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Article

On Parametric Fuzzy Linear Programming Formulated by a Fractal

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Abstract: Fractal strategy is an important tool in manufacturing proposals, including computer design, conserving, power supplies and decorations. In this work, a parametric programming, analysis is proposed to mitigate an optimization problem. By employing a fractal difference equation of the spread functions (local fractional calculus operator) in linear programming, we aim to analyze the restraints and the objective function. This work proposes a new technique of fractal fuzzy linear programming (FFLP) model based on the symmetric triangular fuzzy number. The parameter fuzzy number is selected from the fractal power of the difference equation. Note that this number indicates the fractal parameter, denoting by λ ε [0, 1]. Accordingly, we specify the objective function to the fractional case, utilizing the fractal difference equation. We apply the suggested model in an application under the oil market. Based on the fractal fuzzy set theory, the fuzzy demand, transportations, management, inventory and buying cost are explained and formulated in a unique fractal fuzzy linear programming model. This model is presented to obtain a maximal profit production approach with an evaluation. The costs indicate that the proposed model can bring valued solutions for developing profit-effective oil refinery methods in a fuzzy fractal situation. Some examples are illustrated in the sequel.

Keywords: Parametric fuzzy; linear programming; objective function; fractal; fractional calculus; fractal difference equation

1 Introduction

The idea of a fuzzy linear programming (FLP) problem is the most important method for decision making [1]. In 1986, Carlsson and Korhonen suggested the parametric method of fuzzy linear programming [2]. They presented a parametric model to get the optimal solution. The parameter space is the environment of probable constraint values that clarifies certain mathematical modeling, which could typically be a subset of finite-dimensional Euclidean space. Normally, the parameters are suggested in a formula (or function) in which the condition is a domain of the function. The benefit of the parameter spaces is its capability to create profitable yet flexible strategies. The mathematical representations offer a massive variety of procedures to assess the system.

Recently, researchers have explored the FLP by utilizing the parametric analysis and parametric spaces. Payan et al. [3] presented a linearization development to determine the multi-objective linear fractional programming problem with fuzzy parameters. Ghaznavi et al. [4] categorized the notion of parametric analysis in FLP, though the objective function quantities are parameterized. Ebrahim [5] introduced a training of parametric analysis to optimize the solution of many problems. He carried a PFLP to define the optimal outcome and the fuzzy optimal detached values as a function of parameters, when the fuzzy cost factors are unsettled alongside an original fuzzy cost function. Meanwhile, Hesamian et al. [6] introduced a partial PFLP system for a similar study, which has been further improved by Hesamian et al. [7]. Recently, Zaire et al. [8] formulated a hybrid system concerning non-parametric system and paramedic system simultaneously. Lastly, Singh et al. [9] proposed a parametric analysis of the usefulness in a multi-objective linear programming problem to create the fuzzy set solution.

In this work, we utilize the concept of fractal derivative to present a parametric set for solving a fuzzy linear system. A fractal derivative [10] is a subclass of the local fractional calculus for which the fractal measurement strictly exceeds the topological measurement. This study suggests a new system of fractal fuzzy linear programming (FFLP) model based on the symmetric triangular fuzzy number and, consequently, the objective function. We employ the suggested model in the oil market. Based on the parametric fuzzy set theory, the fuzzy demand and cost have been clarified, and an FFLP model has been developed to obtain a maximal profit production approach. Overall, it is concluded that the model brings valuable information for developing oil refinery methods in a parametric fuzzy situation.

2 The Fuzzy Processing

One or more of the following uncertainties can occur in the refinery industry: the cost of oils alternates depending on the international oil reserve. In such cases, the factors that may increase the oil price include global political issues, military pressure, periodic demand, and/or cold (or hot) issues. Similarly, environmental security problems have tempted dogmatic discussions on valuing rules for oil, which has led to additional uncertainties [11–15]. Consequently, the manufacturing price is diverse, as well as the unit inventory price. Diverse unit transportation price (such as the oil transporter, whether trains or tankers) is utilized to provide oil from the port to the refinery to filling locations. These transporters also face a number of uncertainties. Finally, the management price is the amount of the electricity of the factories, conservation, taxation, and damage across the different stages of oil production, all of which imply a fuzzy cost. Based on the above issues, we present a fuzzy system describing each of the uncertainties.

2.1 Parametric Spaces

Here, we consider a fuzzy demand Δ_t^ρ of the manufacture ρ at the month t as symmetric triangle, fuzzy-valued by the following: (see Fig. 1)

$$\Delta_t^\rho = \{(d,\eta(d)) : \eta(d) = (d,(1-\delta_d)d,(1+\delta_d)d)_t^\rho\} \tag{1}$$

where d indicates the shrinking middle point (a halfway point), $Ld_t^\rho := (1-\delta_d)d$ represents to the left side of d, and $Rd_t^\rho := (1+\delta_d)d$ indicates the right side of d.

