$

$L_2 = h\left[\frac{\alpha_1\alpha_2\left(S_H^n + \frac{K_1}{2}\right)\left(I_R^n + \frac{P_1}{2}\right)}{N_H} - \tau_c\left(I_H^n + \frac{L_1}{2}\right) - r_c\left(I_H^n + \frac{L_1}{2}\right) - \tau_{nc}\left(I_H^n + \frac{L_1}{2}\right) - \delta\left(I_H^n + \frac{L_1}{2}\right) - \mu_H\left(I_H^n + \frac{L_1}{2}\right)\right].$

$M_2 = h\left[\tau_c\left(I_H^n + \frac{L_1}{2}\right) + r_c\left(I_H^n + \frac{L_1}{2}\right) - \gamma\left(R_H^n + \frac{M_1}{2}\right) - \mu_H\left(R_H^n + \frac{M_1}{2}\right)\right].$

$O_2 = h\left[\Lambda_R - \frac{\alpha_1\alpha_3\left(S_R^n + \frac{O_1}{2}\right)\left(I_H^n + \frac{L_1}{2}\right)}{N_H} - \mu_R\left(S_R^n + \frac{O_1}{2}\right)\right].$

$P_2 = h\left[\frac{\alpha_1\alpha_3\left(S_R^n + \frac{O_1}{2}\right)\left(I_H^n + \frac{L_1}{2}\right)}{N_H} - \mu_R\left(I_R^n + \frac{P_1}{2}\right)\right].$

Stage#3

$K_3 = h\left[\Lambda_H - \frac{\alpha_1\alpha_2\left(S_H^n + \frac{K_2}{2}\right)\left(I_R^n + \frac{P_2}{2}\right)}{N_H} + \gamma\left(R_H^n + \frac{M_2}{2}\right) + \tau_{nc}\left(I_H^n + \frac{L_2}{2}\right) - \mu_H\left(S_H^n + \frac{K_2}{2}\right)\right].$

$$L_3 = h\left[\frac{\alpha_1\alpha_2\left(S_H^n+\frac{K_2}{2}\right)\left(I_R^n+\frac{P_2}{2}\right)}{N_H} - \tau_c\left(I_H^n+\frac{L_2}{2}\right) - r_c\left(I_H^n+\frac{L_2}{2}\right) - \tau_{nc}\left(I_H^n+\frac{L_2}{2}\right) - \delta\left(I_H^n+\frac{L_2}{2}\right) - \mu_H\left(I_H^n+\frac{L_2}{2}\right)\right].$$

$$M_3 = h\left[\tau_c\left(I_H^n+\frac{L_2}{2}\right) + r_c\left(I_H^n+\frac{L_2}{2}\right) - \gamma\left(R_H^n+\frac{M_2}{2}\right) - \mu_H\left(R_H^n+\frac{M_2}{2}\right)\right].$$

$$O_3 = h\left[\Lambda_R - \frac{\alpha_1\alpha_3\left(S_R^n+\frac{O_2}{2}\right)\left(I_H^n+\frac{L_2}{2}\right)}{N_H} - \mu_R\left(S_R^n+\frac{O_2}{2}\right)\right].$$

$$P_3 = h\left[\frac{\alpha_1\alpha_3\left(S_R^n+\frac{O_2}{2}\right)\left(I_H^n+\frac{L_2}{2}\right)}{N_H} - \mu_R\left(I_R^n+\frac{P_2}{2}\right)\right].$$

Stage#4

$$K_4 = h\left[\Lambda_H - \frac{\alpha_1\alpha_2\left(S_H^n+K_3\right)\left(I_R^n+P_3\right)}{N_H} + \gamma\left(R_H^n+M_3\right) + \tau_{nc}\left(I_H^n+L_3\right) - \mu_H\left(S_H^n+K_3\right)\right].$$

$$L_4 = h\left[\frac{\alpha_1\alpha_2\left(S_H^n+K_3\right)\left(I_R^n+P_3\right)}{N_H} - \tau_c\left(I_H^n+L_3\right) - r_c\left(I_H^n+L_3\right) - \tau_{nc}\left(I_H^n+L_3\right) - \delta\left(I_H^n+L_3\right) - \mu_H\left(I_H^n+L_3\right)\right].$$