δ_d denotes the spread function formulating as a variance function in terms of selling price (the impulse response point):

$$\delta_d(\mathsf{C}_t^\rho) = a_1 + a_2\mathsf{C}_t^\rho$$

(for some constants a_1 and a_2 in R). Moreover, the symbol η is equivalent to $\eta_\lambda(d) \equiv (d, Ld_t^\rho, Rd_t^\rho)$ representing to the membership grade of the element d in the set Δ_t^ρ and is known as the membership function. By the definition of the spread function, we may describe any value in the above space as follows:

$$\eta(\alpha) = \{0, \alpha \le Ld_t^\rho \frac{\alpha - Ld_t^\rho}{d - Ld_t^\rho}, Ld_t^\rho < \alpha \le d \frac{Rd_t^\rho - \alpha}{Rd_t^\rho - d}, d < \alpha \le Rd_t^\rho$$

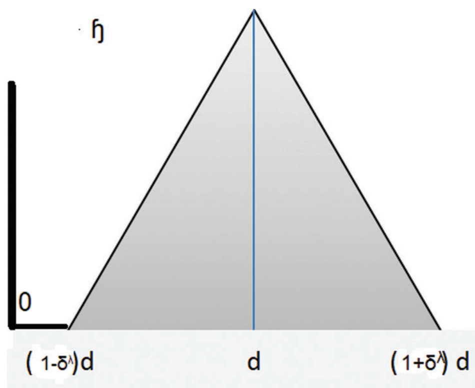


Figure 1: The symmetric triangle fractal fuzzy value of Δ_t^ρ

We proceed to define another item in the parametric space, which is the fuzzy buying cost, as follows:

$$BC_t^\jmath = \{(\flat, \eta_\lambda(\flat)) : \eta(\flat) = (\flat, (1 - \delta_\flat)\flat, 1 + \delta_\flat)\flat)\}, \tag{2}$$

where $\flat$ is the middle price of the type oil $\jmath$, $L\flat_t^\jmath := (1 - \delta_\flat)\flat$ is the left part of $\flat$ and $R\flat_t^\jmath := (1 + \delta_\flat)\flat$ is the right part to $\flat$ of the type oil $\jmath$ at month t. Now, we introduce the uncertain production price as follows:

$$PC_{t,\varphi}^\rho = \{(\wp, \eta(\wp)) : \eta(\wp) = \left(\wp, (1 - \delta_\wp)\wp, (1 + \delta_\wp)\wp\right)\}, \tag{3}$$

where $\wp$ is the rough middle value, $L\wp_{t,\varphi}^\rho := (1 - \delta_\wp)\wp$ is the left part of $\wp$ and $R\wp_{t,\varphi}^\rho := (1 + \delta_\wp)\wp$ is the right part to $\wp$ of the type oil ρ at month t from refinery φ. Similarly, we define the rest of parametric spaces, fuzzy inventory price, fuzzy transportation and fuzzy management value, respectively as follows:

$$IC_{t,\varphi}^\rho = \{(\imath, \eta_\lambda(\imath)) : \eta(\imath) = (\imath, (1 - \delta_\imath)\imath, (1 + \delta_\imath)\imath)\}, \tag{4}$$

$$\top C_{t,\varphi}^\rho = \{(\tau, \eta_\lambda(\tau)) : \eta(\tau) = (\tau, (1 - \delta_\tau)\tau, (1 + \delta_\tau)\tau)\}, \tag{5}$$

$$MC_{t,\varphi}^\rho = \{(\mu, \eta_\lambda(\mu)) : \eta(\mu) = \left(\mu, (1 - \delta_\mu)\mu, (1 + \delta_\mu)\mu\right)\}. \tag{6}$$

2.2 Local Fractional Difference Operator

Yang presented the local fractional derivative (fractal) of the function $g(x)$ of order $(0 < \lambda < 1)$ at the fixed value x_0 as [10]:

$$\Upsilon^\lambda g(x) = lim_{x \to x_0} \frac{D^\lambda(g(x) - g(x_0))}{(x - x_0)^\lambda}, \tag{7}$$

where $D^\lambda(g(x) - g(x_0)) \approx \Gamma(\lambda + 1) D(g(x) - g(x_0))$and the forward difference D formulated as $D(g(x) - g(x_0)) = g(x) - g(x_0)$, and Γ represents the gamma function $\Gamma(n + 1) = n!$.

Now, by using the concept of a fractal, we introduce a generalization of the spread function $\delta_d(\mathsf{C}_t^\rho) = a_1 + a_2\mathsf{C}_t^\rho$ as in the following fractal difference equation

$$\begin{aligned}\delta_d^\lambda(\mathsf{C}_t^\rho) &:= D^\lambda(\delta_d(\mathsf{C}_t^\rho) - \delta_d(\mathsf{C}_0^\rho)) \approx \Gamma(\lambda+1)D(\delta_d(\mathsf{C}_t^\rho) - a_1) = \Gamma(\lambda+1)(\delta_d(\mathsf{C}_t^\rho) - a_1)\\ &= a_2\Gamma(\lambda+1)\mathsf{C}_t^\rho, := \lambda\Gamma(\lambda+1)\mathsf{C}_t^\rho,\end{aligned} \tag{8}$$

where $a_2 = \lambda, \delta_d(\mathsf{C}_t^\rho) > 0, \delta_d(\mathsf{C}_0^\rho) = a_1$ and the fractal $\lambda \in (0,1)$. Note that if $\lambda \in (0,1)$, then we have $\Gamma(\lambda+1) \in (0,1)$. In this place, we note that λ plays a critical role in improving the classical system when $\lambda \approx 1$ (see [11]). In our investigation, we suppose that $\lambda = 0.5$ to get a good result for maximization. We shall use the fractal difference operator δ_d^λ in all spaces given in Eqs. (1)–(6). For example, a fuzzy fractal demand Δ_t^ρ of the manufacture ρ at the month t becomes

$$\Delta_t^\rho = \{(d, \eta_\lambda(d)) : \eta_\lambda(d) = \big(d, (1-\delta_d^\lambda)d, (1+\delta_d^\lambda)d\big)_t^\rho\} \tag{9}$$

where d indicates the shriveled middle point (i.e., halfway point), $Ld_t^\rho := (1-\delta_d^\lambda)d$ represents to the left side of d and $Rd_t^\rho := (1+\delta_d^\lambda)d$ indicates the right side of d. δ_d^λ is given in Eq. (8). Similarly, for the other parametric spaces $B\mathsf{C}_t^\jmath, P\mathsf{C}_{t,\varphi}^\rho, I\mathsf{C}_{t,\varphi}^\rho, \top\mathsf{C}_{t,\varphi}^\rho$ and $M\mathsf{C}_{t,\varphi}^\rho$.

3 Mathematical Modeling System

Under a fuzzy economic situation and constraints on manufacturing ability, inventory and operations, we propose a mathematical model to sustain a master buying and manufacturing proposal such that when the fluctuating demand is achieved, the maximum gain can be attained at an acceptable flat value.

3.1 Objective Function

The aim is to maximize the Π_t which is formulated by

$$Maximize \sum_t \upharpoonleft(\Pi_t - \underline{B}\mathsf{C}_t - \underline{P}\mathsf{C}_t - \underline{I}\mathsf{C}_t - \underline{T}\mathsf{C}_t - \underline{M}\mathsf{C}_t), \tag{10}$$

$$\Pi_t = \left(\sum_\varphi \upharpoonleft \sum_\rho \upharpoonleft \Theta_{\varphi,t}^\rho.\mathsf{C}_t^\rho\right),$$

$$\underline{B}\mathsf{C}_t = \sum_\jmath \upharpoonleft(B\mathsf{C}_t^\jmath.\chi_t^\jmath) = \sum_\jmath \upharpoonleft\Big(\big(\flat, (1-\delta_\flat^\lambda)\flat, (1+\delta_\flat^\lambda)\flat\big)_t^\jmath.\chi_t^\jmath\Big)$$

$$\underline{P}\mathsf{C}_t = \sum_\varphi \upharpoonleft \sum_\rho \upharpoonleft(P\mathsf{C}_{\varphi,t}^\rho.\Theta_{\varphi,t}^\rho) = \sum_\varphi \upharpoonleft \sum_\rho \upharpoonleft\left(\left(\wp, \left(1-\delta_\wp^\lambda\right)\wp, \left(1+\delta_\wp^\lambda\right)\wp\right)_{\varphi,t}^\rho.\Theta_{\varphi,t}^\rho\right)$$

$$\underline{I}\mathsf{C}_t = \sum_\varphi \upharpoonleft \sum_\rho \upharpoonleft(I\mathsf{C}_{\varphi,t}^\rho.I_{\varphi,t}^\rho) = \sum_\varphi \upharpoonleft \sum_\rho \upharpoonleft\Big(\big(\imath, (1-\delta_\imath^\lambda)\imath, (1+\delta_\imath^\lambda)\imath\big)_{\varphi,t}^\rho.I_{\varphi,t}^\rho\Big)$$

$$\underline{T}\mathsf{C}_t = \sum_\varphi \upharpoonleft \sum_\rho \upharpoonleft(T\mathsf{C}_{\varphi,t}^\rho.T_{\varphi,t}^\rho) = \sum_\varphi \upharpoonleft \sum_\rho \upharpoonleft\Big(\big(\tau, (1-\delta_\tau^\lambda)\tau, (1+\delta_\tau^\lambda)\tau\big)_{\varphi,t}^\rho.T_{\varphi,t}^\rho\Big),$$

$$\underline{M}\mathsf{C}_t = \sum_\varphi \upharpoonleft \sum_\rho \upharpoonleft(M\mathsf{C}_{\varphi,t}^\rho.\Theta_{\varphi,t}^\rho) = \sum_\varphi \upharpoonleft \sum_\rho \upharpoonleft\left(\left(\mu, \left(1-\delta_\mu^\lambda\right)\mu, \left(1+\delta_\mu^\lambda\right)\mu\right)_{\varphi,t}^\rho.\Theta_{\varphi,t}^\rho\right),$$

where $\chi_t^\jmath, \Theta_{\varphi,t}^\rho, I_{\varphi,t}^\rho, T_{\varphi,t}^\rho$ are variables.

3.2 Constraint Inequalities

We have the following list of constraints inequalities:

• Invention of manufacturing in a month plus the previous manufacturing inventory must be larger than or equal to the fluctuating demand (variation demand) for the manufacturing. Therefore, we suggest the average by using λ as follows:

$$\left(\frac{1}{\Phi}\sum_{\varphi} ?(\Theta^{\rho}_{\varphi,t} + I^{\rho}_{\varphi,t-1})^{1/\lambda}\right)^{\lambda} \geq \left(d, (1-\delta^{\lambda}_{d})d, (1+\delta^{\lambda}_{d})d\right)^{\rho}_{t}, \tag{11}$$

$(t = 1, \ldots, T, \lambda \in (0,1), \rho = 1, \ldots, P)$;

- The invention of every manufacturing at all refineries should be no less than the economical manufacture quantity of every manufacture at all refineries. Consequently, we obtain the following inequality

$$\Theta^{\rho}_{\varphi,t} \geq \underline{M}^{\rho}_{\varphi,t}, (t = 1, \ldots, T, \rho = 1, \ldots, P, \varphi = 1, \ldots, \Phi); \tag{12}$$