$$M_4 = h\left[\tau_c\left(I_H^n+L_3\right) + r_c\left(I_H^n+L_3\right) - \gamma\left(R_H^n+M_3\right) - \mu_H\left(R_H^n+M_3\right)\right].$$

$$O_4 = h\left[\Lambda_R - \frac{\alpha_1\alpha_3\left(S_R^n+O_3\right)\left(I_H^n+L_3\right)}{N_H} - \mu_R\left(S_R^n+O_3\right)\right].$$

$$P_4 = h\left[\frac{\alpha_1\alpha_3\left(S_R^n+O_3\right)\left(I_H^n+L_3\right)}{N_H} - \mu_R\left(I_R^n+P_3\right)\right].$$

Final stage

$$S_H{}^{n+1} = S_H^n + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]. \tag{16}$$

$$I_H{}^{n+1} = I_H^n + \frac{1}{6}[L_1 + 2L_2 + 2L_3 + L_4]. \tag{17}$$

$$R_H{}^{n+1} = R_H^n + \frac{1}{6}[M_1 + 2M_2 + 2M_3 + M_4]. \tag{18}$$

$$S_R{}^{n+1} = S_R^n + \frac{1}{6}[O_1 + 2O_2 + 2O_3 + O_4]. \tag{19}$$

$$I_R{}^{n+1} = I_R^n + \frac{1}{6}[P_1 + 2P_2 + 2P_3 + P_4]. \tag{20}$$

where h is any discretization parameter and $n \geq 0$.

5.3 Non-standard Finite Difference Approximation

The discretization of the system (1)–(5) under the rules of the non-standard finite difference scheme is as follows [27]:

$$S_H^{n+1} = \frac{S_H^n + h\Lambda_H + \gamma h R_H^n + h\tau_{nc} I_H^n}{1 + \frac{h\alpha_1\alpha_2 I_R^n}{N_H} + \mu_H h}. \tag{21}$$

$$I_H^{n+1} = \frac{I_H^n + \frac{h\alpha_1\alpha_2 S_H^n I_R^n}{N_H}}{1 + h\tau_c + r_c h + h\tau_{nc} + \delta h + \mu_H h}. \tag{22}$$

$$R_H^{n+1} = \frac{R_H^n + h\tau_c I_H^n + r_c I_H^n}{1 + \gamma h + \mu_H h}. \tag{23}$$

$$S_R^{n+1} = \frac{S_R^n + h\Lambda_R}{1 + \frac{h\alpha_1\alpha_3 I_H^n}{N_H} + \mu_R h}. \tag{24}$$

$$I_R^{n+1} = \frac{I_R^n + \frac{h\alpha_1\alpha_3 S_R^n I_H^n}{N_H}}{1 + \mu_R h}. \tag{25}$$

where h is any discretization parameter and $n \geq 0$.

5.4 Convergence Analysis

Considering the functions, A, B, C, D, and E at the system (21)–(25) as follows:

$$A = \frac{S_H + h\Lambda_H + \gamma h R_H + h\tau_{nc} I_H}{1 + \frac{h\alpha_1\alpha_2 I_R}{N_H} + \mu_H h},\ B = \frac{I_H + \frac{h\alpha_1\alpha_2 S_H I_R}{N_H}}{1 + h\tau_c + r_c h + h\tau_{nc} + \delta h + \mu_H h},$$

$$C = \frac{R_H + h\tau_c I_H + r_c I_H}{1 + \gamma h + \mu_H h},\ D = \frac{S_R + h\Lambda_R}{1 + \frac{h\alpha_1\alpha_3 I_H}{N_H} + \mu_R h},\ E = \frac{I_R + \frac{h\alpha_1\alpha_3 S_R I_H}{N_H}}{1 + \mu_R h}.$$