- The manufacturing at all refineries is controlled by the maximum production capability. Thus, we conclude the following inequality

$$\Theta^{\rho}_{\varphi,t} \leq \underline{M}^{\rho}_{\varphi,t}, (t = 1, \ldots, T, \rho = 1, \ldots, P, \varphi = 1, \ldots, \Phi); \tag{13}$$

• The quantity of every category of oil produced should be greater than or equal to the minimum monthly buying amount of every category of oil by agreements. Hence, we obtain the next inequality

$$\chi^{\jmath}_{t} \geq \underline{B}^{\jmath}_{t}, (\jmath = 1, \ldots, J; t = 1, \ldots, T); \tag{14}$$

- The total sum of oil purchases rendering to agreements is at least a positive fraction β (balance parameter) of the total sum of basic oils by the refinery manufacturing. Then as a conclusion, we have the constrained inequality

$$\sum_{\jmath} \underline{B}^{\jmath}_{t} \geq \beta \sum_{\jmath} \chi^{\jmath}_{t}, (\jmath = 1, \ldots, J; t = 1, \ldots, T); \tag{15}$$

• The total of manufactured material transported from a refinery by any type of transportation (trains, pipelines or trucks) is less than or equal to the maximum permissible amount of the transportation

$$T^{\rho}_{\varphi,t} \leq \underline{T}^{\rho}_{\varphi,t}(t = 1, \ldots, T, \rho = 1, \ldots, P, \varphi = 1, \ldots, \Phi); \tag{16}$$

• The overall sum of manufactured material transported from refineries ought to be less than or equal to the refineries' overall manufacturing output. Hence, we get the inequality

$$\sum_{\varphi} ?T^{\rho}_{\varphi,t} \leq \sum_{\varphi} ?\Theta^{\rho}_{\varphi,t}, (t = 1, \ldots, T, \rho = 1, \ldots, P, \varphi = 1, \ldots, \Phi); \tag{17}$$

• The overall sum of manufactured material transported from refineries has to be greater than or equal to the fluctuating (changing) demand for the manufacturing process. Connected to what has been mentioned above, we have the following constrain

$$\sum_{\varphi} ?T^{\rho}_{\varphi,t} \geq \left(d, \left(1-\delta^{\lambda}_{d}\right)d, \left(1+\delta^{\lambda}_{d}\right)d\right)^{\rho}_{t}, t = 1, \ldots, T, \lambda \in (0,1), \rho = 1, \ldots, P, \varphi = 1, \ldots, \Phi); \tag{18}$$

• Overall, the variables must be non-negative

$$\chi_t^{\jmath}, \Theta_{\varphi,t}^{\rho}, I_{\varphi,t}^{\rho}, T_{\varphi,t}^{\rho} \geq 0, (\jmath = 1, \ldots, J, \rho = 1, \ldots, P; \lambda \in (0, 1), t = 1, \ldots, T, \varphi = 1, \ldots, \Phi). \quad (19)$$

4 The Technique

The following steps represent our technique, respectively:

4.1 The Result of the Objective Function

We aim to maximize the objective function $\Lambda = c\chi$, where c indicates symmetric coefficients. By utilizing the symmetric coefficient terms:

$$\left(c, \left(1 - \delta_c^{\lambda}\right)c, \left(1 + \delta_c^{\lambda}\right)c\right) := (c, l_{\lambda}c, r_{\lambda}c).$$

Based on the 3-D parametric space, we represent the objective function by the 3-multi-objective system, as follows:

$$max\Lambda_1 = c_1\chi_1 + \ldots + c_n\chi_n max\Lambda_2 = l_{\lambda}c_1\chi_1 + \ldots + l_{\lambda}c_n\chi_n max\Lambda_3 = r_{\lambda}c_1\chi_1 + \ldots + r_{\lambda}c_n\chi_n, \quad (20)$$

where $l_{\lambda} = 1 - \delta_c^{\lambda}$ and $r_{\lambda} = 1 + \delta_c^{\lambda}$. The next step is to compute the upper and lower bound of Λ, (Λ^U, Λ^L) . Accordingly, we have the following system for the lower bound

$$\Lambda_1^L = min(c_1\chi_1 + \ldots + c_n\chi_n)\Lambda_2^L = min(l_{\lambda}c_1\chi_1 + \ldots + l_{\lambda}c_n\chi_n)\Lambda_3^L = min(r_{\lambda}c_1\chi_1 + \ldots + r_{\lambda}c_n\chi_n), \quad (21)$$

and the following system for the upper bound

$$\Lambda_1^U = max(c_1\chi_1 + \ldots + c_n\chi_n)\Lambda_2^U = max(l_{\lambda}c_1\chi_1 + \ldots + l_{\lambda}c_n\chi_n)\Lambda_3^U = max(r_{\lambda}c_1\chi_1 + \ldots + r_{\lambda}c_n\chi_n). \quad (22)$$

Following the membership function of Λ, we employ:

$$\eta_{\lambda}(\Lambda) = \{0, \Lambda \leq \Lambda^L \frac{\Lambda - \Lambda^L}{\Lambda^U - \Lambda^L}, \Lambda^L < \Lambda \leq \Lambda^U 1, \Lambda > \Lambda^U \quad (23)$$

where we aim to maximize $\eta_{\lambda}(\Lambda)$; or maximize $\gamma = min(\gamma_1, \gamma_2, \gamma_3)$ where $\eta_{\lambda}(\Lambda) \geq \gamma$. This solves the following issue:

$$\begin{aligned} &max\gamma, subjectto(c_1\chi_1 + \ldots + c_n\chi_n) - \gamma_1\left(\Lambda_1^U - \Lambda_1^L\right) \geq \Lambda_1^L(l_{\lambda}c_1\chi_1 + \ldots + l_{\lambda}c_n\chi_n) - \gamma_2\left(\Lambda_2^U - \Lambda_2^L\right) \geq \\ &\Lambda_2^L(r_{\lambda}c_1\chi_1 + \ldots + r_{\lambda}c_n\chi_n) - \gamma_3\left(\Lambda_3^U - \Lambda_3^L\right) \geq \Lambda_3^L\chi_i \geq 0, i = 1, \ldots, n \; \gamma \in [0, 1], \end{aligned} \quad (24)$$

where $\Lambda_k^L, k = 1, 2, 3$ is given in Eq. (21).

4.2 Transportation Problem

We formulate the multiple objective functions $(\Lambda_1, \Lambda_2, \Lambda_3)$ based on Eq. (10) as follows:

$$\begin{aligned} max\Lambda_1 &= \sum_t \dagger(\Pi_t - BC_t - PC_t - IC_t - TC_t - MC_t) \\ max\Lambda_2 &= \sum_t \dagger(\Pi_t - (1 - \delta_C^{\lambda})(BC_t + PC_t + IC_t + TC_t - MC_t)) \\ max\Lambda_3 &= \sum_t \dagger(\Pi_t - (1 + \delta_C^{\lambda})(BC_t + PC_t + IC_t + TC_t - MC_t)), \end{aligned} \quad (25)$$

where $\delta_C = a_1 + a_2 C, a_1, a_2 \in R$. Next, by utilizing Mathematica 11.2, we compute the upper and lower bounds. Accordingly, the system becomes as follows:

$$\begin{aligned}&\sum_t \dagger(\Pi_t - BC_t - PC_t - IC_t - TC_t - MC_t) - \gamma_1(\Lambda_1^U - \Lambda_1^L) \geq \Lambda_1^L\\&\sum_t \dagger(\Pi_t - (1-\delta_C^\lambda)(BC_t + PC_t + IC_t + TC_t - MC_t)) - \gamma_2(\Lambda_1^U - \Lambda_1^L) \geq \Lambda_2^L\\&\sum_t \dagger(\Pi_t - (1+\delta_C^\lambda)(BC_t + PC_t + IC_t + TC_t - MC_t)) - \gamma_3(\Lambda_1^U - \Lambda_1^L) \geq \Lambda_3^L.\end{aligned} \tag{26}$$

Next, we proceed to convert the fuzzy inequality constraints as it is implied by Eqs. (11) and (18):

$$\left(\frac{1}{\Phi}\sum_\varphi \dagger(\Theta_{\varphi,t}^\rho + \boldsymbol{I}_{\varphi,t-1}^\rho)^{1/\lambda}\right)^\lambda - \gamma \boldsymbol{d}(\boldsymbol{1} - (\boldsymbol{1} - \delta_{\boldsymbol{d}}^\lambda))_{\boldsymbol{t}}^\rho \geq \boldsymbol{d}(\boldsymbol{1} - \delta_{\boldsymbol{d}}^\lambda)_{\boldsymbol{t}}^\rho, \lambda \in (\boldsymbol{0}, \boldsymbol{1}), \tag{27}$$

and

$$\sum_\varphi \dagger \boldsymbol{T}_{\varphi,\boldsymbol{t}}^\rho - \gamma \boldsymbol{d}(\boldsymbol{1} - (\boldsymbol{1} - \delta_{\boldsymbol{d}}^\lambda))_{\boldsymbol{t}}^\rho \geq \boldsymbol{d}(\boldsymbol{1} - \delta_{\boldsymbol{d}}^\lambda)_{\boldsymbol{t}}^\rho. \tag{28}$$

Finally, we combine the multi-objective functions to maximize the following:

$$\begin{aligned}&max\, \gamma,\ subjectto \sum_t (\Pi_t - BC_t - PC_t - IC_t - TC_t - MC_t) - \gamma_1\left(\Lambda_1^U - \Lambda_1^L\right)\\&\geq \Lambda_1^L \sum_t \left(\Pi_t - \left(1-\delta_C^\lambda\right)(BC_t + PC_t + IC_t + TC_t - MC_t)\right) - \gamma_2\left(\Lambda_1^U - \Lambda_1^L\right)\\&\geq \Lambda_2^L \sum_t (\Pi_t - (1+\delta_C^\lambda)(BC_t + PC_t + IC_t + TC_t - MC_t)) - \gamma_3\left(\Lambda_1^U - \Lambda_1^L\right)\\&\geq \Lambda_3^L \left(\frac{1}{\Phi}\sum_\varphi (\Theta_{\varphi,t}^\rho + I_{\varphi,t-1}^\rho)^{1/\lambda}\right)^\lambda - \gamma\, d\left(1 - \left(1-\delta_d^\lambda\right)\right)_t^\rho\\&\geq d(1-\delta_d^\lambda)_t^\rho \sum_\varphi T_{\varphi,t}^\rho - \gamma\, d\left(1 - \left(1-\delta_d^\lambda\right)\right)_t^\rho \geq d(1-\delta_d^\lambda)_t^\rho \gamma \in [0,1],\end{aligned} \tag{29}$$

taking in account that inequalities (12)-(17) and (19) are all achieved.