The elements of Jacobian matrix as follows:

$$\frac{\partial A}{\partial S_H} = \frac{1}{1 + \mu_H h}, \frac{\partial A}{\partial I_H} = \frac{h\tau_{nc}}{1 + \mu_H h}, \frac{\partial A}{\partial R_H} = \frac{\gamma h}{1 + \mu_H h}, \frac{\partial A}{\partial S_R} = 0, \frac{\partial A}{\partial I_R} = -\frac{(S_H + h\Lambda_H + \gamma h R_H + h\tau_{nc} I_H)\left(\frac{h\alpha_1\alpha_2}{N_H}\right)}{\left(1 + \frac{h\alpha_1\alpha_2 I_R}{N_H} + \mu_H h\right)^2}.$$

$$\frac{\partial B}{\partial S_H} = \frac{\frac{h\alpha_1\alpha_2 I_R}{N_H}}{1 + h\tau_c + r_c h + h\tau_{nc} + \delta h + \mu_H h}, \frac{\partial B}{\partial I_H} = \frac{1}{1 + h\tau_c + r_c h + h\tau_{nc} + \delta h + \mu_H h}, \frac{\partial B}{\partial R_H} = 0, \frac{\partial B}{\partial S_R} = 0,$$

$$\frac{\partial B}{\partial I_R} = \frac{\frac{h\alpha_1\alpha_2 S_H}{N_H}}{1 + h\tau_c + r_c h + h\tau_{nc} + \delta h + \mu_H h}, \frac{\partial C}{\partial S_H} = 0, \frac{\partial C}{\partial I_H} = \frac{h\tau_c + r_c}{1 + \gamma h + \mu_H h}, \frac{\partial C}{\partial R_H} = \frac{1}{1 + \gamma h + \mu_H h}, \frac{\partial C}{\partial S_R} = 0,$$

$$\frac{\partial C}{\partial I_R} = 0, \frac{\partial D}{\partial S_H} = 0, \frac{\partial D}{\partial I_H} = -\frac{(S_R + h\Lambda_R)\left(\frac{h\alpha_1\alpha_3}{N_H}\right)}{\left(1 + \frac{h\alpha_1\alpha_3 I_H}{N_H} + \mu_R h\right)^2}, \frac{\partial D}{\partial R_H} = 0, \frac{\partial D}{\partial S_R} = \frac{1}{1 + \mu_R h}, \frac{\partial D}{\partial I_R} = 0, \frac{\partial E}{\partial S_H} = 0,$$

$$\frac{\partial E}{\partial I_H} = \frac{\frac{h\alpha_1\alpha_3 S_R}{N_H}}{1 + \mu_R h}, \frac{\partial E}{\partial R_H} = 0, \frac{\partial E}{\partial S_R} = \frac{\frac{h\alpha_1\alpha_3 I_H}{N_H}}{1 + \mu_R h}, \frac{\partial E}{\partial I_R} = \frac{1}{1 + \mu_R h}.$$

Theorem 3: For $n \geq 0$, the eigenvalues of the Jacobian matrix at disease-free equilibrium for the system (21)–(25) lie in the unit circle if $R_0 < 1$.

Proof: The Jacobian matrix at disease-free equilibrium (DFE-E_0) = $(\frac{\Lambda_H}{\mu_H}, 0, 0, \frac{\Lambda_R}{\mu_R}, 0)$ is as follows:

$$|J(E_0)-\lambda|=\begin{vmatrix} \frac{1}{1+\mu_H h}-\lambda & \frac{h\tau_{nc}}{1+\mu_H h} & \frac{\gamma h}{1+\mu_H h} & 0 & -\frac{\left(\frac{\Lambda_H}{\mu_H}+h\Lambda_H\right)\left(\frac{h\alpha_1\alpha_2}{N_H}\right)}{(1+\mu_H h)^2} \\ 0 & \frac{1}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}-\lambda & 0 & 0 & \frac{\frac{h\alpha_1\alpha_2\frac{\Lambda_H}{\mu_H}}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h} \\ 0 & \frac{h\tau_c+r_c}{1+\gamma h+\mu_H h} & \frac{1}{1+\gamma h+\mu_H h}-\lambda & 0 & 0 \\ 0 & -\frac{\left(\frac{\Lambda_R}{\mu_R}+h\Lambda_R\right)\left(\frac{h\alpha_1\alpha_3}{N_H}\right)}{(1+\mu_R h)^2} & 0 & \frac{1}{1+\mu_R h}-\lambda & 0 \\ 0 & \frac{\frac{h\alpha_1\alpha_3\frac{\Lambda_R}{\mu_R}}{N_H}}{1+\mu_R h} & 0 & 0 & \frac{1}{1+\mu_R h}-\lambda \end{vmatrix}=0.$$

$$\lambda_1=\left|\frac{1}{1+\mu_H h}\right|<1, \lambda_2=\left|\frac{1}{1+\gamma h+\mu_H h}\right|<1, \lambda_3=\left|\frac{1}{1+h\mu_R}\right|<1.$$

$$|J(E_0)-\lambda|=\begin{vmatrix} \left(\frac{1}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}\right)-\lambda & \frac{\frac{h\alpha_1\alpha_2\frac{\Lambda_H}{\mu_H}}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h} \\ \frac{\frac{h\alpha_1\alpha_3\frac{\Lambda_R}{\mu_R}}{N_H}}{1+\mu_R h} & \frac{1}{1+\mu_R h}-\lambda \end{vmatrix}=0.$$

$$P_1=Trace\ of\ J=\left(\frac{1}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}\right)+\frac{1}{1+\mu_R h}$$

$$P_2=Determinant\ of\ J=\left(\left(\frac{1}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}\right)\left(\frac{1}{1+\mu_R h}\right)\right)-\left(\frac{\frac{h\alpha_1\alpha_3\frac{\Lambda_R}{\mu_R}}{N_H}}{1+\mu_R h}\right)\left(\frac{\frac{h\alpha_1\alpha_2\frac{\Lambda_H}{\mu_H}}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}\right).$$

Lemma 1: For the quadratic equation $\lambda^2-P_1\lambda+P_2==0$, $|\lambda_i|<1$, i = 1, 2, 3, if and only if the following conditions are satisfied:

(i). $1+P_1+P_2>0$.

(ii). $1-P_1+P_2>0$.

(iii). $P_2<1$.

Proof: The proof is straight forward.

Theorem 6: For $n\geq 0$, the eigenvalues of the Jacobian matrix at endemic equilibrium for the system (21)–(25) lie in the unit circle if $R_0>1$.

Proof: The Jacobian matrix at the endemic equilibria $E_1=(S_H^*, I_H^*, R_H^*, S_R^*, I_R^*)$ as follows:

$$JJ(E^*)=J\left(S_H{}^*, I_H{}^*, R_H{}^*, S_R^*, I_R^*\right)=$$

$$\begin{bmatrix} \frac{1}{1+\mu_H h} & \frac{h\tau_{nc}}{1++\mu_H h} & \frac{\gamma h}{1+\mu_H h} & 0 & -\frac{(S_H{}^*+h\Lambda_H+\gamma hR_H{}^*+h\tau_{nc}I_H{}^*)\left(\frac{h\alpha_1\alpha_2}{N_H}\right)}{\left(1+\frac{h\alpha_1\alpha_2 I_R^*}{N_H}+\mu_H h\right)^2} \\ \frac{\frac{h\alpha_1\alpha_2 I_R^*}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h} & \frac{1}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h} & 0 & 0 & \frac{\frac{h\alpha_1\alpha_2 S_H{}^*}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h} \\ 0 & \frac{h\tau_c+r_c}{1+\gamma h+\mu_H h} & \frac{1}{1+\gamma h+\mu_H h} & 0 & 0 \\ 0 & -\frac{\left(S_R^*+h\Lambda_R\right)\left(\frac{h\alpha_1\alpha_3}{N_H}\right)}{\left(1+\frac{h\alpha_1\alpha_3 I_H{}^*}{N_H}+\mu_R h\right)^2} & 0 & \frac{1}{1+\mu_R h} & 0 \\ 0 & \frac{\frac{h\alpha_1\alpha_3 S_R^*}{N_H}}{1+\mu_R h} & 0 & \frac{\frac{h\alpha_1\alpha_3 I_H{}^*}{N_H}}{1+\mu_R h} & \frac{1}{1+\mu_R h} \end{bmatrix}$$

.