5 Document of the Development

We applied the above mentioned model in the Iraqi Patrol Company (IPC). IPC is the major oil manufacturing firm in Iraq. This company has one part located in Kirkuk city in northern Iraq, and another in Basrah city in southern Iraq. While there are four sub-companies in the middle of Iraq: Al-Doura, Al-Samawa, Al-Najaf and Al-Diwaniya (all have four yields) correspondingly. In this study, we dealt with the primary materials, specifically Gasoline (Ga), Kerosene (Tk), Gas oil (Go) and Fuel oil (Fo). The types of plain oils were limited to the four types mentioned above, and the planning and manufacturing period was set to T = 12 months (between 2018 and 2019).

To ensure an acceptable oil source, the IPC must come to an agreement with industrial oil countries. Relevant to our work, each group of basic oil j must be delivered in every month t. Fig. 2 depicts the stages of our procedure.

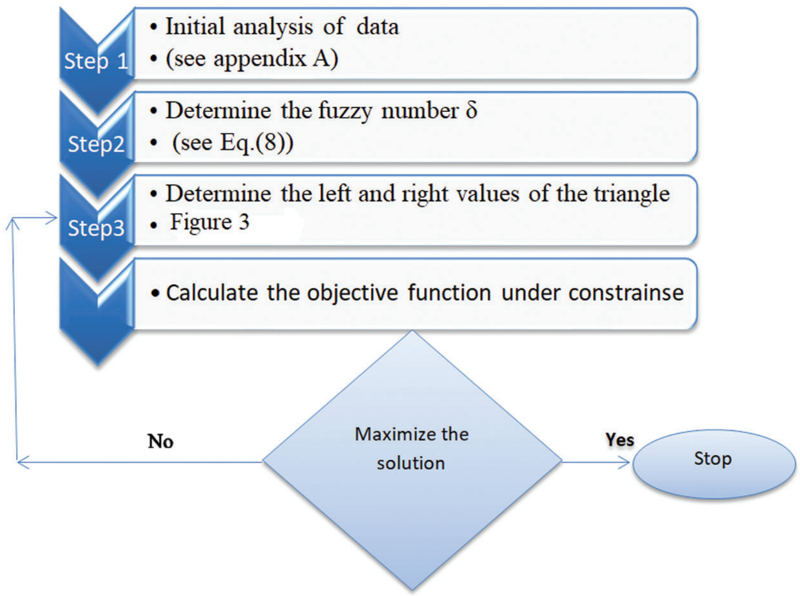


Figure 2: Steps to maximize the solution and find the best interval of Υ

5.1 Data Analysis

To find the optimal solution, we collected our data as in Appendix A from its sources. Fig. 3 provides symmetric triangle values based on the parameter values $0 < \lambda < 1$. The tables involve three values of $\lambda : 0.2, 0.5, 0.8$; which implies three different values of γ. It is evident that there is a relation between γ and λ which can be recognized by the equation $\gamma = 1 - \underline{\lambda} = \lambda$. Each product has its own parameter fuzzy value λ. For example, to determine $(\Lambda_1, \Lambda_2, \Lambda_3)$ for the production costs, we search for the maximum value in all refineries (which is 28.8 in the Al-Samawa refinery). Then by setting the three values $\lambda : 0.2, 0.5, 0.8$, we determine the parametric fuzzy number of the product by using (8) as follows:

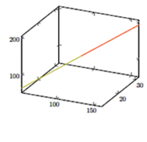
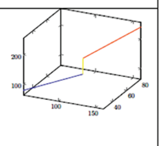
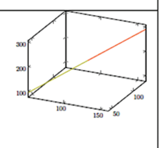
For $\lambda = 0.2$, we have $\lambda\Gamma(1 + 0.2) = 0.2 \times 0.91 \approx 0.2 \rightarrow \underline{\lambda} = 1 - 0.2 = 0.8$. Therefore, we obtain two parametric fuzzy numbers corresponding to $28.8 : (23.04, 51.84)$ (see Fig. 3). This new method provides many advantages, such as high accuracy and stability of the data. Using the information in Section 3 and Section 4, we follow the steps:

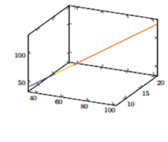
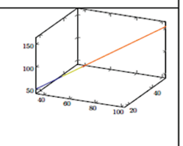
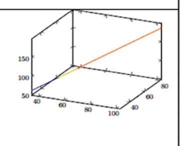
• Step 1: we obtained the upper bounds Λ_3 and lower bounds Λ_2 of the objective function (see the second and third column of the first matrix in Fig. 3, respectively)

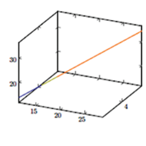
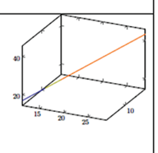
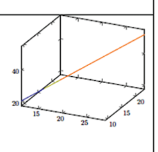
• Step 2: we determine the vector $(GaTkGoFo)$ by employing the values in step one. This vector represents the interval or the best value of γ; for instance, the value $\gamma = 0.5$ implies the proper evaluation for the left and right amounts of the triangle.

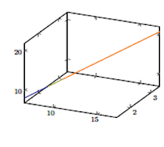
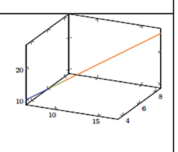
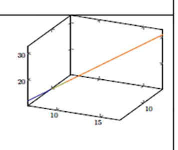
• Step 3: Using the vector in step two, we estimate the interval or the value of $\gamma \in [0, 1]$ to maximize the problem in (24). As we realize that the value of $\gamma = \lambda$ (the fractal parameter). Based on the analysis shown in Fig. 3, the most accurate analysis is given when $\lambda = 0.5$.

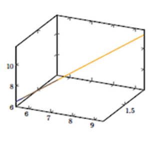
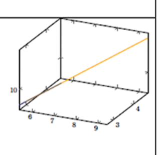
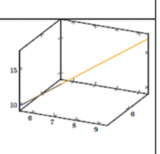
• Step 4: By employing the vector in step two, we calculate the interval or the value of $\gamma \in [0, 1]$ to maximize the problem in (24). In this place, we confirm that by using Mathematica 11.2, solving system (24) implies that the exact value $\gamma \approx \frac{4}{5}$ maximizes the system.

	Maximum solution	Objective function	Graph
$BC[\bar{\lambda}=0.2]$	$\begin{pmatrix}160.5 & 32.1 & 192.6\\107.5 & 21.5 & 129\\107.2 & 21.44 & 128.64\\52.7 & 10.54 & 63.24\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.8\\0.8\\0.8\\0.8\end{pmatrix}$	
$BC[\bar{\lambda}=0.5]$	$\begin{pmatrix}160.5 & 80.25 & 240.75\\107.5 & 53.75 & 161.25\\107.2 & 53.6 & 106.8\\52.7 & 26.35 & 79.05\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.5\\0.5\\0.5\\0.5\end{pmatrix}$	
$BC[\bar{\lambda}=0.8]$	$\begin{pmatrix}160.5 & 128.4 & 288.9\\107.5 & 86 & 193.5\\107.2 & 85.76 & 192.96\\52.7 & 42.16 & 94.86\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.2\\0.2\\0.2\\0.2\end{pmatrix}$	

	Maximum solution	Objective function	Graph
$MC[\bar{\lambda}=0.2]$	$\begin{pmatrix}102.8 & 20.56 & 123.36\\57.9 & 11.58 & 69.48\\47.7 & 9.54 & 57.24\\33.9 & 6.78 & 40.68\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.8\\0.8\\0.8\\0.8\end{pmatrix}$	
$MC[\bar{\lambda}=0.5]$	$\begin{pmatrix}102.8 & 51.4 & 154.2\\57.9 & 28.95 & 86.85\\47.7 & 23.85 & 71.55\\33.9 & 16.95 & 50.85\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.5\\0.5\\0.5\\0.5\end{pmatrix}$	
$MC[\bar{\lambda}=0.8]$	$\begin{pmatrix}102.8 & 82.24 & 185.04\\57.9 & 46.32 & 104.22\\47.7 & 38.16 & 85.86\\33.9 & 27.12 & 61.02\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.2\\0.2\\0.2\\0.2\end{pmatrix}$	

	Maximum solution	Objective function	Graph
$PC[\bar{\lambda}=0.2]$	$\begin{pmatrix}28.8 & 5.76 & 34.56\\16.8 & 3.36 & 20.16\\14.6 & 2.92 & 17.52\\11.4 & 2.28 & 13.68\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.8\\0.8\\0.8\\0.8\end{pmatrix}$	
$PC[\bar{\lambda}=0.5]$	$\begin{pmatrix}28.8 & 14.4 & 43.2\\16.8 & 8.4 & 25.2\\14.6 & 7.3 & 21.9\\11.4 & 5.7 & 17.1\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.5\\0.5\\0.5\\0.5\end{pmatrix}$	
$PC[\bar{\lambda}=0.8]$	$\begin{pmatrix}28.8 & 23.04 & 51.84\\16.8 & 13.44 & 30.24\\14.6 & 11.68 & 26.28\\11.4 & 9.12 & 20.52\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.2\\0.2\\0.2\\0.2\end{pmatrix}$	

	Maximum solution	Objective function	Graph
$IC[\bar{\lambda}=0.2]$	$\begin{pmatrix}17.2 & 3.44 & 20.64\\9.7 & 1.94 & 11.64\\8.5 & 1.7 & 10.2\\6.7 & 1.34 & 8.04\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.8\\0.8\\0.8\\0.8\end{pmatrix}$	
$IC[\bar{\lambda}=0.5]$	$\begin{pmatrix}17.2 & 8.6 & 25.8\\9.7 & 4.85 & 14.55\\8.5 & 4.25 & 12.75\\6.7 & 3.35 & 10.05\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.5\\0.5\\0.5\\0.5\end{pmatrix}$	
$IC[\bar{\lambda}=0.8]$	$\begin{pmatrix}17.2 & 13.76 & 30.96\\9.7 & 7.76 & 17.46\\8.5 & 6.8 & 15.3\\6.7 & 5.36 & 12.06\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.2\\0.2\\0.2\\0.2\end{pmatrix}$	

	Maximum solution	Objective function	Graph
$TC[\bar{\lambda}=0.2]$	$\begin{pmatrix}9.4 & 1.88 & 11.28\\5.6 & 1.12 & 6.72\\5.4 & 1.08 & 6.48\\6.7 & 1.34 & 8.04\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.8\\0.8\\0.8\\0.8\end{pmatrix}$	
$TC[\bar{\lambda}=0.5]$	$\begin{pmatrix}9.4 & 4.7 & 14.1\\5.6 & 2.8 & 8.4\\5.4 & 2.7 & 8.1\\6.7 & 3.35 & 10.05\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.5\\0.5\\0.5\\0.5\end{pmatrix}$	
$TC[\bar{\lambda}=0.8]$	$\begin{pmatrix}9.4 & 7.52 & 16.92\\5.6 & 4.48 & 10.08\\5.4 & 4.32 & 9.72\\6.7 & 5.36 & 12.06\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.2\\0.2\\0.2\\0.2\end{pmatrix}$	