Hence, the largest eigenvalue of the Jacobian is less than one, ultimately remaining will also lie in the unit circle when $R_0 > 1$. Thus, endemic equilibrium is stable.

5.5 Computational Approximations

By using the values of the parameters as presented in Tab. 1. The diagrams for the system (1)–(5) for disease-free equilibrium (DFE) and endemic equilibrium (EE) plotted with MATLAB software as follows:

Table 1: Value of parameters

Parameters	Values
$\Lambda_{\mathbf{H}}$	0.8
$\Lambda_{\mathbf{R}}$	0.8
$\mu_{\mathbf{H}}$	0.8
$\mu_{\mathbf{R}}$	0.8
$\alpha_{\mathbf{1}}$	1.00166 (DFE) 3.00166 (EE)
$\alpha_{\mathbf{2}}$	1.0004 (DFE 3.0004 (EE)
$\alpha_{\mathbf{3}}$	0.1
$\tau_{\mathbf{c}}$	0.7
$\tau_{\mathbf{nc}}$	0.9
$\mathbf{r_c}$	0.2
δ	0.133
γ	0.220

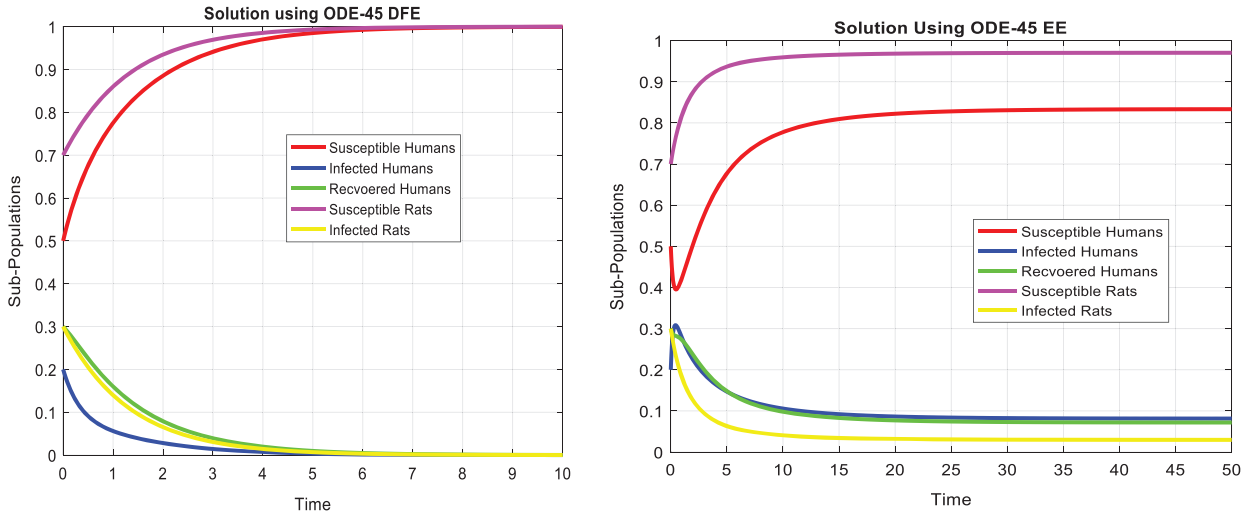

Figure 1: Combine graphical behaviors of the Lassa fever disease (a) Sub-populations at disease-free equilibrium (DFE) (b) Subpoulations at endemic equilibrium (EE)

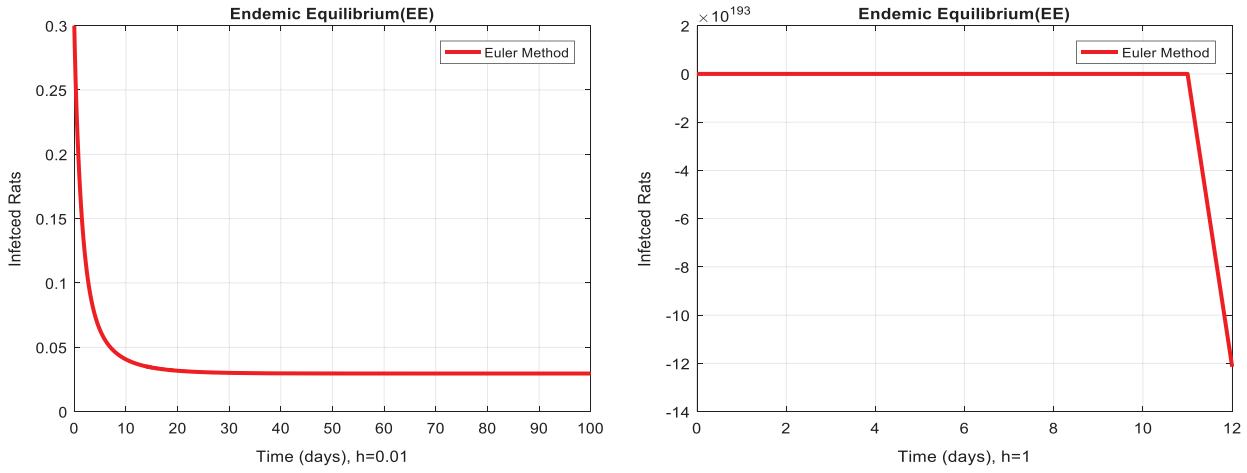

Figure 2: Euler method for the behavior of infected rats at different time-step sizes (a) Infected rats at h = 0.01 (b) Infected rats at h = 1

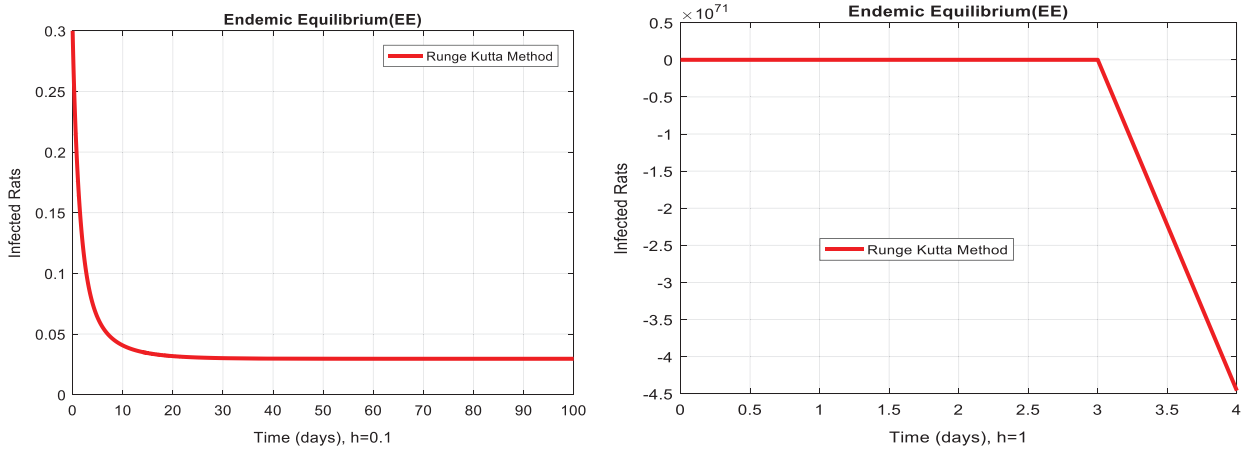

Figure 3: Runge Kutta method for the behavior of infected rats at different time-step sizes (a) The behavior of infected rats at time step size h = 0.1 (b) The behavior of infected rats at time step size h = 1