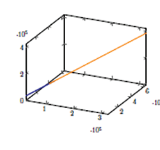
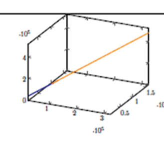
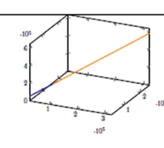
	Maximum solution	Objective function	Graph
$D[\bar{\lambda}=0.2]$	$\begin{pmatrix}98484 & 19696.8 & 118180.8\\25286 & 5057.2 & 30343.2\\82562 & 16512.4 & 99074.4\\321893 & 64378.6 & 386271.6\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.8\\0.8\\0.8\\0.8\end{pmatrix}$	
$D[\bar{\lambda}=0.5]$	$\begin{pmatrix}98484 & 49242 & 147726\\25286 & 12643 & 37929\\82562 & 41281 & 123843\\321893 & 160946.5 & 482839.5\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.5\\0.5\\0.5\\0.5\end{pmatrix}$	
$D[\bar{\lambda}=0.8]$	$\begin{pmatrix}98484 & 78787.2 & 177271.2\\25286 & 20228.8 & 45514.8\\82562 & 66049.6 & 148611.6\\321893 & 257514.4 & 579407.4\end{pmatrix}$	$\begin{pmatrix}Ga\\Tk\\Go\\Fo\end{pmatrix}=\begin{pmatrix}0.2\\0.2\\0.2\\0.2\end{pmatrix}$	

Figure 3: From the top, Selling block costs, Management block costs, Product costs, Inventory costs, transportation costs and demand costs with $\underline{\lambda}$ = 0.2; 0.5; 0.8 respectively

5.2 Impact Analysis

Since the hypotheses and incomes are time irregular, several mathematical tests can describe the impact of uncertainty of the recent model. By shifting both demand and the price factors, as well as the estimate of the systems (see Fig. 3) with various symmetric triangular fuzzy number, the demand *D* and the price factors can be examined. The graphs in Fig. 3 indicate the date of the corresponding matrix $(\Lambda_1, \Lambda_2, \Lambda_3)$; The data show that the manufacturers' order conferring to income action is unaffected by rises in the selling cost.

6 Discussion and Conclusion

We point out the following facts for our data that is collected in Appendix A.

• *Modeling system*: The fuzzy system can compute variables such as the selling and demand. This system has the ability to consider the relation between the selling amount and demand is investigated in three cases (real, upper and lower). The consequence limits the accumulative profit rule of gathering, buying amount, and selling value at the same average, while minimizing both at the same rate gives the least profit. The significance of a membership takes back to first principles of elastic and broader requests; it also provides a current systematic method with beneficial outcomes.

• *Study case*: This analysis illustrates that when there is more manufacturing yield, the total price will be lower, and vice versa. Nonetheless, the manufacturing design must trade-off demand and output, and the maximum market demand controls it. A study on IPC recognized uncertain market demand and different prices in the indefinite setting. The dynamics and outcomes indicate that the developed system is capable of delivering valuable data for increasing profit-effective oil refinery approaches under different settings.

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Appendix A

Month	Products of Al-Doura refinery				Products of Al-Samawa refinery				Products of Al-Najaf refinery				Products of Al-Diwaniya refinery			
	Gasoline Ga	Kerosene Tk	Gas oil Go	Fuel oil Fo	Gasoline Ga	Kerosene Tk	Gas oil Go	Fuel oil Fo	Gasoline Ga	Kerosene Tk	Gas oil Go	Fuel oil Fo	Gasoline Ga	Kerosene Tk	Gas oil Go	Fuel oil Fo
1	86318	22162	72363	282128	17093	6672	17591	59760	16642	7876	15532	52344	14557	5257	12977	43771
2	96442	24762	80850	315217	19166	7501	19724	67192	18702	8851	17456	58826	16264	5874	14498	48902
3	98484	25286	82562	321893	19573	7640	20143	68430	19139	9058	17863	60199	16505	5961	14714	49629
4	96264	24716	80701	314636	19236	7508	19797	67254	18982	8984	17716	59704	16400	5923	14620	49311
5	94843	24351	79510	309993	18723	7308	19268	65458	18475	8744	17244	58112	16007	5781	14270	48130
6	84187	21615	70576	275162	16579	6471	17062	57964	16711	7909	15598	52564	14180	5121	12641	42636
7	76727	19699	64322	250781	15269	5960	15713	53382	14738	6975	13756	46357	12926	4668	11523	38868
8	75218	19312	63057	245846	15163	5897	15549	52824	14860	7033	13870	46742	12821	4630	11429	38550
9	81256	20863	68120	265584	16473	6430	16953	57593	16135	7636	15060	50752	14014	5061	12493	42137
10	94488	24260	79212	308831	18811	7342	19359	65767	18824	8909	17570	59210	16324	5896	14552	49084
11	95465	24511	80031	312025	19077	7446	19633	66696	19069	9025	17798	59979	16445	5939	14660	49447
12	85963	22071	72065	280967	17394	6789	17901	60813	17270	8174	16119	54322	14768	5334	13167	44407
Highest and lowest values	98484 75218	25286 19312	82562 63057	321893 245846	19573 15163	7640 5897	20143 15549	68430 52824	19139 14738	9058 6975	17863 13756	60199 46357	16505 12821	5961 4630	14714 12641	49629 38550