5.6 Comparison Section

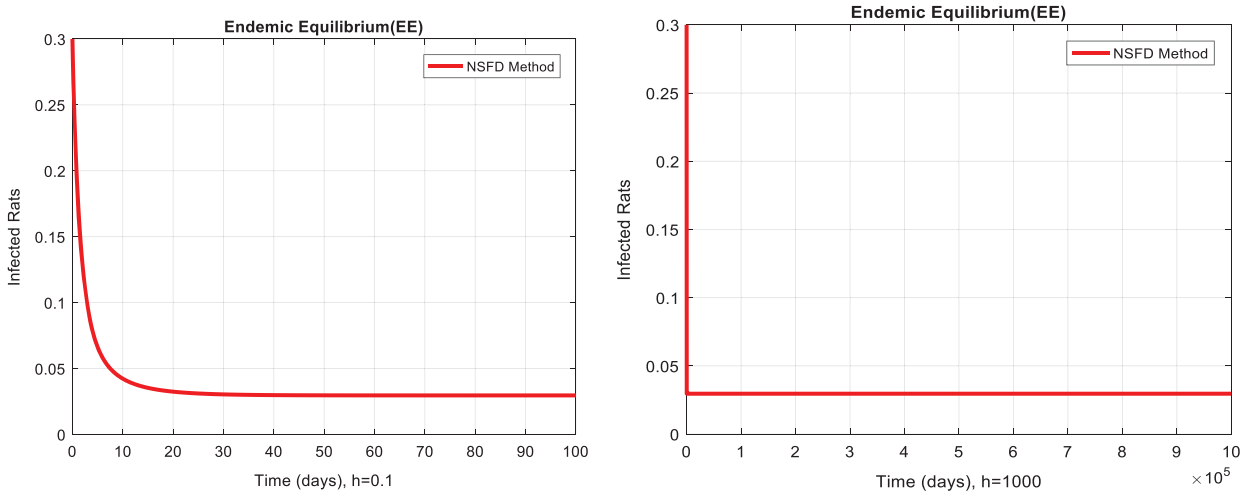

Figure 4: NSFD method for the behavior of infected rats at different time-step sizes (a) The behavior of Infected rats for EE at h = 0.1 (b) The behavior of infected rats for EE at h = 1000

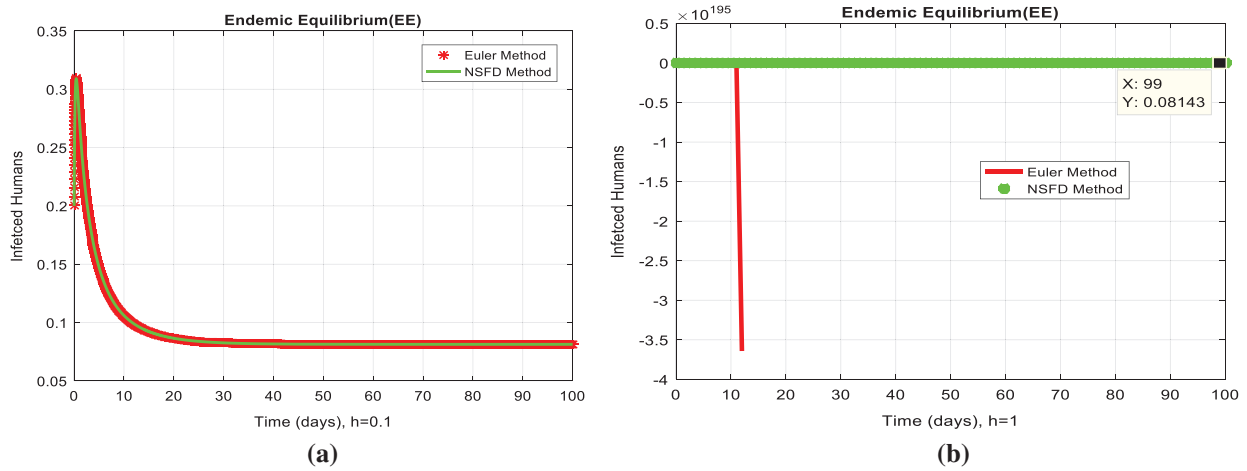

Figure 5: (Continued)

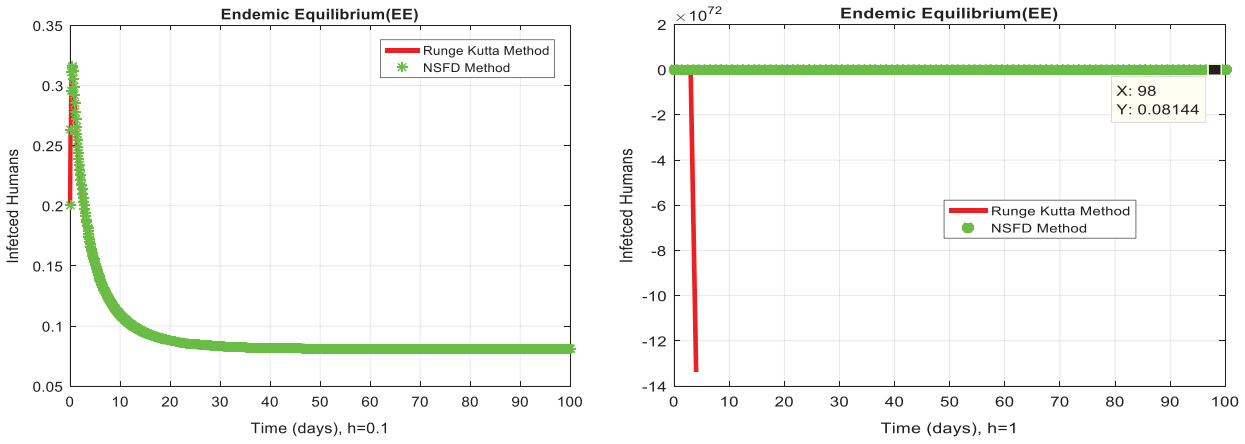

Figure 5: Combine graphical behaviors of NSFD with Euler and Runge Kutta methods at different time-step sizes (a) Comparison of Euler and NSFD at h = 0.1 (b) Comparison of Euler and NSFD at h = 1 (c) Comparison of Runge Kutta and NSFD at h = 0.1 (d) Comparison of Runge Kutta and NSFD at h = 1

6 Results and Discussion

We investigated the transmission dynamics of Lassa fever disease in humans and rats through the study. The critical point is modeling, terminology related to epidemiology, and Lassa fever disease. Dynamical analysis of the model is investigated. Computational analysis, including well-known methods, is presented. Mostly, methods are valid for only tiny time step sizes. But inappropriately flop for huge time step sizes like Euler and Runge Kutta. Our proposed scheme (NSFD) remains convergent for step sizes like $h = 100$. Furthermore, Tab. 2 shows the efficiency of the numerical methods.

Table 2: Comparison analysis of methods at different values of h

h	Euler	RK-4	NSFD scheme
0.01	EE = Convergence DFE = Convergence	EE = Convergence DFE = Convergence	Convergence
0.1	EE = Convergence DFE = Convergence	EE = Convergence DFE = Convergence	Convergence
1	EE = Divergence DFE = Divergence	EE = Divergence DFE = Divergence	Convergence
100	Divergence (method failed)	Divergence	Convergence

7 Conclusion

The non-standard finite difference scheme was designed for the communication dynamics of Lassa fever disease. Unfortunately, the earlier methods, like Euler and Runge Kutta of order 4^{th}, are unsuitable because they depend on time step size. So, Euler and Runge Kutta are tentatively convergent. When we increase the time step size, the graph of Euler and Runge Kutta gives variation in result from time to time they display divergent. The new well-known numerical scheme, like the non-standard finite difference scheme

independent of time step size. The NSFD scheme is a comfortable tool on behalf of dynamical properties like stability, positivity, boundedness and shows the exact behavior of the continuous model. The graphical behavior of ODE-45, Euler, Runge Kutta, NSFD schemes and comparison of schemes are given in Figs. 1a, 1b, Figs. 2a, 2b, Figs. 3a, 3b, Figs. 4a, 4b and Figs. 5a–5d respectively. In the end, we could extend this idea to all types of nonlinear and complex models. In the future, we could develop our analysis into fuzzy epidemic models and many other types of modeling as given in [27–31].

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